

Materials Science and Engineering
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Lecture - 03
Crystal systems and structures: Lattice


Hello, friends one of the most important and to understand the main material behavior is crystal systems and structure ok. So, if we understand the crystal structures, we will be able to understand the material behavior in a much better way and actually a lot of properties depend upon that what is the crystal structure of the material ok. So, today's lecture we will consider we will talk about first we will talk about a lattice and then we will come to the crystal systems and crystal structures ok.

So, when we want to understand the crystal systems or structures basically the lattice is the framework, which you can use to understand that what do we mean by crystal system and structures ok. Lattice in loose sense you must have used a lot of time that whenever you want to see you see a regular pattern you say that this is a lattice ok.

For example, nowadays you must have seen on the pavement you put those tiles some very nice shape tiles and they are put next to each other you must have seen the or you must have seen the, the honey bees they the, the, the structure they make which is a very nice hexagonal lattice

So, the lattice is there in the nature and you must have seen it ok. So, the basis for crystal systems and structure is start it starts from the lattice ok, so, first lets understand the lattice.

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Auguste Bravais

Topics covered

- Lattice – 2D and 3D
- Unit vector and cell
- Lattice + Basis = Crystal

Bravais, A. (1850). "Mémoire sur les systèmes formés par les points distribués régulièrement sur un plan ou dans l'espace" [Memoir on the systems formed by points regularly distributed on a plane or in space]. *J. Ecole Polytech.* **19**: 1–128.

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The this idea was actually proposed by Bravais that is why they are called Bravais lattices. So, in this particular lecture we will cover the lattice in 2 D and 3 dimensions, 2 dimension and 3 dimensions and a unit vector and cells and then we will talk about lattice and plus basis to make crystals, and then what are the different crystal systems and crystal structures ok.

So, the Bravais proposed that that the idea of lattice and using that idea of lattice he proposed that there are can be only 7 crystal systems and 14 crystal structures. So, you can understand and this work he did in 1850 and you can understand that in at that time he could predict what will be the crystal system and structure in all the natural or manmade materials which are possible ok, and they are still the the there is no change in that there it is still holding and he could do that only by doing geometry ok.

So, he must have a spent a lot of time and lot of thinking and of course, him did not have any distractions of TV, and your io your smart phones and all that which we nowadays have all these distractions ok. So, he did not have that; obviously, and because of that I think he could have devoted lot of time and energy in this I concept, and using geometry only he proposed it only these many crystal systems and structures are possible. And with all the fancy tools which we have nowadays it is still holding with we don't have any other then what he proposed in 1850.

So, the and ideas were very nice and elegant. So, starting from 2 d lattice that what do we mean by lattice ok, the important properties of a lattice is that a lattice has exactly same surroundings ok.

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2D- lattice

Important properties of a lattice –

- A lattice has exactly same surroundings (or scenery if you like !!) when looked from any lattice point, in a particular direction
- It should have translation symmetry
- and rotation symmetry

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This is very important or if you if you like you can call it as a same scenery is there. So, if you take any lattice point and if you see around that lattice point other lattice points if you see the surrounding has to be same if I take this lattice point and suppose I take maybe another lattice point after ten lattice points and if I also check surrounding around that lattice point, between this and the new one there should not be any change it should be same the scenery has to be same ok.

So, the surrounding has to be exactly same of course, you have to look only from 1 direction if you want we are looking from another direction the surrounding will change, but from that direction if you see any lattice point you choose any lattice point the surrounding around that lattice point will be is exactly same.

So, from looking from 1 direction if I choose any lattice point and it k there can be infinite lattice points ok. So, it is a imaginary concept you can extend, extend it to the length of the universe if you want you choose any lattice point and around that the surrounding has to be same. So, this is the first property it should have , how you can make a lattice is a very simple thing the most important property it should have is what we call as translation symmetry ok.

