Mechanical Behavior of Materials
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Lecture - 63
Fatigue - IV

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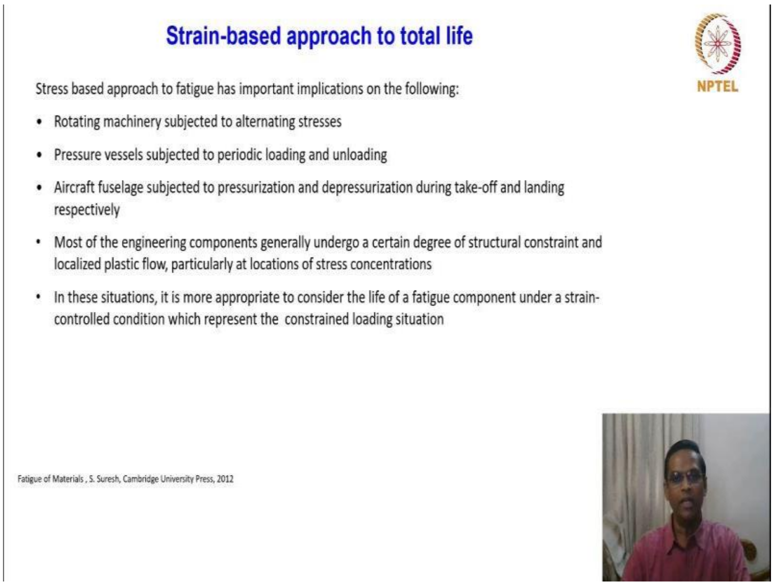

Hello I am Professor S. Sankaran in the department of Metallurgical and Materials Engineering. We will just summarize some of the quantities which we have already learned. So, we started with the stress-based approach of fatigue which has got an important implications in rotating missionaries subjected to alternating stress pressure vessels subjected to periodic loading and unloading.

Aircraft fuselage subjected to personalization and depreciation during take-off and landing of respectively. However, most of the engineering components generally undergo certain degree of structural constraints. So, this is very different from the about three points and localized plastic flow particularly at locations of stress concentrations. So, wherever the components you know they undergo certain degree of structural constraint and localized plastic flow we need to go for the other way of approach.

That is stress based approach takes care of this kind of loading. And this constrained situation it is more appropriate to consider the life of the fatigue component under the strain control condition which, represent the constraint loading situation. So, here why I mean I am trying to kind of you know justify why we look at stress-based approach as well as strain-based approach.

Though, we have seen both of them already now I just lined up summarizing and then bringing into a kind of combined discussion.

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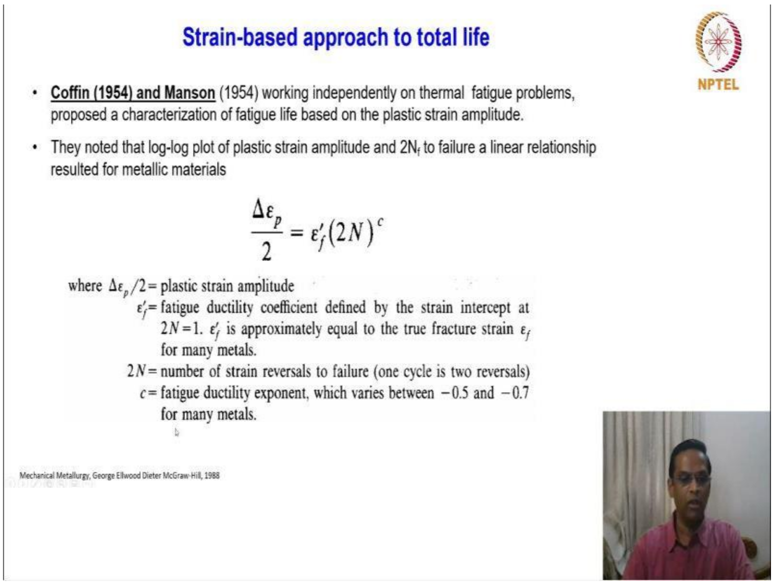

So, the strain-based approach to total life is generally analysed by a coffin mansion, this is called Coffin Mansion plot we normally refer in the community. Coffin and Mansion working independently on thermal fatigue problems proposed a characterization of fatigue life based on the plastic string amplitude. They noted that log-log plot of amplitude and $2N_f$ to failure a linear relationship resulted for a metallic material.

Very, very important point. People work if you want a character is a plastic strain, I mean stain-controlled fatigue this is the first plot everybody will look at. So, which is

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon_f'(2N)^c$$

Where Δε/2 is a plastic strain amplitude. ε'_f prime is a fatigue ductility coefficient defined by the strain intercept at 2N = 1.

