

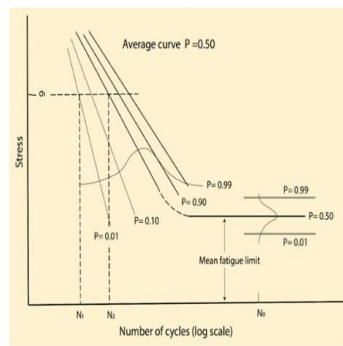
Mechanical Behaviour of Materials
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Module - 11
Lecture - 61
Fatigue - II

Hello, I am Professor S. Sankaran in the department of Metallurgical and Materials Engineering.

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Statistical Nature of Fatigue



Representation of fatigue data on a probability basis

Mechanical Metallurgy, George E. Dieter, McGraw-Hill, 1988

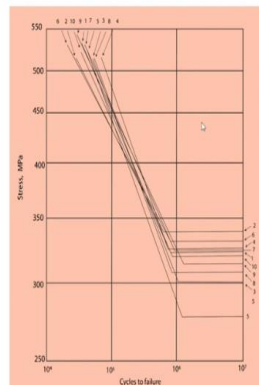
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So, what does it mean? And you cannot talk about the fatigue behaviour of any material by simply conducting one or two tests.

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Statistical Nature of Fatigue



Summary of S-N curves, each based on 10 specimens, drawn from the same bar of steel

Mechanical Metallurgy, George E. Dieter, McGraw-Hill, 1988

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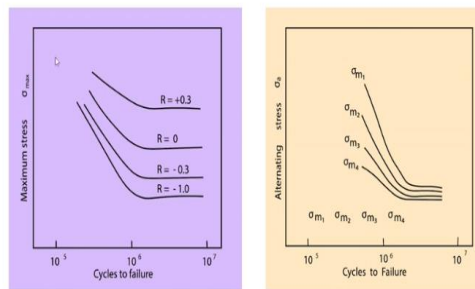


So, that is an idea. And in connection to that argument, you see here as this is a typical S-N curves, each based on ten specimens, drawn from the same bar of the steel. So, each one is based on the ten specimens. That means, you see that the range of endurance limit one can obtain here. So, you cannot say that taking one fatigue sample tested like this and then this is a fatigue limit. Obviously, that is not going to be the reality.

And also you will see the lot of scatter here in this; and this is the same bar of a steel. And we assume that this is not under environment or any other variables are fixed. So, this is something you have to understand.

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Two methods of plotting fatigue data when mean stress is not zero



Mechanical Metallurgy, George E. Dieter, McGraw-Hill, 1988

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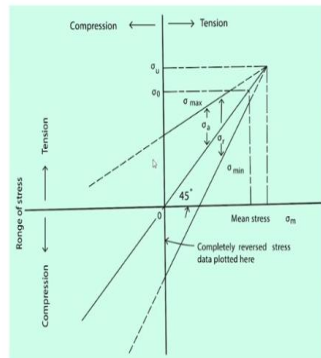


Two methods of plotting fatigue data when the mean stress is not zero. We know what is mean stress; $(\sigma_{\max} + \sigma_{\min}) / 2$; this is mean stress. When the mean stress is not zero, that means R value is increasing. So, when you look at the R value, - 1; -0.3; R = 0; and R = +0.3. That means, the mean stress keeps on increasing. So, with the increase in the mean stress, what do you see? You see the higher fatigue limit.

So, that is one way of looking at it. With the increase in the mean stress or you can see that the R value increasing to more positive, then also you will see that the fatigue limit is higher for a given material. And you can look at in the other way which is, if you plot for the same data as an alternating stress versus number of cycles to failure, then you see that σ_1 and σ_2 and σ_3 be; the highest mean stress will come down here and the lowest mean stress will exhibit the higher fatigue limit. So, this is the other way of looking at it.

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Goodman diagram



Mechanical Metallurgy, George E. Dieter, McGraw-Hill, 1988

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And there is also another way of looking at this effect of mean stress, which is proposed by Goodman; so, it is called Goodman diagram. What you are seeing here is, this is actually a four quadrant. This is a compression tension in this axis; and this is tension compression in this axis. So, range of stress is given here. And what you are seeing in this is a completely reversed stress data plotted here; this straight line.