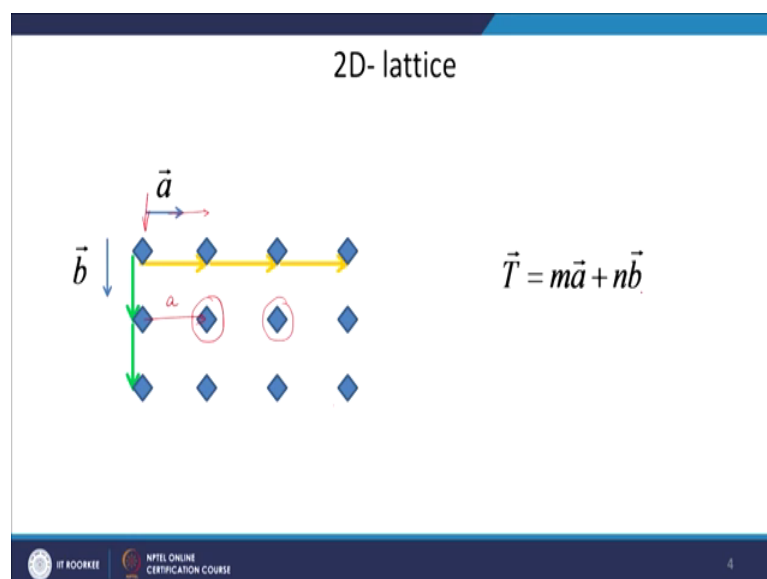
So, basically if you have we start with 1 lattice point if I you give you 2 translation vectors in 2 dimensions, then you will be able to get all the lattice points and up to infinity ok, again you can take a take example of the pavement ok. So, if I want to fill the pavement there is a pavement or any porch is let us say and I have to fill the whole area ok.

So, what I will do I will put first tile, then I will put the next tile. So, by 1 translation I have put the next tile. So, from the center of the tile if you see from this center to the next tile center there is a you can say it is a vector is there then I put another tile. So, like that if I keep putting or I can keep translating the vector then you will get the whole area covered and you have a lattice.

So, in the lattice also the most important property is it should have a translation ok; that means, by translation of a 1 unit vector I should get the next lattice point, another translation of that same unit vector I should get the next lattice point and so on. And that is why there surrounding is same because it has a translation symmetry. So, everywhere the translation has to be same the translation vector has to be same, another very important property it should have is called rotation symmetry.

We will not cover or not dwell into rotation symmetry too much, but we will use translation symmetry to understand the lattice and to construct a lattice ok. So, let, let us start with the with a lattice point like that.

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So, this is a lattice point and we want to create a 2 dimensional lattice ok, and suppose I have defined a unit vector a in this direction and suppose I so when I translate it by a , in that direction I have should get the next lattice point.

Similarly, if I keep translating I should get the next lattice point, by the same vector a similarly I can have another vector b ok, in another direction and if I keep translating by the vector b in another direction I should get the next lattice point. So, if I do a vector translation of this a vector it should have been little bit larger here ok.

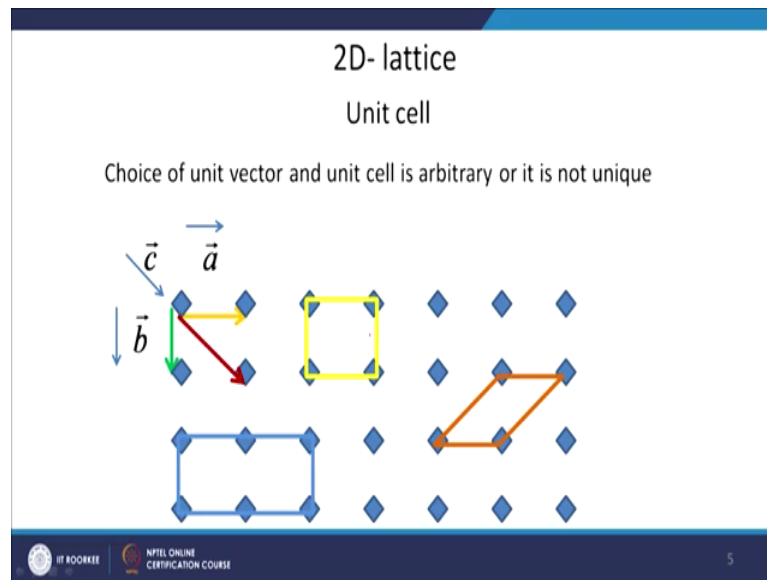
So, by doing a translation of a I should get the next lattice point in this direction, in the direction of a if I keep translating by a vector b in this direction, in the direction of b I should get the next lattice point and if I keep doing that I will fill the whole area here and I should get the lattice point.

