Mechanical Behaviour of Materials
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Lecture - 08
Elastic Stress - Strain Relations, Part-I

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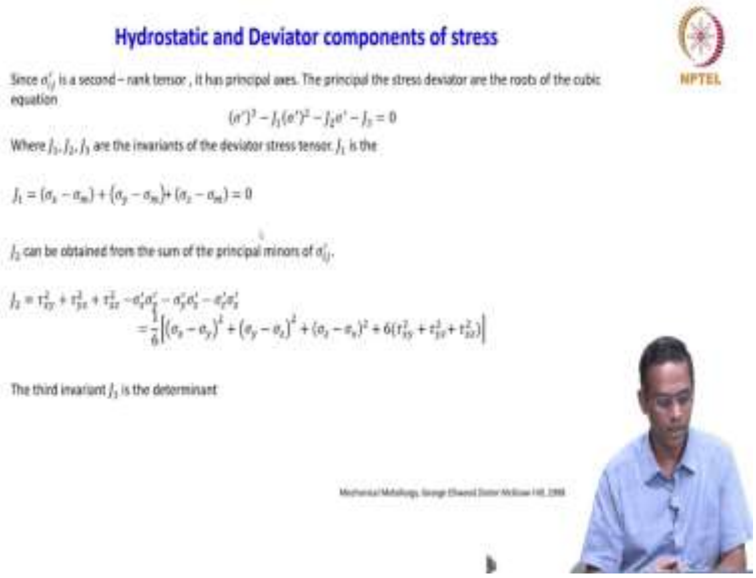

Hello, I am Professor Sankaran in the department of metallurgical and materials engineering, since σ_{ij}' is a second rank tensor, which already we have stated it has the principal axis the principal stress deviator are the roots of the cubic equation. This also we know. We have we have looked at stress and then strain now; it is a stress deviator because they are all tentorial quantities. So, nothing new to us.

So, here instead of I_1, I_2, I_3 here it is J_1, J_2, J_3 or invariants of the deviator stress tensor and J_1 is simply this and the J_2 is this. This is principal some of the principal minors that that we have already seen and J_3 is the determinant, let me go back.

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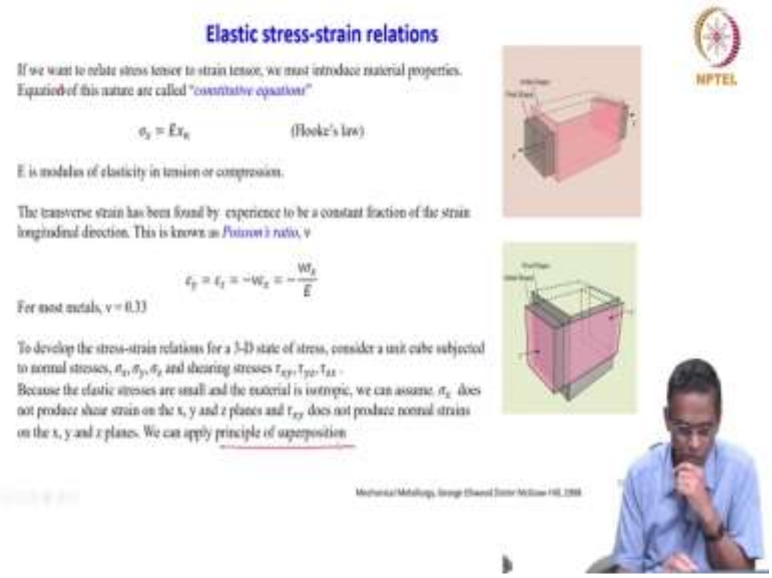

J $_3$ is that determinant total determinant. So, this particular invariant I am showing because some of the yielding theories, so, there are there will be looking at several theories yielding theories. So, some of the yielding theories will directly note I mean, they will simply quote this J $_2$ invariant J $_1$ invariant now, that time, if you are not familiar with this, then it is a little confusing. So, now that you know, J $_2$ is it is a invariant of the stress deviator matrix that we know.

So, J $_1$ is known. So, once you remember all this, these three equations and their invariants then it is very easy to appreciate later on. So, we have spent considerable time on the description of the stress and description of the strain and some important I would say the properties of these tensors and their parameters. Now, we will slowly get into relations between stress, elastic stress and elastic strain, please remember, we are looking at only small strains and which is purely elastics.

