Mechanical Behaviour of Materials
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Lecture - 06
Strength of Materials – a short overview part – IV

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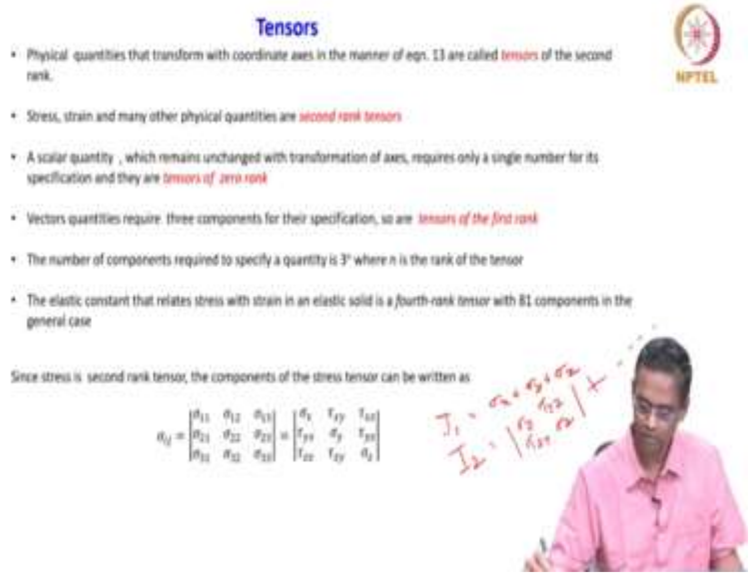

Hello, I am Professor Sankaran in the department of metallurgical and materials engineering. I just wanted just a very basic point which you almost might have already studied just to recall these physical quantities that transform with the coordinate axis in a manner of equation what are we have just seen one coordinate to other are called tensors of second rank. The physical quantities what are the physical quantities we measured? We looked at stress that is the physical quantities.

So, the physical quantities such as stress, strain that transform with the coordinate axis in a manner like what we have called tensors of the second rank. So, the stress, strain and many other physical quantities are the second rank tensors are important. So, keep that in mind. The one good thing is about this is once you see that stress can be treated as a second rank tensor, then there is a lot more avenue to do mathematical manipulation, that is idea. All the tensor quantities, all tensor manipulation you can bring in and then try to see how much clarity we can bring in.

Because such a complex quantity, it is not like a vector which requires only 3 components to describe, but, once it moves to the tensor form, it is a little more complicated. So, a scalar quantity which remains unchanged with the transformation of axis requires only a single number for its specification and they are tensor of zero rank. So, any scalar quantity is not going to change they are going to remain unchanged for any transformation of coordinates. So, they are tensors of zero rank.

Just I am mentioned vector quantities request only three competences for their specifications. So, they are tensors of a first rank. So, a vector is tensors of first rank and the stresses you know tensors of second rank, this is important. So, the number of components required to specify a quantity is3^n, n is the rank of the tensor and we are going to see in few classes the elastic constant that relates to stress with the strain in an elastic solid is a fourth rank tensor with 81 components in the general case.

So, we use elastic constants Young's modulus and some things for less. So, that relates the stress and strain for a pure elastic material as it is a fourth rank tensor that means, 3^4 , which is 81 components are required to specify the state of stress do not describe the state of stress whatever. So, but we will be seeing this when we go to a specific cubic system, how this elastic constants are evaluated.

So, stress is a second rank tensor the components of the stress tensor can be written as like this. So, we can write σ_{ij} is a tensorial representation is equal to mod of σ_{11}, σ_{12}, σ_{13}, and σ_{21}, σ_{22}, σ_{23}, σ_{31}, σ_{32} and σ_{33} or it can be simply a σ_x, $\tau_{xy,}$, τ_{xz} and so on. So, once it is in the form of the matrix then it is easy for us to relate this with some other properties as well.

