Mechanical Behaviour of Materials

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Lecture - 07
Strength of Materials – A Short Overview Part – V

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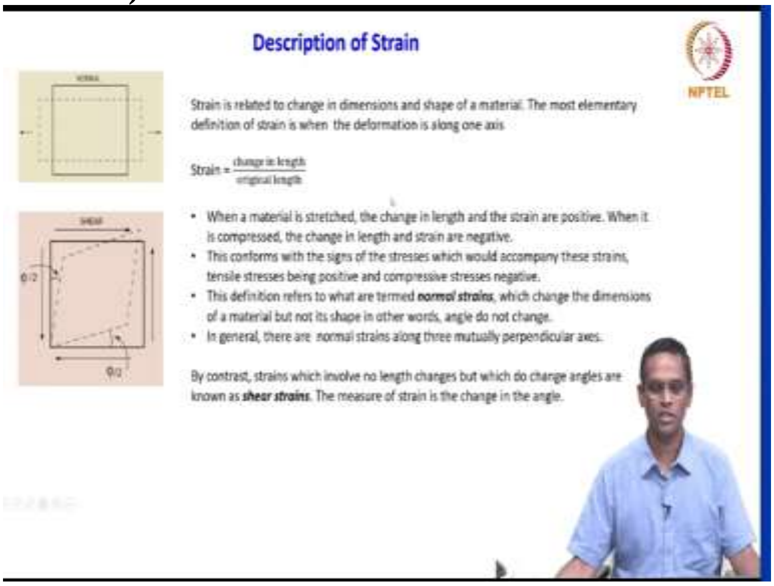

Hello I am Professor S. Sankaran in the department of metallurgical and materials engineering. We will continue our discussion on the description of strain what we have just looked at yesterday. I will quickly review what we have just started with. So, this is exactly what I have shown in the slide, we have just gone through, similar to strain stress whatever the description we have seen how we are going to see the similar kind of a description a strain at a point.

This is what I just started yesterday and we went through some normal classification of the strains. And then we also looked at what is a normal strain and shear strain and it is exactly the similar terminology as we have seen in normal stress and here it is normal strain, shear stress and shear strain similar geometrical conditions will be considered here as well.

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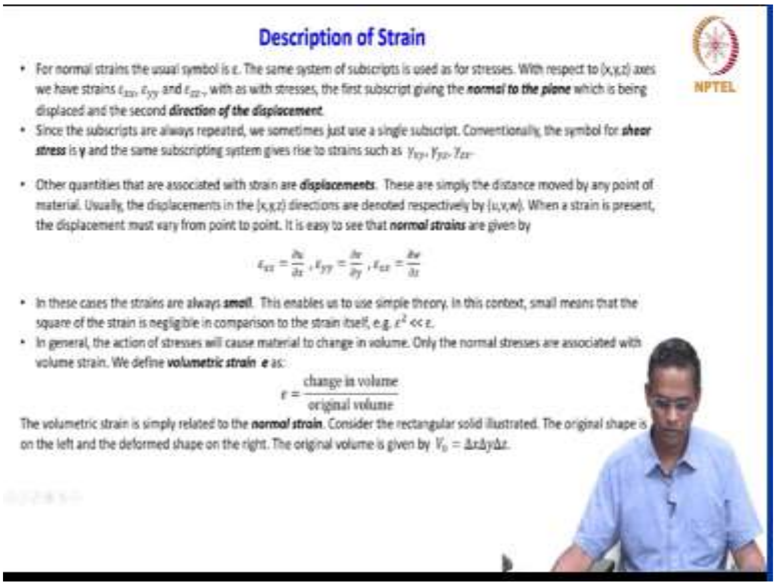

So, I quickly skip this slide because we have already gone through this and one point I just want to mention here is before we get into the details of this normal and shear strain, there is another term we have seen that is called volumetric strain where which is measure which is denoted as e which is a change in volume divided by original volume. And this volume of exchange is also simply related to normal strain and that is exactly I have stopped yesterday and then we will just see from that point. So, we are going to consider the rectangular solid as an illustration.