So, what you have to look at here is, there are two primary line you have to concentrate. So, this is σ_{\max} which you know; and this is a σ_{\min} , which cuts through or the compression region, and it just goes beyond. And what do you need to understand is, as the mean stress is increasing and this σ_{\max} and the σ_{\min} curve is going to converge; it is trying to converge.

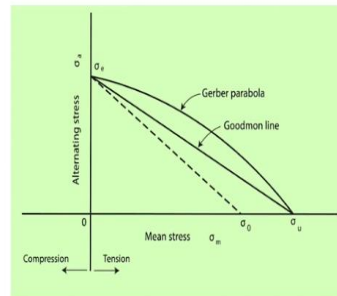
And it is exactly converging at a point where it reaches the material's ultimate tensile strength. So, this is the beauty. So, you will get the convergence at the ultimate strength of the material. And if you want to take the data just below the yield points and then you can stop up to here. And it comes here; this is the point. So, what is that we are trying to understand?

So, you have the range of stress σ_{\min} and σ_{\max} available for the fatigue cycling at the lower mean stress. You have a huge range is available with mean stress is low. As the mean stress is increasing, the range available between the σ_{\max} and σ_{\min} keep on decreasing till the yield point. So, that is one way of looking at it. Or this is another way, nice way of looking at the effect of mean stress.

There are lots of calculations people do by analysing the fatigue data, but I did not want to get into the details in the interest of time, but one primary idea is to look at the effect of mean stress.

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Alternative method of plotting the Goodman diagram



Soderberg relation:

$$\sigma_a = \sigma_e \left[1 - \frac{\sigma_m}{\sigma_y} \right]$$

Goodman relation:

$$\sigma_a = \sigma_e \left[1 - \frac{\sigma_m}{\sigma_u} \right]$$

Gerber relation:

$$\sigma_a = \sigma_e \left[1 - \left(\frac{\sigma_m}{\sigma_u} \right)^x \right]$$

Mechanical Metallurgy, George E. Dieter McGraw-Hill, 1988

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There is another way of plotting the Goodman diagram is this. Instead of that kind of a plot, you will be having an alternating stress, σ_a versus σ_m . And this is a yield strength line; and this is a Goodman line; and this is a Gerber parabola. So, you will have the equations, linear equations argument in terms of alternating stress and endurance limit. And this is Gerber relations, which is given by this. So, these are all the corresponding plots.

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Cumulative damage

- **Over stressing:** Testing at a stress above the fatigue limit for some number of cycles and subsequently running the test at another stress level generally it results in reduction in life.
- **Under stressing:** Testing at a stress level below the fatigue limit for large number of cycles and then testing at a higher stress. It results in increase in fatigue limit due to localized strain hardening at sites of possible crack initiation.

Miner's Rule:

Damage caused by n number of cycles at different stress levels

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \dots + \frac{n_k}{N_k} = 1 \quad \text{or} \quad \sum_{j=1}^{j=k} \frac{n_j}{N_j} = 1$$

Fails when load interaction and sequence effects are considered

Mechanical Metallurgy, George E. Dieter McGraw-Hill, 1988

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So, we will now talk about another important aspect called cumulative damage. In the fracture mechanics, we discussed about overloading. So, here we are talking about over

stressing. Testing at a stress above the fatigue limit or some number of cycles and then subsequently running the test at another stress level, generally it results in a reduction in life. So, you know the fatigue limit of the material, but initially you subject the material to very high loads for particular number of cycles, which is much above the fatigue limit.

And then, you subject them to lower stress cycling. Then the fatigue limit is going to be reduced or the number of life cycles to failure is going to be reduced. Under stressing: Testing at a stress level below the fatigue limit for a large number of cycles and then testing at higher stress; it results in increase in the fatigue limit due to localised strain hardening at sites of possible crack initiation.

