Mechanical Behaviour of Materials
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Module - 11
Lecture - 59
Fracture Mechanics - XI

Hello, I am Professor S. Sankaran in the department of Metallurgical and Materials Engineering.

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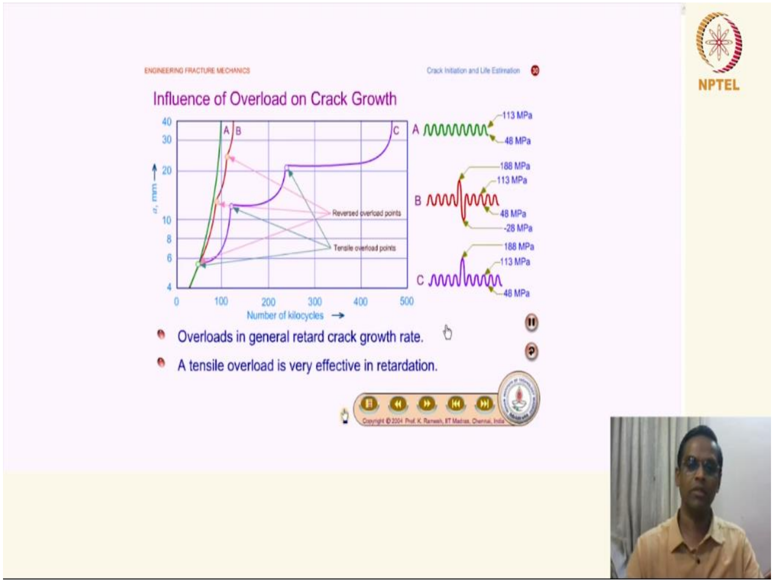

Hello everyone, welcome to this lecture. And today we will look at the influence of overload on the fatigue crack growth. In the last class, we looked at the different crack growths or mechanisms starting from plasticity induced crack closure or plastic zone size induced crack closure, and then followed by the oxide induced crack closure, the toughness induced crack closure, the second face particles act as a bridge between these two crack surfaces, which we have identified that know, during more two kind of failure, how the wedging takes place and how the crack growth is retarded or crack closure is facilitated and so on.

And one of the important idea of which we were discussing yesterday is the plastic zone size develop ahead of the crack tip, affect significantly the crack growth rate. That is something which you are trying to understand. So, today we will try to study that aspect, that is the influence of overload on that crack growth. And what you are seeing here is, in this plot is crack length versus number of kilocycles.

And this is a normal fatigue cycles with the amplitude shown here, the maximum and minimum load and is shown in the crack length A. And what is shown here in the B is, yes, a fatigue cycling is done with this amplitude and suddenly the amplitude is increased in the positive and negative direction; and then it back to the normal cyclic. So, this kind of an overload also seem to have retardation; small retardation you are able to witness because of this overloading.

And what is shown in the third curve is only the amplitude of the positive load is increasing potentially and there is no second and the negative side. That no compressions only in a tension side. So, that creates a significant retardation of the crack growth. That is a tensile overload points; you can see that. So, this is fully a reversed overload points, the previous case. The tensile overload points significantly retard the crack growth. Overlords in general retard crack growth rate. A tensile overload is very effective in retardation. So, this is something which you are seeing.

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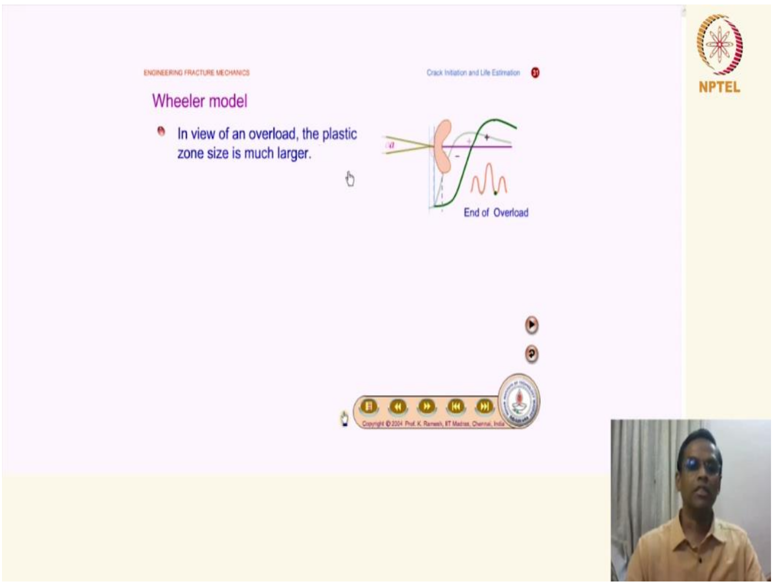

So, what Wheeler has observed is this. It is a Wheeler model. In view of overload, the plastic zone size is much larger. So, this is what shown in this schematic animation.

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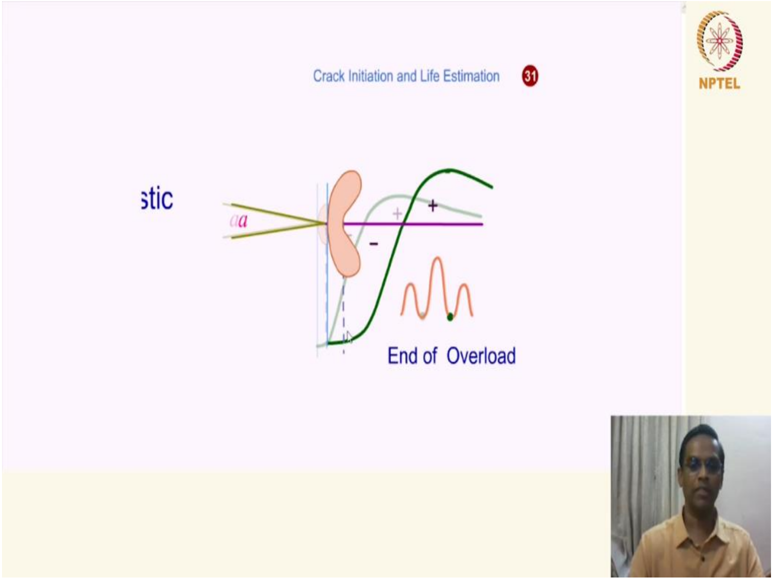

The original crack length is a; and then, you can see that this plastic zone size is small. So, the incremental crack growth takes place within this small plastic zone region. And that is after the regular amplitude cycle here. Once the amplitude of the cycle is increased at the end of these overload cycle, then you can see that, (()) (04:15) in the plastic zone size ahead of the crack tip.

And then, what happens is, this crack has to now go through the larger plastic zone's region and which will be retarded. So, that is the basic physics behind the retardation. So, you can also see that the residual stress developed ahead of this crack tip also significantly higher as compared to the previous case. That is what is shown here. So, we will complete this animation now. So, small plastic zone size, there is now bigger plastic zone size. And now we will see how it changes to.

