Materials Science and Engineering

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Lecture – 26
Engineering and True Stress and Strain

Hello friends. So, we have already discussed about tensile test and what kind of properties we get. And we have also discussed in general about what do we mean by stress, what do mean by strain and we have discussed little bit about elastic deformation. So, maybe sequence can be little bit you may feel that it is if we are discussing elastic deformation before tensile test, but it can be argued that elastic deformation understanding is what we were trying to do from the bonds which are there between the atoms.

So, then we discussed about the tensile test and what distance curve we get from the tensile test. And that you have different type of deformation elastic and plastic and plastic and plastic also we saw you can have uniform and non uniform plastic deformation and so on. Now stress strain also when we start discussing in terms of tensile stress strain curve, we want to define it in terms of two different stress and strain curve and these or stress and strain values which are engineering and true.

So, we have we are now trying to be very specific that what do we mean by straight stress and strain in terms of mechanical properties or in terms of a test using the tension doing a tension test.

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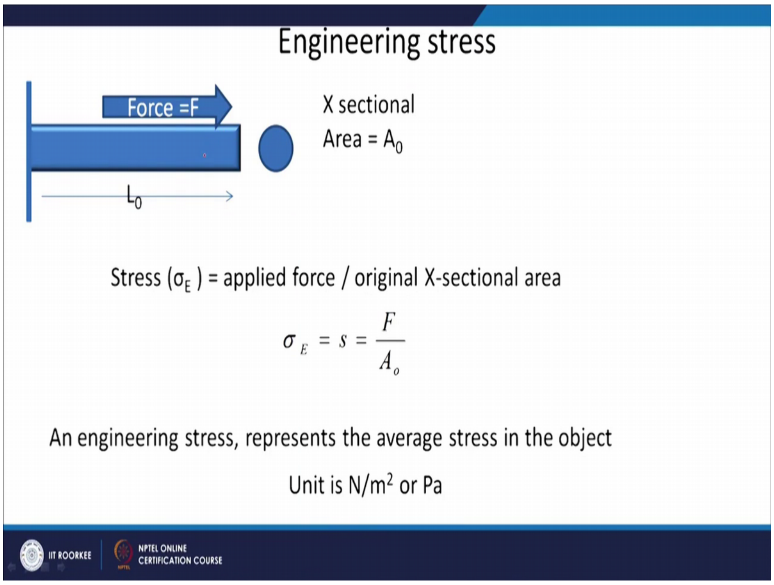

So, first let us define the engineering stress; so, basically if this is cross sectional area is there for a for example this sample and I am applying a force and it is kind of being in it fixed at a particular point here and you are applying a force. And this is the cross sectional area of this sample and that is given by a naught here then the stress which is the engineering stress here which will be denoted by a sigma E is applied force divided by the original cross sectional area.

So, force can be applied or it can be the response of the material to the deformation. So, you if you deform the material the response of the material can be in form of resisting force or resisting stress. So, that is force divided by the original cross sectional area will give you the engineering stress. So, we are now very specific that which area we are talking about here that I am talking about the original or initial.

So, when we started the experiment whatever was the original cross sectional or initial cross sectional area of the material; we are talking about only that area divided by force of forces divided by that area. Of course, already we have talked about the units here; so, an engineering stress represent the average stress in the object whatever is the average cross sectional area and divided by the forces divided by this cross sectional area will give you the engineering stress.

So engineering strain if you want to define; so, when we are again trying to deform the material.

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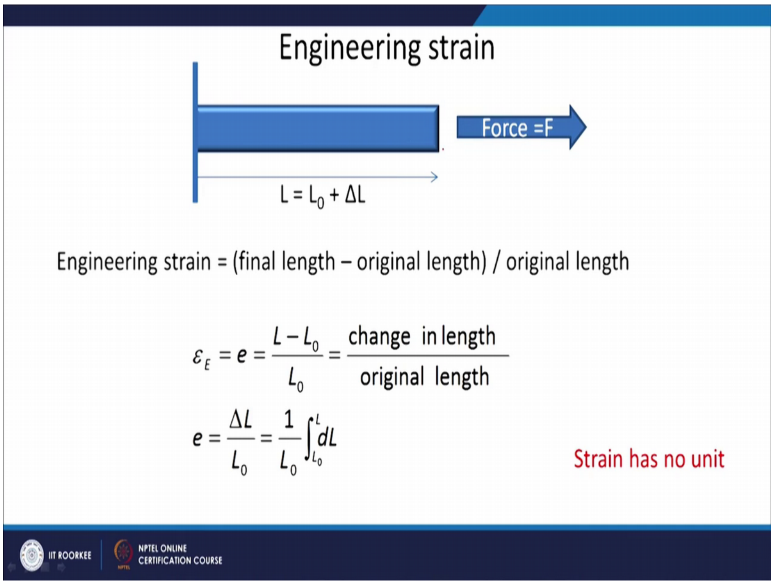