ε'_f is approximately equal to the true structure strain ε'_f for many metals. 2N is number of strain reversals to failure one cycle is two reversals and c = fatigue ductility exponent which varies between – 0.5 to – 0.74 for many metals. So, this is a very important relation as for the strain-based approach to total life.

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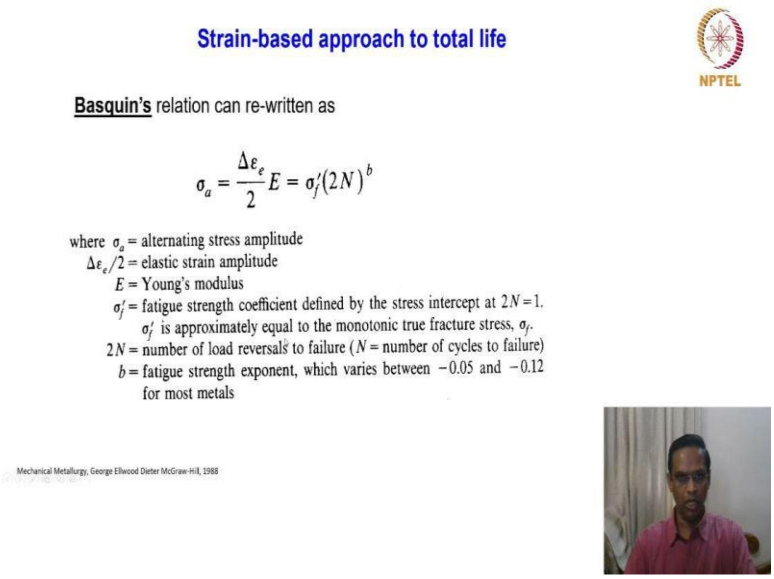

And then, we know that stress-based relation which is given by the Basquin equation which we have seen earlier which reads like this

$$\sigma_a = \frac{\Delta\varepsilon_e}{2}\,E = \sigma_f'(2N)^b$$

So, this σ_a is alternating stress amplitude this is as elastic strain amplitude E young's modulus σ'_f is fatigue strength exponent. And so, this is similar to σ_f and b is the fatigue strength exponent. So, now we have these two equations you will see if you put together how does it looks like.

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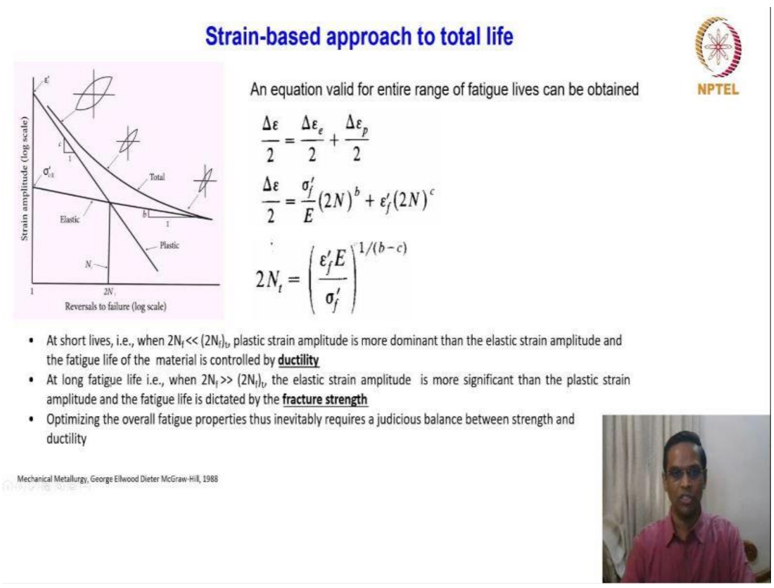

So, this is what is shown here an equation value for entire range of fatigue lives can be obtained by this kind of an equation. So, one is you know strain based another stress bases. So, $\Delta\varepsilon/2 = \Delta\varepsilon e/2 + \Delta\varepsilon_p/2$. So, if you can just substitute this from Basquin and coffin mansion plot you get this equation and then you have the a common expression for this enough which is two in a total in total.

And this plot very nicely illustrates you know the coffin mentioned plot versus Basquin plot this is a combined curve you can see that. A short life that is when $2N_f < <(2N_f)_t$ plastic strain amplitude is more dominant than the elastic strain amplitude. And the total fatigue life of the material is controlled by ductility. See this is how you should appreciate how to read these equations right at long fatigue life.