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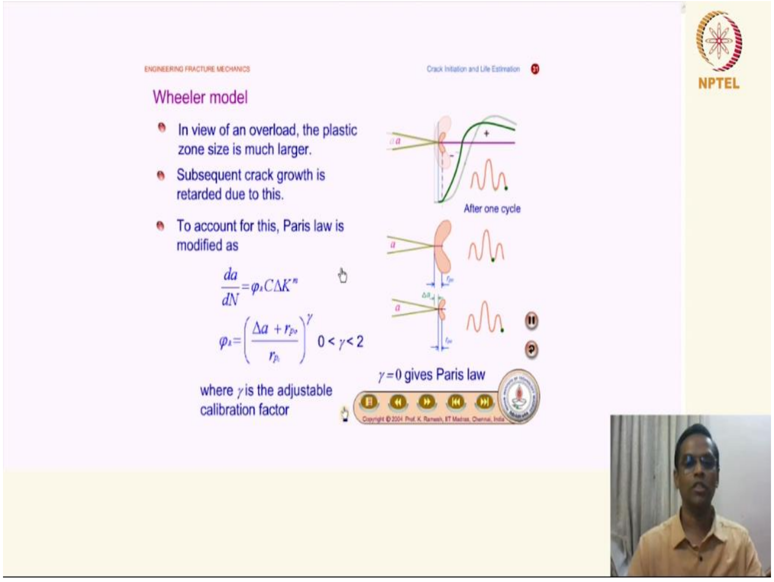

It is subsequently; you can see, it has gone back to its original plastic zone size, but which is interestingly within the larger plastic zone size. So, that is what it is. So, the crack retardation force will be much small in the small plastic zone size. That is the idea. So, subsequent crack growth is retarded due to this. And to account for this, Paris law is modified as

$$\frac{da}{dN} = \varphi_R C \Delta K^m$$

Where

$$\varphi_R = \left(\frac{\Delta a + r_{pc}}{r_{p0}}\right)^{\gamma}$$

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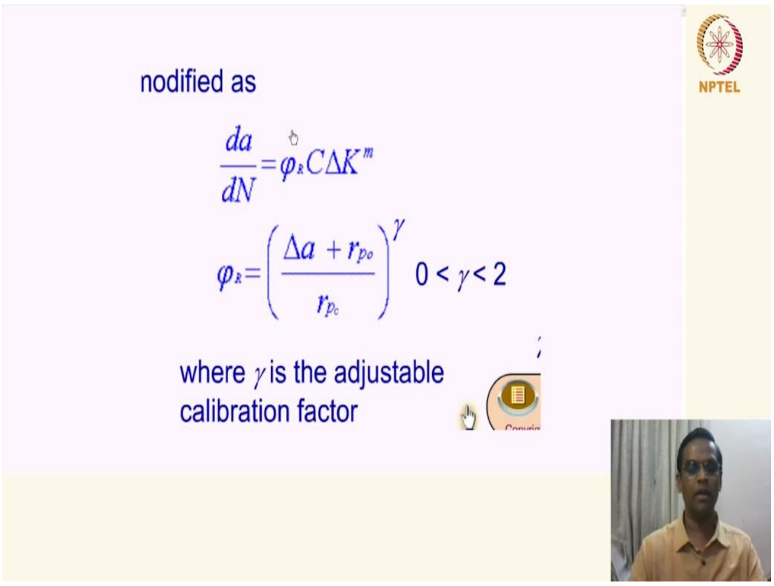

This is a typographic error. This is a plastic zone size r p per cycle, pc; divided by r_{p0}; that is plastic zone size of the overload; so, which is an overload. r_{p0} should be here. So, this factor will be less than 1; so, the crack growth rate is retarded. So, that is how we should read this equation. So, γ is the adjustable calibration factor which is less than and between 0 to 2. And when γ is equal to zero, then it becomes a Paris law. So, what you are seeing here is again the; at the end of the overload, the plastic zone size is the much bigger. And in end of the normal cycle amplitude, this is much smaller.

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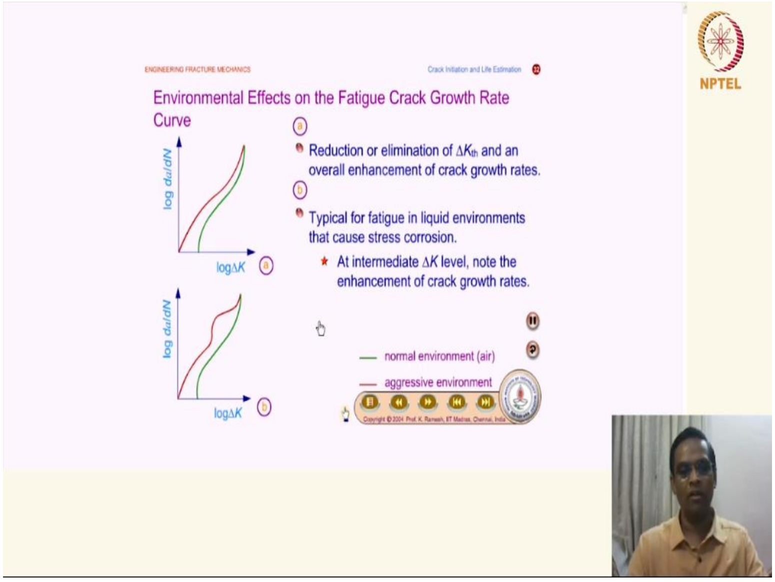

So, now, we will turn our attention to the environmental effects on the fatigue crack growth rate curve. And this is da/dN versus ΔK. And what you are seeing is the reduction or elimination of the ΔK threshold and an overall enhancement of the crack growth rates. So, what you see here is, this point is ΔK threshold. It has been shifted to the origin. That means, there is no ΔK threshold exists, because the aggressive environment has eliminated the threshold.

That means, the crack will start growing instantaneously as compared to the normal environment. That is how we should look at it. And this is for the one situation. And the typical for the fatigue in the liquid environments; and that causes the stress corrosion. So, this is also your characteristic of the liquid environment and stress corrosion. And there is another situation also, you can see that.

It is not just ΔK threshold is 0; and also we can see the sudden increase in the fatigue crack growth rate. Rate is also significantly amplified. And then you can see that effect here also. At intermediate ΔK level, note the enhancement of the crack growth rates. This is what is shown here. So, this is a normal environment and then aggressive environment. So, these are very important effect of environment on the fatigue crack growth rates.