So, your now length L is will be equal to the initial length L naught and whatever is the elongation or deformation which is delta L. So, initial length will be L naught that is my initial length which has become L after deformation L.

So, after deformation this has become L. So, L is equal to L naught plus whatever is the deformation delta L. So, in a engineering strain if I want to know that will be equal to final length minus original length divided by original length. So, basically delta L I already we have seen the delta L divided by L naught. So, delta L contains the final length L minus L naught that is my going to be my delta L divided by original length.

So, in the in some in the mathematical terms I can write it as epsilon E sub E or I can just say that with a small e which I am saying that it is engineering strain that will be equal to L minus L naught upon L naught. So, that is change in length upon original length. So, delta L upon L naught as I told you or in terms of if you want to integrate over all the deformations then it will be 1 by L naught integration from L naught to L into d L.

So, suppose if I am doing the deformation in increments.

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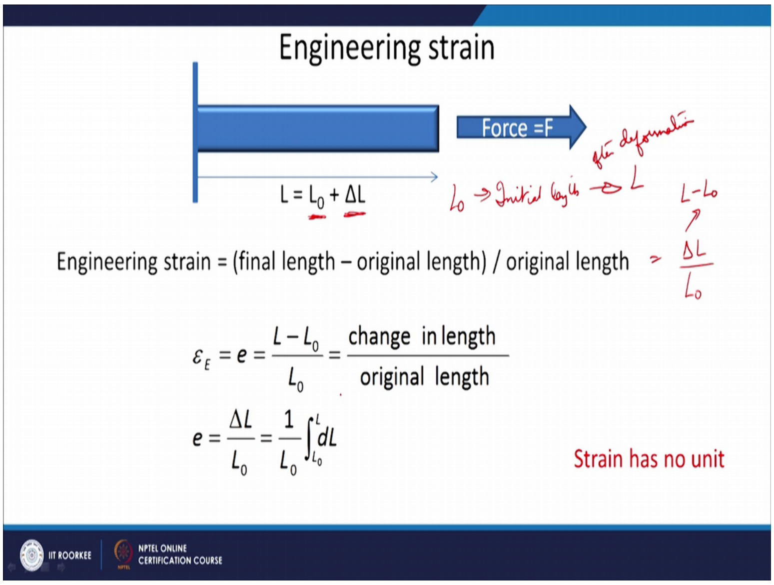

I can just do an integration like this to get the total strain in the engineering strain and the material. Important thing to notice that strain has no unit because we are dividing by length by length. So, there is not going to be any unit for strain engineering strain they also is expressed in form of fraction or percentage. So, either we will saying we will be saying 0.2 percent; if you remember this is what we said when we were trying to find out the yield strength of a stress strain curve which is a continuous stress strain curve.

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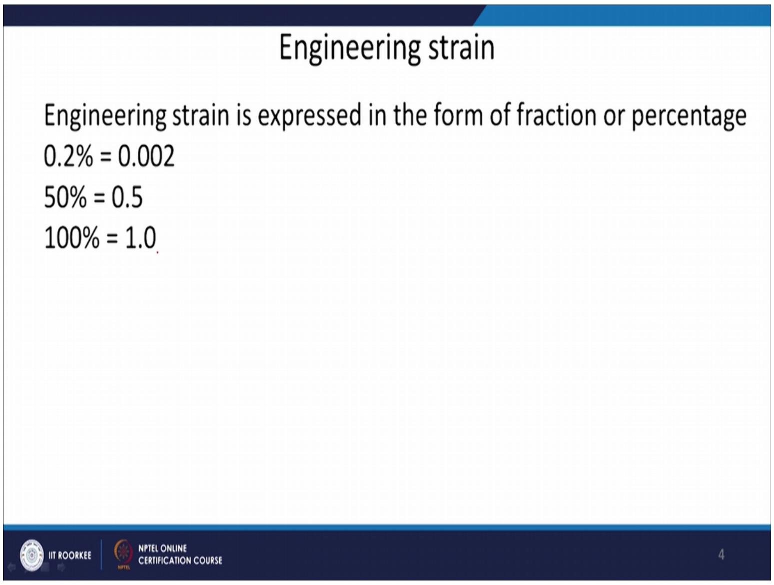