For example, the first invariant of this matrix is I_1 we were talking about 3 invariants. So, the first invariant is I_1 is what? I_1 will be $\sigma_x + \sigma_y + \sigma_z$. So, I_2, is what is summation of principal minors for example, if you take what is principal minors? So, if you take this so it will be σ_y, τ_{yz}, τ_{zy}, σ_z plus so on it will keep on going like that. So, I_3 will be the solution of the entire determinant.

So, that will be there, these are the quantities we can relate we have already seen, that is why I am not going to repeat it I already showed what the invariants in the 2, 3 slides before. So, all that I want to emphasize here is stress is tensor of rank 2 and it can be written like this. So, this is on some of the basic idea, I want to just flash it and then move around.

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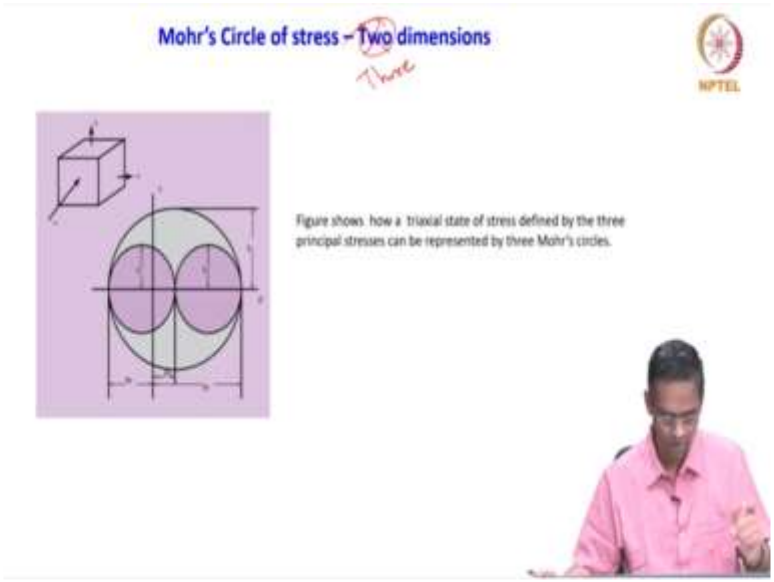

So, again coming back to this Mohr's circle of two dimensions we have seen, but this is going to be three dimensions now, this is not two, this is three. So, the Mohr's circle also can be visualized in three dimensions. So, what you are seeing in this Mohr's circle is how a triaxial state of stress defined by the three principal stresses can be represented by three Mohr's circles. So, what is that we are seeing? So, this is the member which is subjected to two triaxial state of stress.

That means, two normal stress of equal magnitude, the third one is in a compression mode, these two are in tension mode. So, three dimensional state of stress is plotted in Mohr's circles. So, what we are seeing here is you have σ_1 that is the highest principal stress and σ_2 is here and σ_3 is completely compression, σ_3 is compression. So, σ_2 is tension, σ_1 is tension and talking about shear stress that τ_2 is highest and this is a τ_1 and this is τ_3. So, τ_3 and τ_1 and for any combination of stresses can be brought into; this simple representation, a graphical representation that is the beauty of this representing the Mohr's circle.

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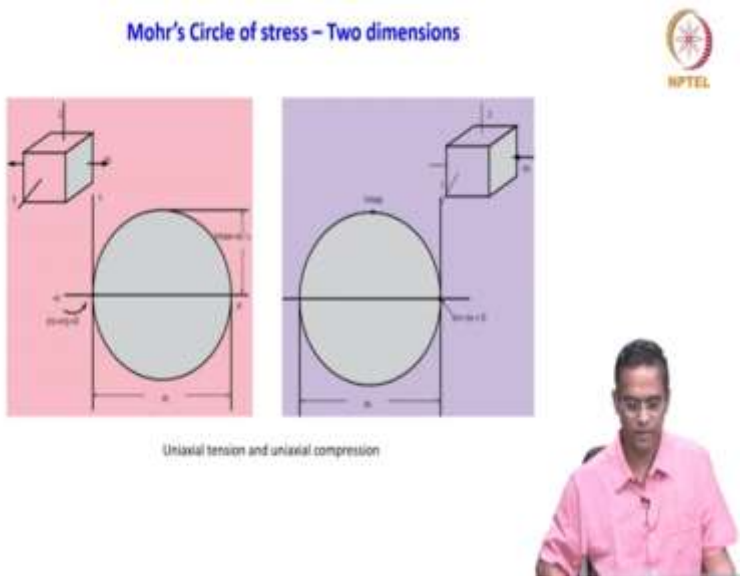