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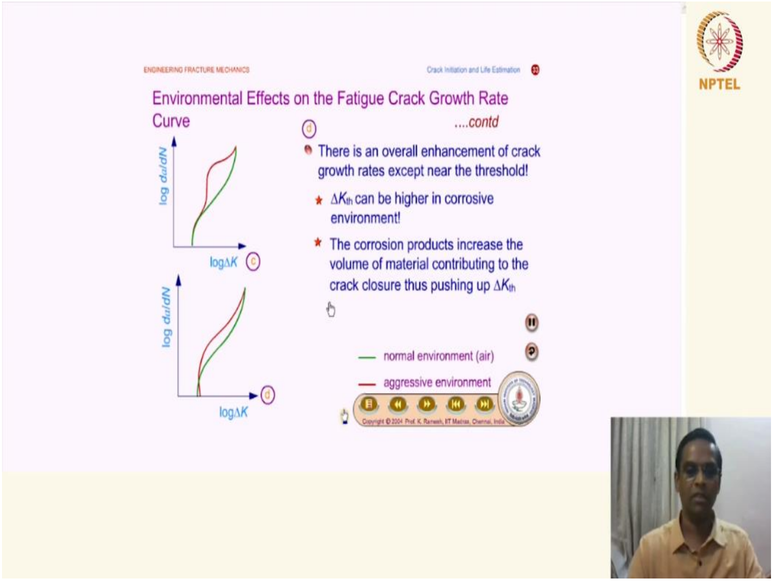

So, what you are seeing is another very typical crack growth rate increases like this. And this could be because of one of the environmental effects such as gaseous hydrogen. The environment has to be gaseous hydrogen. And there is an overall enhancement of the crack growth rates except near the threshold. And ΔK_{th} can be higher in the corrosive environment.

And the corrosion product increases the volume of the material contributing to the crack closure, thus pushing up the ΔK. So, you can see that sometimes the corrosion products also help in pushing up the ΔK here. Here it has a beneficial effect by the corrosion products at the crack tip. So, this kind of positive action also sometimes visualised.

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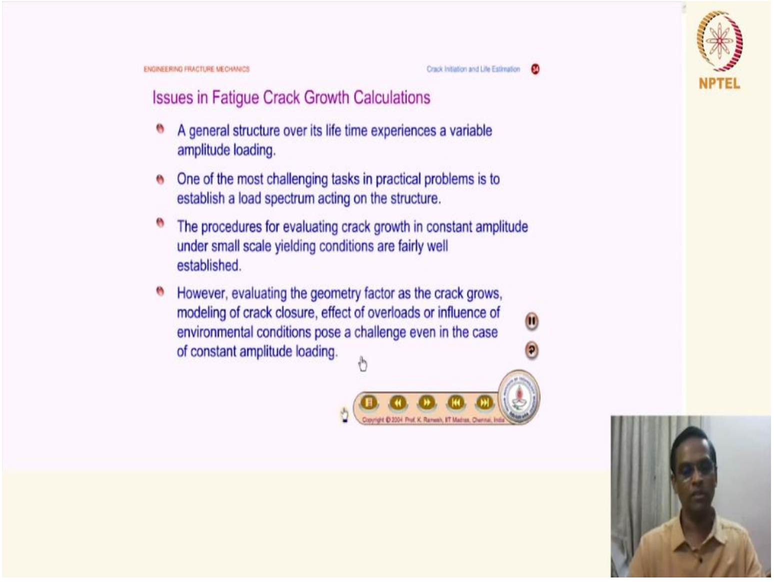

So, now look at some of the primary issues in fatigue crack growth calculations. In general structure over its lifetime experiences a variable amplitude loading. One of the most challenging tasks in practical problems is to establish a load spectrum acting on the structure. And the procedures for evaluating the crack growth in a constant amplitude under small scale yielding conditions are fairly well established.

However, evaluating the geometry factor as the crack grows, modeling of the crack closure, effect of overloads, influence of environmental conditions pose a challenge even in the case of constant amplitude load. So, these are all some of the issues with the fatigue crack growth.

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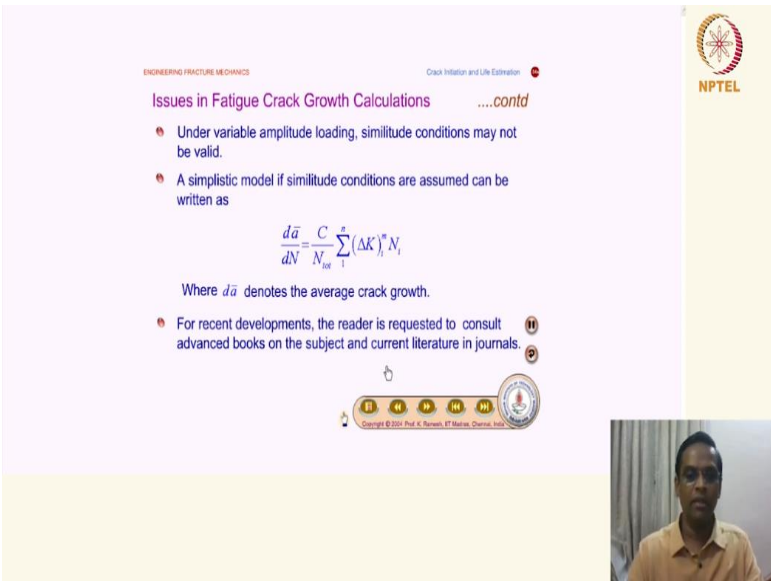

And another issue is, under the variable amplitude loading, similitude conditions may not be valid. And a simplistic model, if the similitude conditions are assumed can be written as; this is some modification for different situations. So, it becomes like a cumulative effect. So, some model has been developed. And we will not get into the details; just to give you an idea of what happens in a different kinds of situations, how the fatigue crack growth is calculated. Just I want to give the perspective.

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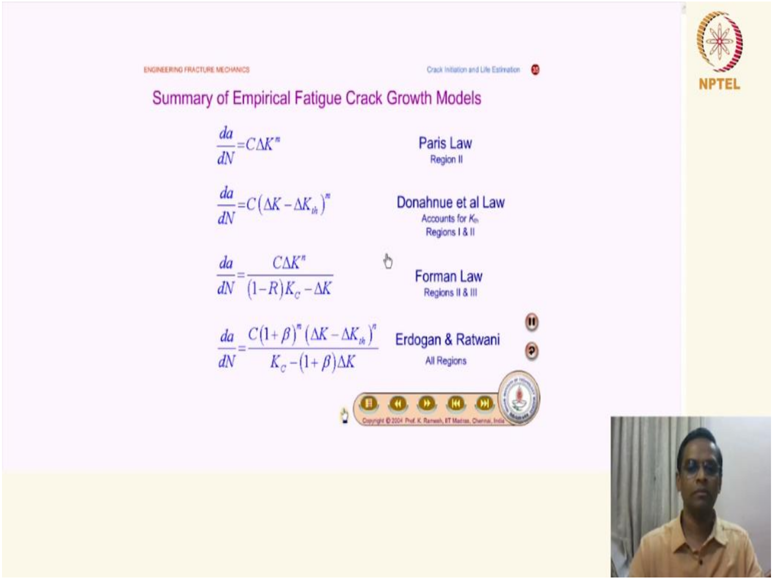

And if you look at the summary of all the empirical fatigue growth models tabulated here, I have just flashed it in the beginning also, just to keep it as a summary, we will stop it here.