So, we found out the yield strength after a 0.2 percent of permanent strain or plastic strain; in fraction it will be 0.002; 50 percent will be equal to 0.5; 100 percent will be equal to 1. So, you can express it in different forms fractional forms or in percentage.

Now, the question is that when I am deforming a material initially when you are in elastic part the deformation is very small and you will not be able to notice that, but if I am going into plastic deformation what will happen? This is my some bar with this cross sectional area; I am applying the force and after some deformation now the cross sectional area has reduced and the length has increased ok.

So, if a question is posed here what is conserved here? So, what is conserved here is volume. So, whatever is the increase in the length the decrease in the cross sectional area will be proportional and that will be can be founded by volume? So, my cross sectional area is A into initial length is L naught that will be equal to A the final cross sectional area and a final length; so, length will increase. So, cross sectional area has to will decrease so, that the overall volume constancy will be going to be there; so, volume is going to be conserved.

So in this case my length is increasing my cross sectional area is reducing. Now the problem or the question which we want to pose here is that when I am finding out the engineering stress; I said the force divided by the original cross sectional area A naught, but now during the deformation my area A itself is changing. So, you can say that area has reduced; so, my cross sectional area has come down. So, the stress which we are calculating from engineering stress relationship and that is not correct that is not the stress which the material is experiencing. The material is experiencing the stress which is given by force divided by the instantaneous area let us say I call it as a i.

So, because my both area and length are changing it is it going to affect our calculation of stress and strain during subsequent deformation? So, that is what I am trying to answer here that because my cross sectional area is reducing; the stress calculation which I am doing from engineering stress expression is not going to be the actual stress with the on which the material is experiencing.

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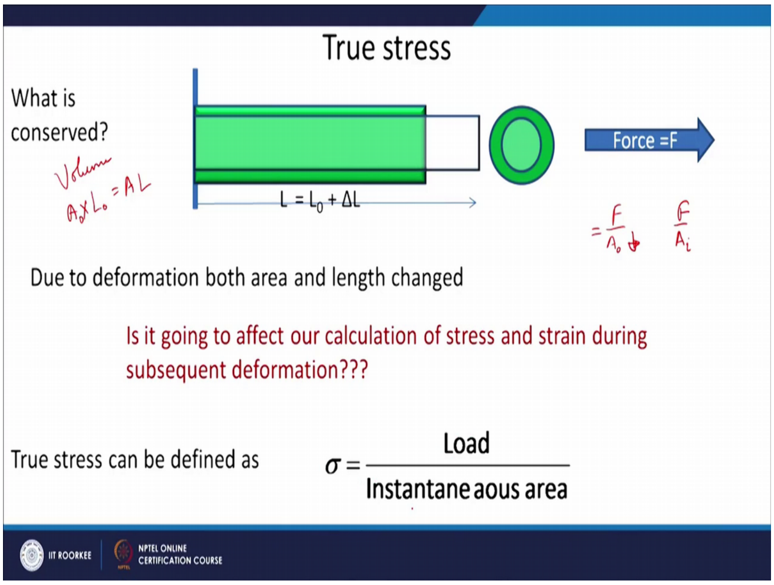

What material is experiencing is because of the cross sectional area A i which is now force divided by A i.

So, that is what we call as true stress; so, true stress is when I am dividing the force by the instantaneous area at that moment. So, true stress can be defined as load divided by instantaneous area; similarly true strain because length is going to change. So, engineering strain we defined as a delta L by L naught which was written in integration form like this the story true strain is calculated as a change in length in reference to instantaneous length. So, whatever we discussed in case of stress that will be true in case of the strain also. So, for example, for the first case right now the change in length is delta L divided by the L naught that is my going to be my engineering strain ok.