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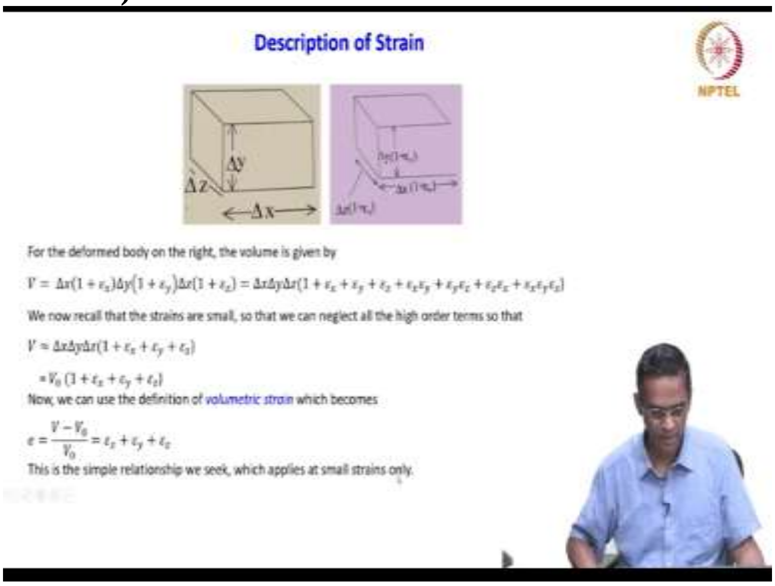

The original shape is like this has the dimension of Δ x and Δ z and Δ y. And this is after the deformation. So, after deformation what happens the Δ x becomes Δ x + εΔ x , so (1 + ε_x) and 1 + ε_y and 1 + ε_z that is the increment in the strain in the all three mutual perpendicular directions. So, how are we going to write the equations like this for the deform body on the right hand side for this geometry what we are seeing here.

The volume change or the volume is given by since the change in volume is itself is given as a measure here, so we directly write the volume of this deformed body, which is Δ x times (1 + ε_{x}), Δ y(1 + ε_{y}) , Δ z(1 + ε_{z}) which can be rewritten like this. So, if you recall that the strains are small this is what we have just described in the beginning some of the relations what we have just seen before also these relations are valid for a small strain.

So, here also there is we will be working this condition so, that we can neglect the higher order terms or if you just consider only this first down. So, then what we will get is something like this, V is approximately equal to Δ x* Δ y and Δ z times 1 + ε_{x} + ε_{y} and ε_{z} and which can be rewritten like this V_0 (1 + ε_{x} ε_{y} and ε_{z}).

$$V = V_0\,(1 + \varepsilon_x + \varepsilon_y + \varepsilon_z)$$

So, this is a kind of a definition of volumetric strain which becomes

$$e = \frac{V - V_0}{V_0} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

. So, this is a volumetric strain. And what we have to remember is these relationship they are all applied at small strains only.

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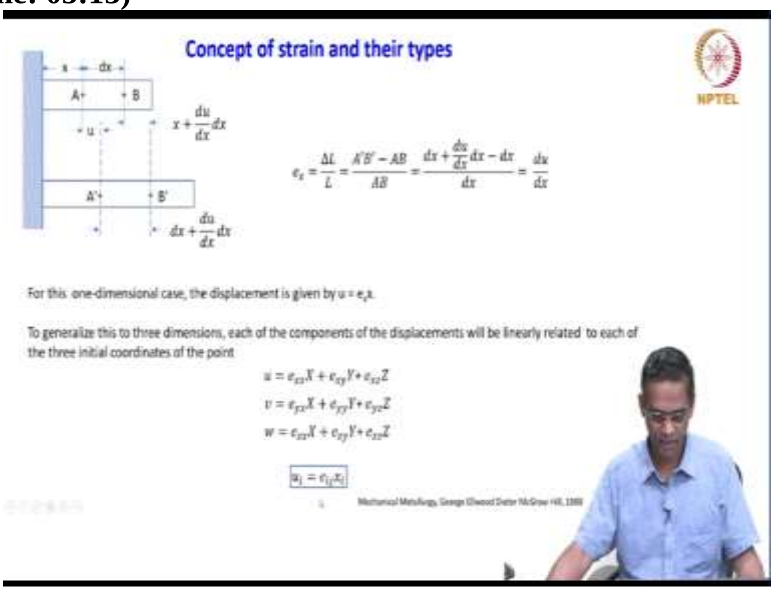

We are talking about elastic strain, but even there it is a small strain so, now we will focus our attention on details, how we mathematically look at this strain and are mathematically described in terms of small equations like what we have done in a stress at a point. So, similar thing we can just try to get a physical meaning and this is the fixed edge on this left hand and then you have the member which is fixed from one end and you can see that the two points on this member A,B.

The distance between A and B is dx and the distance between A and fixed end is x so, this is the undeformed region or undeformed member and this bar is just pulled in this one direction x direction. So, what we are seeing here is the displacement. So, the point A has moved from A to A′ so, that is displacement is U and point B is moved here to B ′ the displacement is x du / dx times dx. So, what does it mean? So, it is not the equal displacement that is because here it is considered the B which is here is far away from the fixed end.

