**Mechanical Behaviour of Materials**
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**Lecture - 09**
**Elastic Stress – Strain Relations Part - II**

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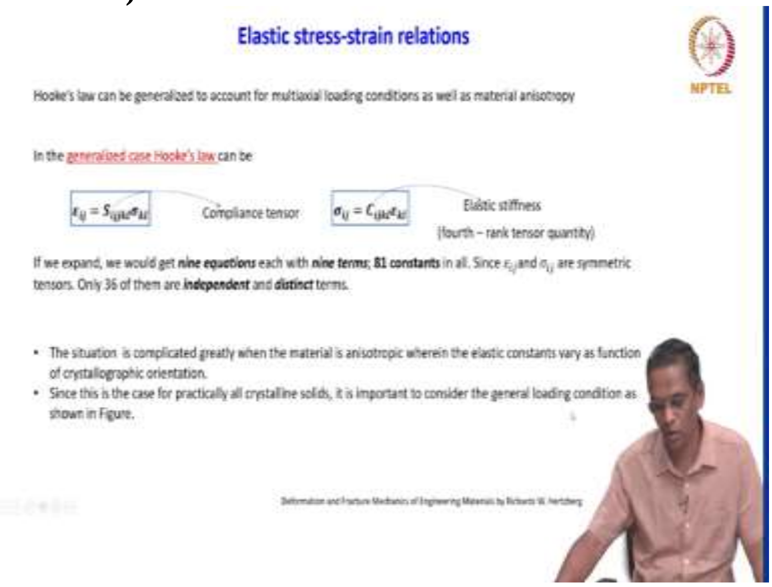


Hello, I am Professor Sankaran in the department of metallurgical and materials engineering. Hello everyone let us continue our discussion on this introduction to the mechanical behaviour of materials. If you look at what we have seen so, far we looked at the stress at a point and we also looked at a strain at a point in much more detailed manner. And then we also looked at the plane strain condition plane stress condition and so on. So, then we moved on to elastic I would say stress strain relations.

On the basis of linear elasticity theory, we looked at the most of the simple relations and constitutive relations and then from the constitutive relations, what are the other relations you can make use I mean you can derive involving various material constants like Young's modulus, shear modulus, bulk modulus and so on. So, we will just continue that and so one of the constitute law we have just seen is Hooke's law I said that we have also looked at this Hooke's law in a more generalized form.

So, this generalized Hooke's law can be used to account for multiaxial loading condition as well as material anisotropy, very important point. Suppose, we can clearly show that in a multiaxial condition for example, any material subjected to different type of loading, I will just show up just in few slides later I will show some general state of stress in our day to day life examples. So, in such conditions, this generalized Hooke's law only is being used.

So, I will just recall this again generalized Hooke's law, it can be written like this in a tentorial notation, $\varepsilon_{ij} = S_{ijkl}$, there is a type of here it is ijkl not double j, ijkl and $\sigma_{kl}$ where $S_{ijkl}$ is compliance tensor. Similarly we can write for the stress

$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$

where $C_{ijkl}$ is elastic stiffness if you recall, we have just gave us some introduction about tensor quantities and this elastic stiffness we just described that as a fourth rank tensor quantity this is what it is?

So, we now know how many constants it will have we just looked at the formula $3^n$ so, this is a fourth rank tensor quantity. Suppose if you expand we would get nine equations each with the nine terms. So, total mod 81 constants that is what it is elastic to stiffness in all. Since $\varepsilon_{ij}$ and $\sigma_{ij}$ are symmetric tensors only 36 of them are independent and distinct terms.

So, we have also introduced this terminology symmetric tensor so, you do not have to worry about it we have just had some introduction about all these terms. So, we will just see how these constants are evaluated and this is for isotropic materials all this relations. The situation is complicated greatly when the material is anisotropic where in the elastic constants vary as a function of crystallographic orientation.

So, this is very important point we have to remember then the material exhibits anisotropic that means, even in real material you take cubic systems you all know that we mark different directions in a cubic system whether it is simple cubic or body centred cubic, phase centred cubic, you all my term undergone an exercise how to mark directions using Millar indices. So, if you recall all this you know we also talked about material our not material atom populated in each type of plane.

For example, (1 0 0) what is the plane density atomic packing density and if you look at (1 1 1) what is the atomic packing density and so on (1 1 0) and so on. So, in each direction the packing density is going to be different and similarly, you are the stiffness constant is also going to be slightly different. So, that is why we are talking about anisotropic material or isotropic nature of the material. So, since this is the case for practically for all crystalline solids, it is important to consider the general loading condition as shown in figure.