Now, for subsequent deformation when the length is again increasing to some other value; if I divide it again by the initial length the strain which I am going to get is not the correct strain because in between my length has increased to L. So, for subsequent deformation I should use the actual length of the material to get the strain and that is what is the problem.

So, now for engineering for true strain what we will do we will do? we will also take the instantaneous length rather than the original length. So, now, the summation if you want to look it will be L 1 minus L naught. So, there will be like initially the length is L naught and the in subsequent deformation, it is becoming L 1, L 2, L 3.

So, what we are saying that for each of these increments. So, it will be L 1 minus L naught divided by L naught then L 2 minus L 1 divided by L 1 then L 3 minus L 2 divided by L 2 that should be the summation. If you compare this with the with the engineering strain in that the summation will be like this L 1 minus L naught divided by L naught plus L 1 minus L 2 minus I know it will be L 2 minus L 1 upon L naught plus L 3 minus L 2 upon L naught and so, on.

So, in all case you will have in the denominator you will have L naught that is what we have taken out and basically you are doing summation over here. So, that is what is the integration we are using in d L; so, if I took this type of summation for true strain what will happen my length is now continuously changing. So, a first increment it was L naught second increment L 1, L 2 and so, on.

So, it will be now d L by L and the integration is lower limit L naught to L. So, integration of this type will give you quantity in terms of basically. So, integration here will give you Lon L naught to L. So, if I put it will become log Lon L minus Lon L naught and in logarithmic terms it will become Lon L upon L naught. So, this is what you will get in case of true strain.

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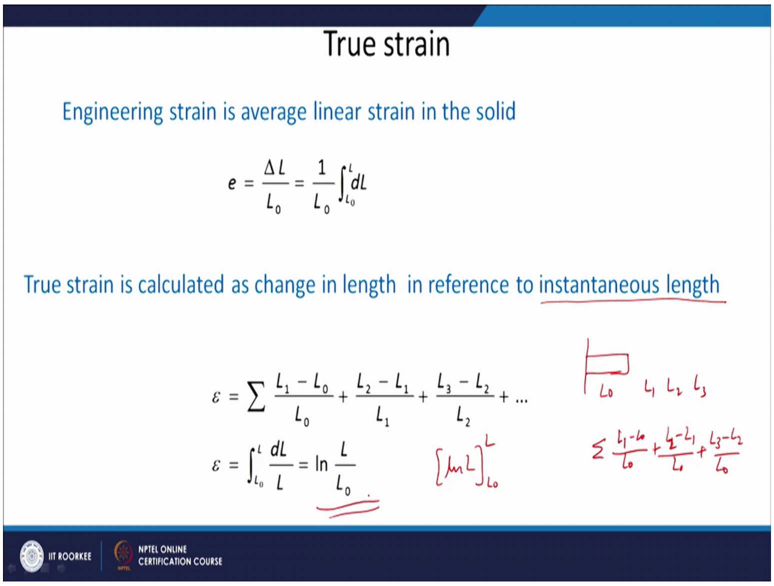

In that is why we call this strain as logarithmic strain also; now what is the advantage of true strain calculation that is very important. So, let us to engineering strain calculation in increments. So, your initial length is L 1 then it increased to L 2 after first increment then it increased to L 3 after the third second increment and so, on. And this is from 1 to 2; it is delta L 1 to from 2 to 3 it is delta L 2 3 and so, on.

So, engineering strain if I want to calculate between 1 and 2; it will be delta L 1 2 divided by L 1 e 2 3 will be equal to delta L 2 3 divided by L divided by L 2 and so, on. And if I want to do the whole deformation in one go for example, rather than doing in this increment suppose I want to take this particular deformation from directly from L 1; the initial length to the final length L 3, then I will say that the strain is equal to e 1 3 delta L 1 three divided by L 1 which will be equal to L 3 minus L 1 upon L 1.

Now, the thing which we want to compare here whether this two increments are equal to this one single increment; see in terms of dimension in both the cases dimension of the my material is going to same going to same; so, L 1 to L 3. So, that is one quantity which is there which cannot be changed you know that length has increased from L 1 to L 3. In one case we have done this in increments, in second case we have done it in one go from one from L 1 to L 3, but if you compare if you do a summation here then the summation of these two you can do a summation very simple L 2 will be multiplied here L 1 will be multiplied here and bottom you will have L 1 L 2 and so, on.