So, when we say elastic deformation then it can be compression as well as tension. So, you take these two examples, this is an initial shape of the member body and which has been subjected to tensile deformation that means, the length increases the width decreases. Similarly, this member is subjected to compression then the length decreases the height increases. So, that is something normally we expect this is when pure elastic deformation.

So, if you want to relate a stress tensor to the strain tensor we must introduce material properties the equation of this nature are called constitutive relations or constitutive equations, but what is this constitutive equation we are familiar with this is well known Hooke's law

$\sigma_x = E x_n$

So, this is your linear elastic deformation. So, E is the material property which is modulus of elasticity in tension or compression in a linear fashion.

So, the transfer strain has been found by experience to be a constant fraction of the strain in the longitudinal direction this is known as Poisson's ratio. So, this means, you can express this

$\boldsymbol{\varepsilon}_y = \boldsymbol{\varepsilon}_z = - \nu\ \sigma_x / \boldsymbol{\varepsilon}_x = \sigma_{x/E}$

So, this is a simple relation first time we are writing a relation between as strain versus stress in a linear.

For most metalS ν is 0.33. So, this is for a single dimension. So, if you are interested in developing this relation for 3D state of stress and then you should consider a similar cube, where we have just looked at for description of a stress at a point. Similar geometry you can imagine, where all the normal stresses σ_x σ_y and σ_z and where the shear stresses were marked.

So and before we get into the relation of the stress strain in a 3D state of stress, there are some assumptions, what are the assumptions because the elastic stresses are small and the material is isotropic very important, you have to pay attention to these details, if these are the conditions for material is isotropic and the elastic sources are small, then we can assume σ_x does not produce a shear strain on the x, y and z planes and τ_{xy} does not produce normal strains on x y and z planes we can apply principle of superposition.

So, under this conditions the σ_x will not have influence on shear strain and shear strain will not have influence on normal strain then we can use a principle of superposition for what is that?

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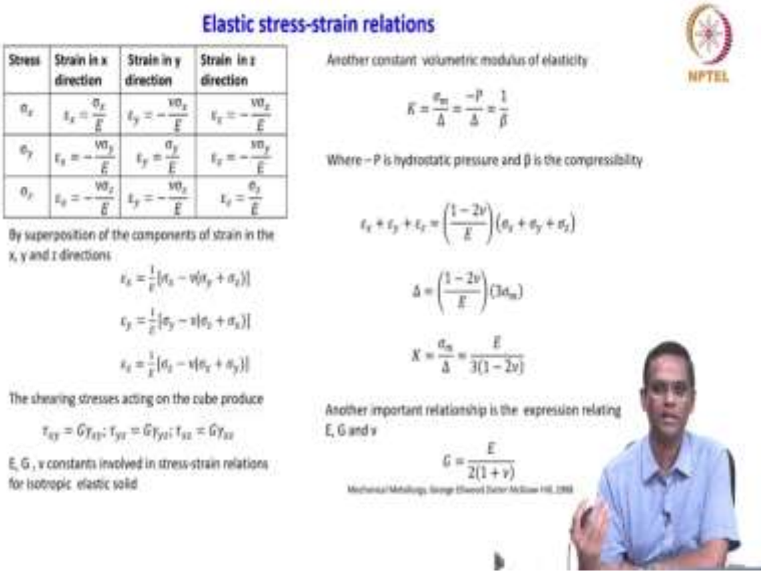

So, this is a table 3* 3 matrix you can create using similar what we have just written there for a stress in x direction σ_x, we can write a strain in x directions

$\boldsymbol{\varepsilon}_x = \sigma_x / E$,

but the strain in y direction can be returned

$\boldsymbol{\varepsilon}_y = -\nu\, \sigma_x / E$

$\boldsymbol{\varepsilon}_z = -\nu\, \sigma_x / E$

So, what does it mean we are now looking at the effect of stress in the x direction, but what its effect on or what kind of strain it will produce in y direction and z direction can be expressed with the simple relation?

That is what you have to look at it. Similarly, you look at the strain the stress system x normal stress that is σ_y is a normal stress in y direction, the strain in y direction is simply given by σ_y / E, but the effect of this on x direction that is if you want to see what kind of $\boldsymbol{\varepsilon}_x$ it will produce or what kind of $\boldsymbol{\varepsilon}_z$ it will produce just by the normal stress which is acting y direction, then we get these kind of relations.

So, that is the beauty of this superposition of the strain components in the x, y and z directions. So, we can just simply write like this. So, whatever I have just put in the table, we can just write it like this

$\boldsymbol{\varepsilon}_x = 1 / E\, [\sigma_x - \nu(\sigma_y + \sigma_z)]$

So, that is what it is. So, these kinds of relations can be written for three axes. So, this is called a generalized Hooke’s law.