So, this is just exactly opposite to what we have done in the over stressing. So, you subject the material to very large number of cycles at much below the fatigue limit, where it forms a localised strain hardening sites ahead of the crack initiation. So, the localised strain hardening delays the crack initiation. It is something to do with the; like we just mentioned about the fatigue limit also, a material which is subjected to stain ageing or work hardening will exhibit sharp fatigue limit; something like that.

So, when the material is subjected to different type of loading and how to accommodate that total life? How to understand the total life? So, that is why this Miner Rule is looked at, it is called Palmgren Miner Rule. And damage caused by n number of cycles under different stress levels are given like this. So, this is $(n_1/N_1) + (n_2/N_2)$. So, something like it goes up to (n_k/N_k) . This total, the fraction of this different load cycles is equal to 1.

So, which is written like this

$$\sum_{j=1}^{j=k} \frac{n_j}{N_j} = 1$$

This is called a Miner's Rule. So, you can look at some of the numericals using these relations; then you will appreciate it better. This is also another way of studying the load interaction and sequence effects. So, you can make the material or subject the material to over stressing for some cycling and then under stressing for some cycling. And then you can do it several times and then see how the material sustains or the material life; so, that way of looking at is.

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Changes in the surface properties



Fatigue limit decreases with following:

- ✓ Decarburization of surfaces of heat treated steels
- ✓ Soft aluminum coating applied to a stronger age-hardenable Al-alloy sheet
- ✓ Electro polishing of steels

Fatigue limit increases with following:

- ✓ Nitriding
- ✓ Carburizing
- ✓ Flame and induction hardening

Effect is more in cases where the stress gradient exists. Crack initiation at the interface between hard and soft phases



The another important point now we are interested is changes in the surface properties. And fatigue limit decreases with the following: Decarburisation of surface of heat treated steels; soft aluminium coating applied to a stronger age-hardenable aluminium alloy sheet; electro polishing of steels. So, all these are going to modify the surface of the material which is going to affect the fatigue limit.

In fact, these are all the factors which are going to decrease the fatigue limit. And what are the factors which will increase the fatigue limit? So, surface treatment like nitriding, carburising, flame and induction hardening; these are the surface treatments which are going to increase the fatigue limit. The effect is more in cases where the stress gradient exists. The crack initiation at the interface between the hard and soft phases; it is something like what we have just looked at the residual stress developed ahead of the crack tip in the fracture mechanics. The same aspects we are discussing here.

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Surface residual stress



- Residual stress arises when the plastic deformation is not uniform in not uniform throughout the cross-section of the part being deformed
- In general, for a situation where part of the cross section is deformed plastically while the rest undergoes elastic deformation, the region which was plastically deformed in tension will have a compressive residual stress after unloading, while the region which was deformed plastically in compression will have tensile residual stress when the external force is removed
- For many purposes residual stresses can be considered identical to the stresses produced by an external force.
- Addition of residual compressive stress, which exists at a point on surface to an externally applied tensile stress on that surface decreases the likelihood of fatigue failure at that point



So, what are the important factors as far as surface residual stresses are concerned, which are connected to fatigue? That is what I want to just emphasis in this point. Residual stress arises when the plastic deformation is not uniform throughout the stress; I mean, cross-section of the bottom being deformed; the part being deformed. There is a typo here; is not uniform throughout the cross-section of the part being deformed.

So, that is something you have to remember. In general, for a situation where part of the cross-section is deformed plastically, while the rest undergoes elastic deformation, the region which was plastically deformed in tension will have a compressive residual stress after unloading, while the region which was deformed plastically in compression will have a tensile residual stress when the external force is removed.

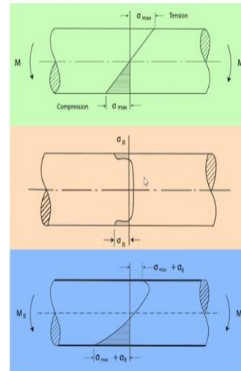
We have pictorially seen this aspect in ahead of the tractive in the fracture mechanics chapter. So, a very nice animation we have shown the same idea, how the residuals; in fact, we showed the residual stress profile ahead of the tractive; but here, we are talking about surface; general definition. For many purposes, residual stresses can be considered identical to the stresses produced by an external force.