That is when $2N_f >>(2N_f)_t$ the elastic strain amplitude is more significant than the plastic strain amplitude and the fatigue life is dictated by the fracture strength. So, which fatigue test or what type of fatigue study will give what type of material behaviour something like that. Optimizing the overall fatigue properties that inevitability requires a judicious balance between strength and liquid.

So, this is a well-known challenge, even today people work on different modern materials still this is the strength versus ductility is a huge challenge.

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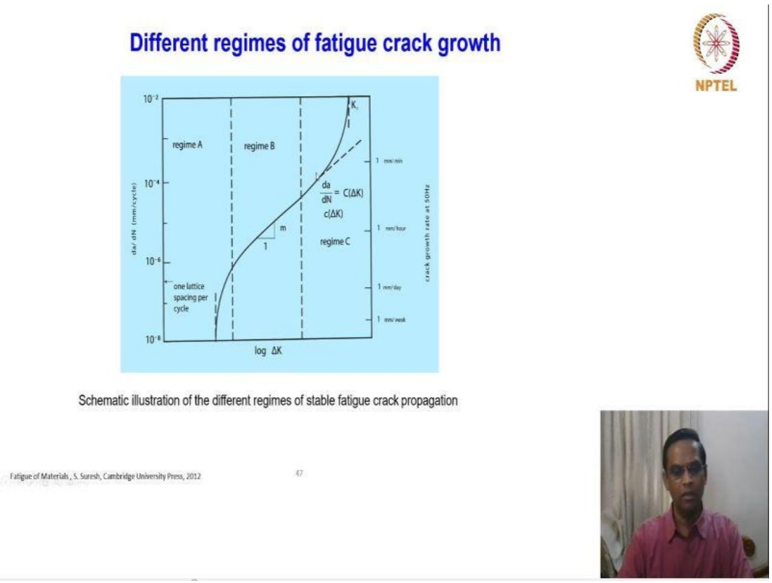

Now we will now look at the little bit on fatigue crack growth mechanisms. Though we have seen this already in the fracture mechanics chapter elaborately, I just want to touch upon some of these small features or important points which we have missed in that chapter. So, this is you know now very well about this fatigue crack growth which is divided into three regions regime A regime B regime C.

And this is described I mean regime B is described by the Paris equation and regime 1 is you know the important parameter design parameters ΔK threshold is we have discussed. And then regime 3 the material can reach the K_1C. So, that is how it is so in fact this particular equation Paris law is the link between fatigue and fracture mechanics and very important engineer, I mean design parameters this equation gives. Though it is an empirical relation but it is very important relation.

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And some salient features concern to near threshold fatigue crack growth I want to just mention. The stress intensity factor range approaches the fatigue crack growth threshold ΔK an operational definition for ΔK_0, ΔK_0 or ΔK threshold the same here is commonly used in terms of maximum crack growth rate typically 10^{-8} mm based on the accuracy of the crack monitoring system and the number of elapsed cycles.

Structural engineers are often faced with the task of designing components which can withstand very high frequency low amplitude loads over fatigue lifetimes as high as 10^{-10} to 10^{-12} load reversals. For example, a high-speed turbine rotor operating at 3000 revolutions per minute may be subjected to 10^{10} site stress cycles over a typical lifespan of 20 years.

Suppose if you take this as an example if the fatigue crack growth were to occur in this rotor at seemingly insignificant rate of 3 x 10^{-9} mm per cycle. That is 100 of an atomic spacing per cycle this would still represent a total crack advance of 30 mm during the life of the router, which is sufficient to pass a catastrophic failure. So, now this clearly demonstrates why we need to worry about the threshold fatigue threshold you know ΔK values that is one idea.

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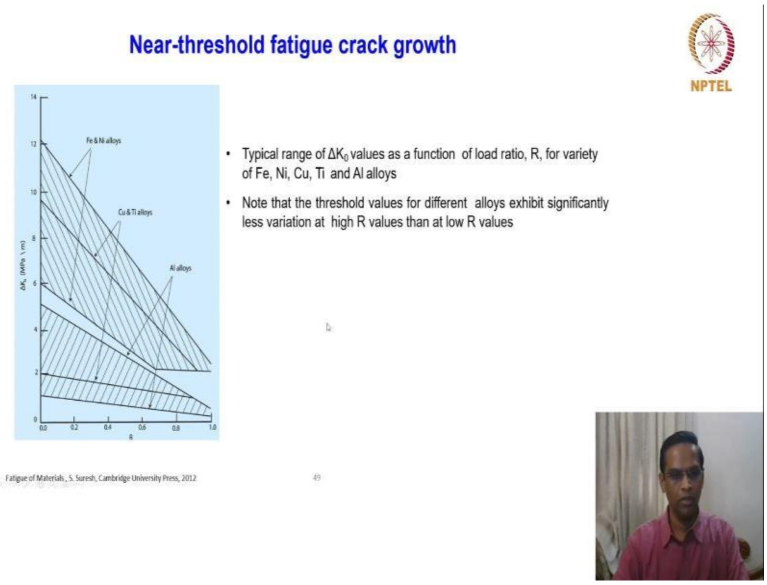