It is that means the stress strain relation in three dimensions. So, the shearing stresses acting on the cube produce that is

$\tau_{xy} = G\gamma_{xy}$,

shear stress and shear strain and this is G is the shear modulus. Similarly, in yz and xz is that directions. So, you have three constants now. E Young's Modulus, G shear modulus, ν Poisson's ratio these are the constants involved in the stress strain relations for isotropic elastic solids.

This is very important this relationships are fully valid for isotropic elastic solids and then we can now look at the other relationship where this elastic constants have another constant volumetric modulus of elasticity what is that so, which is

$K = \sigma_m$(mean stress) / Δ = $-P / \Delta$ = $1/\beta$

What is this -P is a hydrostatic pressure and $\beta$is the compressibility.

So, these are some general relations these stress strain components have. So, we can just try to put this into perspective how we can write this again another relationship you have K can be related like this and then and it basically what we are trying to see is we are giving trying to give another relation for volumetric modulus which is can also be written like this.

$$K = \frac{\sigma_m}{\Delta} = {}^{E}/_{3(1-2\nu)}$$

So, all these relations will be handy if you try to solve some small problems in the day of elasticity, we will try to use it when it comes especially in fracture problems and I know fracture mechanics for example, look at some of the, you know the stress and strain states and we will be using all this relations. So, another important relationship is the expression relating Young's modulus, shear modulus and ν is

$G = E / 2(1 + \nu)$.

These all some standard relations.

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So, now, I am coming to another important topic or I would say, we are looking at fundamental. So, I am introducing another terminologies plane strain and plane stress. So, we so far we have looked at a strain in one dimension strain in three dimensions stress in one dimension stress in three dimensions, but we did not talk about a plane strain or plane stress. So, the question is why do we do that?

The problem is most of the engineering problems are practical in a practical situations becomes too complex. So, if you make it three dimensional problem or reduce it to one if you reduce one dimension, the mathematical treatment can be as simpler you can try to solve completely that is a primary reason. So, the many problems can be approximated by two dimensional approach, we assuming that the stress or the strain varies only in the single plane usually XY plane.

There are two ways to create such approximations by assuming that the force normal to the plane is 0 the plane strain this is called plane strain assumption, if you assume that the force normal to the plane is 0, this is plane strain assumption. The two approximations we shall continue or when shall outline the plane stress type of analysis first. We simply take the equation of generalized Hooke's law and put $\sigma_x = 0$ then we have just developed the equations for us a generalized Hooke's law where you put simply $\sigma_x = 0$.

Then we get this equation. So, I have shown three quations and if you just plug in σ_x equals 0, then you will have this set of equation this is valid for, this is a plane stress type equation.

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So, this is exactly for shear strain, so, 0 forces in the z direction implies these two components become 0 therefore, only the first of the two equation is nontrivial. So, this is a condition for the plane strain problems or plane stress problems. These two equations what we have shown is planes just formation for a plane strain analysis is a little more complex by putting $\boldsymbol{\varepsilon}_z = 0$ the Hooke's law gives an expression for σ_x is this.

And this can be substituted in the first two equations of the generalized Hooke's law involving σ_x and σ_y only then this equation becomes like this. So, it looks a little complex, but actually it is not if you just plug in this values there, then you yourself will find out that this is simple to manipulate, but what is important is the new terms which will emerge that is what you have to focus by just that is why this plane stain analysis bit complex.

But, you can just appreciate by looking at this expressions so, just plug in this assumptions into this and then you will get finally this expression

$$\varepsilon_x = \frac{1-\nu^2}{E}\left(\sigma_x - \frac{\nu}{1-\nu}\sigma_y\right)$$

We can simplify this we can define a new parameter called the plane strain modulus that is

$E' = E / 1 - \nu'^2$ and a quantity ν' such as

$\nu' = \nu / 1 - \nu$.

Then we can write the equation as $\boldsymbol{\varepsilon}$ x = 1 / E′ times $\sigma_x - \nu' \sigma_y$. $\varepsilon_z = 0$ and they both shear strain equation still applies in a plane strain. So, why I have brought this particular derivation is because, you will see that wherever the problems are described, suddenly you will see that you know this 1 / 1 - ν term comes out 1 / 1 - ν square terms will come in most of the fracture mechanics problems, but people just do not pay attention to this it will come how.

