Modeling of Tundish Steelmaking Process in Continuous Casting

Prof. Pradeep K. Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 13
Characterization of Flow in Actual Systems

Welcome to the lecture on Characterization of Flow in Actual Systems. So, we talked about the idealized systems and we talked about the plug flow ideal plug flow and also the ideal mixed flow purely mixed flow, but then actually in actual system you know the flow is in between these 2 extremes and we must have the models you know which needs to be followed.

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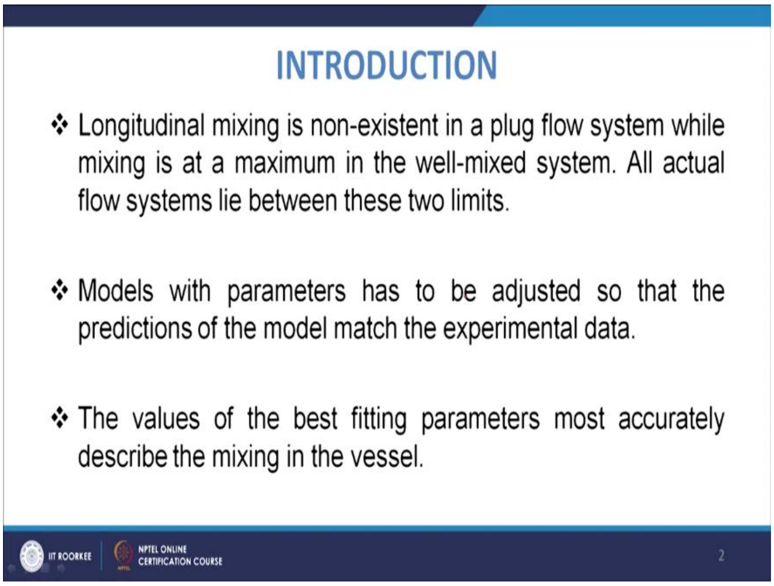

So, the longitudinal mixing is non existence as we discussed in the plug flow system ideal system we have seen that it is though that system in which there is no longitudinal mixing. And in the case of well mixed system it is mixing is the maximum and these 2 are basically the 2 extremes and the actual flow systems that lie between these 2 limits.

So, you will have the models with parameters and you will have to have the adjustments. So, that when you do the experimental analysis then the prediction of that model will should match with the experimental data. So, how to develop you know those models and how to do the adjustments you know that needs to be seen.

Now for that there are different models and you know certainly we will be talking about the values of the best fitting parameter which will be most accurately describing the mixing in the vessels. So, that is regarding the you know output characteristics. So, in that we are going to have the study of the longitudinal dispersion model.

So, in actual there will be dispersion even in the longitudinal direction whereas, in plug flow we have assume that there is no longitudinal dispersion. So, that tracer which is you know coming at the inlet. So, you will have after sometime it will go appearing at the outlet there is no longitudinal mixing.

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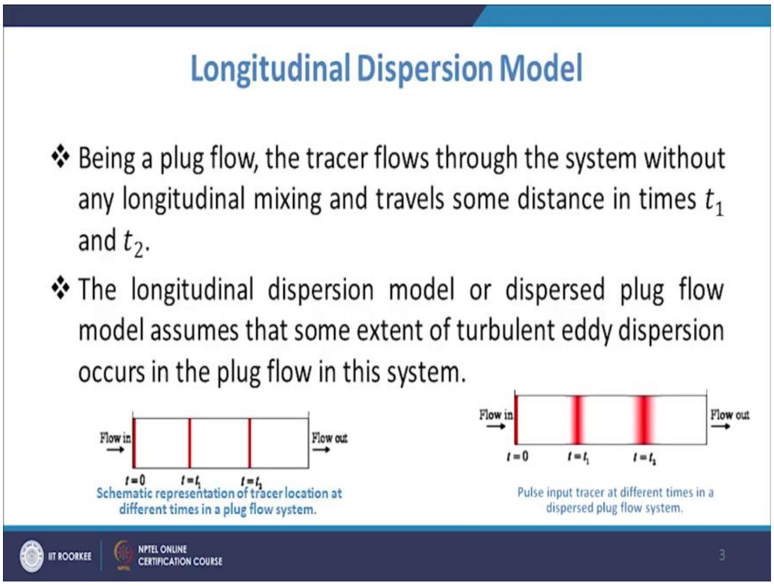

Whereas, when we talk about the dispersion model so, in the longitudinal dispersion model you know in that the tracer will flow through the system with certain longitudinal mixing. So, in plug flow there is no longitudinal mixing and travel some distance in time t_1 and t_2. So, as you see that it will be you know it may t=0.