So, that displacement is more in B′ so, the total displacement are the distance between the A ′ and B′ = dx + du / dx times dx. So that is how it is represented. So, for that if you look at the linear strain here we are talking about in one direction that is x direction here. So, the linear strain is $\varepsilon_x = \Delta L / L$, this is something in fact even the very beginning or introduction of this course we have just seen change in length by original length. So, similar to that is a linear strain.

So, if you write it in terms of according to the geometry what is shown here it is A ′ B ′ - AB /AB and you can just substitute these measures into this equation then what you see is du / dx. So, simply what you get the linear strain along the direction x is measured here as du / dx that is a displacement. Displacement with respect to X direction that is linear strain x this is for one dimensional case and the displacement is given by $u = e_x X$ so, that is the displacement.

To generalize these to three dimensions, each of these components of the displacement will be linearly related to each of the three initial coordinates of the point. So, what is that it will be written like this. So, you will have these components from e_{xx} in the X direction and e_{xy} I mean this is Y direction and this is from Z direction. So, you can also correlate this it is almost similarly, I know it is quite familiar to us.

This kind of equation we have already seen for the stress σ_{xx} σ_{xy} and σ_{xz}. So, here it is e_{xx} e_{xy} and e_{xz} so, what is that it is simply the normal strain and the shear strain. So, for the three directions are displacements in the three perpendicular directions and coordinates. So, that is how you should look at it. So, generally you can write $u_i = e_{ij}$ in x_i it is a general notation.

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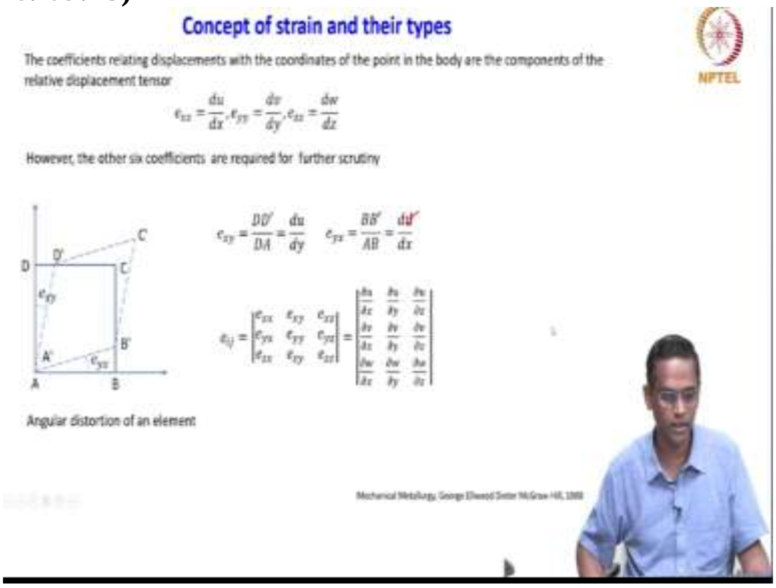

So, the coefficients relating displacements with the coordinates of the point in the body are components of the relative displacement tensors. So, you can see that e_{xx} was shown to be du / dx and e_{yy} is dv / dy and e_{zz} = dw / d z. However other 6 coordinates are required for the further scrutiny so, this is only a normal strain. So, it is not complete description, that description is not entered here. So, we need further six coefficients which correspond to other shear component.

So, we can also look at this diagram, which we have already seen in fact, so, just for the completion of this derivation I brought it here again. So, you see that the square member has been deformed in the form of angular distortion or I would say the deformation is angular distortion of this element here. So, what is this deformation here? The deformation is the member A B C D has been deformed into A ′ B ′ C ′ and D ′ and this deformation is not linear deformation but an angular deformation or called angular distortion.

So, this is the shear stress or shear strain here it is not stress here shear strain e_{yx} and e_{xy} these are the strains we are interested. So, what is that how do we find that for e_{xy} for example, the displacement along the x direction. So, this is what it is e_x and e_y the same nomenclature follows here to subscripts. So, it is an x direction, so, DD′ divided by DA. So, the DD length is here which is parallel to x and this is DA which will give you du / dy that is a shear strain.

Similarly, you have e yx that will be BB′ this is a y direction and divided by AB this distance will give you e_{yx} that is dv / dx they should be dv / dx not there was a correction here. No, no it is correct only I am sorry, I am confusing here. So, let me we will come to that later, because we are talking about the shear strain not be normal strain here. S, now it will be clear here. So, we can put it all these components into the form of a matrix for a three dimensional strain.