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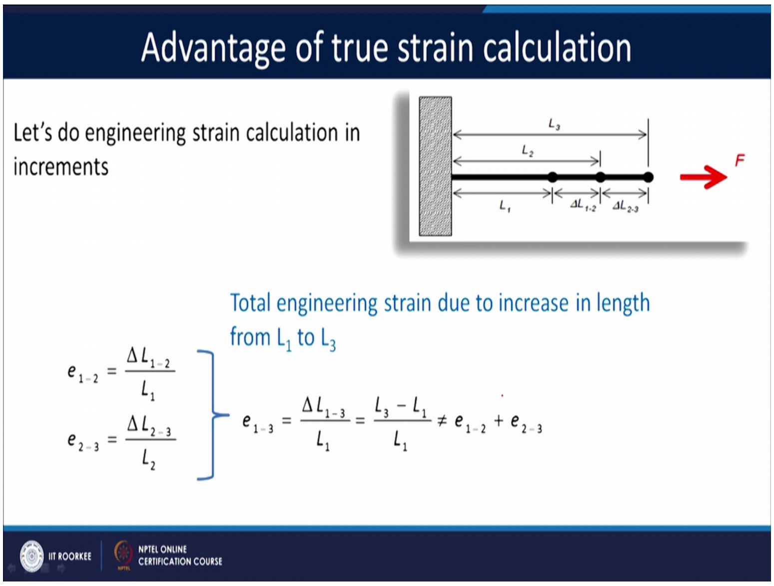

Now, that quantity is not equal to this is not equal to L 3 upon minus L 1 upon L 1 or in terms of delta L; if you want to write it will not be equal to delta L 1 3 upon L 1. So, when we are were taking engineering strain in increments and if you go in one short, the strain in both the cases the calculation of a strain in both the cases or the value of strain in both the cases is not going to be same. So; that means, there is some problem in engineering calculation of engineering strain because if my length is going to be same the during both the cases strain should come same and that is where the importance of true strain calculation comes ok.

Now, let us see you do the same calculation with true strain again we are doing first case in increment L 1 then from L 1 to L 2, from L 2 to L 3 with that is written here epsilon 1 2. So, it will be log L 2 by L 1 epsilon 2 3 log L 3 by L 2 and if we Y Y 1 to do in one shot it will be epsilon 1 3 will be equal to Lon L 3 by L 1; we are going directly from L 1 to L 3 and this we are going from L 1 L 2 from L 2 to L 3. So, in increment here we are going in one step.

Now, we have to see that whether this is equal to this or not. So, I am doing a summation here epsilon 1 2 plus epsilon 2 3; I am writing in logarithmic form log L 2 by L 1, this is plus log L 3 by L 2. So, if I expand this it will be log L 3 L 2 minus log L; L 1 plus Lon of L 3 minus Lon of L 2.

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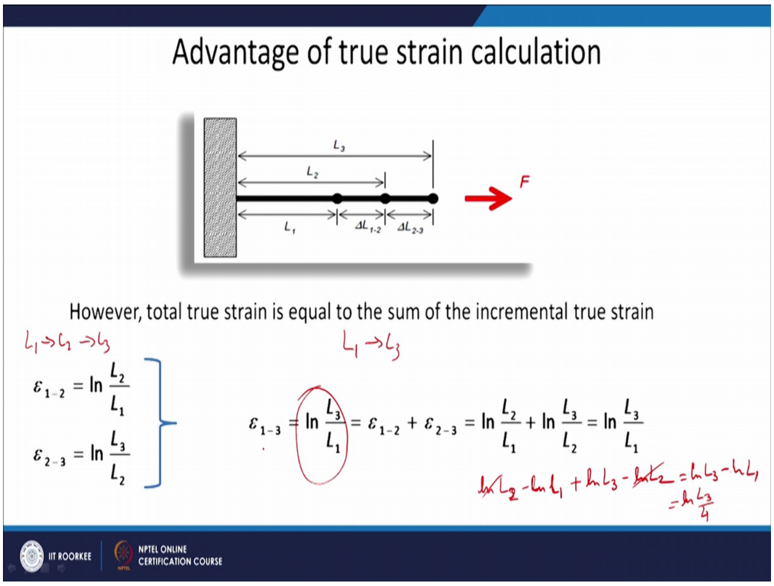

So, if you see this I will cancel both it is negative it is positive and now this I can write as Lon L 3 minus Lon L 1. What do you mean by Lon L 3 minus Lon L 1 L 1? They say the Lon L 3 upon L 1 which is same as what we have got in the first only one single step. So, with true strain the importance of true strain calculation is that I can if I do some strain in increments. And if I do the same deformation in one step the amount of strain will be same if we do calculation using true strain and that is very important in material processing where sometime deformation are done in stages.