So, this is how it will be, if you just work it out, then you know that, so, that is because we are manipulating the Young's modulus. Young's modulus will have some different effects in the plane strain condition and that is why we are doing this. So, that kind of you know alertness will be useful, instead of simply looking at the expression, what is given in the textbook, you can just derive by our self and then see and then get familiar with the terms is that is the purpose of this slide.

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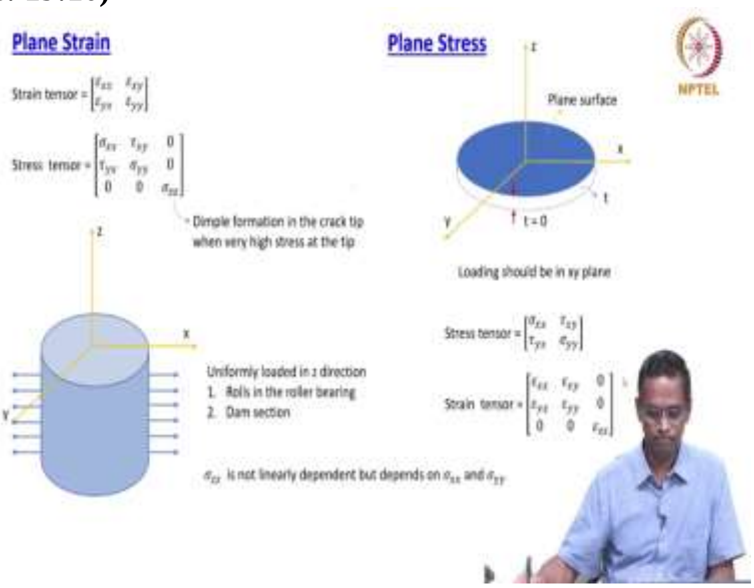

Otherwise, there is no significance here. So, now, in general in a plane stream, you can write strain tensor like this **ε** $_{xx}$, **ε** $_{yy}$ and **ε** $_{xy}$ and normal strain and shear strain, normal stress and shear stress. So, please remember in a plane strain condition, strain tensor is 2×2 matrix for a plane strain conditions stress tensor is 3×3 matrix. So, you get this is not 0. So, this is very important. We will see when we discuss, In a fracture mechanics concepts so, it will be you know dimple formation in the crack tip when we when very high stresses at the crack tip.

So, we I will show some examples when these kind of examples particular examples when we discussed that subject. So, this is a schematic for plane strain and real time examples are roles in the roller bearings and dam section, they are all considered to be a plane strain condition and what is plane stress? This is a schematic for a plane stress loading should be only in xy plane and this is a stress tensor and this is a strain tensor.

So, the stress tensor again it is for a plane stress it is 2×2 matrix, but not the strain tensor strain tensor is 3×3. So, ε_{zz} is not 0. So, this is again, very important point to remember. So, that is, that these are all the introduction to this stress strain relations, I just want to review and these are all some of the fundamentals given in all the textbooks, but some of this fatigue differences, small details, people do not pay attention, then they struggle to get the idea.

So, that is idea that is why I just wanted to review this, though it is simple. So, from I will stop here today. See, one important point is I have flipped the slides pretty fast and if you start writing on the blackboard, it is always good. Especially, when we write just too many equations on a slide and that too, if you just flip it through just by reading, it is very difficult to follow.

So, what I request all of you to do is if you look at any of the expressions you try to write, firstly, look at the video and then try to write of your own in your notes and then see whether you are getting this expression, by your own derivation. Small relations, but then you have to practice that there is no other go. Even if I tried to do it on the blackboard and it will be it will take a lot of time.
But these are these important things you have to do and if you have any difficulties, you can always write to us and when we have the interaction section, we can get some of the clarifications or even sent by email and so on. So I do not want to dump too much of information in every lecture as well, I want to go slow, but at the same time, I want to give enough background before I start the subject in detail.

So, I will stop here from next class onwards, I will do an elastic anisotropic again, we are still in fundamentals, we are discussing the fundamentals, it will run through a few more classes before I cover other topics which is required as a background for this course. So, the next class I will discuss about anisotropic and elastic anisotropy and then how the real time crystal structures will have the elastic properties.

So, then we will slowly get into properties of materials and how the elastic deformation and then we will be using all this concepts are terminologies, definitions, whatever we have used so far and that will be useful. That is the purpose of these preliminary lectures in the beginning. So, I will stop here and then we will continue in the next class.