So, you can see that $\partial u / \partial x$ and $\partial u / \partial y$ and $\partial u / \partial z$ and here $\partial v / \partial x$ $\partial v / \partial y$ and $\partial v / \partial z$. So, I was correct so can correct this now, so it will be $\partial v / \partial x$. So, here again e_{zx} e_{zy} and e_{zz} can be given by this $\partial w / \partial x$ $\partial w / \partial y$ and $\partial w / \partial z$. Similar to this we can just if you take a three dimensional figure and then it can be simply worked out like this for this. So, similar to what we have seen in the stress.

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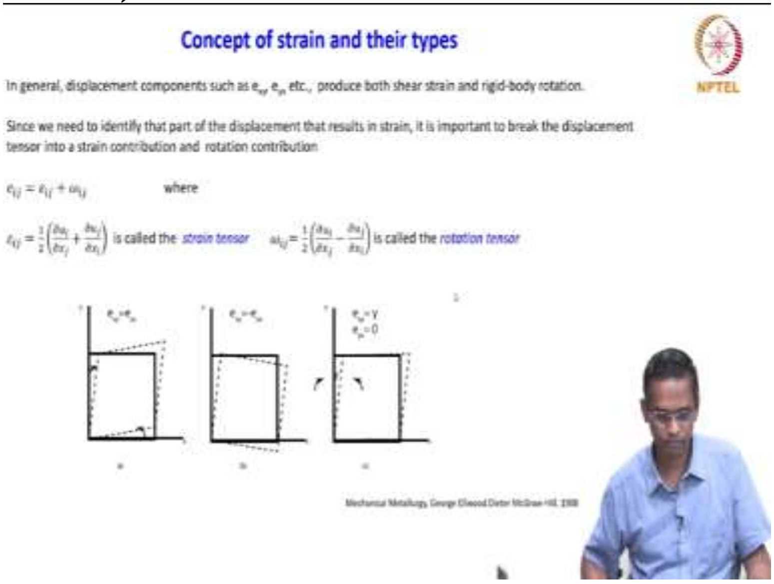

So, what we are now seeing so, far is linear or strain so, also the strain which has got two components, what are those two components? The displacement component such e_{xy} and e_{yx} etc, produce both shear strain and rigid body rotation, very important. So, we are to differentiate these two things shear strain and rigid body rotation. So, look at this figure a, b, c the one which we have just now seen it is simply an angular distortion.

But here what we are seeing is it is not a distortion here it is simply a rotation and here it is a simple shear just a simple shear. So, how do we distinguish these things? So, since we need to identify that part of the displacement that results in strain it is important to break the displacement tensor into shear contribution and rotation contribution. So, we are going to look at this displacement tensor and we are going to break down into strain contribution and rotation contribution like what we have seen in the stress tensor.

Similarly, we will look at the displacements tensor and also later we will also see that you know the strain tensor and we will also try to decompose. So, this is a notation here $e_{ij} = \varepsilon_{ij} + \omega_{ij}$ where $e_{ij} = 1/2\ \partial u_i / \partial x_j + \partial u_j / \partial x_i$ is called the strain tensor and $\omega_{ij} = 1/2\ \partial u_i / \partial x_j - \partial u_j / \partial x_i$ is called the rotation tensor. So, this e_{ij} can be decomposed into this strain contribution and rotation contribution. So, this is very important.

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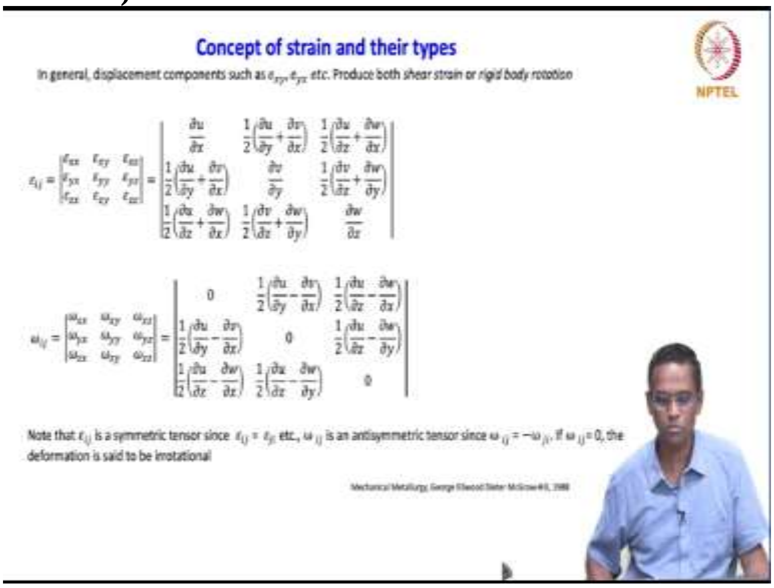

So, in general the displacement components such as e_{xy} and e_{yx} etc.produce both shear strain and rigid body rotation. So, we can just bring in we can just replace all this shear component and rotation component into the matrix 3* 3matrix that is here it is a shear strain matrix. So, you can see that we know that how you get individual component like this and this is what we have just shown.