You cannot think that residual stresses will be always less than yield stress or ultimate tensile stress or so on. So, there could be instances where residual stress also will have the similar magnitude, very high magnitude as good as yield stress as well as ultimate tensile stress. Addition of residual compressive stress which exists at a point on a surface to an externally

applied tensile stress on that surface decreases the likelihood of fatigue failure at that point. So, this point is very important.

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Surface residual stress



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And we will just look at the residual stress development profile in this schematic. What is shown here? This is a shaft which is being subjected to bending. So, bending moment is given us this. So, the surface is subjected to tension. The bottom layer is subjected to compression. So, this is a maximum compression, σ_{max} . And this is a σ_{max} tension. So, that is a profile.

So, this is a shaft or bar which has not subjected to any prior deformation or the residual stresses were not developed prior to the deformation. Now, we will do typically a surface treatment on this same shaft, typically a sharpening type of a surface treatment which can produce a surface residual stress with a profile like this, σ_R . So, what will happen after that?

The same bending, if you do after this surface treatment, then what is that we have seen? What is that now we are seeing? Now, the $\sigma_{max} + \sigma_R$; what you are seeing is $\sigma_{max} + \sigma_R$ is this; and that is a tension. And this is $\sigma_{max} + \sigma_R$ is a compression. So, this reduction in the magnitude of the σ_{max} in tension is because of the compensating residual stress already present in the surface.

So, that is how you have to understand. So, the surface compressive residual stress significantly reduces the σ_{max} tension on the surface. So, that way, it will be less effective in initiating the crack or propagating the crack on the surface. So, that is the idea.

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Cyclic deformation in ductile solids

Fatigue of Materials, S. Suresh, Cambridge University Press, 2012

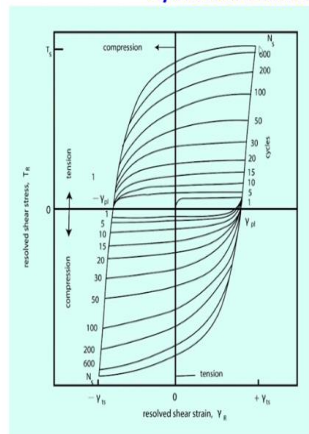
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Now, whatever we have just looked at so far, which is a stress life approach; people also refer that as a high cycle fatigue. Now, we will look at the cyclic deformation in ductile solids, specifically to the ductile solids because most of the fatigue phenomenon now we are going to see, it is basically stress life approach in the previous case; now it is a strain life approach. We are going to control the strain of the applied load, through the applied load. Strain is being controlled here. And then, look at the life. For earlier, it was a stress controlled fatigue; this is a strain controlled fatigue.

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Cyclic strain hardening in single crystals



Fatigue of Materials, S. Suresh, Cambridge University Press, 2012

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- Schematic illustration of typical variations in resolved shear stress, as function of the resolved shear strain for an FCC single crystal oriented for single slip and fatigued at a fixed value of resolved plastic shear strain amplitude.
- After N_f cycles the hysteresis loops saturate with a peak tensile stress of at a total applied shear strain



So, cyclic strain hardening in single crystal: So, the initial description like what we had in the tension; we looked at the single crystal behaviour initially. Similarly, in cyclic deformation also, we will first describe the single crystal behaviour; then we will compare that with the

polycrystal materials. So, what you are seeing here is very interesting plot. So, this is a four quadrant access you have.

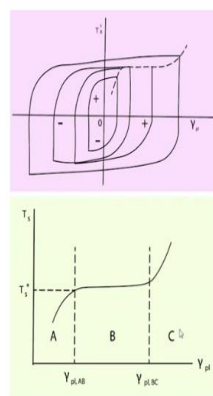
This side is a tension; this is a compression; and this is tension compression. And the cyclic load starts from here; one; and then it goes like this, one; and reaches five; come back, five; reaches ten; come back, ten. So, like that, it will keep on going like this. So, what you are seeing is, as the number of cycle increases and the material is undergoing strain hardening. So, the applied stress keeps on increasing.