So, that is how you can just use that makes use of them. So, to take some few examples, this uniaxial tension, and you see that there is only one axis where σ_1 is acting. So, this is how it is represented. So, it is very simple if you have σ_1. So, the $\sigma_1$is the maximum and this is a $\sigma_{1/2}$ that is a maximum shear stress and σ_2 and σ_3 will be 0, if it is a uniaxial tension. And the same opposite uniaxial compression, so, it comes to a complete negative axis σ_3. So, and then this is a max and σ_1 and $\sigma_2 = 0$. So, this is this way of representing.

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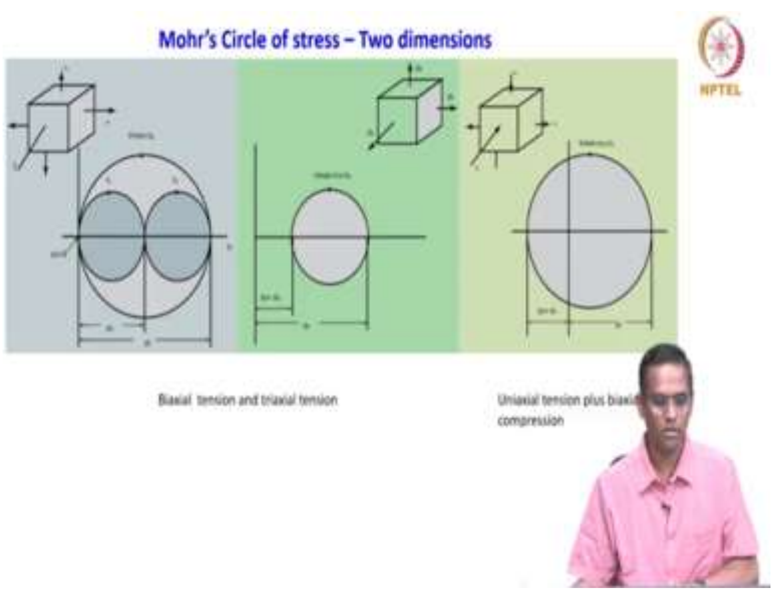

And you can now represent biaxial tension and tri axial tension. So, biaxial tension is, this is an element of equal magnitude σ_1 and σ_2. So, you have this σ_3 is 0 here. So σ, you can see this is the maximum and then this is σ_2 in tension and this is $\tau_{3\ maximum}$ and $\tau_{1\ maximum}$ and this is

$\tau_{2\ maximum}$ shear stress maximum is here, this is τ_1, τ_3. So, what is that, even though it is σ_3 is 0, but $\sigma\ \tau_3$ is not 0.

So, this is how we have to remember. Similarly, for triaxial tension, so this is an element where it is subjected to triaxial tension. So, σ_2 and σ_3 they are equal in magnitude. So, σ_1 is the highest here and this is σ_2 and σ_3 and this is a τ_{max}. And uniaxial tension plus biaxial compression, so uniaxial tension this direction biaxial compression in 2 orthogonal directions, so, how it is represented? So, you see that σ_1 again the highest and σ_2 and σ_3 are equal but in compression.

So, you have τ_{max} is here which is $\sigma\ \tau_2 = \tau_3 = \tau_{max}$. So, what you can appreciate by looking at all these examples is any combination of stress can be visualized you can just resolve this you know normal stress versus shear stress and also as a function of any angle orientation right, you can just easily represent this that is why it is useful we can just look at the it basically representation. But instead of looking at all these complex equations, if you can just put this in one graph, then it is easy to visualize. So, that is an advantage that is about the Mohr's circle.