So, we can simply plug into this value and then you will see that we will get two, 3*3 matrix for shear strain as well as rigid body rotation for these two components. So, this is just to give you an idea that these two quantities are also a tensor quantities. Note that ε_{ij} is a symmetric tensor since $\varepsilon_{ij} = \varepsilon_{ji}$ etcetera while ω ij is an anti symmetric tensor please note that $\omega_{ij} = -\omega_{ij}$ if $\omega_{ij} = 0$ then the deformation said to be irrotational. so, very useful terminologies and relations.

So, when we say that the deformation also can be linear or it can be angular distortion involving rotation and so on, and when that can be easily represented by this rigid body rotation tensor. So, this is useful it looks it is a very simple idea, but then when you put it in a form of mathematical equation it gives you much more clarity.

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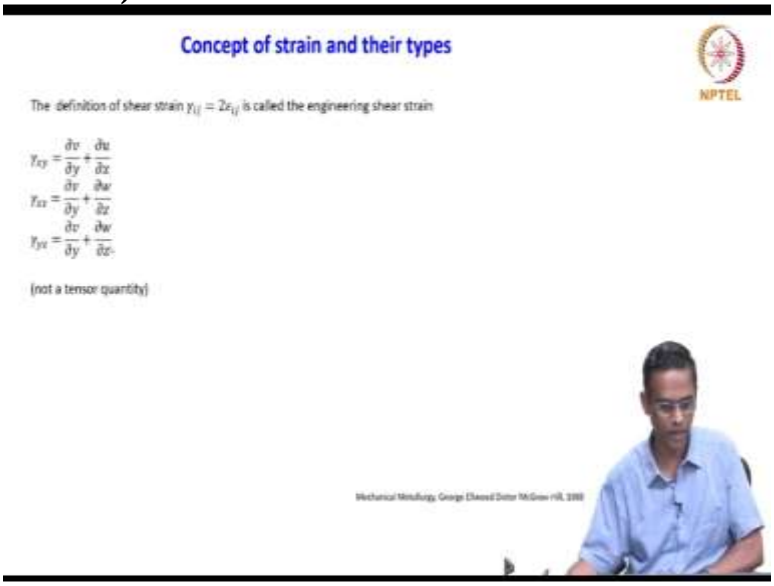

So, that is about the total strain, now we talk about the definition of shear strain which is γ_{ij} = 2 times ε_{ij} is called engineering shear strain. So, we will get into the details of this and as we proceed, but let us now see what is this engineering definition of this engineering shear strain. So, here it is

$\gamma_{xy} = \partial v / \partial y + \partial u / \partial x$ and

$\gamma_{xz} = \partial v / \partial y + \partial w / \partial z$ and

$\gamma_{yz} = \partial v / \partial y + \partial w / \partial z$.

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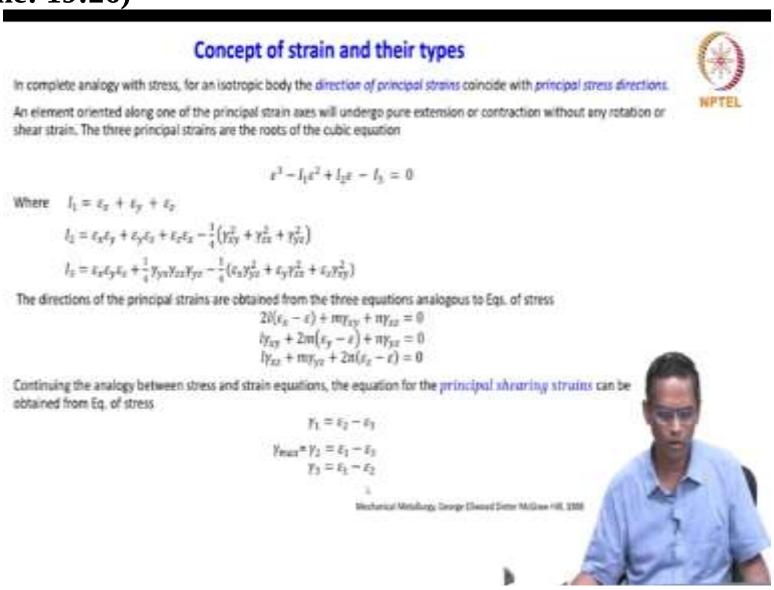

And it is not a tensor quantity that is small information. So, now, what we are going to do is in a complete analogy with the stress for isotropic body of the direction of the principal strains coincide with the principal stress directions. So, similar to stress description, where we were analysing the principal plane and the principal stress and so on here again the direction of principal strain and will be considered similar to what we have just seen for the stress.