So, the calculation of strain is going to be same whether you do in stages or if you do in one step and that is why true strain calculations are used in all the material processing industries. Now you can also relate the engineering stress or strain with the true stress and strain. So, here we are a finding out the relationship between engineering and true strain. So, if I want to write a engineering strain it will be delta L by L naught; the initial or original length which I can write as L minus L naught upon L naught L is your final length minus initial length divided by initial length which I can write like L upon L naught minus 1.

So, I take L upon L naught minus L upon L naught upon L naught which will be actually 1 and I can take this one here. So, it will become e plus 1 equal to L by L naught and now we know that what is epsilon; the true strain is Lon of final length divided by the initial length.

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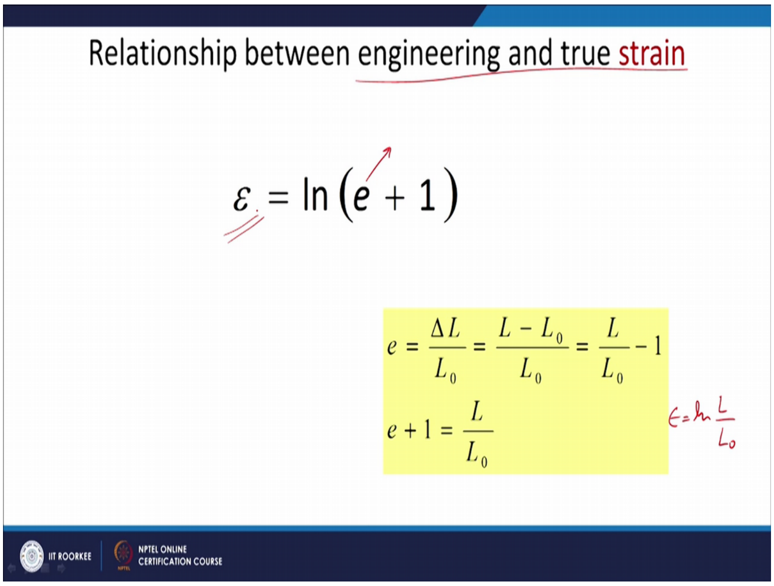

So which will look like this; so, I can replace it here. So, now, my engineering this is the true strain is basically logarithmic of e plus 1.

So, whatever you calculation you do in engineering strain you add 1 into that and take the logarithmic of that will give you the true strain. Now relationship between engineering and true stress again let us take the in engineering stress I will write it as P by A naught for true stress I will write it as P by A which is instantaneous area if I divide and multiply by A naught in this case. So, it will become P upon A naught into A naught by A; A naught by A can be written at as L by L naught; because again volume conservation I as I told you that because in plastic deformation volume is going to be conserved.

So, I can write it like this and we already know that L by L naught is equal to e plus 1 which we did earlier in the for engineering stress strain calculation; sorry engineering and true strain calculation. So, L by L naught will be equal to e plus 1; so, I will be replacing here A naught by A by e plus 1 my P by A naught is equal to equal to the engineering stress. So, this is what I have written now the engineering stress into e plus 1 ok.