So, it is here and that t_1 and t_2 also the it is maintaining it is identity so, that is basically the plug flow ideal plug flow. However, when we talk about the actual case so, in that you know you will have the dispersion when there is dispersion in the longitudinal direction. So, it is also known as dispersed plug flow.

So, it will be assuming that there will be certainly some extent of turbulent eddy dispersion which will be occurring in the plug flow in this case. So, what you see that, you see at $t =$

0 this is how the tracer was and at $t = t_1$ you see that there is certainly some dispersion in the longitudinal direction and at $t = t_2$ when it has gone.

So, you will have again the dispersion so, it will the tracer will be looking like this you know. So, it is because of the longitudinal dispersion you know that is occurring you know in the actual case. So, similarly if you go for even the step input.

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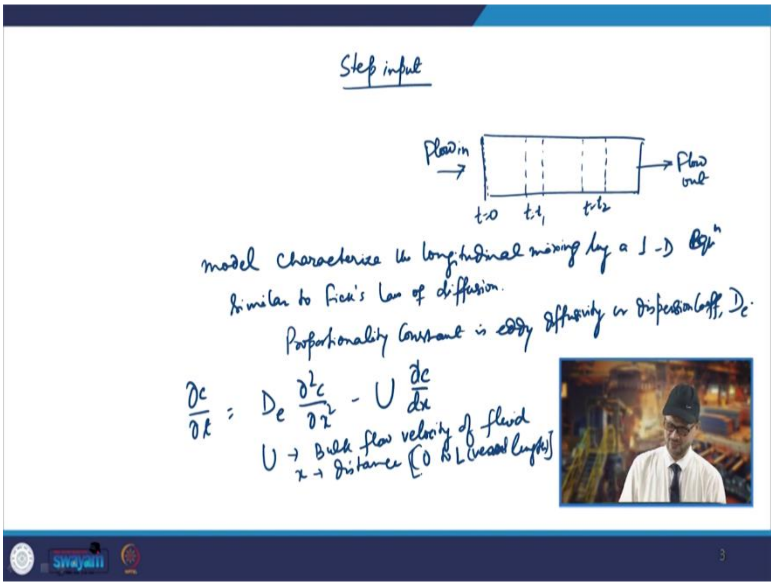

So, that was the case in the case of pulse input, but if you go for the step input also. So, in the case of step input if you see if you are you know you know flow is like going here. So, this will be flow in and this you will have the flow out. So, in that case also if you.

So, you will have $t = 0$ here and I mean ideally you know you will have after so, when you have the normal you know ideal plug flow in that case when you go for step inputs after some time you know that is your residence time. So, you will have automatically the concentration will rise.

So, your dimensionless concentration will come to 1 whereas, if you go for the dispersed plug flow. So, in the will disperse plug flow if you come to $t = t_1$. So, at t equal to there will be certain dispersion. So, this will be $t = t_1$ similarly you will have a $t = t_2$you will have this way the case.

So, accordingly it will move even in the case of step input. So, what happens that when we talk about these models. So, here you have the longitudinal mixing and that can be represented you know that will be the model will characterized this longitudinal mixing you know that it will be characterized by one dimensional you know equation similar to the fixed law of diffusion.

So, there will be diffusion that is going on in the longitudinal direction. So, it will be similar to that you know 1 dimensional fixed law of diffusion. So, here there will be again and you will have the constant of proportionality and this constant of proportionality will be known as the eddy diffusivity.

So, you will have so, this model which we will be using. So, that will be the model characterizing the longitudinal you know dispersion or longitudinal mixing and this is by the 1-D, 1 dimensional fixed law of diffusion 1 dimensional equation that is similar to the fixed law of diffusion.