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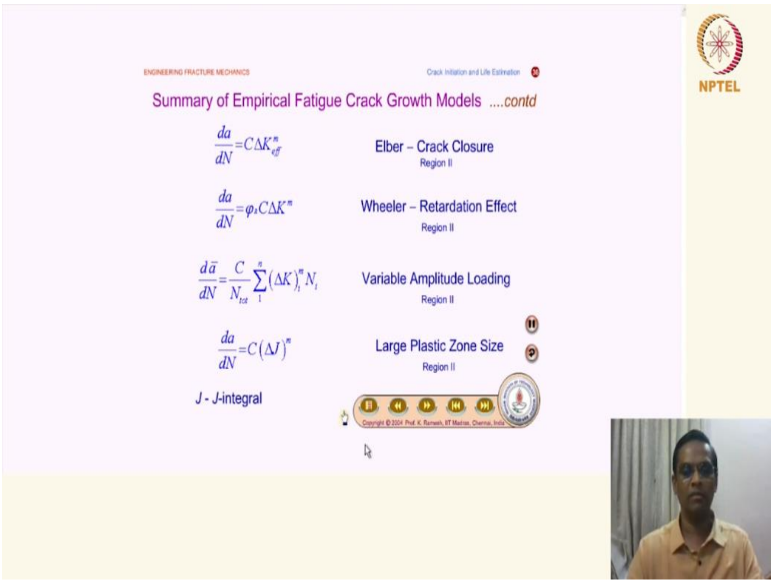

And this is for the crack closure models, where K effective has been incorporated into this Paris law equation. And this is proposed by Elber; this is proposed by Wheeler; and then or a variable amplitude loading; and there is a last large plastic zone and instead of ΔK, ΔJ has been introduced, which we will discuss in the next few slides; J-integral.

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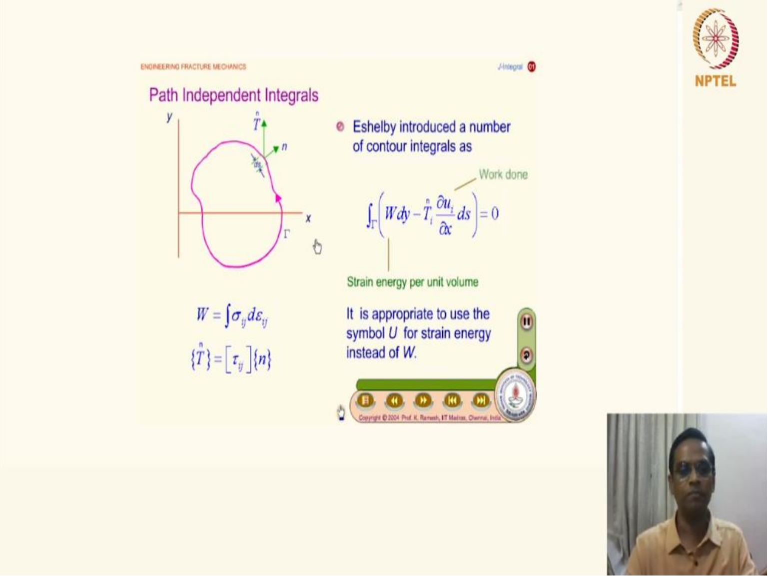

We will now turn our attention to the J-integral; one last factor toughness parameters in the table. And before we get into the J-integral, there we need to know something about path independent integrals. And Eshelby introduced a number of contour integrals. And what is contour integral? It is schematically shown in red. This is a contour, basically a symbol of a line integral.

So, what you are seeing here is

$$\int_{\Gamma}\left(Wdy - T_i^n \frac{\partial u_i}{\partial x} dS\right) = 0$$

So, the parameter what we are seeing here is, we have already seen in the theory of elasticity, but they have been recasted in a different form; that we have to understand first. So, Wdy is nothing but the strain energy per unit volume. And $\frac{\partial u_i}{\partial x}$ is work done. So, strain energy minus the work done is normally considered as a potential energy in theory of elasticity.

And the change of potential energy with respect to crack length and incremental crack length is something related to our strain energy release rate; something of that kind. So, that idea has been simplified after several modifications in a simple equation like this. So, in a nutshell, if you look at the idea is something related to the strain energy release rate. So, that is what we have to appreciate this equation.

And W can be written as

$$\int \sigma_{ij} d\varepsilon_{ij}$$

And

$$\{T^n\} = [\tau_{ij}]\{n\}$$

This is a Cauchy formula. This also we have seen already. And it is appropriate to use the symbol U for the strain energy instead of W. So, this is a contour, as I just mentioned in the beginning; the line contour. And this is an outward normal; and this is a traction vector, T^n. So, we will see now. And the traction vector here is written in the integral notation; so, it can be expanded to xy and so on. So, that is how it is. We will see how it written.

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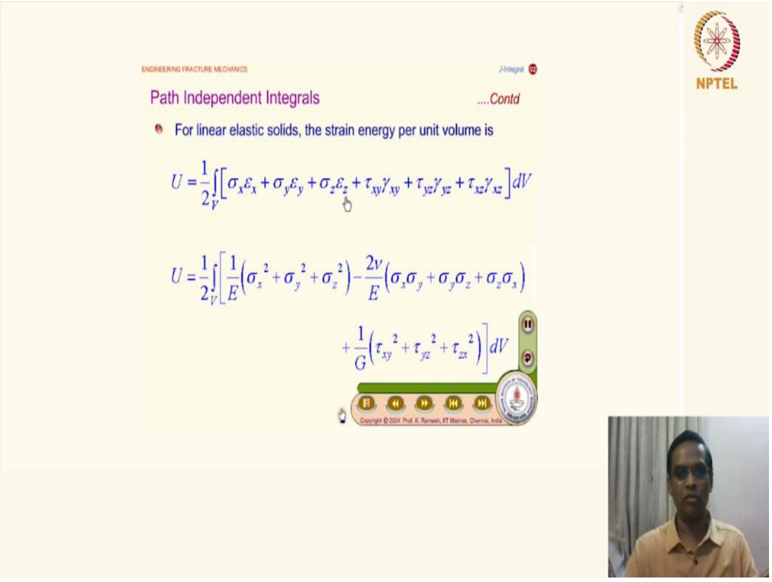

For linear elastic solids, the strain energy per unit volume is written like this. This equation you have already seen. And this is the whole expression for the linear elastic solids. What we have to understand is the J-integral is used in LEFM and J-integral is also used in the non-linear fracture mechanics; and J-integral is also used in elasto-plastic fracture mechanics. So, that is how we have to see how it is being used.