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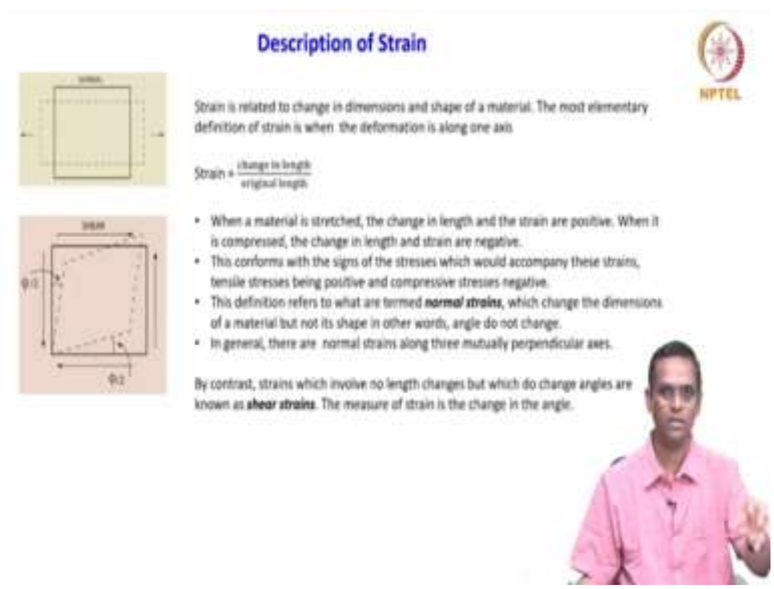

And so far we have just discussed about the description of a stress. Now, I want to move on to a description of strain. So, again, it will be more detailed, but you will realize that as we move along in the course, these are all very important, but people normally tend to ignore these small details, so it is better to spend some time on this and then have a complete grip but then get into the subject.

So, like stress, strain is related to change in dimensions and shape of material, the most elementary definition of strain is when the deformation is along one axis. So, like this you have a normal component and which is being pulled in both; directions or one direction in one axis in both directions. And the strain is given a very simple definition strain is equal to change in length by original length in the beginning also we have just returned average strain.

We talked about average strain and here we are talking about a strain at a description of a strain at a point, we are also we are making a very simple introduction we are not giving any specific coordinates at a point but just a general description then we will get into the details. So, when a material is stretched that change in length and thus the strain are positive when it is compressed the change in length and the strain are negative.

So, you can do when you say change in length, it could be uniaxial tension or compression both operation involve the change in the shape. So, it could be a positive strain or it could be a negative strain. So, we can this confirms with the signs of stresses, which would accompany the strains tensile stresses being positive and compressive stresses are negative, it is a conventional way of putting it and as I just mentioned this definition refers what are called normal strains.

Which change that dimensions of a material but not its shape, in other words, angle they do not change very important. So, when you say normal strain, we are not talking about angle. So, it changes the dimensions of a material but not its shape. On the other hand, there are normal strains along three mutually perpendicular axes. So, similar to stress, we also have the normal strain along the mutually perpendicular axis.

The moment you bring these have a change in this what is that changing here it is again a shear stress and rotation. So, the shear stress brings what change into this basic member. So, it gives an angular change $\phi/2$ here and here with respect to this line and with respect to this line there is an angle change. So, when there is by contrast strain which involve no length changes, but which do change angles are known as shear strains.

So, normal strain and shear strain, these are all very certain definitions, but it is useful if you pay some attention what are the details involved because these things you can relate later on you know for example, if you say deformation. So, what causes deformation what type of is this just a simple link change or you know, whether it is a slip we have used lot of terms. So, these small details only bring in clarity later. So, that is why we are paying attention to this.