And if you look at the threshold values for typical alloy structural alloys you know this is ΔK versus R this is a stress ratio. You can see that typical range of ΔK values as a function of a load ratio for a variety of copper nickel, sorry iron nickel copper titanium aluminium alloys and note that threshold values for different alloys exhibit significantly less variation at high R values than at lower values.

So, that is quite obvious at low R values the you know the delta threshold range is quite high as the R increases that means you know it goes higher mean stress we have seen in the beginning of the lecture. It all comes down the ranges comes down this is something you have to know.

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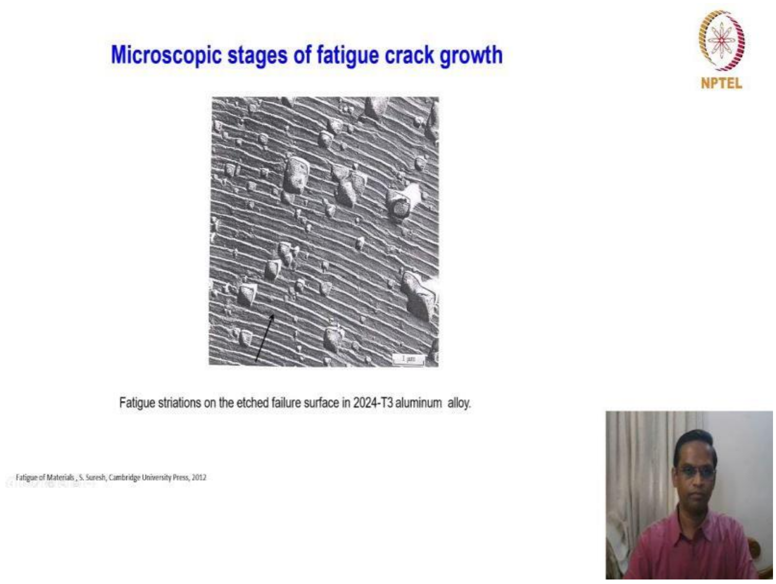

Now we get into this crackdown mechanisms what you are seeing is a very nice picture showing the fatigue striations on an etched failure surface in 2024-T3 aluminium alloy. This is also a book cover of this book by professor Suresh. So, very nice so striations are characteristic of fatigue crack growth which you know these particular striations get generated with the material in the crack growth the regime of two, this is what we have seen in the last chapter also.

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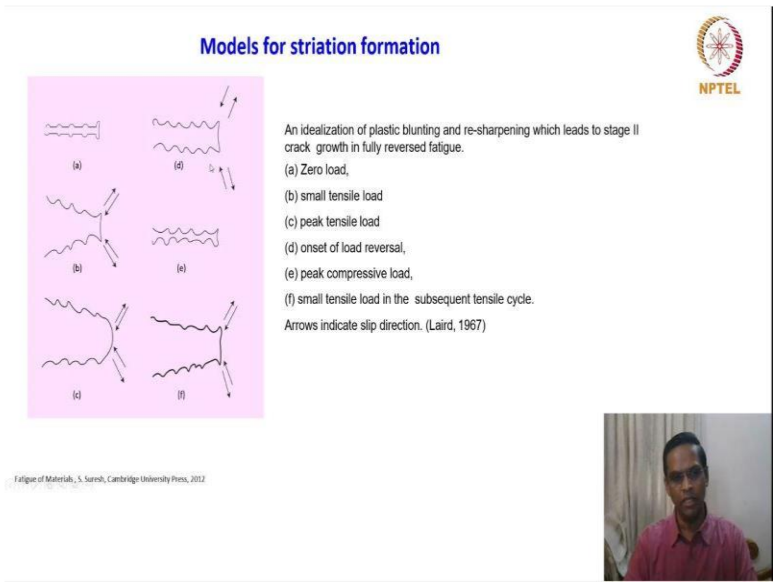

Just to give you one more model there are several models reported in the literature but, one model I just want to discuss which I did not discuss in the fracture mechanic's chapter. So, this is a very nice schematic which is describing this and idealization of plastic blunting and the resharpening which leads to stage 2 cracked growth in a fully reversed fatigue. So, this is a model for the striation formation in this stage 2 of the crack growth curve.