So, the element oriented along one principal strain axes will undergo pure extension or contraction without any rotation or shear strain, the three principal strains are the roots of the cubic equation. So, what we are seeing in the stress is the same cubic equation for the strain as well $\varepsilon^3 - I_1\varepsilon^2 + I_2\varepsilon - I_3$.

I_1, I_2, I_3 are invariant coefficients you can see that similar to stress, it can be written like that.

So, just for the completion I have brought this and also to emphasize the fact that it will be a completely similar to what we have seen in stress. The directions of the principal strains are obtained from the three equations analogous to the equations of the stress. So, if you recall, we wrote a similar equations for the stress by looking at the force equilibrium. So, similar thing can be drawn for the strain as well.

So, continuing the analogy between stress and strain equations, the equation for the principals shearing strains can be obtained from the equation of stress similar to what we have written there, we can also get it from the for the string as well. So, if you recall this all these equations exactly will match like what we have derived for the stress. So, I am just going little fast in this, please have a revise this.

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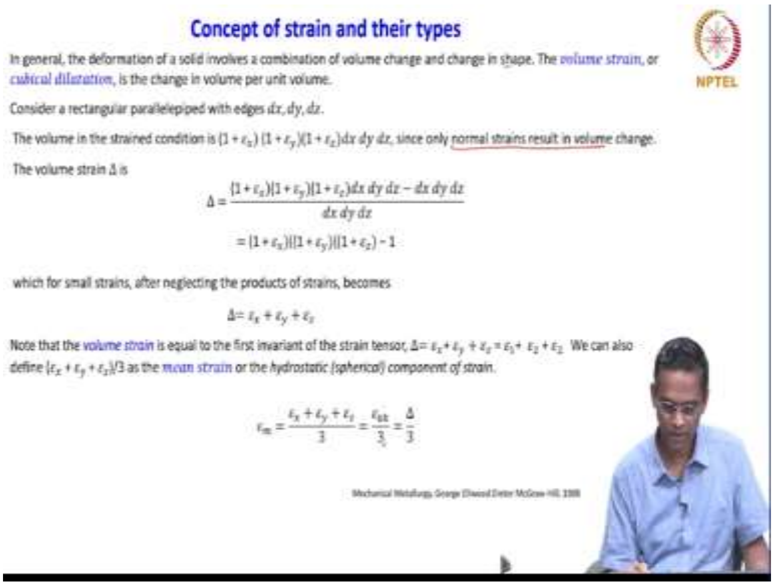

And then it will be clear but just for sake of time I just want to rush it through. So, here comes another important aspect the, in general the deformation of a solid involves a combination of volume change and change in shape. So, the volume strain or the cubical dilatation is a change in volume per unit volume. So, let us consider the volume of a rectangular parallelepiped with the edges dx, dy and dz.

So, the volume in the strain condition it is exactly similar to what just we have developed this equation in the beginning of this class, the same equation here. The deformed body only the normal strains results in the volume change. So, this is an important point only normal strains results in the volume change. So, the volume strain Δ is equal to that is $\varepsilon_x + \varepsilon_y + \varepsilon_z$ what you are seeing, can we written it like this. So, and then you will get this equation this we have already seen.

So, I am just skipping that point and this is again valid for small strains and then we are neglecting the higher terms and so on and then we get the $\Delta = \varepsilon_x + \varepsilon_y + \varepsilon$ z. So, note that the volume strain is equal to the first variant of the strain tensor that is the $\Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z$, which is again equal to $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$. We can also define similar to mean stress here it is a mean strain $\varepsilon_x + \varepsilon_y$ and $\varepsilon_z / 3$.

The mean strain or the hydrostatic component of the strain very important. So, like we have just looked at the hydrostatic stress and then here it is hydrostatic component of strain. So, which can be done like this as a

$$\varepsilon_m = \varepsilon_x + \varepsilon_y + \varepsilon_z / 3$$

or this written sometimes with this notation ε $_{kk}$, mostly in tentorial representation which is also written as Δ / 3.