It goes up to n number of cycles. So, that is called N_s . And it goes to kind of saturation. That is why it is called N_s . So, after some time, it saturates and then stays in the same loop for a long time. So, this is a γ_{pl} ; that is plastic strain, completely a plastic strain; the positive; and this is negative. So, what we are now talking about is a strain controlled fatigue, especially plastic strain; so, resolved shear strain, γ_R .

So, this is a schematic illustration of typical variations in resolved shear stress. As a function of resolved shear strain for an FCC single crystal oriented for single slip and fatigued at a fixed value of resolved plastic shear strain amplitude. So, we are talking about this amplitude. After N_s cycles, the hysteresis loop saturates with the peak tensile stress. So, what is peak tensile stress here? This is the peak tensile stress. Wherever it saturates, this is called peak tensile stress of a total applied shear strain.

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Cyclic saturation in single crystals



(a) Hysteresis loops with resolved shear stress at saturation plotted against the resolved plastic shear strain

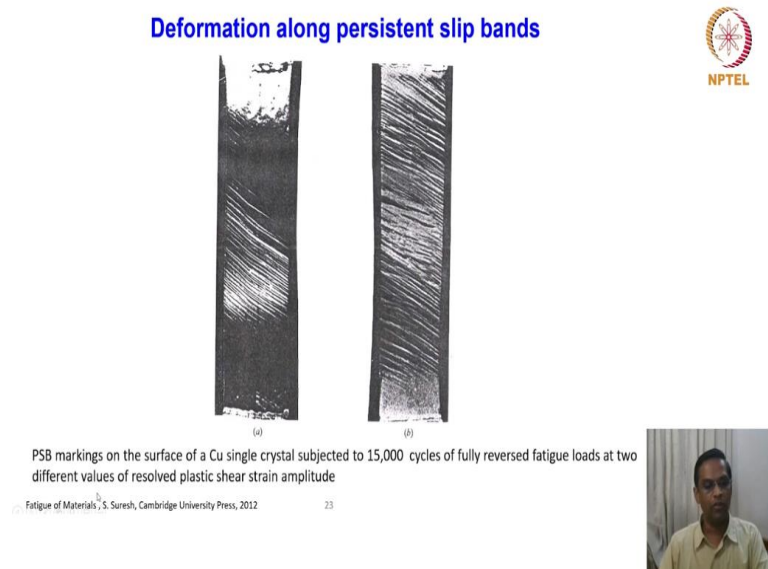
(b) A schematic diagram showing different regimes of the saturation stress-strain curve



So, what is shown here? That is for the complete data description. Now, what we are trying to do is, this hysteresis loops with a resolved shear stress at a saturation plotted against the resolved shear strength. So, the saturated, the hysteresis loops are plotted. So, you get to see that all these corners of these hysteresis loops are connected. So, this is nothing but a stress-strain curve; so, which is what taken out and then plotted here for the clarity; a schematic diagram showing the different regimes of the saturation stress-strain curve.

So, this is a saturated stress-strain curve. That is why it is called T s star. So, which is divided into three regions, A, B, C. And then, people look at like some other; like we have look at a peak curve also like that; the different stages of a peak curve. Here also, the cyclic stress-strain curve has been divided into three regions and looked at what happens to the microstructure and so on.

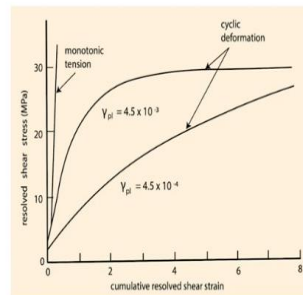
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So, these are all some of the slip band which is spread across the complete gauge. This is also called persisted slip band, because these slip bands not the surface slip bands which is a through thickness slip bands. So, it will persist even if you polish the surface; that is why it is called persisted slip bands. So, these markings on surface of the copper single crystal subjected to 15,000 cycles of fully reversed fatigue loads at two different values of resolved plastic shear strain amplitude.

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Cyclic strain hardening in single crystals



Resolved shear stress plotted against cumulative resolved shear strain for copper single crystals subjected to tension and fatigue



So, that now comes the important point. This is a resolved shear stress versus resolved shear strain plot. So, you have a monotonic tension, and this is cyclic information at two different, shear strain level. So, what you are seeing is monotonic tension; the amount of plastic strain which is accommodated is, it is completely very small as compared to cyclic deformation. This is something you have to appreciate; very important and fundamental in nature.