And if you see the, the surrounding as I was telling you suppose you choose this lattice point if I check the surrounding ok. So, you have 1 lattice point here another here and the distances they look very same then what I suppose I choose this lattice point again the surrounding is same. So, for each lattice point the surrounding is same and that is because you have this translation symmetry.

So, this translations vector can be easily represented by a mathematical equation like this that m into a you go in this direction n into b you go in b direction you will get the whole translation, if this vector is the unit vector then m and n becomes unity ok. So, it is simply the translation in by a and b vector in the direction of a and b respectively then by doing that I can get the full lattice.

Now, what should be the choice of unit vector and unit cell ok, that is an arbitrary choice there is no unique way of saying that this should be the unit vector or this should be the unit cell. So, the unit cell is the cell, which kind of represent that particular lattice if I you choose a unit cell it is a representation of the whole lattice So, that is the idea of a unit cell.

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So, suppose again as I we did earlier that a in this direction and b in this direction will construct the whole lattice ok, I can also construct the lattice by instead of choosing b I can choose another vector which is c ok, and if I do that also then I should get the next lattice point and so on ok. So, the idea here is that there is no unique way of defining a unit vector I can choose any vector as a unit vector and still I can construct the whole lattice.

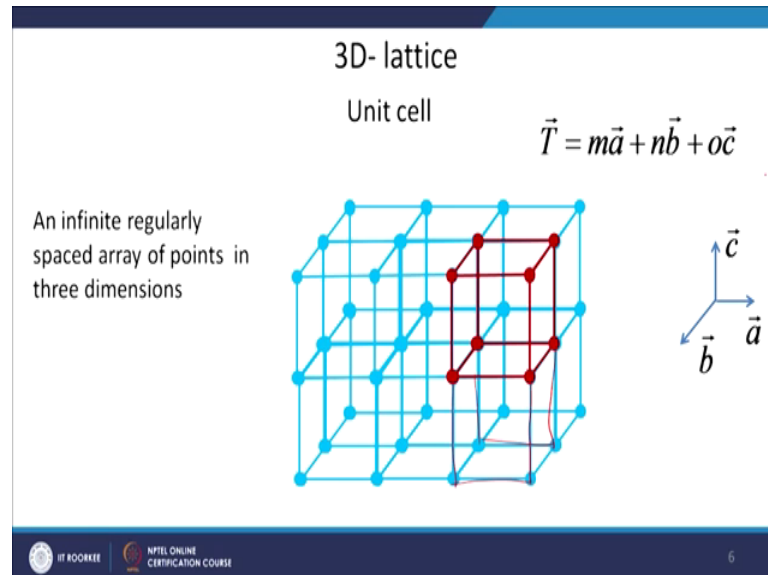
Similarly, the unit cell I can choose any unit cell for example, covering these 4 lattice points I have chosen a unit cell like this, or I can choose a unit cell like this may, but in this case now it is covering more than or it is covering 6 here you 6 lattice points, or I can choose a unit cell like this also.

Ah reputation of this unit cell will also give you the lattice. So, the choice is arbitrary you have to choose it according to your own idea of the lattice the basic of choosing a unit cell is that it should represent the lattice ok. So, if you see this particular lattice it looks like a square lattice ok; that means, a and b vectors are same their length is same. So, both the vector have the same distances or same magnitude.

So; that means, that if I choose a square unit cell for this that is actually able to represent this particular lattice ok. So, choice can be anything out of these three, but still I will choose this is square lattice because it is able to bring out the property of the lattice, that

it is a square lattice it has a square symmetry. Now, coming to 3 dimension lattice again the same thing can be done in 3 dimensions ok.

