**Mechanical Behavior of Materials**
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**Module No # 12**
**Lecture No # 56**
**Fracture Mechanics – VIII**

Hello I am professor Sankaran in the department of metallurgical and materials engineering.

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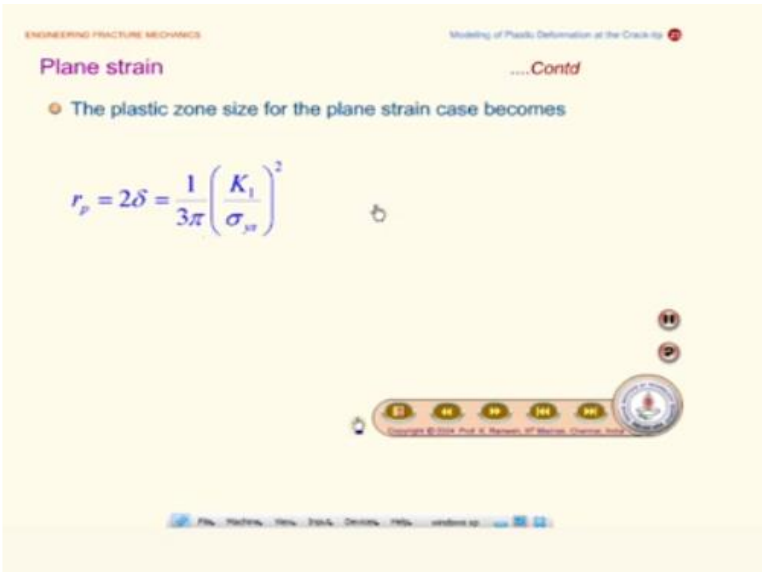


So the plastic zone size for a plane strain cases becomes finally $r_p = \frac{1}{3\pi}\left(\frac{K_1}{\sigma_{ys}}\right)^2 = 2\delta$.

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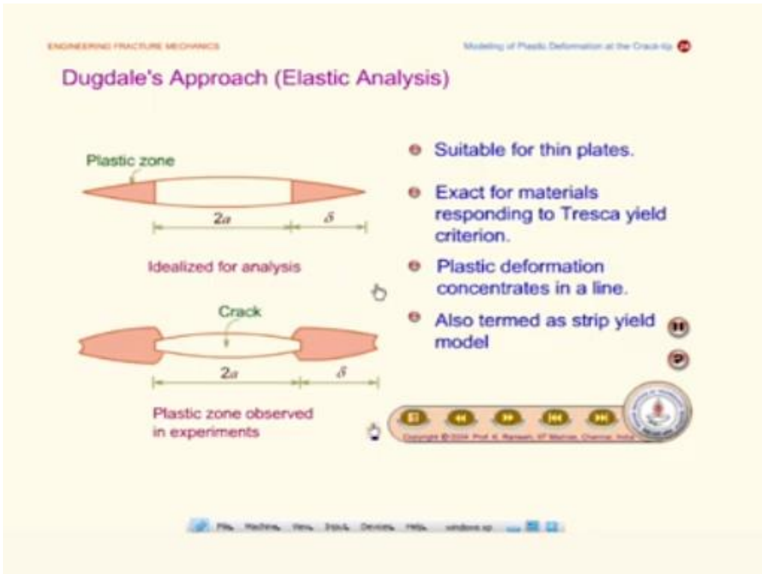


And then we can look at the another model, which is very important model because this model assumes the entire length of the plastic zone as a crack length. So this is what we are going to see so this is called elastic analysis and this schematic is shown here, what is proposed by Dugdale for this attractive plastic zone and what people have actually observed in experiment is this schematic is shown here. So, this kind of analysis suitable for thin plates, exact for materials responding to Tresca yield criterion and the plastic deformation concentrates in a line. Also termed as strip yield model so these are all some of the yield salient features connected to this Dugdale model and then we will see how it is been carried forward.

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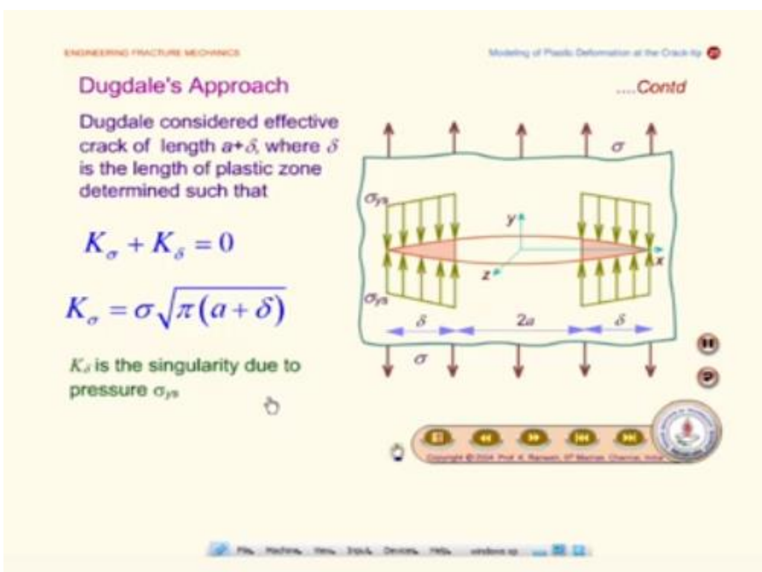

So, this is a central crack in an infinite plate subjected to uniaxial loading the crack length is now you see that 2a, and this plastic zone is δ and δ. So you have 2 δ + 2 K, so what is shown here so though this is plate is subjected to uniaxial tension here. This particular plastic zone at the 2 tips are subjected to some compression by the compression, I mean $\sigma_x$ , $\sigma_{yx}$ that type of stresses you know acting in the opposite direction to the far field stress.