So, what is shown here is what is suppose you consider the crack like this and which is at a zero load. And if the small tensile load is increasing then this crack becomes like this it opens up and sorry here this is not, sorry about this I am supposed to show this is a small tensile load and then the moment it reaches the peak tensile load and what you have to observe is it is a crack blunting.

So, this is that blunting occurs and then what happens in a d, onset of load reversal. So, it becomes you know that means the crack opening displacement comes down and what happens to this blunt surface is again becoming a two sharp edges, because of the reversible loading. And this is a peak tensile and this is a peak compression. So, peak compression what happens is very interesting to note.

So, you compare this crack length and this crack length, the crack length is extended by this mechanism the peak by comp by compressing further these sharp edges become further sharper and becomes another blue edge and it is ready to go back to this model again. So, again small tensile load in the subsequent and tensile cycle lead to this. So, this becomes this but you can see that the length has been increased compared to this one.

So, arrows show; indicate slip direction and this is proposed by a Laird and this one particular mechanism is a model mechanism for the striation formation. Because striation is the; characteristic of fatigue loading alone that is why I just want to bring this idea so, for the completion.

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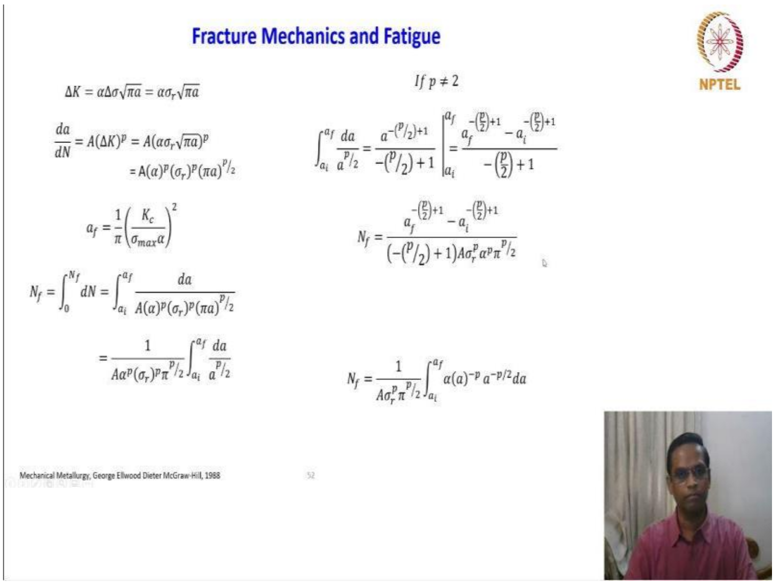

And one more important aspect is the as I just mentioned this Paris law connects the fracture mechanics pretty and with this, we will be able to calculate you know the life of the fatigue specimen from this relation just to I want to show this equation how we can understand this. So, this is a Paris equation and you can see that you can rewrite this equation like this. And we know that to the crack length is obtained by the stress intensity factor equation which is

$$a_f = \frac{1}{\pi}\left(\frac{K_c}{\sigma_{max}\alpha}\right)^2$$

So, α is the material parameter and if you integrate this expression and then you can integrate from the initial crack length, I mean this is the initial loading to final cycle initial cycle to final cycle

$$N_f = \int_0^{N_f} dN = \int_{a_i}^{a_f} \frac{da}{A(\alpha)^p(\sigma_r)^p(\pi a)^{p/2}}$$

So, what is shown here is σ_r is the stress reversal complete reversal.

So, it is Δσ is replaced like a σ_r and in some books, it is p is the exponent is written as m but here it is in this reference it is given as a p. So, we can just go ahead with that so after integration after rearrangement then you have to finally reach this integral value integral

$$\int_{a_i}^{a_f} \frac{da}{(a)^{p/2}}$$

So, if you perform this integral for the value of not equal to 2.

The p value normally it is 3 to 4 for most of the matrix but in this particular case it is derived for if $p \neq 2$ then this it has to go in this way. And if you substitute this in carry out the integration and then substitute the limits, then you get this final expression and what you have to remember is this is not you know taking care of the crack length. The crack length versus the α dependence on the crack length is ignored here.

But if you consider α as a function of crack length then this equation has to be used for calculating the enough. So, we will solve some numerical in the tutorials then you will appreciate this, so what I want you to do know is through this Paris equation we will be able to calculate the life of the component subjected to fatigue loading. So, this is one demonstration.