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So, in now instead of 2 dimension it is an infinitely regularly spaced array of points in 3 dimensions. So, you can see a nice array of lattice points is there ok, and 1 of the unit cell is carved out of that which is able to nicely represent this particular lattice and in this case it is a cubic unit cell because it has a nice cubic symmetry to it ok, all the 3 lengths are equal ok. So, that is why we have chosen this one, we could have chosen unit cell like this also where this is extending up to here instead of ending here.

So, this could have been a unit cell, but it is not a giving you a nice representation of the lattice because it has a nice symmetry, where a is equal to b equal to c. So, a unit cell which has a cubic dimension that should be the right unit cell for this lattice. So, unit cell can be chosen arbitrarily, but you should choose it which represent that particular lattice.

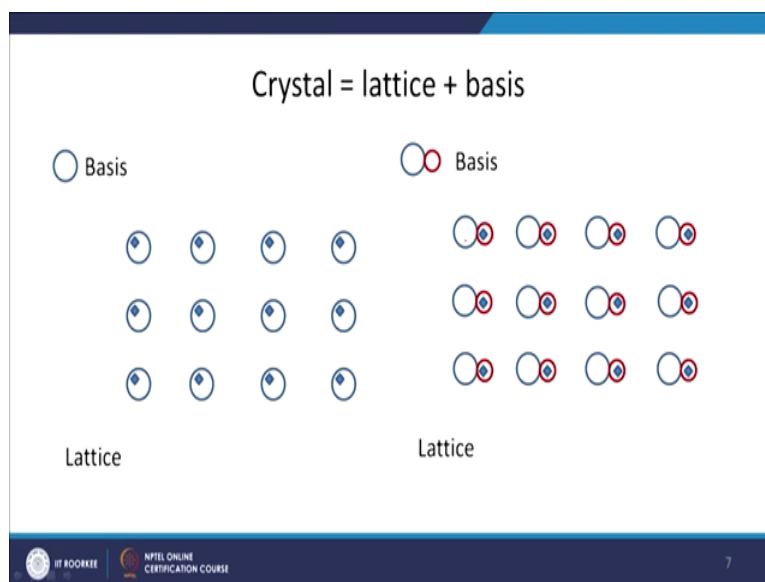
So, abc is in these 3 direction and now my translation vector can be instead of 2 dimension it has to be in 3 dimension. So, m times a n times b and o times c will give you the next unit or next lattice point and so on. Now, till now we have not talked about the crystals or crystal structures or atomic arrangement, we have not talked about all these things we have only talked about lattices ok.

So, lattices can be anything it is a 3 dimensional space and you start putting a lattice point here, and now you define that after this much translation I should get the next lattice point. In a this much translation I should get the b this much translation I should get the c and if I keep repeating, I will have I can have full or fill the whole space here in this room. And the choice can be anything for abc and depending upon that you will have the whole space filling arrangement for this 3 dimensional space.

So, we have not talked about anything about crystal or crystal structure or atomic arrangement also s and so on. So, you have to keep these 2 ideas separate 1 is lattice which is just a 3 dimensional space filling arrangement that how I can arrange in this 3 dimension lattice point. So, that it will fill this room or you fill this 3 dimensional space.

Now, if I start putting atoms or molecules or iron on this lattice point then I am building a atomic crystal structure and then I am talking about materials, till that time it is it is a simple geometrical arrangement of points in 3 dimension , which has a translation symmetry and we satisfy the, the requirement of a lattice.