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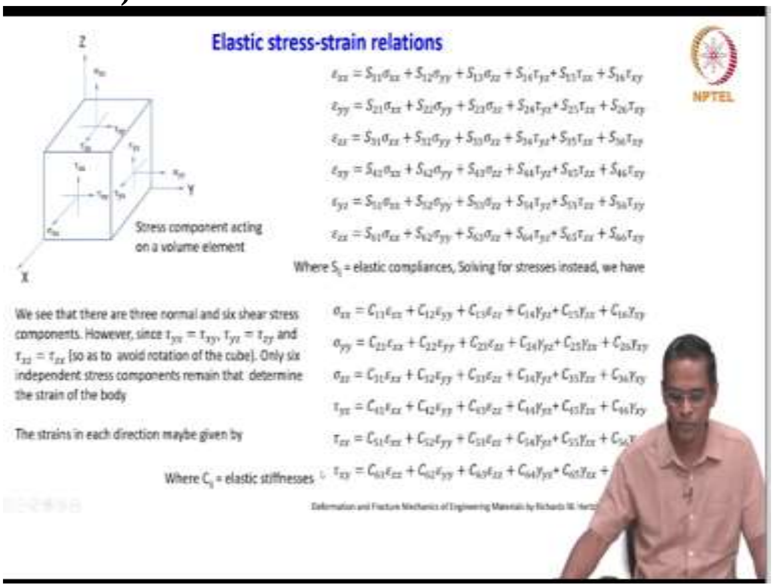


So, we have already seen this figure is most familiar to you now so, this is a general case where we describe the state of stress in three dimension we can consider this again. So, you have the 3 mutually orthogonal, normal stresses and then shear stresses are marked on the respective plane. And we see that there are 3 normal and 6 shear stress components. However, since $\tau_{yx} = \tau_{xy}$ and $\tau_{yz} = \tau_{zy}$ and $\tau_{xz} = \tau_{zx}$ so, as to avoid rotation of the cube, only 6 independent stress components remain that determine the strain of the body.

So, this also we have already seen just we are going to this recall, because we are going to see what is this? So, the strains in the each direction may be given by these kind of equation. So, I am just giving you these equations not to just scare you or something, but you should understand what is that we are talking about 81 constants how it is coming to 36 and then how it becomes you know 6 normal stresses and shear stress and so on.

And how this equivalents or realized so, you have this strains on each direction and you see that it is each direction normal stress and the shear stress components. Totally you have about 6 independents just components. So, all 6 are here so, it is 6 is for each direction. So, where this $S_{ij}$ you can see that $S_{11}$, $S_{12}$ and so on here $S_{11}$, $S_{21}$, $S_{31}$ and so on they are called elastic compliances. And you can express the same thing for a stress because that is what linear elastic theory comfortably shows that.

So, in terms of stress, these are the equations and what is $C_{ij}$? This is elastic stiffness. So, we can now think of using this relation for a real system.

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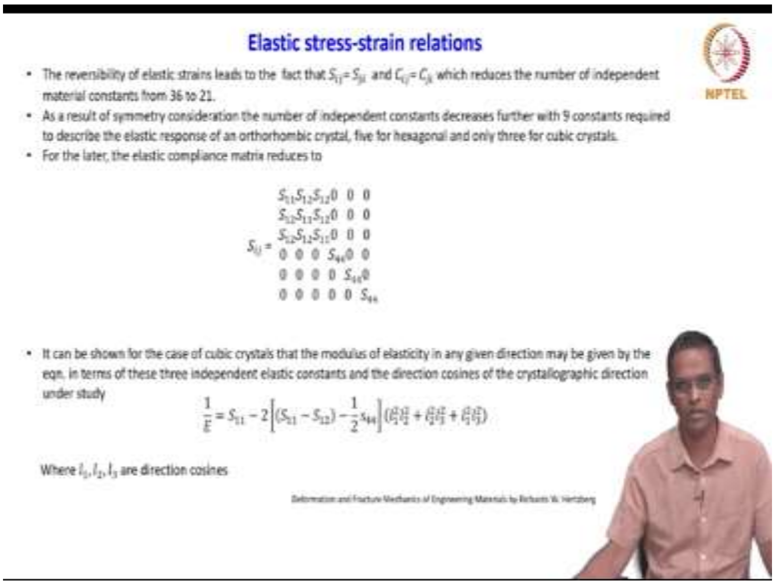


So, the reversibility of elastic strain leads to the fact that $S_{ij} = S_{ji}$ and similarly, $C_{ij} = C_{ji}$ which reduces the number of independent material constants from 36 to 21. So, we have also looked at the similar reversibility in the shear stress condition. Similarly, in elastic stiffness and compliance coefficient also the reversibility rule is applied and which further reduces the independent constants from 36 to 21 this is the point you have to remember.