So, as you know you will have in the fixed law of diffusion also you have the diffusion constant. So, here also you will have a proportionality constant and the proportionality constant will be known as the eddy diffusivity or we also call it as the dispersion coefficient that is D_e. So, in this case whatever you know constant of proportionality we use that is known as the you know dispersion coefficient and is this dispersion of this tracer which is there in the continuous flow system.

So, that will be resulting because of this eddy diffusivity eddy diffusion as well as the bulk flow. So, if you talk about the unsteady state you know concentration. So, that equation; so, that equation can be written as the $\frac{\partial c}{\partial t}$. So, that will be change of the concentration, that will be $D_e \frac{\partial^2 c}{\partial x^2} - U \frac{\partial c}{\partial x}$. So, that is you know the unsteady state equation for the concentration that can be written.

So, that D_e is basically the dispersion coefficient. So, here U is the basically bulk flow velocity. So, this is bulk flow velocity of the fluid and x is certainly the you know distance in that x coordinate. So, x will be you know distance and it will be varying from 0 to the length. So, that is L. So, it will be 0 to L, L is the vessel length. So, this way you will have these terms.

So, if you try to further write this equation in the dimensionless form. So, t should be written in terms of θ and similarly you can have x will be replaced by the term y. So, y can be taken as something x divided by the characteristic length or length of the vessel L.

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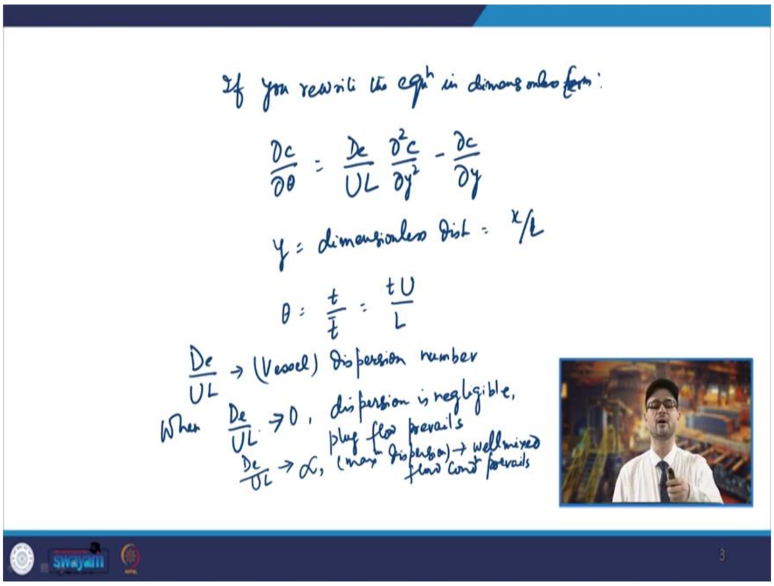

So, if you find. So, if you rewrite this equation. So, you can rewrite in dimensionless form. So, in that case you can you know dimensionless terms you can see you know term. So, you can write $\frac{\partial c}{\partial \theta}$where the theta will be you know dimensionless time.

So, then you will have $\frac{D_e}{UL}\frac{\partial^2 c}{\partial y^2} - U\frac{\partial c}{\partial y}$. So, here you are what you have done is you have non dimensionalized. So, you have non - dimensionalized by you know in the so, y will be the dimensionless distance and that will be equal to $\frac{x}{L}$.

So, you have divided x by the vessel length. So, that is y and θ is certainly $\frac{t}{\bar{t}}$. So, that will be$\frac{tU}{L}$. So, this way you know you can have the equation that is you know the in the non dimensional form. Now if you see this equation in this equation you have the term that is $\frac{D_e}{UL}$. So, this $\frac{D_e}{UL}$ it is a term that is dimensionless group and this term is known as the dispersion number or vessel dispersion numbers.

So, we call, so call it sometimes as the dispersion number or the vessel dispersion number. So, if you look at this term this is nothing, but this is the inverse of the Peclet number term.

So, you know in Peclet number you have the convection ratio of convection term to diffusion term whereas, in this case you have diffusion term to convection term.

So, this is basically so, the inverse of the you know Peclet number term and you know this is. So, it will be talking about the extent of longitudinal dispersion. So, it will be talking about the longitudinal dispersion in the sense that if this it is value is more so, longitudinal dispersion is more. So, accordingly your values will be you know coming.