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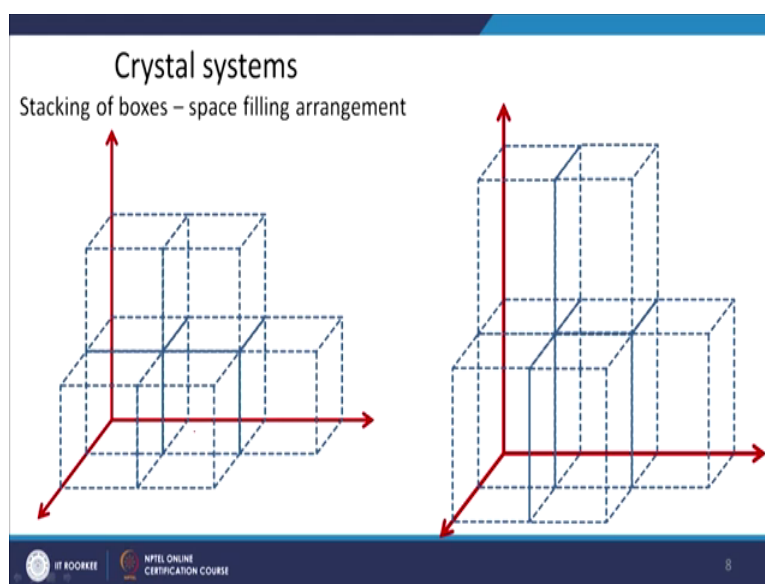
So, when you put a basis on the lattice point then it becomes a crystal ok. So, the crystal must have a lattice and a basis and basis can be anything it can be as 1 single atom , it can be a molecule 2 2 atoms combined sitting on the lattice point or it can be an ion which has a some positive or negative charge.

So, that I am representing here with 2 different type of basis in this first example the basis is simple 1 blue circle, in another 1 it is a blue circle combined with a red circle with 2 different diameters ok. So, now I will start arranging the, the basis on the lattice point ok, arrangement can be any way the circle can be such that that my lattice point is at the center of the circle or in this case it is little bit in a offset position ok. So, anything can be possible, but for all the basis the condition should be same. So, you can see that it is offset, but the offset is of the same type in all the conditions or for all the lattice points ok.

So, now I have put a basis, so you can consider that I have put an atom, so now it has become a crystal. So, lattice plus basis makes a crystal this is another arrangement of these 2 circles which is a basis ok, and I have done an arrangement like that. So, the arrangement is such that the lattice point is coming within the red circle here. So, that will be same for all the lattice points it, it cannot be that for 1 point the blue circle the lattice point come within the blue circle then it cannot be a crystal.

So, the now this is another crystal with suppose you can consider that it is a molecule here 2 atoms are joined there and that is put at the lattice point. So, like that for different crystals you can put different basis or for the layer same lattice also to get different type of crystals.

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Now, as I told you that Bravais said that there are only 7 crystal systems are there and there are fourteen crystal structures ok. So, how this concept of first let us see how this concept of lattice came, so why this seven systems are only the possibilities ok, to understand that you have to see that how I can have a space filling arrangement.

So, 4 times this is example this is a 3 dimensional space a room is there if I have to fill it this room what I should do ok, basically if I take example of boxes if I key start keeping boxes 1 above another I will be able to fill the whole space ok. And that is how B also started that what can be the different ways or how what can be the different type of boxes which are available which can give me this space filling arrangement ok.

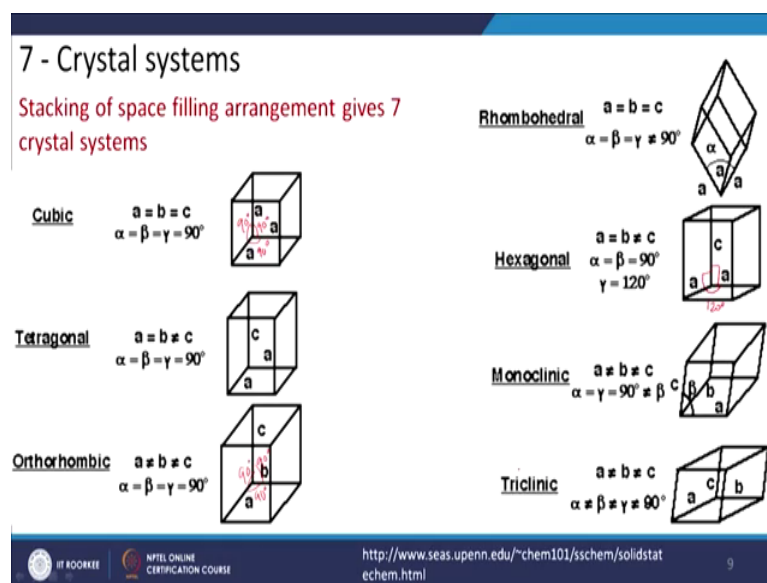
So, you can consider that how understand this lattice from the staking of the boxes ok. So, I am just giving you an animation here to have a space filling arrangement ok. So, this is a 3 dimensional space here 3 vectors are there and I am putting a sta doing a staking ok. So, I took a box such that the 3 dimensions are same $a=b=c$ is same.