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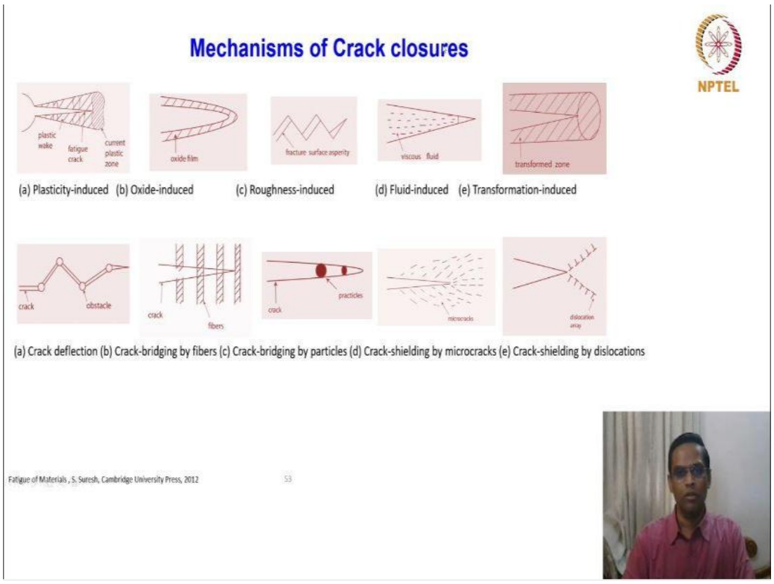

The other point I want to just summarize is mechanisms of crack closures. We have already seen this in the fracture mechanics just to recall. So, whatever is you know we have already discussed views one of the important crack closure mechanisms is the plastic zone size developing ahead of the cracker. So, we have seen this is a plasticity induced crack closure see as you move ahead the plastic zone size bigger and bigger than the cracked resistance or get closed sometimes.

Then oxide film induced crack closure, roughness induced by closure especially in a mole two kind of situation and fluid induced required closure is there and transformation induced crack closure. Also, we have seen that is in fact we will just see the one classical example for this transformation induced plasticity you will see as a case study. And crack deflection mechanisms by obstacles or second phase particles.

And crack bridging by fibres this is for composites materials and crack bridging by particles you know we said that bridging action in the fracture mechanics chapter. Similar to that and crack shielding by micro tracks this also we will see now in another one of two slides there is a case study you will see. And crack shielding by dislocation this is also possible. So, this is several mechanisms which support the crack closure behaviour.

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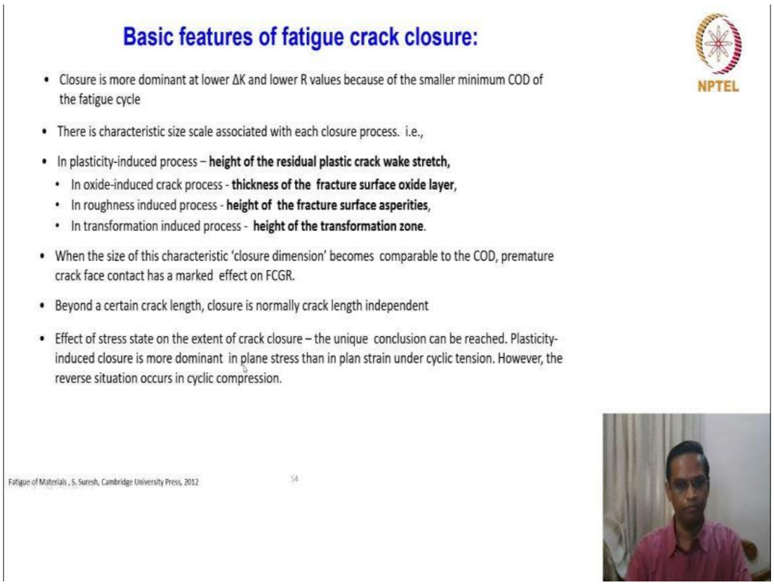

Some of the basic features are closure is more dominant at lower ΔK and lower R values because of the smaller minimum COD of the fatigue cycle. And there is a characteristic size scale associated with each closure process. That is if the plasticity induced process is considered then height of the residual plastic crack wake stretch that is to be very important parameter for the crack closure.

If it is an oxide induced crack closure causes then the thickness of the fracture surface oxide layer that will play a major role for the crack closure process. And if it is a roughness induced process then height of the fracture surface asperities. And if it is a transformation induced process then the; height of the transformation zone which is going to determine the correct closure effectiveness.