As a result of symmetry consideration the number of independent constants decreases further with 9 constants required to describe the elastic response of an orthorhombic crystal, 5 for hexagonal and only 3 for cubic materials. So, life becomes simpler once you bring in lot more parameters which can be you know, similar in all orientations for example, symmetry when you consider the symmetry, it gives further reduction in number of independent constants for example, for hexagonal system it is 5 and only 3 for cube system.

So, for the later systems the elastic compliance matrix reduces like this instead of 6 by 6 it is still remains 6 by 6, but then you have a lot of components becomes 0 because of the symmetry consideration. So, it is $S_{ij} = S_{11}$ $S_{12}$ like this only these two stiffness constants of course, three, $S_{11}$ $S_{12}$ and $S_{44}$ these are the three constants which is able to describe the anisotropic nature of the crystal in a cubic crystal.

So, it can be shown that for the case of cubic crystals that the modulus of elasticity in any given direction may be given by the equation in terms of these three independent elastic constants and the direction cosines of the crystallographic direction under the study. So, this is the one expression we can use for finding out I would say the anisotropy are the describing the elastic modulus in a simple cubic system.

$$\frac{1}{E} = S_{11} - 2[(S_{11} - S_{12}) - \frac{1}{2} S_{44}](l_1^2 l_2^2 + l_2^2 l_3^2 + l_1^2 l_3^2)$$

So, what I was trying to tell you by giving all these things we just got introduced to the linear, you know elasticity theory and then how it describes the stress strain and its relation and how it validates for isotropic material then what happens to anisotropic material and how this elastic constants are evaluated just give you a feel of just you know showing equation.

But just get have a grip of know how this equations are useful in real systems so, that is what my intention. So, later probably you can do some problems involving this we can calculate elastic modulus for particular given direction for different crystal systems then you will appreciate this concept much more easily.

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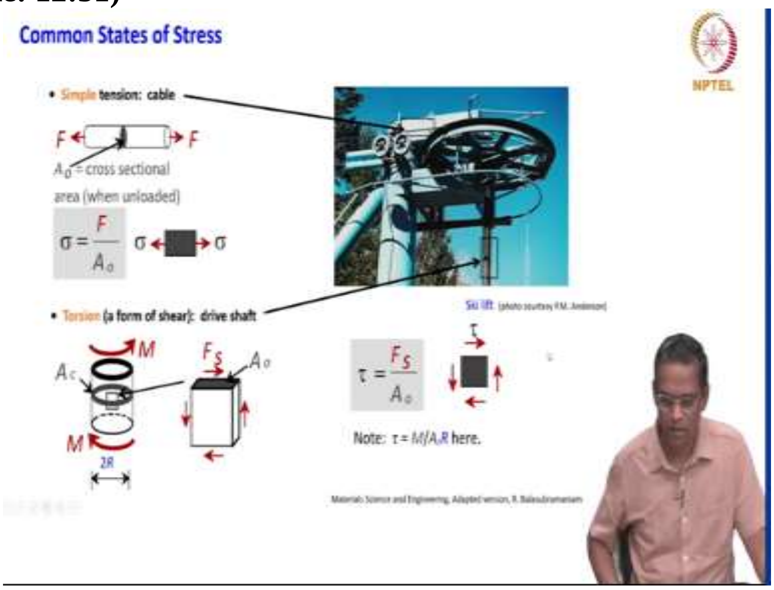


Having gone through a lot of expressions for stress and strain and relation so on now when they come to reality, I am just showing some of the real time applications where we can show the common states of stress, I have shown some very colourful picture here and what is this equipment this is a sky lift basically a sky lift we will just go through one or two competence here and then try to understand what is the state of stress for example, you see the rope here which goes through this pulley and right this wheel.

Similarly, this rope and there is a rotating shaft here, which rotates of course when the wheel is rotating this rod also will rotate, but we can just try to get some grip of what is the state of stress here for example, this particular rectangular box which is showing this rope what kind of stress it undergoes? It undergoes simple tension it just pulls the dry will just rotates in this direction. So, it just pulls so, the rope is being subjected to as simple tension.