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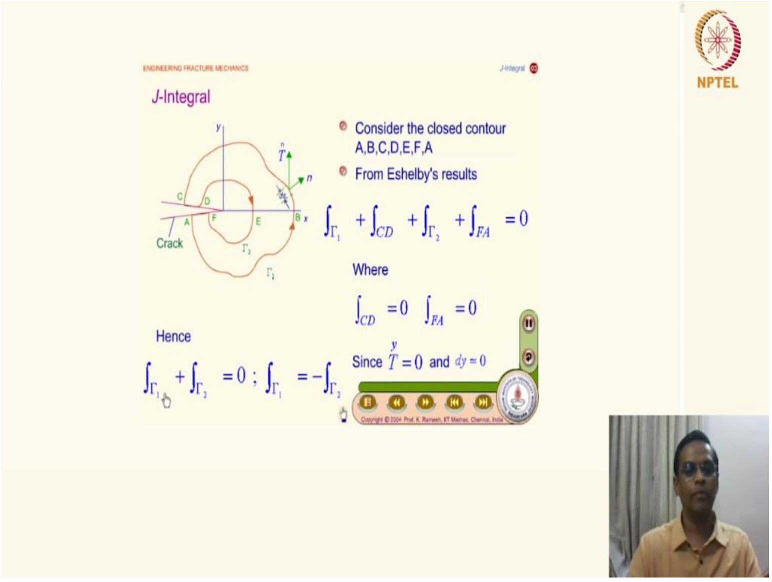

So, what is J-integral? So, what you are seeing here is a crack. The important point to note here is, the crack is not loaded. So, the contour one goes like this; the other contour comes in the opposite direction. And then, it is a closed contour. What you have to appreciate is, it is closed contour A, B, C, D, E, F; it will get closed here. And then, it starts with one face of the crack and then it ends with the other face of the crack, and the crack is not loaded.

So, in between, it is a free surface that you all know. So, this is considered a closed contour A, B, C, D, E, F, A. And from Eshelby's results, the integral, this line 1, that $\int_{r1}$ +$\int_{CD}$ + $\int_{r2}$ + $\int_{FA}$ = 0. So, the total value of this integral system is supposed to be 0. That is the Eshelby's assumption. $\int_{CD}$ = 0; $\int_{FA}$ = 0; because the traction in the crack surfaces is 0, because it is a free surface. So, when you substitute this and then the traction y = 0 and dy = 0 on the crack surfaces, so, you get the interrelationship between line $\int_{r1}$ and $\int_{r2}$; and it is written like this.

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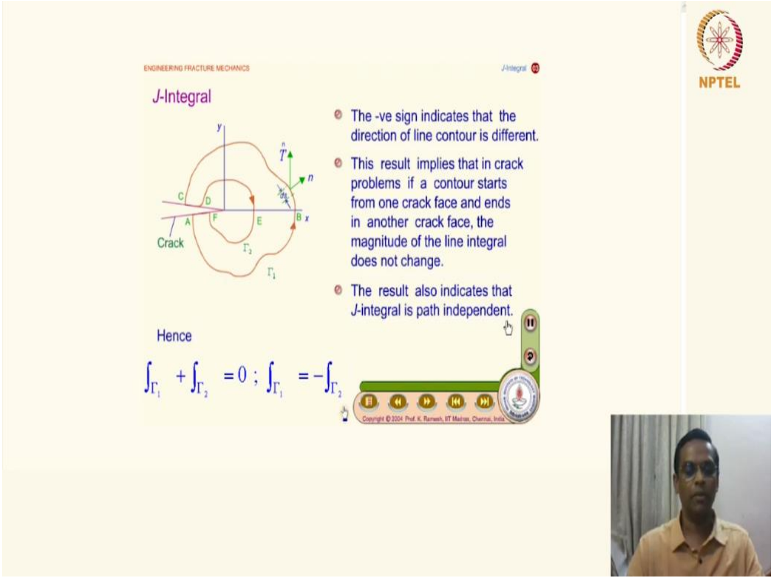

The negative sign indicates that the direction of the line contour is different. So, this goes like this; this goes like this. This result implies that in crack problems, if a contour starts from one crack face and ends in another crack face, the magnitude of the line integral does not change. The result also indicates that J-integral is path independent.

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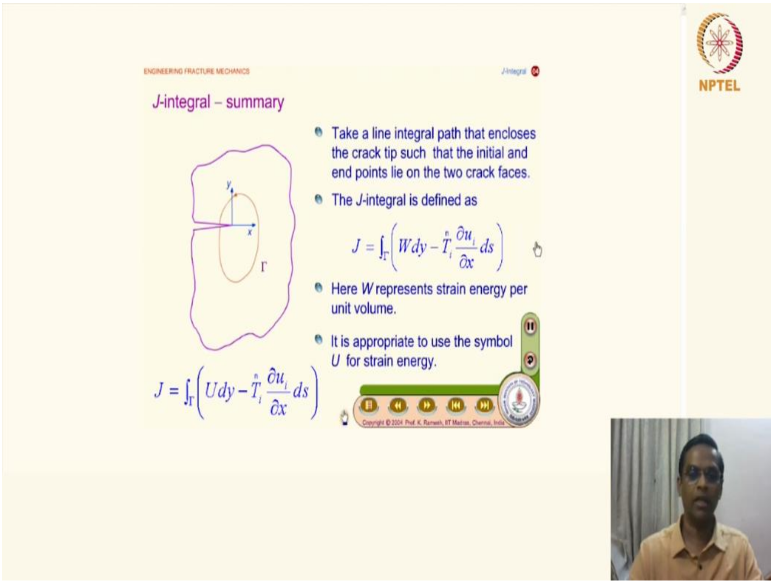

So, let us summarise the basic assumptions or features of a J-integral. So, there is a crack in the body. And then, the contour is drawn from one face to the other face of the body. Take a line integral path that encloses the crack tip such that the initial and end points lie on the two crack surfaces. The J-integral is defined as

$$J = \int_{\Gamma} \left(W dy - T_i^n \frac{\partial u_i}{\partial x} dS \right)$$

where W represents the strain energy per unit volume. It is appropriate to use the symbol U for a strain energy. So, we can rewrite this expression like

$$J = \int_{\Gamma} \left(Udy - T_i^n \frac{\partial u_i}{\partial x} dS\right)$$

We can rewrite like this.

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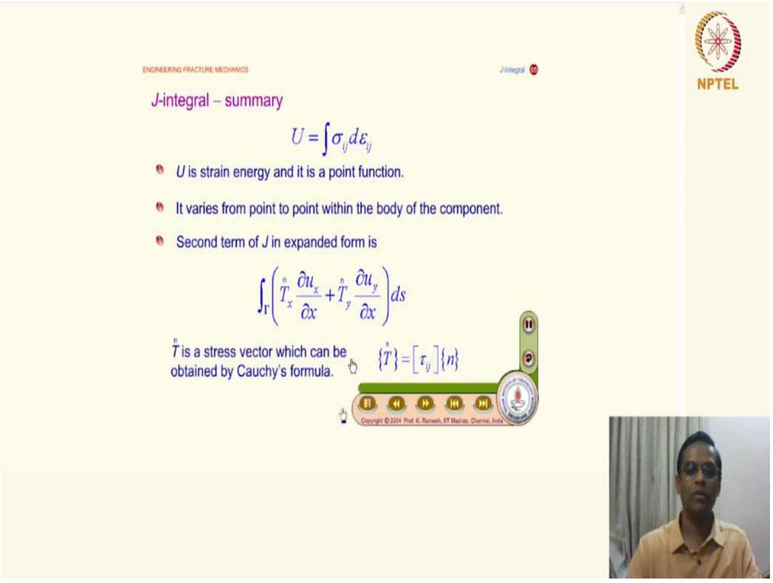

And what we can now look at is, U we can write like this:

$$U = \int \sigma_{ij} d\varepsilon_{ij}$$

U is a strain energy and it is a point function. It varies from point to point within the body of the component. The second term of J in an expanded form:

$$\int_{\Gamma} \left(T_x^n \frac{\partial u_x}{\partial x} + T_y^n \frac{\partial u_y}{\partial x}\right) dS$$

T^n is the stress vector which can be obtained by Cauchy's formula, like we mentioned before; so, thus we know already.