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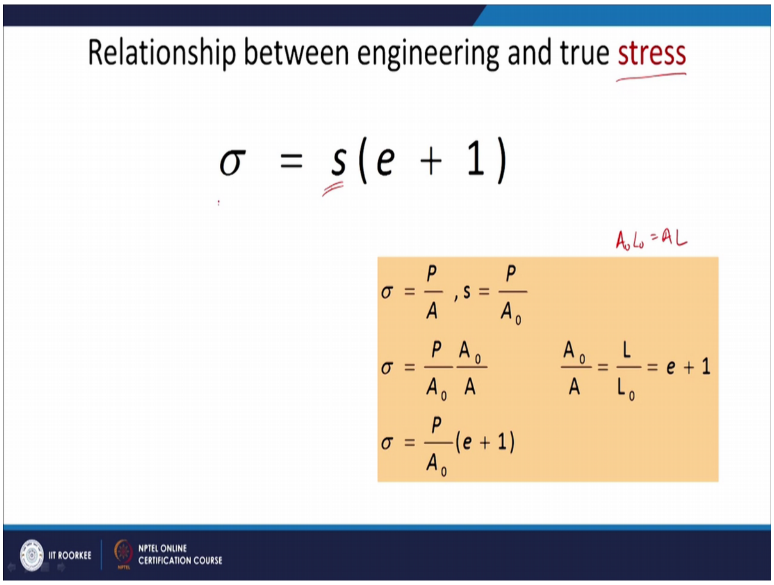

So, this is what will be the relationship between the engineering and true stress and the previous one is between the engineering and true strain. So, if you know the engineering calculation also if you have done, you can easily find out the counterpart to stress true strain values from that. And now let us see what will be the effect on the stress strain curve because of that.

So, already we have seen this red part is your engineering stress strain curve which is what we have we get from the machine. So, in from machine you get the data in terms of force versus elongation; sorry elongation versus force. So, elongation will be on x axis and force will be on y axis this is what I get from the machine elongation and here it will be force. So, elongation if I divide by original length then it will be equal to engineering strain force divided by original cross sectional area or inner area will give me the engineering stress. So, we can get that whatever is the curve.

Now, as I told you can find out the true stress strain curve from that. So, now, as I told you that when I am deforming length is increasing; so, what I am seeing is that length is increasing and area is reducing that my stress values are more than the engineering stress. But since my length is increasing the length therefore, in the strain calculation it will be delta L by L. So, if L is continuously increasing; that means, my strain value will be lower the true strain value will be lower. So, you can see that the points are shown by back arrows here.

So, stress is going up, but strain is coming down or is showing lower value than the engineering strain. So, engineering strain will be more true strain for the corresponding a elongation will be less, but engineering stress will be low and true stress will be high for the same amount of deformation. So, my curve goes like in this side, but up upward side.

So, this is how is true stress stress values or the true stress stress curve curve will be there and this is how the engineering stress strain curve will be there. After making actually the situation becomes just opposite now my, you can see that my; the strain value is more the true strain value is more than the engineering strain. And similarly my of course, the stress value will always be high compared to the true stress value will always be higher compared to the engineering stress ok.

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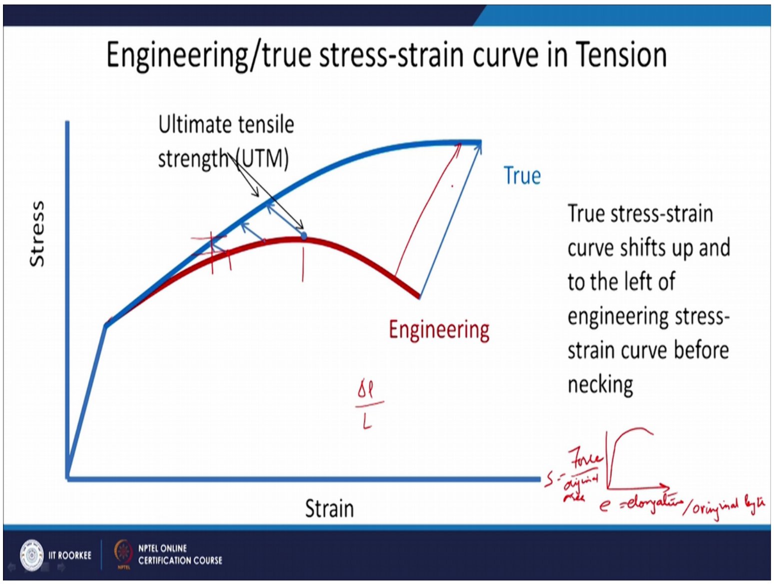

So, after making basically one thing to note is that we will take only the area and length in the necked region not the overall gauge length to for to the calculation of true value. So, that you have to consider that up to making as I told you have a uniform elongation. So, I can take the whole gauge length and the cross sectional area to find out the true stress strain values and so, on.