So, this is what we describe as simple tension and the wire will have a cross section the cable will have a cross section $A_0$. And we know this expression, but simple expression

$$\sigma = F / A_0$$

and a simple tension. What about this shaft? Suppose this rotating shaft this is torsion it undergoes simple torsion form of shear. So, we know now what is shear component of a stress? So, how do we describe them? This is what we describe them.

This is a physical picture drive shaft of diameter 2R and this is the moment shear and this is the cross section and this is the notation we have already seen for how to represent the shear stress. So, this kind of notation is if you recall this is a positive shear stress so, shear stress again shear force divided by area. So, this is a simple I just want to give a feel of a real time application sky lift.

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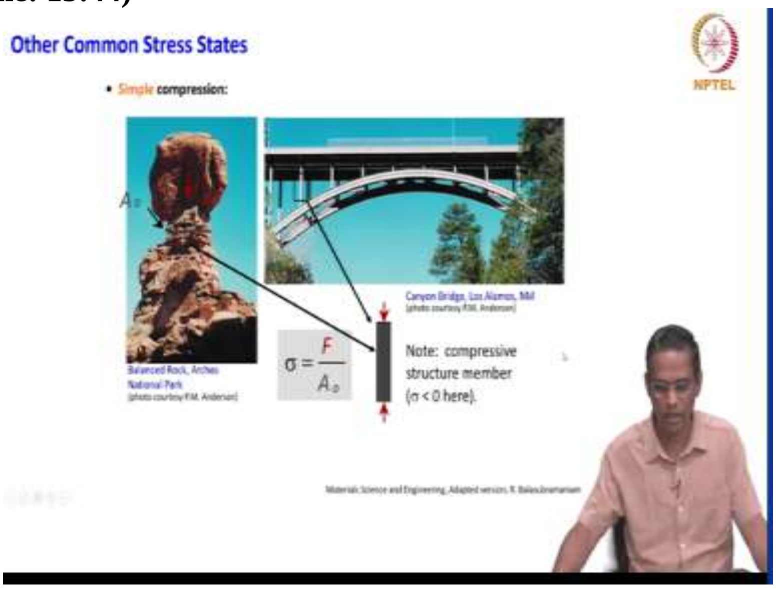


And next one, we will show some simple compression. You can see that this is a bridge is a hanging bridge, we will we can look at this small iron strips which is being connecting these two big structures, a curved structures and this linear structure and there is another example it is a huge rock sitting on this small pileup of kind of a cliff here, which is there in this national park and very nice photographs.

But we can just think about an element from this region and this rod, what kind of stress it undergoes, it undergoes a simple compression. So, we can represent this by this so, compressive stress or this is a negative stress so, that is why we look at this.

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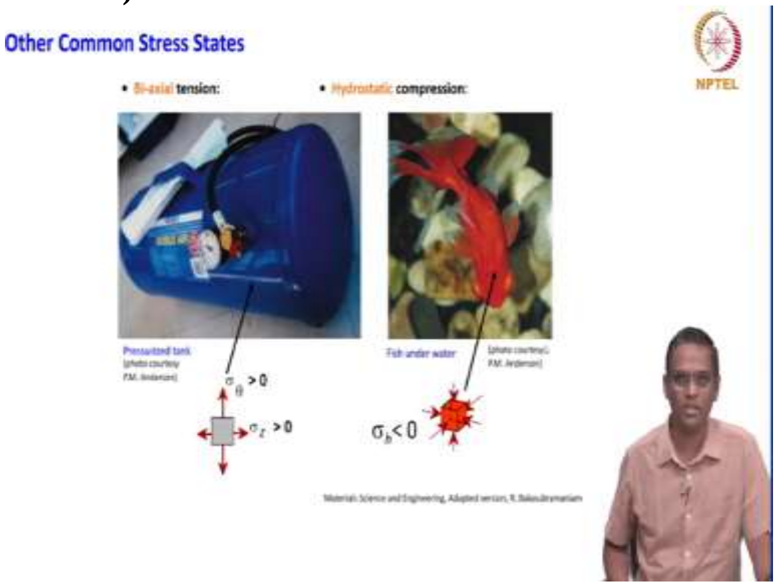


And this is; what is this? This is a pressurized tank so, just think about what kind of stress it can experience. So, pressurized tank experiences this kind of a stress force, what is this? They are all positive stress but in both directions that means Bi -axial tension. That is the real time and what is this? This fish in the fish tank fish and water whatever it is what it undergoes very nice picture. So, this is the symbolism what does it mean? It undergoes a triaxial state of stress what kind of stress? It is a compressive stress. But in all directions, we also have another name for this if it is there is no shear term what is that called you just you have to recall we have just shown the matrix also. So, this is hydrostatic stress, but it is in the compression mode so hydrostatic compression. So, like that you can just connect day to day applications or whatever, you come across you just think about what type of stress it undergoes.