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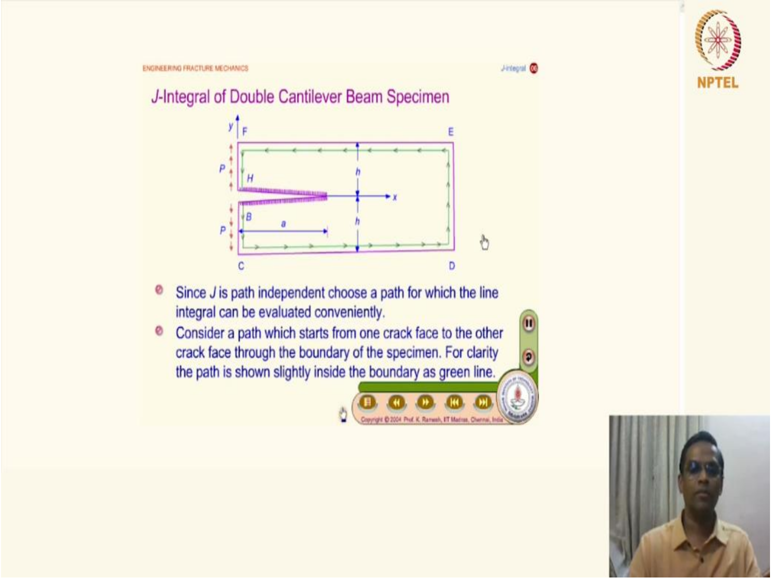

And we can just verify this J-integral for a double cantilever beam specimen of this kind. So, what is shown here is the same contour is drawn in this. So, what we have to appreciate here is, all these surfaces are free surfaces. So, we will see how it is useful. Since J is a path independent, choose a path for which the line integral can be evaluated conveniently. Consider a path which starts from one crack face to the other face through a boundary of specimen. For clarity, the path is shown slightly inside the boundary as a green line. So, as such, you can choose this as a boundary. And they are all free surface.

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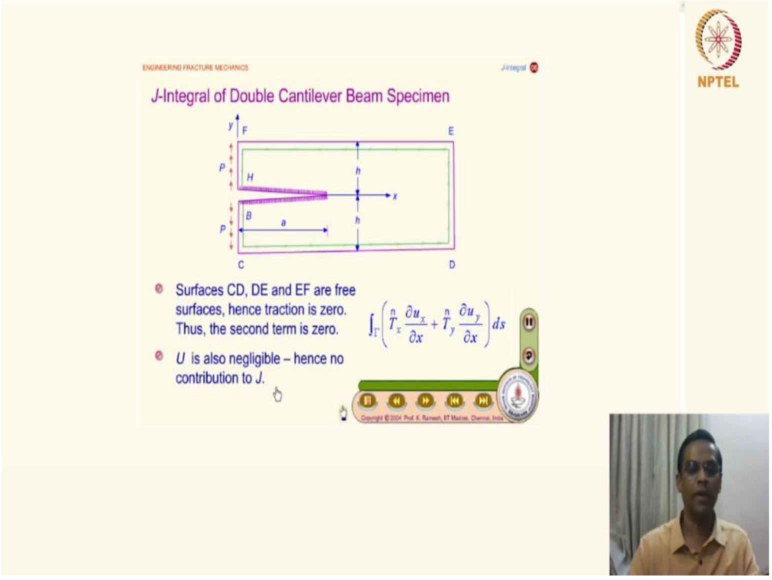

So, how do we write the equations for this? Surfaces CD, DE, EF are free surfaces; hence, the traction is zero. Thus, the second term is zero. So, this one is zero. U is also negligible; hence, no contribution to J from this side; that is what it is.

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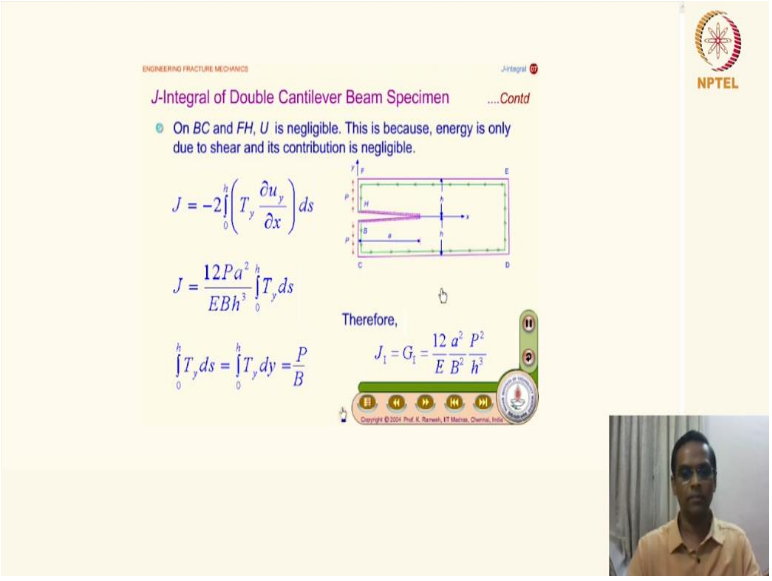

And what is remaining? On BC and FH, U is negligible. This is because, energy is only due to shear, and the contribution is negligible. And if you look at this other point is only these two faces. So, we have to write it like for these two faces, which is written like this

$$J = -2\int_0^h \left(T_y \frac{\partial u_y}{\partial x}\right) ds$$

And this particular double cantilever beam problem, the strength of materials gives me a solution as the slope of the beam.

This, from the strength of material solution, you can directly take it; and which is given like this. So, what we have to evaluate is only $\int_0^h T_y\, ds$, which is nothing but P/B. This is the value of this integral. So, you can write it like this. So, what we find out, as the J is also equal to G in a fully elastic material. For elastic material, J is nothing but a strain energy release rate.