So, it is not the molecular property of the system basically, but it will be depending upon the flow condition of the system so, depending upon the flow condition this $\frac{D_e}{UL}$ that will be changing. Now if you talk about you know the this value of this dispersion number. So, depending upon the different value of the dispersion number you will have the different you know shape of the F or the C curve.

So, that can be seen and more you know it is more pronounced effect can be seen in the case of C curve instead of F curve. So, when your $\frac{D_e}{UL}$ that will be approaching to 0. So, it will be limiting case that will be towards the plug flow whereas, if it is approaching towards infinity then it will be representing the well mixed flow. So, that is how the significance of these 2 you know these 2 extreme values of $\frac{D_e}{UL}$ carries.

So, you know if you. So, when your $\frac{D_e}{UL}$ when this is approaching towards 0 it means the dispersion is negligible. So, D_e term is you know that is a negligible. So, in this case you know so, once it is negligible, you will have the prevalence of the plug flow. So, plug flow prevails on the other hand if your $\frac{D_e}{UL}$ is very very large.

So, if the $\frac{D_e}{UL}$ is approaching towards infinity it means dispersion is very very high and the maximum possible. So, maximum dispersion and that will lead to the well mixed flow conditions. So, that is how you know your there is the significance of this number $\frac{D_e}{UL}$ that is your dispersion number. And you can we can have we will see that how the by changing this dispersion number your there will be change in the shape of the F curve or the C curve.

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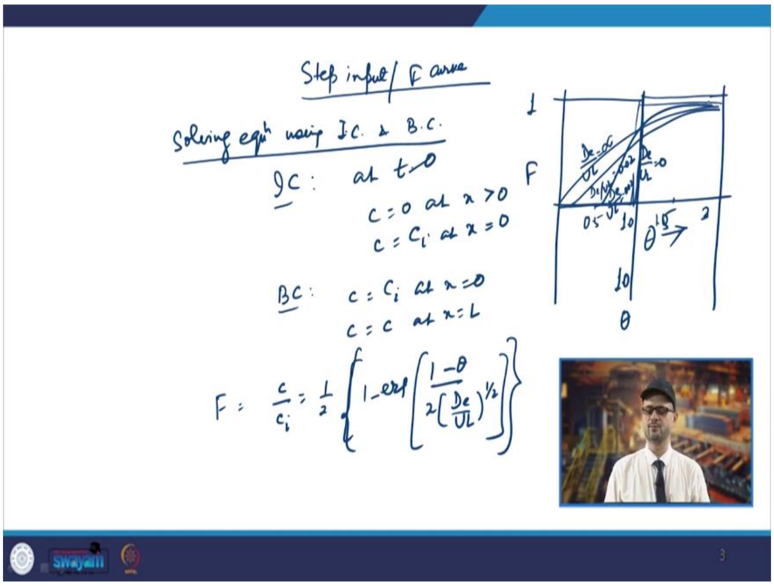

So, if you talk about the F curve. So, if you take the step input or F curve. So, the equation which we have seen earlier this was the equation. So, this was the concentration equation now this equation you know can be solved and you will have to have the initial as well as the boundary condition.

So, this equation if you solve using the initial as well as the boundary condition. So, solving equation using initial condition and boundary condition. So, if you have the initial condition so, you will have initial conditions are like at t=0 you will have c=0 at x>0 and you will have $c = c_i$ at x=0. So, initially you will have that condition because you have you are putting the tracer there.

So, and then if you have the boundary conditions also and the boundary condition will be like you will have $c = c_i$ at x=0 and then if you go to x=L. So, that will be c at x=L. So, if you solve I mean in this with this initial and boundary condition that particular equation then you are getting the solution of that equation as $F = \frac{c}{c_i}$s that is your F value and this value will be $\frac{1}{2}[1 - erf[\frac{1-\theta}{2(\frac{D_e}{UL})^{1/2}}]]$.

So, this is how you know if you take this as. So, this you know becomes the value of F when you solve that concentration equation that was $\frac{\partial c}{\partial t}$will be you know that was $\frac{\partial^2 c}{\partial x^2} - U\frac{\partial c}{\partial x}$. So, this now if you see.