It will also give a very good you know, feel for this subject if you start thinking or connecting you will not find it more mathematical equation and so on so, it will be interesting. So, now that we have described the kind of you know state of stress and strain and we have looked at simple relationships.

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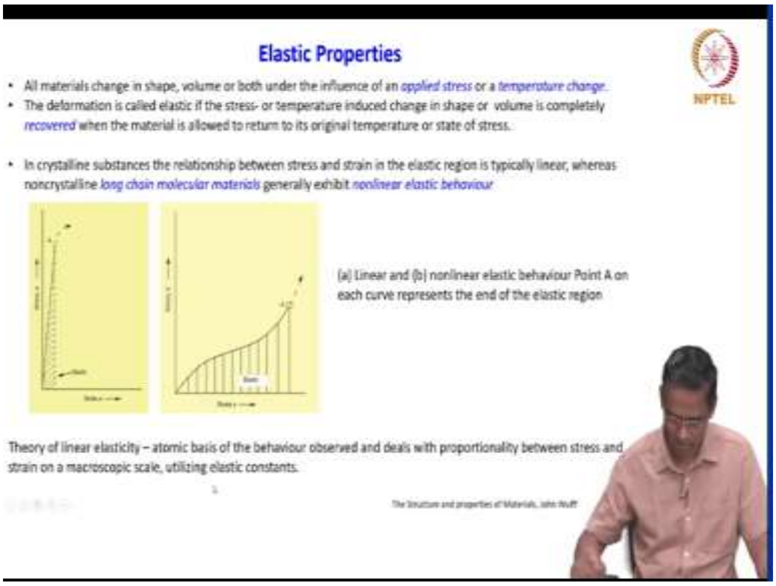


And now, we go to properties because at the end of the day, with all this knowledge, we should try to understand and relate these theories to understand the properties of the material, that is our aim. So, first we look at elastic properties, we have now gone through elasticity now, we will look at elastic properties what are the elastic properties, you all have some idea about elastic properties, in a day to day life.

You have n number of things, which we use from every day, every material. It will undergo a lot of elastic deformation. It takes some simple things like what I demonstrated in the introduction video. All materials change in shape, volume are both under the influence of an applied stress or a temperature change. So, this is what we try to describe even if you remember when I showed this content more curved.

The potential wide diagram we talked about two things one is potential energy and other is we related that curve to the force and also we brought temperature into the discussion that is because that also can influence inter atomic distance that is very important. That is why applied force and temperature change can bring change in shape, volume are both any material very important.

The deformation is called elastic if the stress or a temperature induced change in the shape or volume is completely recovered, when the material is allowed to return to its original temperature or state of stress. So, this also you know very well now, any material any type of force you apply irrespective of state of stress or if you remove the force, if it retains its original position then it is completely recovered elastically got recovered elastic energy got recovered something like that.

And similarly, the temperature we can see in the future class some of the examples even a temperature dependent behaviour material, for example shape memory, they are all related to this.

So, in crystalline substances, the relationship between stress and strain in an elastic region is typically linear very important, look at the statement carefully, in crystalline substances the relationship between stress and strain in elastic region is typically linear. Whereas, non-crystalline that is long chain molecular materials generally exhibit nonlinear elastic behaviour. So, it is not necessarily that stress word is inversely proportional to strain that is a linear relation may not be valid for all kinds of material. And now, you know why we just started with a different kind of bonding and structure and so on. Now, you see, the moment you change from crystalline to non-crystalline or long chain molecular materials.

For example, a polymer, this linearity is no longer exist, so they exhibit nonlinear elastic behaviour. This is exactly shown in this two stress strain diagram so, stress versus strain and the material undergoes linear response to the stress up to the point A, that means beyond this point A, the material will no longer behave elastically that is the limit. Similarly, you see that it is an interesting curve so, it is not linear but it is still an elastic region, it all the way up to point A, it undergoes elastic deformation, but it is nonlinear.

So, non-crystalline long chain molecular material will exhibit the elastic behaviour but not linear behaviour. So, this is what theory of linear elasticity try, to explain this on atomic basis of behaviour observed and deals with the proportionality between stress and strain on a macroscopic scale utilizing elastic constants. So, this we will see what is that.