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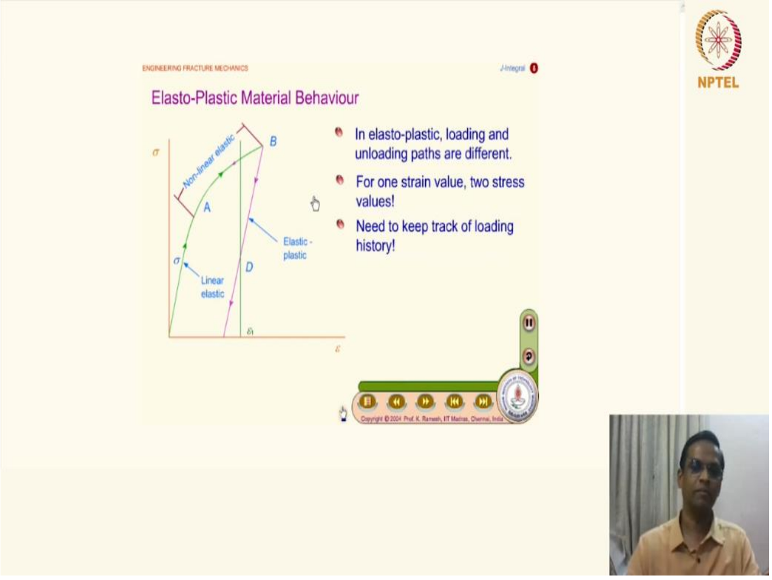

But how we can apply to the non-linear problem or elasto-plastic problem is; that is what we are going to see now. So, in elasto-plastic material behaviour, what is the issue? So, if you look at the stress-strain diagram, up to yield point or as long as it is linear elastic, it can come back; if you remove the load, you can trace the path. Even in the non-linear elastic mode, if the load is removed, the path can be traced.

The moment you cross the elastic limit and if you go to the plastic limit, then they trace the unloading path is different. Suppose if you take this particular line for a given strain, that could be two stress values. For this particular strain value, you have two stress values. So, this is the complication in the elasto-plastic deformation. That is what is shown here. In elasto-plastic, loading and unloading paths are different. For one stain value, two stress values are obtained. Need to keep the track of loading history. That is going to be a challenge. That is why this elasto-plastic is complicated.

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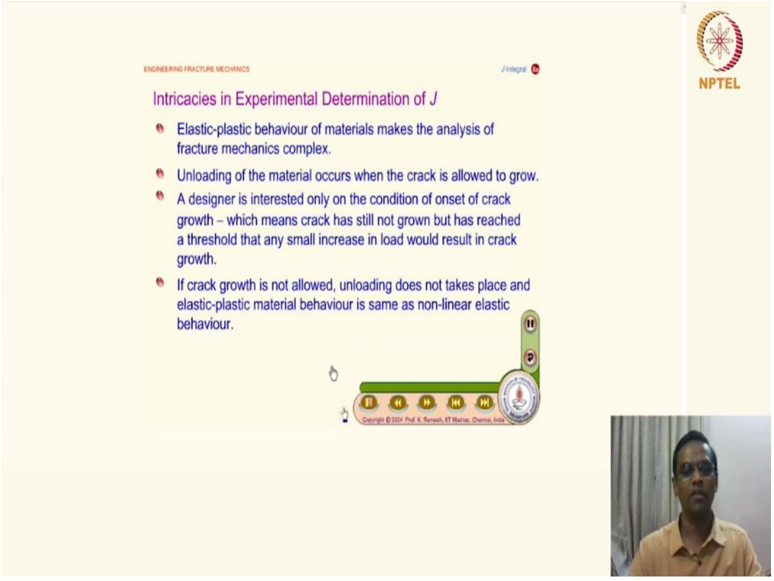

So, we will see some of the intricacies in experimental determination of J. Elasto-plastic behaviour of material makes the analysis of fracture mechanics complex. Unloading of material occurs when the crack is allowed to grow. A designer is interested only on the condition of onset of the crack growth, which means crack has still not grown but has reached a threshold that any small increase in the load would result in crack growth.

If crack growth is not allowed, unloading does not take place and elastic-plastic material behaviour is same as non-linear elastic behaviour. So, these are some of the important points to be noted as far as determining the J.

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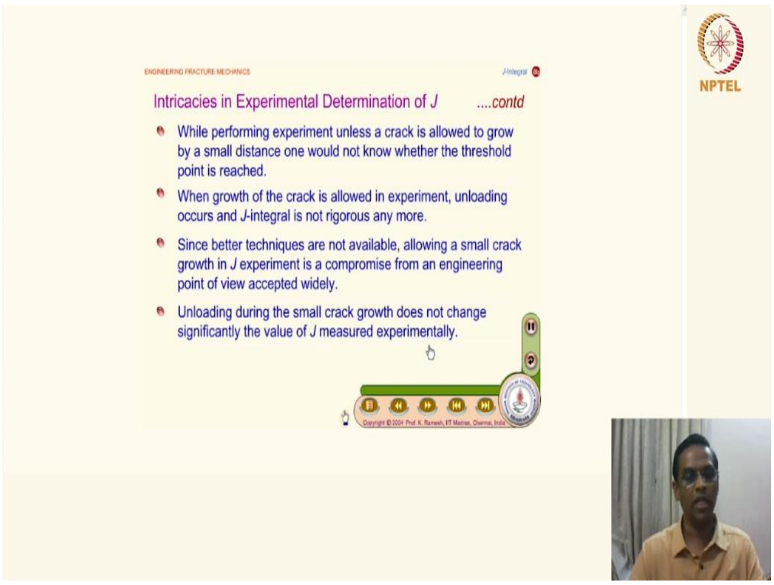

While performing the experiment, unless a crack is allowed to grow by small distance, one would not know whether the threshold point is reached. When the growth of the crack is allowed in experiment, unloading occurs and J-integral is not rigorous anymore. Since better techniques are not available, allowing a small crack growth in J-experiment is a compromise from an engineering point of view accepted widely. Unloading during small crack growth does not change significantly the value of J measured experimentally. So, these are all some of the points to be noted.

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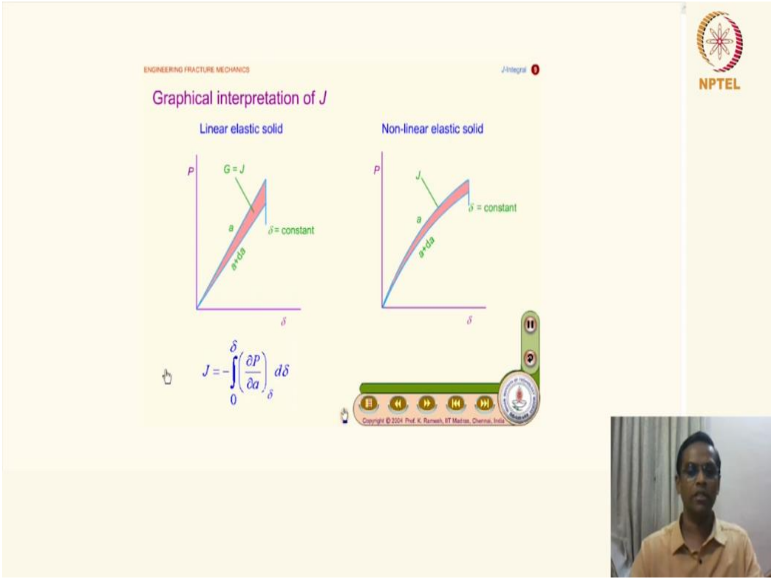

And then, we now look at the graphical interpretation of J for the linear elastic solid, the load versus displacement curve. So, as we just mentioned, as far as the elastic solid is concerned, the J is equal to G, energy release rate. For the non-linear elastic solid also, J is the area within this enclosed region. J can be defined as J is equal to $-\int_0^{\delta} \left(\frac{\partial P}{\partial a}\right)_{\delta} d\delta$ So, this is the definition of J.