So, it is a cube and I am putting over 1 another and I am trying to the saying that the whole 3 dimensional space can be filled by doing that ok. I can have another type of box ok, which is not a cube it is a cuboid and then I have another way in which I can do this space filling.

So, you can understand and you can now have different boxes ok. So, what Bravais did is by doing all this geometry he found out that only there are 7 possible ways to do this space filling arrangement or 2. So, there are only 7 type of boxes to fill this whole 3 dimensional space ok, and he started putting it on one another and saw that this is satisfying my condition ok.

So, you cannot have a box which is a spherical box, or you cannot have a box with which has a curvature because then you will have some areas which will not be filled ok. So, you have to have arrangement which is a space filling arrangement and this gives you a space filling arrangement so, by doing this he proposed that there are only seven crystal systems ok which you can have, so starting with a very simple system or box and that is it cubic ok.

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So, in cubic a is equal to b is equal to c and $\alpha = \beta = \gamma = 90^\circ$ so; that means, the 3 dimensions are same and the angle between these 3 axis is 90° , most one of the most simple arrangement you can think of.

Now, you start complicating the, the arrangement. So, the first complication which you can bring is that a should be equal to b , but the c dimension is not equal. So, a equal to b , but c is not equal to both of these; however, the angles are still 90° ok.

So, when you have this arrangement it is called tetragonal ok. So, these 2 example I have shown in the earlier slide, slide to you again bring more complications I can say that a is not equal to b not equal to c ; that means, that all the 3 are not equal to each other. So, a is different b is different and c is different.

Now, , but the angles are still 90° to each other. So, angle between these 3 axis is still 90° all are 90° , here also all angles for 90° . So, in this all these 3 cases angles are 90° in cubic all axis for same in tetragonal 1 axis was different in orthorhombic all the 3 axis are different angles are still 90° .

So, this is the most you can say general example from which you can get cubic or tetragonal, by putting some constraints then comes now we say that and the dimensions are same a is equal to b is equal to c α is equal to β is equal to γ angles are also equal, but they are not equal to 90° ; that means, angle α can be

anything other than 90 degree beta can be anything other than 90 degree and gamma also can be another anything other than 90 degree.

But all the angles whatever it is let us say it is 60 degree then alpha beta gamma all are sixty degree ok, equivalent or equal to 60 degree for example, then comes hexagonal. So, again a is equal to b, but is not equal to c alpha and beta are equal to 90 degree, but gamma is not equal to 90 degree. Now it is equal to 120 degree, so this angle here is 120 degree, but these 2 angles are 90 degree to each other.

So, between a and c it is 90 degree between b and c, which is again is shown as a because both are equal. So, between b and c is also it is 90 degree, but between a and b it is 120 degree though it doesn't look like a hexagonal to you, it doesn't look like a hexagonal, but I can show you that how it will be hexagonal in the next slide.

Then comes the monoclinic, now a is not equal to b not equal to c. So, now all the 3 dimensions are not equal alpha and gamma are equal to 90 degree, but not equal to beta ok. So, 2 angles are equal to 90 degree, but are not equal to beta.