So, this is nothing, but the equation which will be relating to the dimensionless time with the dimensionless concentration F and if you plot for the different you know the for the different $\frac{D_e}{UL}$ that is dispersion numbers. So, what you see is that if your there is no dispersion we had seen that when you have no dispersion in that case it leads to the ideal plug flow.

So, what will happen this is theta and this is value of 1. So, your it will be coming like this. So, it will come here and then at this point itself it will go. So, this value is 1 this is F and this is θ So, this is θ, this is 1, this is 0.5, this is 0 points this is 1.5 and this is 2 like that.

So, this flow will be showing. So, here if it goes like this is $\frac{D_e}{UL}$ is 0, now if as the $\frac{D_e}{UL}$ will be increasing now the error extreme value will be $\frac{D_e}{UL}$ as completely you know for completely well mixed flow. So, that is for $\frac{D_e}{UL}$ infinity and that will be that model will be going like this. So, this is the equation for $\frac{D_e}{UL}$ equal to infinity, now you will have the equations. So, you will have one extreme at the plug flow, another is the well mixed flow.

So, otherwise you get the you know the like curves like this or you get the curves like this. So, you know as the so, from 0 it will be increasing. So, it will be 0.00. So, for this it will be $\frac{D_e}{UL}$ it will be 0.002, here $\frac{D_e}{UL}$ will be 0.02. So, this is by the researchers they have found experimentally that for the different values of the $\frac{D_e}{UL}$ as it is 0 it will be the plug flow and as it is going towards infinity you will have the value that is for the well mixed flow it was shows like this.

Because dispersion has just started here itself whereas, for 0 it has not happened and at the mean residence time of θ equal to 1 that is a theoretical mean residence time all has come out and with concentration of 1. So, that is what the meaning of the you know dispersion number or the vessel dispersion number is significance is.

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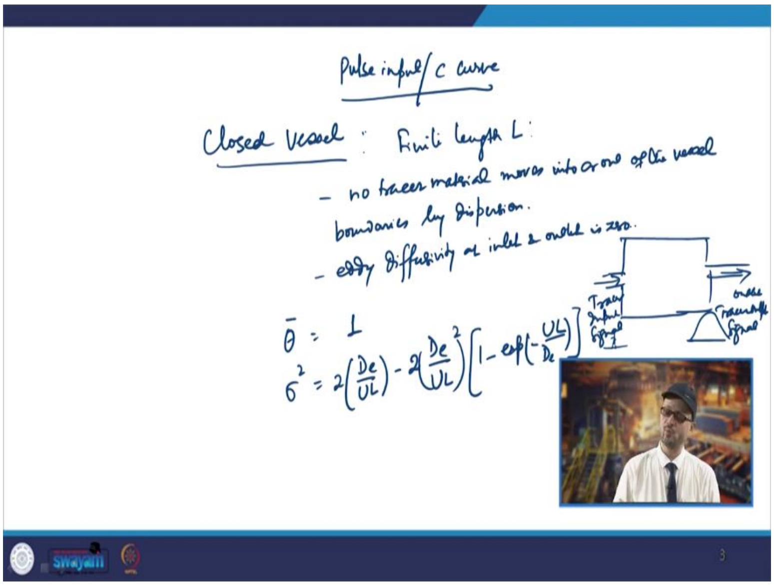

Now if we try to see for the pulse input so, if you go for the pulse input or you analyze the C curve. So, we have to further solve that equation that is $\frac{\partial c}{\partial t} = D_e \frac{\partial^2 c}{\partial x^2} - U \frac{\partial c}{\partial x}$so, that was this equation.

So, if you solve this equation so, you will have again for the pulse input you will have the different boundary conditions and you will have to solve. So, when we talk about the pulse input so, we are talking about the different you know cases and if we take suppose the closed vessel.

So, you may have the case of closed vessel or the open vessel, now in the closed vessel you will have the input from at the input as well as the are the output is no dispersion. So, the tracer will come it will go out where there will be a dispersion going on in the vessel whereas, in the open vessel means you will have the you know these stream which is open.