When the size of this characteristic closer dimension becomes comparable to COD crack opening displacement premature crack face contact has marked which is in a no effect of crack growth fatigue weight. So, any of this dimension has to be much smaller compared to COD then the premature fracture will not take place. Otherwise, this will be more effective it will show marked effect on the fatigue crack growth rate.

Beyond certain crack length closure is normally cracked length independent. Effect of stress state on the extent of crack closure the unique conclusion can be reached plasticity-induced pressure is more dominant in the plain stress than the plain strain under the cyclic tension. However, the reverse situation occurs in the cyclic compression. So, this is one important effect of state of stress on the cracked closure.

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So, now I just want to briefly discuss the fatigue of brittle solids very highly brittle solids. Most of the aspects we have discussed are confined to metals and alloys. And let us see some of the interesting ideas about brittle solids.

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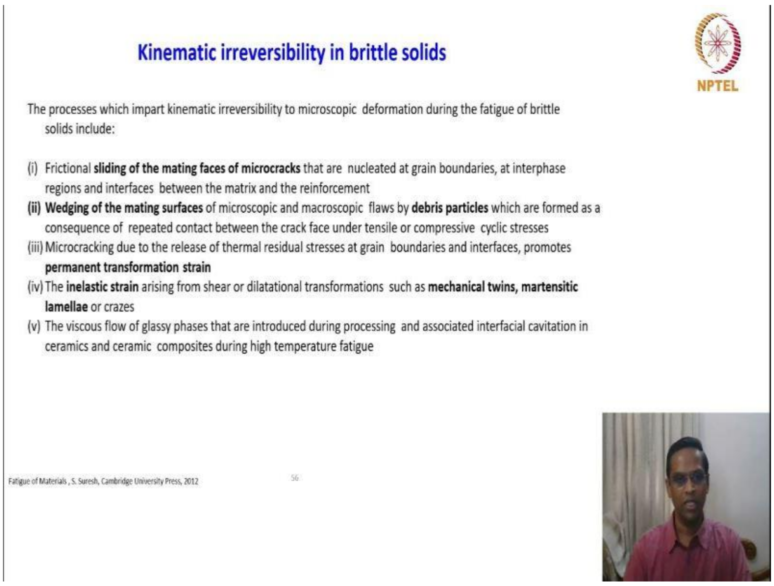

In brittle solids we consider the kinematic irreversibility. What is that? The processes which import kinematic irreversibility to microscopic deformation during the fatigue of brittle solids include; a frictional sliding of mating phases of microcracks. So, we are now going to talk about microcracks in fact even Griffith considered you know microcracks as the you know basic units in the glass.

So, similarly same approach here we are talking about frictional sliding of mating phases of micro cracks that are nucleated at the grain boundaries at interface regions and interfaces between matrix and the reinforcement. Wedging of the mating surfaces of microscopic and microscopic flaws by debris particles which are formed as the; consequence of repeated contact between the crack face under the tensile and compressive cyclic stresses.

And micro cracking due to release of thermal residual stresses at grain boundaries and interfaces and promotes permanent transformation strength. The inelastic strain arising from the shear or the dilatational transformation such as mechanical twins martensitic lamellae or crazes. The viscous flow of glassy faces that are introduced during processing are associated with interfacial cavitation in ceramics and ceramic composites during the high temperature fatigue.

So, all these aspects the very important aspects will contribute to the kinematic irreversibility. So, this is how you should connect these events all these events we will be we know we are not talking anything new here most of these things we are discussed in the form of crack closure. But in a brittle solid all these small, small micro structural events contributes to the kinematic irreversibility how it affects the fatigue behaviour that is how we should discuss now.

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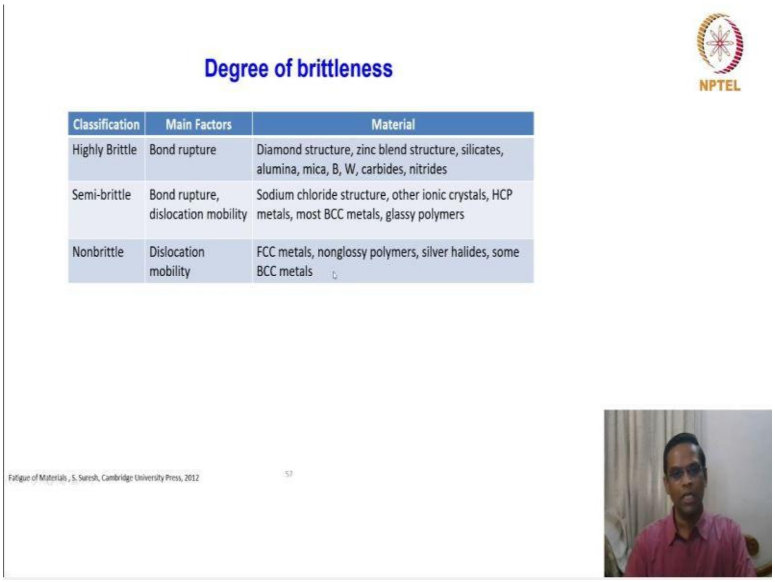