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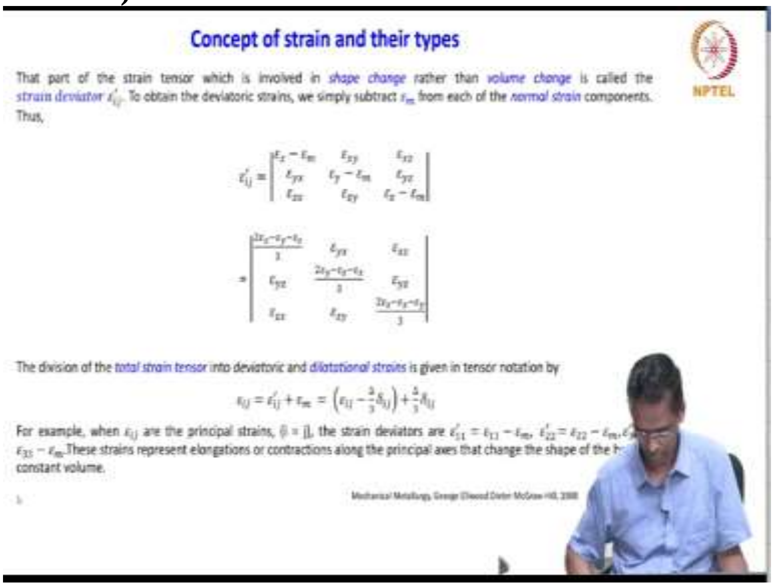

So, the part of strain tensor which is involved in shape change rather than volume change is called strain deviator, see these particular terminology you know the stress tensor which is involved in a shape change or volunteer they are very important in the deformation or theory of plasticity or metal forming and healing theory and so, on that is exactly we are emphasizing this. So, you will appreciate when we refer this term, when we looked at the theory of plasticity and deformation or any metal forming processes.

So, we will refer this particular strain which a strain deviator or stress deviator which is responsible for the building process or forming process. So, that is exactly as an introduction if you are getting to the if you are familiar with these terms and the corresponding equation then it will be easy to appreciate later without any confusion that is why we are seeing all these things in detail. So, the strain tensor can be responsible for shape change or volume change.

So, to obtain the deviatoric strain, we simply subtract ε m is a mean strain from each of the normal strain components. So, it is like this tensor matrix is exactly the similar to the stress matrix. So, the deviatoric strain is

$$\varepsilon_{ij}' = |\varepsilon_x + \varepsilon_m|$$

that means we are just simply subtracting the mean strain from the normal strain component. So, these are all normal string components like $\varepsilon_1\ \varepsilon_2\ \varepsilon_3$ something.

So, if you can just expand this ε m and if you rewrite this equation, this matrix will look like this. The division of the total strain tensor into deviatoric and dilatational strains is given in the tensor notation by this. So, it is

$\varepsilon_{ij} = \varepsilon_{ij}{}' + \varepsilon_m$ which is equal to ε_{ij} - Δ / 3 times Δ ij + Δ / 3 times small Δ ij. So, this is a tentorial notation.

So, for example, when ε ij are the principal strains i = j, the strain deviator are

$\varepsilon'_{11} = \varepsilon_{11} - \varepsilon_m$

similarly,

$\varepsilon'_{22}{}' = \varepsilon_{22} - \varepsilon_m$

and so, on. These strains represent elongations are contractions along the principal axes that change the shape of the body at the constant volume very important. So, the shapes change strains.

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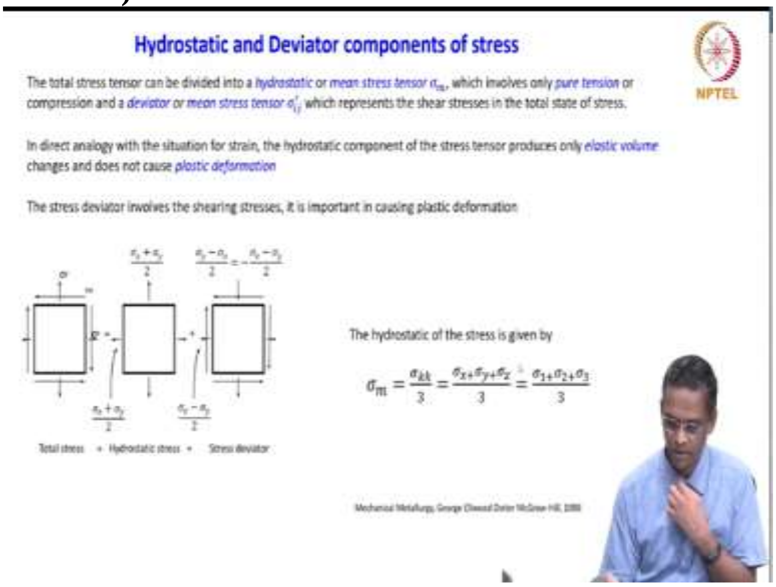

And so, finally we will just look at one more component are the topic of hydrostatic or deviator component of the stress this particular topic we did not touch. Now, that we have just looked at hydrostatic component of a strain and the deviatoric strain, now we will also see the similar component here. The total stress tensor can be divided into hydrostatic and mean stress tensor which involves only pure tension or compression and a deviator or mean stress tensor which represents the shear stresses in that total state of stress.