In monotonic tension, the accommodation of the plastic strain is negligibly small as compared to the cyclic deformation; it is shown at the two different plastic strain level, but you can see that, the amount of shear strain it is accommodated during cyclic deformation is huge as compared to monotonic tension; this is what important. This is again for a data for a copper single crystal subjected to tension and fatigue.

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Cyclic strain hardening in single crystals



The mean spacing of the edge dislocations in the veins, can be related to the trapping distance, d_{trap}

$$d_{trap} \leq \frac{\mu b}{8\pi(1-\nu)\tau_s^*}$$

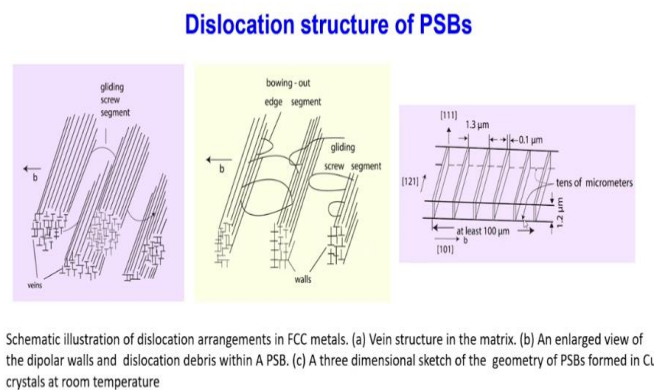
- A transmission electron micrograph of matrix veins structure in single crystal of Cu fatigued to saturation at 77.4 K.
- The plane of figure is primary glide plane and Burgers vector is along $[101]$



So, what kind of microstructure it develops when you accommodate such a large plastic strain in the material? These are all the dislocation bundles. This is a TEM micrograph matrix veins structure. So, the description is veins structure, but actually these are all dislocation bundles in a copper fatigue to saturation at 77.4K. The planar figure is primary glide plane and the Burgers vector is along [101].

So, this is some analysis, dislocation analysis given. So, the bundles are like veins; it runs through the material; that is why it is called vein structure. The mean spacing of the edge dislocation in the veins can be related to the trapping distance. There is some quantification about the dislocation activity in this type of microstructure. Please remember, this is a microstructure of the single crystal of copper fatigue to saturation.

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And there is some schematic about this PSBs. What are PSBs? And then, in order to give you a visual perspective, some models are given. So, you know what is edge dislocation. And these are all bundles of edge dislocations. And gliding screw segments which is in between; you can see that. And there are dislocations bowing out. And with edge segments, screw segment; and what is dislocation wall?

They are all, people try to characterise the deformed structures by giving several different nomenclatures. And people have also looked at the kind of a systematic structure and try to understand them. So, this is nothing but a schematic illustration of dislocation arrangements in FCC metals; please understand. This is also studied only in FCC metals. And vein structure in matrix and enlarged view of dipolar walls and dislocation debris in PSB. A 3-

dimensional sketch of the geometry of PSBs formed in copper crystals at room temperature. This is how the persisted slip bands are envisaged in an FCC metals.

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Cyclic saturation in single crystals



- Macroscopically the saturation of the hysteresis loops corresponds to a state where an equilibrium is achieved between bundles of edge dislocations and the surrounding matrix piled by screw dislocations
- Under these conditions, fine slip markings are observed on the free surfaces
- This slip concentration process is nucleated at the beginning of region B in the cyclic stress-strain curve and it is intensified as the applied plastic strain is increased
- Earlier investigations (Humphrey (1903 and Gough (1933)) showed that fatigue crack initiated along these bands where the slip was intense.
- These slip lines are termed "persisted slip bands" (PSBs) by Thomson, Wadsworth and Louat (1956) who found these in Cu and Ni.
- Why the name?