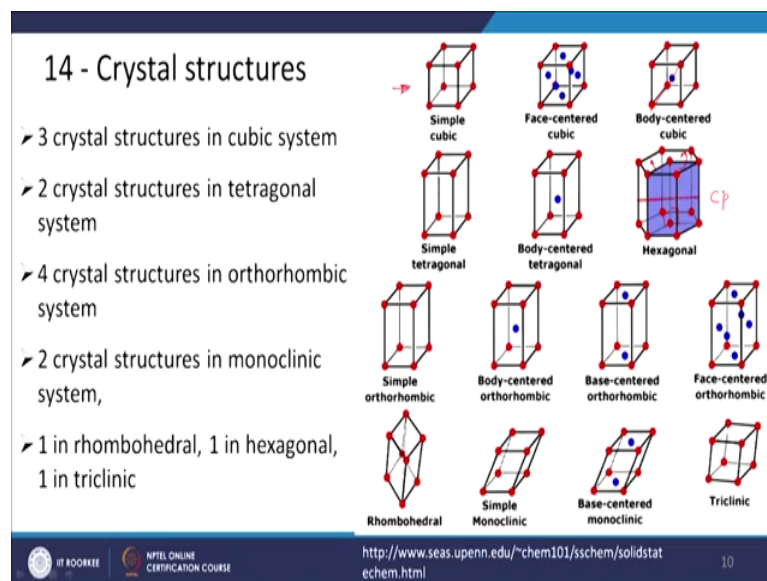
So, now, beta we have not put any condition, so beta can be any angle in the hexagonal case we said the third angle to be 120 degree, but here we are saying the third angle can be anything. So, we are not putting any condition there, so it can take any angle whatever it like.

Then the least symmetrical is the triclinic, so none of the dimensions are equal none of the angles are equal and of course, they are not of course, equal to ninety degree also. So, a angle between the 2 axis can be anything, any 2 axis can be anything their dimension can be anything, and usually you find this kind of crystal system for geological samples for example, rocks. So, rocks also have the same idea is what we understand in materials ok.

So, very interesting concept ah, but problem with the geological samples or rocks is that lot of these rocks have this triclinic symmetry and one of the most difficult to understand because nothing is same all are different ok. So, very difficult to understand anything with the triclinic crystal system.

So, these are the only the seven system ok, so how you got the seven system by doing this staking of boxes ok. So, you make box of this all these different shapes you start putting it over one another and you will see that there is a nice space filling arrangement is there. So, from that he could understand that there can be only these seven ways within which you can do that.

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Now, from this seven comes 14 crystal structures ok, so now, he started putting lattice points at different places and he said that in cube for example, there are 3 different arrangement of lattices are points are possible, one is that all at corners you put the atoms or in this face centered cubic you can put at the face centered position also on the faces of the cube , you at the center of the faces you can put 1 atom or you can have a body centered cubic condition, where you can put the lattice point at the center of the cell.

So, in cube you can have 3 different type of arrangements are possible in tetragonal you have simple one of course, at the corner simple is everywhere. So, simple at the corner and then the next one is the body centered tetragonal. So, there is no face centered tetragonal here or base centered tetragonal here this is what is the greatness of that guy that he could do all kind of permutation and combination and then he said that if you try a face centered body center it can be represented by a much easier or much simpler lattice arrangement ok, which is already he has defined ok. So, that is why those lattice arrangements are not possible

So, but body centered 1 is a unique 1 that cannot be shown by any other arrangement. So, all these are unique arrangement you cannot show any 1 of them with any other arrangement. So, that is what is the greatness of that guy, that he could see that or he could work out all these arrangements.

Then as I told you in case of hexagonal I showed you only this much part of the cell that a and b are same c is not same as in b, but between a and b the angle is 120 degree between a and c and b and c the angle is 90 degree ok. So, this was the unit cells shown, but if you repeat this cell 3 times. So, 1 is shown here if you take this 1 and put here like this and third time you take this 1 and put here then it will give you a hexagonal unit cell. So, that is why it is called hexagonal in that I reputation of this he gives you a hexagonal unit cell.

Then orthorhombic all lot of possibilities are there you have simple orthorhombic on the corner positions body centered orthorhombic, orthorhombic where the lattice point is at the center of the this unit cell, base center orthorhombic. So, 2 unit cell at the base and face centered orthorhombic where the unit cell sorry lattice points are at the face centered position ok. So, all these are lattice points.