But after making it will be non uniform deformation. So, you will take the area of the neck region and also the length where the necking is taking place. And that is why you can see that my now true strain values are more than the engineering strain values because my length has reduced quite a lot.

Now, these are some relationships just to clarify to you for engineering stress at every point my I will be having cross sectional areas A naught and forces whatever my machine is going to provide me. In true stress I am going to take force what my machine is going to provide me and the cross sectional area will be a the instantaneous cross sectional area which can be given by this up to before making you will have this after making as I told you have to take the area of the necked region.

Engineering stress in case of engineering strain all the everywhere you will be dividing only by L naught; in case of true strain you will be dividing it by the instantaneous a length or area in terms of area you can write it like this. And in terms of length it will be L upon L naught; so, it is just opposite inverse relationship is there. So, epsilon e is equal to L I by L naught or A naught by A i and this is equal to this already you have seen this, but in case of after making as I told you have to take the area of the neck.

Now, as I already told you that true stress true strain values are used in metalworking operation. Because there I will do or deform the material in stages; I cannot deform material in one step, but for design purposes for a material for a mechanical guy who is designing a structure they need only the engineering stress and strain then do not need to stress and strain and most of the handbook also gives only the engineering stress and strain.

Why we will not use the true stress strain curve? If you see here in the initial elastic part my engineering and the true stress and strain curve are over overlapping each other.

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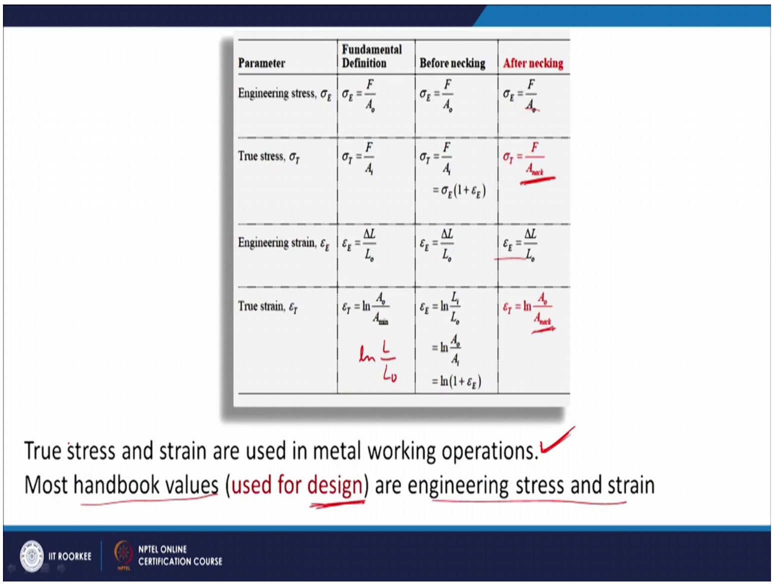

Parameter	Fundamental Definition	Before necking	After necking
Engineering stress, σ_E	$\sigma_E = \frac{F}{A_o}$	$\sigma_E = \frac{F}{A_o}$	$\sigma_E = \frac{F}{A_o}$
True stress, σ_T	$\sigma_T = \frac{F}{A_i}$	$\sigma_T = \frac{F}{A_i} = \sigma_E(1+\varepsilon_E)$	$\sigma_T = \frac{F}{A_{neck}}$
Engineering strain, ε_E	$\varepsilon_E = \frac{\Delta L}{L_o}$	$\varepsilon_E = \frac{\Delta L}{L_o}$	$\varepsilon_E = \frac{\Delta L}{L_o}$
True strain, ε_T	$\varepsilon_T = \ln\frac{A_o}{A_{min}}$ $\ln\frac{L}{L_0}$	$\varepsilon_E = \ln\frac{L_i}{L_o} = \ln\frac{A_o}{A_i} = \ln(1+\varepsilon_E)$	$\varepsilon_T = \ln\frac{A_o}{A_{neck}}$

There is no difference, they are same there is not going to be any change in the value between the true and the engineering up to elastic limit and for a design engineer he will be only designing the material in this range. So, for them it is of no use to know the true stress strain value they only are concerned with the engineering stress strain values and this is what most of the handbooks are going to report.

So, with that I want to thank you this is a very important lecture to understand the difference between these two parameters. So, I hope you might have appreciated that why we want to do a calculation of true stress and strain.

Thank you.