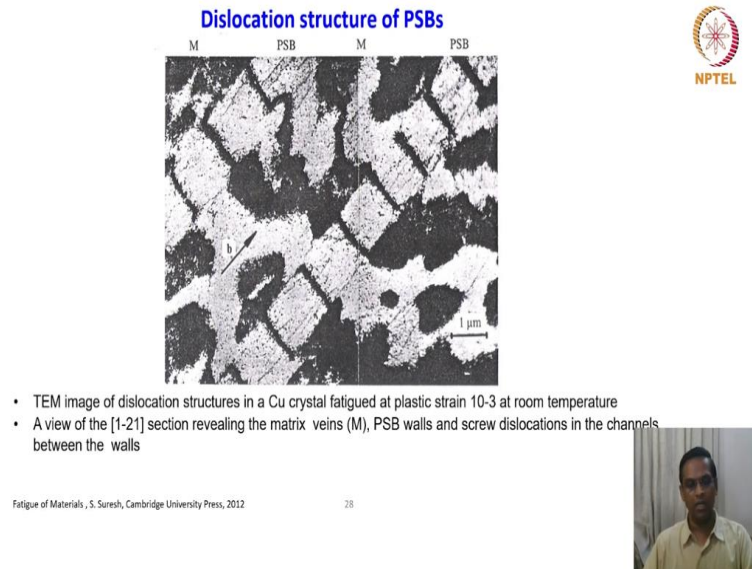
Macroscopically the saturation of the hysteresis loops corresponds to a state where an equilibrium is achieved between bundles of edge dislocations and surrounding matrix piled by screw dislocations. So, why a single crystal reaches a saturated stress or saturated hysteresis loop? So, one of the explanation is that there is an equilibrium is reached between the bundles of edge dislocation and surrounding matrix piled up by screw dislocation.

Under these conditions, fine slip markings are observed on the free surfaces. This slip concentration process is nucleated at the beginning of region B. We are talking about the cyclic stress-strain curve where we just showed A, B, C. So, people are trying to explain that. The slip concentration processes is nucleated at the beginning of region B in the cyclic stress-strain curve and it is intensified as the applied plastic strain is increased.

Earlier investigations by Humphrey and Gough showed that fatigue crack initiated along this bands where the slip was intense. So, wherever you see the very intense slip lines, that is where the crack initiation sites. That is how people are finding it out. These slip lines are termed persisted slip bands by Thomson, Wadsworth and Louat who found these in copper and nickel.

And why the name? Because these bands will be spread through thickness completely. So, even if you remove by polishing, you will keep seeing this bands; that is why it is called the persisted slip band.

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And you see the typical dislocation structures. And this is again a TEM image in a copper crystal fatigued at plastic strain of 10^{-3} at a room temperature. A view of $[1\bar{2}1]$ section revealing the matrix veins, PSB walls and screw dislocation in the channels between the walls. So, this is a PSB. You can see that. And there are veins; you can see; matrix veins.

So, what you have to understand is the fatigue microstructures are unique and they are not similar to a microstructure deformed under tension or compression. So, this is unique microstructure.

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Dislocation structure of PSBs



- The dislocation found within PSBs is considerably different from that of the matrix
- The matrix contains, about 50% by volume, vein-like structures consisting of dense arrays of edge dislocations.
- On the other hand PSB structure is generated due to the mutual blocking of glide dislocations and the formation of parallel wall (ladder) structures which occupy about 10% by volume of the PSBs (Laufer & Roberts, 1966, Woods, 1973, Mughrabi, 1980)

Fatigue of Materials, S. Suresh, Cambridge University Press, 2012

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The dislocation found within the PSB is considerably different from the dot matrix. The matrix contains about 50% by volume, vein-like structures consisting of dense arrays of edge dislocations. On the other hand, PSB structure is generated due to the mutual blocking of glide dislocations and the formation of parallel wall, the ladder structures which occupy about 10% by volume of the PSBs. This is what I have just shown in the previous slide. Basically, this is a description of what we have already seen.

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Formation of labyrinth and cell structures



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Then formation of labyrinth and the cell structures. So, in the next lecture, we will talk about all this further dislocation cell structure and formation and so on, and try to lead this. And we will also look at how this single crystal deformation can be related to or extended to polycrystal deformation in the next class. I will stop here. Thank you.