Before we discuss the mechanisms, we will also define what is the degree of brittleness. We can classify highly brittle, semi brittle and non-brittle materials. And in a highly brittle material the main factors are bond rupture, example is diamond structures zinc blend structures liquids, alumina, mica, boron, tungsten, carbides, and nitrates. In semi brittle materials the main factors which is going to characterize the failure is bond rupture dislocation mobility.

For example, sodium chloride structure other ionic crystals, HCP metals, most BCC metals, and glassy polymers. They are coming under cellular classification and the non-brittle is dislocation mobility FCC metals, non-glassy polymers, silver halides and BCC matrix. So, the degree of brittleness is measured by these parameters and factors and these are all the material examples for that.

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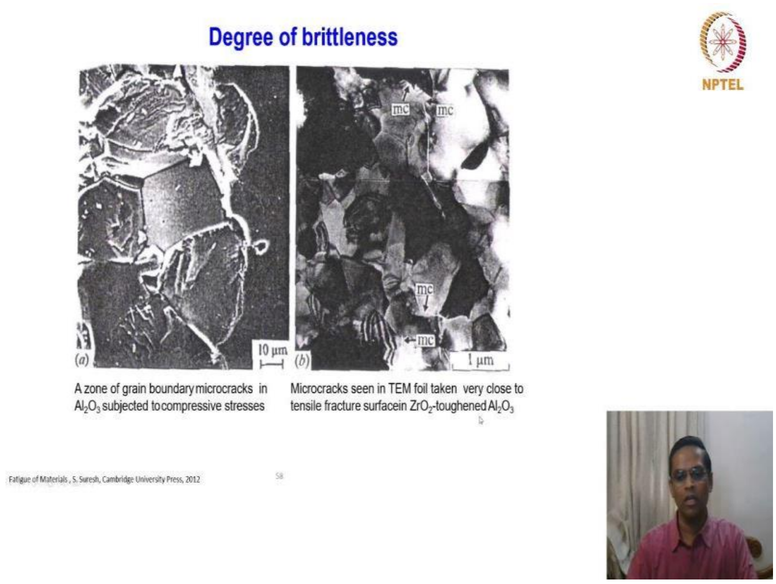

One example is shown and partograph is shown as an example, a zone of grain boundary microcracks in the alumina subjected to compressive stresses. So, you see that all the boundaries are highly cracked completely cracked micro cracks in alumina. And this is in a TEM foil taken very close to the tensile fracture surface in zirconia toughened alumina. So, you can see that all micro cracks are marked with the features in the green borders.

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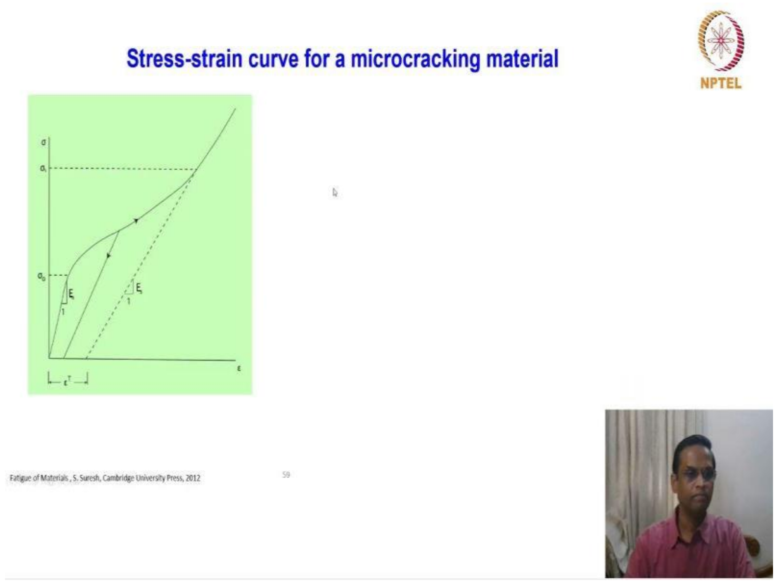

So, how do we understand this stress strain curve for a micro cracking material? Very interesting idea.