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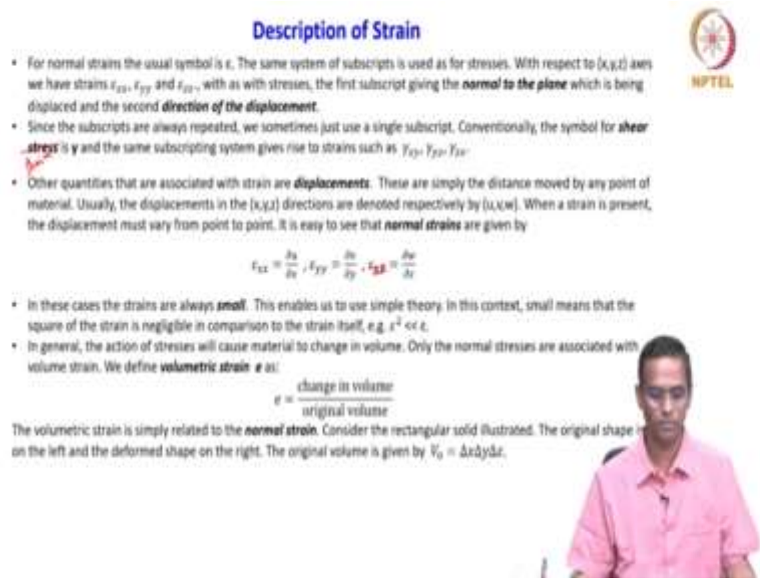

And for normal strains, the usual symbol is ε. The same system of subscripts is used as for stresses with respect to x, y, z axis we have strain ε_{xx}, ε_{yy} and ε_{zz} with as with the stresses the first subscript giving the normal to the plane which is being displaced and the second subscript indicates the direction of displacement similar to what we have described in the stresses strain also have a 2 subscripts to describe.

And since the subscripts are always repeated, we sometimes just use single subscript conventionally the symbol for shear stress is γ and the same is subscripting system gives rise to strain such as γ_{xy}, γ_{yz}, γ_{zx}. So, now we are looking at shear stress not shear stress it is a strain we are discussing about strain sorry this strain there was a type of shear strain here other quantities that are associated with the strain are displacements.

So, what are displacements? So, we need some time to demonstrate this. These are simply the distance mode by any point of the material usually the displacements in the x, y, z directions are denoted respectively by u, v, w. When a strain is present the displacement must vary from point to point it is easy to see that normal strains are given by $\varepsilon_{xx} = \partial_u / \partial_x$, $\varepsilon_{yy} = \partial_v / \partial_y$, ε_{xx} is going to ∂_w / ∂_z this is again a type of here ε is that is zz.

So, the displacements are for normal strains are given like this. In this case, the strains are always small. This enables us to use simple theory in this context small means that the square of the strain is negligible in comparison to that strain itself. For example, ε^2 is far less than ε. So, that is the idea. In general, the action of stresses will cause material to change the volume only the normal stresses are associated with the volume strain,

We define volumetric strain e as change in volume divided by original volume. So, we are bringing now, little more details as compared to a normal strain or we will later on we will also see that, you know engineering strain, we will come to that but then even before going into other details, normal strains, now we are talking about volumetric strain. The volumetric strain is simply related to the normal strain, considered the rectangular solid illustrated the original shapes on the left inside and the deformed shape is the right side.
So, the volume of the original volume is given by the V naught, Δ x Δ y and Δ z. So, this descriptions I will go with some little more illustrations I do not want to mean overload today, I will just stop here, we will come out with some more descriptors and then we will continue the discussion on describing the strain in much more detail with a little simple mathematical formulas and we will also bring some clarity in nomenclatures.

For example, normal strain we are talking about our average strain that what is that normal strain then what is the shear strain we know the now volumetric strain we are talking about. So, we will bring in a lot more clarity once you bring some geometry and then describe in terms of some basic equations, then that clarity also will come because please remember ultimately we will be experimentally measuring only the strain.

So, I said that stress cannot be physically measured. So, it is better to bring in little more clarity in this domain before we get into other relations. So, that is the purpose of this introduction and fundamentals and we are spending too much time on this because it will give a lot more clarity and confidence as you go along. So, we will continue in the next class. Bye.