In direct analogy with the situation for strain, the hydrostatic component of the stress tensor produces only elastic volume change so, this is to be noted. So, the hydrostatic component of stress tensor produces only elastic volume change and does not cause plastic deformation. So, we will refer exactly this particular component in while we discussing the yield phenomenon.

So, the stress deviator involves in shearing stresses, it is important in causing plastic deformation. So, the stress deviator we have to identify how it looks like. So, just to make it more, simple this schematic is given here to the total stress which involves here it is a biaxial stress with the shear strain around it and it can be decomposed into a pure hydrostatic stress plus stress deviator. So, the hydrostatics stress is given by like this

σ_{mean} = σ_{kk} similar to ε_{kk} / 3 here it is $\sigma_x + \sigma_y + \sigma_z$ divided by 3 and this is how it is written.

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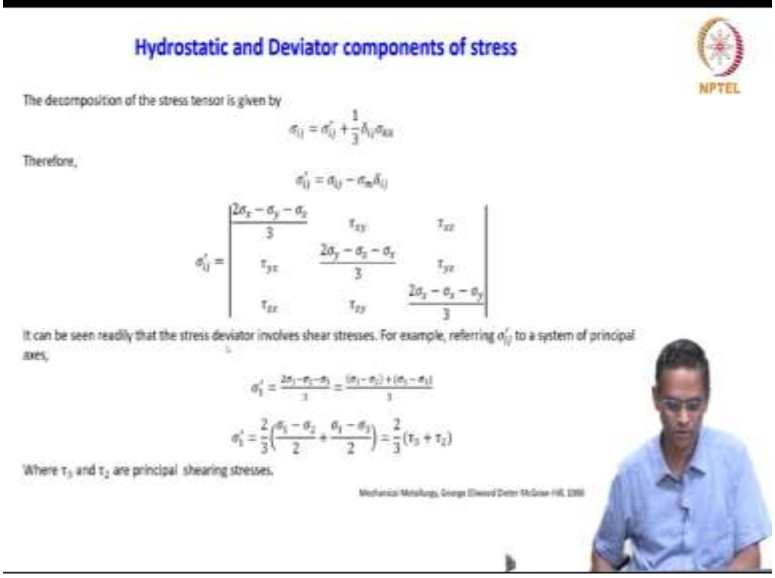

So, in the tentorial notation, the decomposition of the shear stress is given by $\sigma_{ij} = \sigma_{ij}' + 1/3\,\Delta_{ij}$ times σ_{kk}. So, you rearrange this to get the stress deviator then you takes this form. So, we can now put that directly into the matrix stress tensor matrix. So, here it is a deviatoric stress is equal to this is each what should I say the principal stresses or subtracted from the mean stress or hydrostatic stress component, then the matrix will look like so, this matrix we are all familiar so, I am just going quickly.

So, it can be seen readily that the stress deviator involves shear stresses for example referring σ_{ij} to a system of principal axes. So, we can

$\sigma_1{}' = 2(\sigma_1 - \sigma_2 - \sigma_3)/3$

which can be written like this and we can rewrite this into this form

$\sigma_1' = 2/3(\sigma_1 - \sigma_2/2 + \sigma_1 - \sigma_3/2)$

$\sigma_1' = 2/3(\tau_3 + \tau_2.)$

So, this is also very familiar to us, we have already seen what is this? These are all two shear stresses τ_3 and τ_2. If you recall, we have in fact shown that actually the plane directions in a previous class we are shown. So, you just compare these two where these shear stresses are acting. Exactly, they are σ_3 and σ_2 we have shown these two, perpendicular now, they are bisecting the 2 normal stresses we have shown clearly in that.

So, since we have already familiar with this, so it is we are able to appreciate this concept that the stress deviator involves shear stresses. So, this is very nice that, you know it will become so kind when we discuss plasticity problems. The stress deviator which involves shear stress is only this kind of stress deviator is responsible for the plastic deformation. So, that is quite obvious from this equation.

So, the τ_3 and τ_2 are the principal shear stresses we know that.