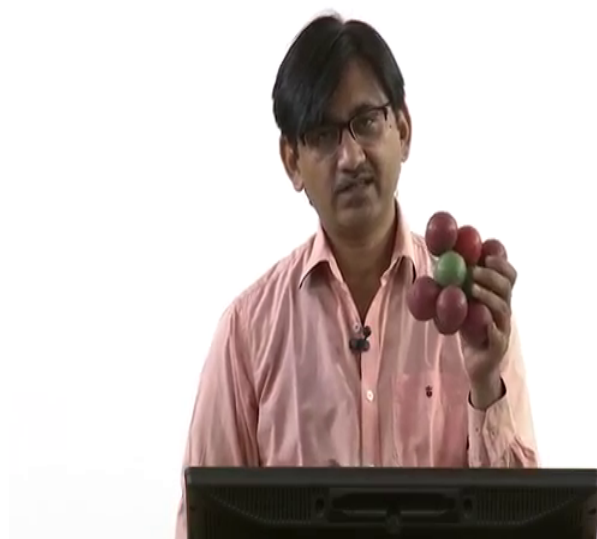
Then you have rhombohedra only 1 possibility monoclinic has 2 possibilities simple monoclinic has been centered monoclinic and of course, triclinic can be only 1 type. So, there are 3 system which has only 1 possibilities 1 is rhombohedral 1 in hexagonal and 1 in triclinic

So, now of course, some of the these crystal structures for example, cubic 1 you must have done in your twelfth class physics also our chemistry ok, and you must have calculated the linear density planar density ok, and you must have also done that what are the dimensions of this face diagonal or body diagonal ok.

So, that I am not covering here just before finishing this off actually in hexagonal there is 1 arrangement is not shown here this is not a closed packed hexagonal ok. There is another hexagonal which is called close packed hexagonal structure which I have brought this ball model here. So, you can see that these, these are the 6 atom at the bottom.

So, you can see here also 1, 1, 2, 3, 4, 5, 6 and 1 at the center. So, 1, 2, 3, 4, 5, 6 and 1 at the center this is the base this is the top which is shown here 1 again 1, 2, 3, 4, 5, 6 and 1 at the center which you can see here. But you can also have another set of this same arrangement at the center of the cell here, at the center of this cell and which is shown here ok.

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So, when you have this arrangement it is called a closed pack arrangement why we call it as a closed pack arrangement this is the closest packing you can have, for example, between the 3 balls here there is another ball sitting in the cavity of that and with the spherical objects this is the best packing you can have ok. So, this is called a closed pack arrangement in hexagonal system and where the, you have this kind of arrangement.

So, you will have another set of the same arrangement, in the center which is like this ok. So, this is a closed packed hexagonal structure, if you want to see a same ball model for FCC it will look like this. So, 4 corner atoms and 1 in the center of the face ok. So, this is the face and 4 corner atom and 1 at the face centered position.

So, the if you see the how the atoms are touching each other or what is the equilibrium distance between the atoms, the equilibrium distance will be in the face diagonal in case of a centered cubic structure whereas, in the this direction you will see that there are some gap is there; that means, the bonds are stretched in these directions, where is in the face diagonal the it is having a equilibrium distance ok. In case of body centered cubic it

will be along the face diagonal along the body diagonal, it will have a equilibrium structure equilibrium is spacing other places it will be a stretched bonds ok.

So, this is a example of a face centered cubic structure and this is for an hexagonal closed pack structure. So, with that I am finishing this lecture in which we have covered the lattice crystal systems and crystal structures ok, and the detail of these we will do or use lot of the ideas of this crystal structure of the especially we will be using example of cubic systems. So and simple cubic also you don't see much.

So, the most of the metallic material will be either in body centered cubic structure or face centered cubic structure or hexagonal closed pack structure ok. So, we will be taking only these 3 in more you will be using in in to understand the material behavior so with that way I.

Thank you for having patience and understanding this concept little bit difficult, but I think you will be able to catch it ok.

Thank you.