So, dispersion taking placed here also at the inlet as well as at the outlet also. So, if you look at the you know in a closed vessel. So, in the closed vessel you will have again. So, continuous casting operation the tundish can be taken as the closed vessel because your tracer inlet is from one side that is from ladle it is entering into the tundish and then the die is coming out through the tundish outlet.

So, the closed vessel by definition we must know that in the case of closed vessel which has a finite length of L and in which there is you know no tracer movement no tracer material moves in to or out of the vessel boundaries by dispersion. So, that is the definition of the closed vessel.

Now in this case the eddy diffusivity which will be there at the inlet and outlet that will be 0. So, your eddy diffusivity at inlet and outlet is 0. So, what we do is again you solve that concentration equation with the you know boundary condition in the closed vessel and in that case you get you know you get the mean as well as the you know variance of the curve. So, you what you see is you get the family of curve.

So, you know you get mean $\bar{\theta}$ as 1 and the you know variance σ^2 of this family of curve which you get. So, you if you solve it numerically so, you get the σ^2 as $2\frac{D_e}{UL} - 2(\frac{D_e}{UL})^2[1 - exp(-\frac{UL}{D_e})]$. So, that is you know the numerical solution that has been reported for when you solve for in the closed vessel.

So, closed vessel will be characterized by this kind of system. So, you will have the vessel here and you will have inlet here and then you will have outlet here. So, from here you will have inlet and from here you will have outlet. So, you will have tracer output signal from here.

So, at this point you will have tracer output signal and this is your input single signal. So, this is your you know tracer input signal and this is your tracer output signal. So, that is how you see that you know in this case what you see that your mean residence time that is $\bar{\theta} = 1$.

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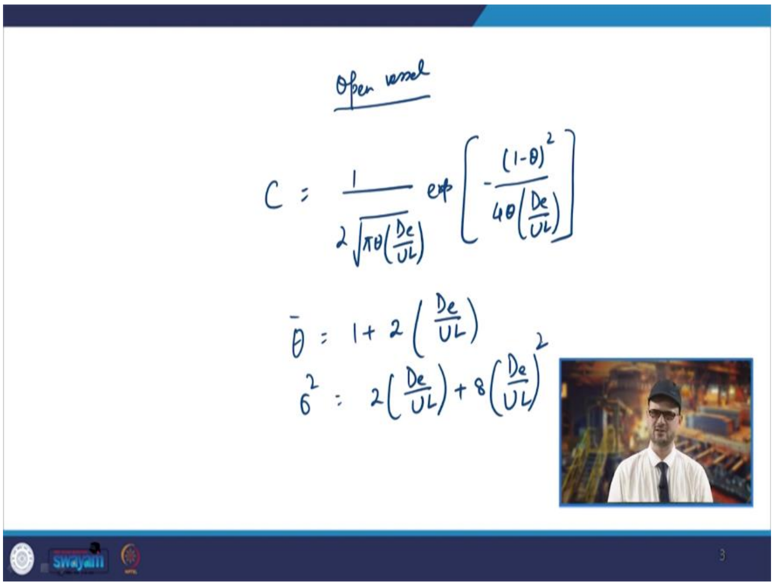

Now, if you take for the open vessel so, in the case of open vessel so, you know it has no discontinuity you know at the location of the tracer injection or the tracer concentration measurement. Now in this case you will have the experimental selected experimental length you know so that will be your open vessel.

So, in that in basically in the infinite vessel so, what you do. So, in the case of open vessel you know where the C curve has been found this is found to be $\frac{1}{2\sqrt{\pi\theta(\frac{D_e}{UL})}} exp[-\frac{(1-\theta)^2}{4\theta(\frac{D_e}{UL})}]$. So, this is by the researches that and that is reported by Sahai and Emi.

So, this is the solution of that equation 4.13. And in this case if you try to find the mean and variance of the family of the curve. So, the mean and variance will be coming as $1 + 2(\frac{D_e}{UL})$ and the variance is coming as $2(\frac{D_e}{UL}) + 8(\frac{D_e}{UL})^2$.

So, you know this is how you know the family of curves can be drawn for the you know pulse input and if you look at the dispersive model with a small amount of dispersion which is that consideration being taken away. So, if you consider the case I mean we are going to have the case for the dispersion model where for the dispersion number value is small.