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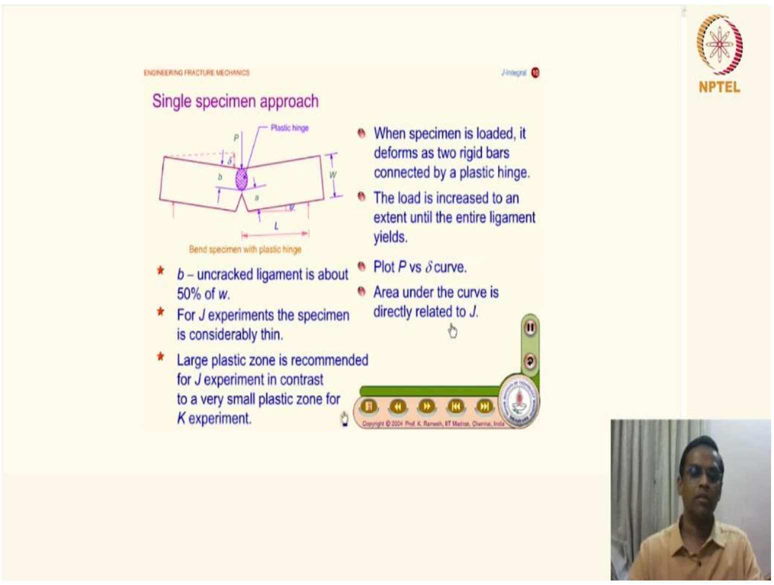

And we can just look at the simple experiments. The single specimen approach where you can see that b, the uncracked ligament is about 50% of the w. For J experiments, the specimen is considerably thin. Large plastic zone is recommended for J experiment in contrast to very small plastic zone for K experiment. When the specimen is loaded, it deforms as two rigid bars connected by a plastic hinge. So, it is shown here like this. This is the plastic hinge. And plot P versus δ and area under the curve is directly related to J. That is how the J experiments are conducted.

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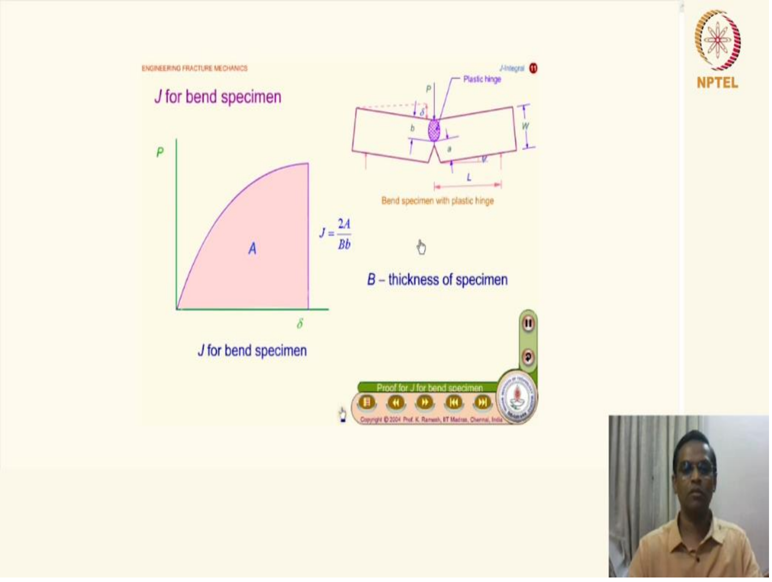

So, we can just look at this graph schematic, P versus δ. This is for J for a bend specimen. So, this is the area under this curve. J is related to 2A/Bb; B is the thickness of the specimen. And other dimensions are given here. A is area; B is this distance; the plastic zone distance.

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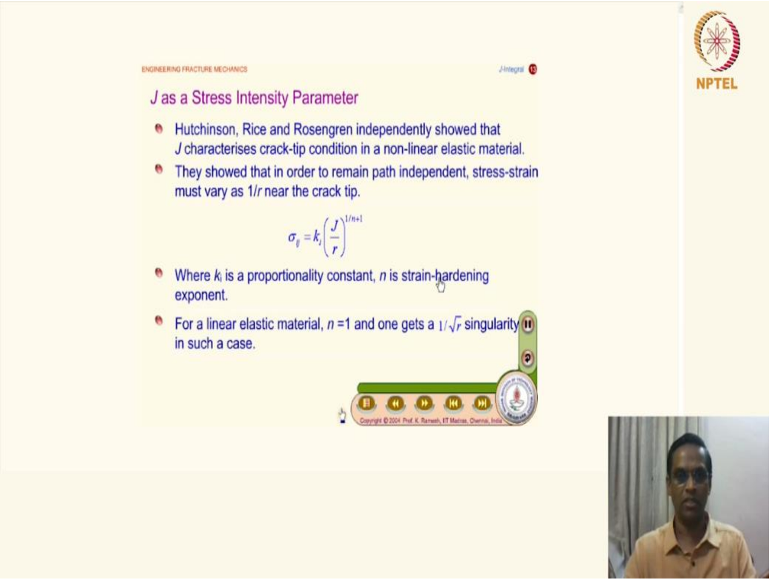

So, J as a stress intensity parameter: And Hutchinson, Rice and Rosengren independently showed that J characterises crack tip condition in a non-linear elastic material. They showed that in order to remain path independent, stress-strain must vary as 1/r near the crack tip. It is given like this. Where k_i is a proportionality constant, n is strain-hardening exponent. For a linear elastic material, n = 1; and one gets a $1/\sqrt{r}$ singularity in such cases.

So, I just would like to stop here. What we have realised is J is also a stress intensity parameter for the completely elasto-plastic material. And it has been shown that it is very useful parameter even though it produces a significant plasticity at the crack tip. And even for the small crack growth is a compromise in the J-experiment, but nevertheless the experimental results are useful. That is what people have reported in the literature.

So, I will stop here. And my intention is just to give you what is J-integral and how it has been characterised and how it is compared with the other fracture parameters. And beyond that, I do not want to discuss in this course, because it is not in the scope of this syllabus. And I urge all of our students who are interested in getting more information about the other parameters or details about these experiments, please refer the other course by Professor K. Ramesh of Applied Mechanics; their course on fracture mechanics. I recommend you to follow that. I will stop the discussion here and then we will now continue with the next lecture. Thank you.