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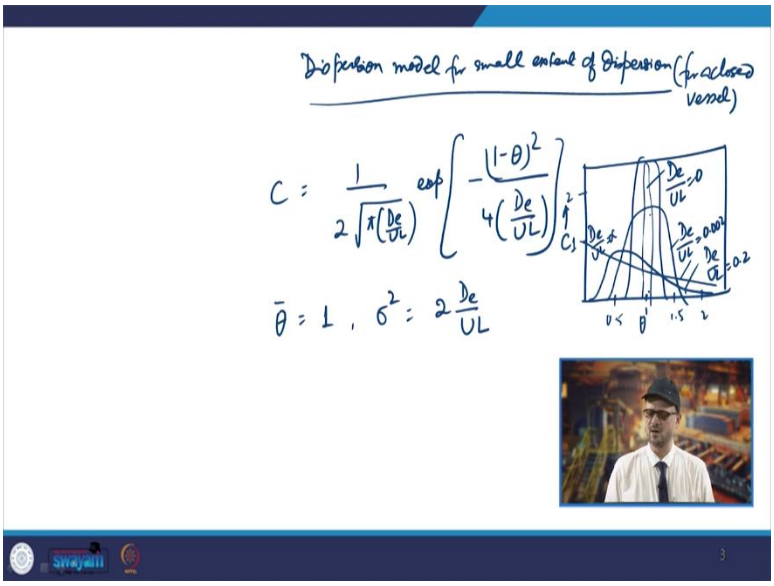

So, dispersion model for small extent of dispersion. So, where there is a small dispersion so, in that case what happens and for a closed vessel so, the closed vessel which is normally for the case of tundish flow. So, in that case the C curve is normally having a Gaussian type of you know profile. So, Gaussian type of RTD will be there when you have a low dispersion and small dispersion in the.

So, your C profile will be like the Gaussian dispersion I mean Gaussian type of profile and in that case if you do the solution of that concentration equation. So, that equation if you solve it that has been found to be as C equal to $\frac{1}{2\sqrt{\pi\theta(\frac{D_e}{UL})}} exp[-\frac{(1-\theta)^2}{4(\frac{D_e}{UL})}]$..

So, that is what has been reported I mean found out that this is how your concentration you know a family curve looks like and for that your mean and variance. So, if you find a mean and variance that mean becomes 1 and variance becomes $2(\frac{D_e}{UL})$. So, if you try to you know draw the these family of curves and for the different value of$2(\frac{D_e}{UL})$.

So, you know for the large and small value of the dispersion in a closed system. So, once you draw then your C curves look like looks somewhat like this as you know. So, as you know that when you have you if you might have you can recall that in the case of well mixed flow. So, completely ideal mixed flow your curve starts from here. So, if you have this is as 1 and this is 2 and, so in the case of a well mixed flow it will go like this.

So, this is for the you know maximum $\frac{D_e}{UL}$. So, your so, for this case $\frac{D_e}{UL}$ will be infinity. So, that is representing the well mixed flow, now if you have the plug flow so, now, in this case this is theta. So, this is 0.5 this is 1, this is 1.5 and this is 2. So, in the case of you know the plug flow as you know. So, this it will move like this.

So, this will be $\frac{D_e}{UL}$ that is 0. So, this do dispersion. So, that is your $\frac{D_e}{UL}$ is 0; however, you will have the curves coming like this. So, and you have also curves in going like this. So, if you see here this curve is corresponding to where this curve will be $\frac{D_e}{UL}$ that will be equal to 0.2 and if you look at the another curve so, you will have you may go like this. So, this will be $\frac{D_e}{UL}$ will be 0.002 or so.

So, like that you will have the variation in the family of these curves and as you go towards the $\frac{D_e}{UL}$ values smaller and smaller. So, you will have the confinement and for the 0 value your you know value your curve comes at θ that value equal to 1.

So, this different values you know represent the different kind of dispersion different extent of dispersion and the family of these C curves can be drawn. So, this is about you know the actual you know flow characterization which takes place. We will be talking about the other aspects like about the calculation of the mean residence time and then it is mean and as well as the variances in our coming lectures.

Thank you very much.