So, we will see how to handle this. Dugdale consider the effective crack of length a + δ where δ is the length of the plastic zone determined such that $K_\sigma + K_\delta = 0$. So how do we understand this $K_\sigma$ this is a you know this is related to the crack singularity and $K_\delta$, it is the opposite. You know the cohesive force you know which is compressed by the magnitude of $\sigma_{yx}$ by this order.

We see this animation again $K_\delta$, is you know that is σ into π a + δ $K_\delta$ , is the singularity due to the pressure $\sigma_{ys}$. So, I am saying it is a compression so it is considered as a pressure how do we carry this forward?

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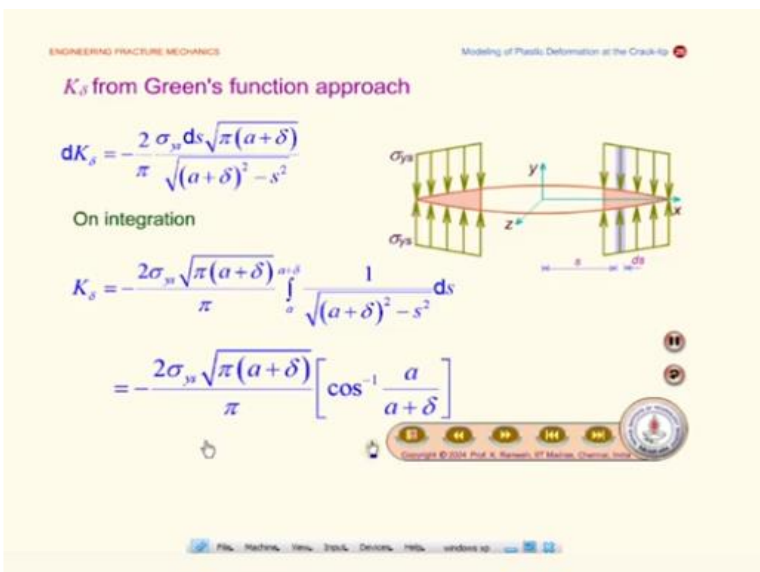


So $K_\delta$ from Green’s function approach so we will see what is that so the Green’s function what is given here is basically the increment crack plastic zone and crack growth in the plastic zone is considered as a ds and this portion is taken as s. And then d $K_\delta$, is a given by this expression and then this is done I mean this expression is for the incremental growth ds. So if you integrate to the entire length then you will get the $K_\delta$.

So that is idea but if you look at this expression it is -2 π because of it is under the compression. So, this basically you know this K expression is taken from the wedge type of loading proposed again by Westergaurd stress function this expression was derived. We did not see it in detail but you can go and refer in the textbooks and the same the wedge type of loading I mean at the 2 ends type of loading type, I mean the Westerguards stress function is given and that the stress intensity factor was derived like this for the tensile load. But here it is negative sign because of it is a compression mode so this is considered like a wedge type of loading and which reads like that $\sigma_{ys}$ into d s into π in to a + δ divided by square root of a + δ whole square – s square. So now you can integrate this we have already done this kind on integration in the previous sections. So, if you look that and if you look at the table of integral you will find the answer for this.

So, we are not getting into the details just if you look at all the steps like some of the steps are you know skipped here just applying the limits and then you will get the final expression like this.

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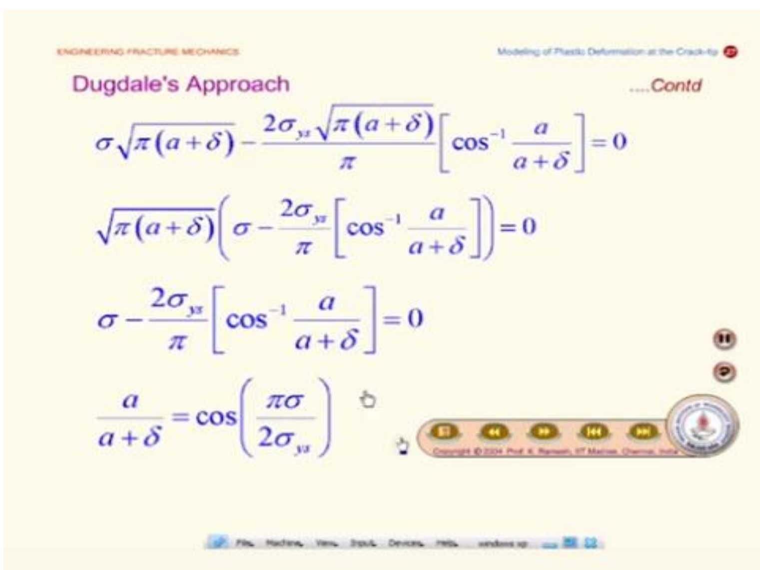


So, from there now you proceed you rewrite this expression like this form where $\sigma\sqrt{\pi(a+\delta)} - \frac{2\sigma_{ys}\sqrt{\pi(a+\delta)}}{\pi}\left[\cos^{-1}\frac{a}{a+\delta}\right] = 0$. You should take out all this π into square root of π into a + δ then we can re-catch this equation like this since it is equal to 0 this can be knocked off and then you can get this expression from this expression. You can rewrite this expression like this $a/(a + \delta) = \cos(\pi \sigma/2 \sigma_{ys})$. So, this is one final expression you get so from there we have to solve for the $\delta$ so how do we do that? So that is what we are going to see now.

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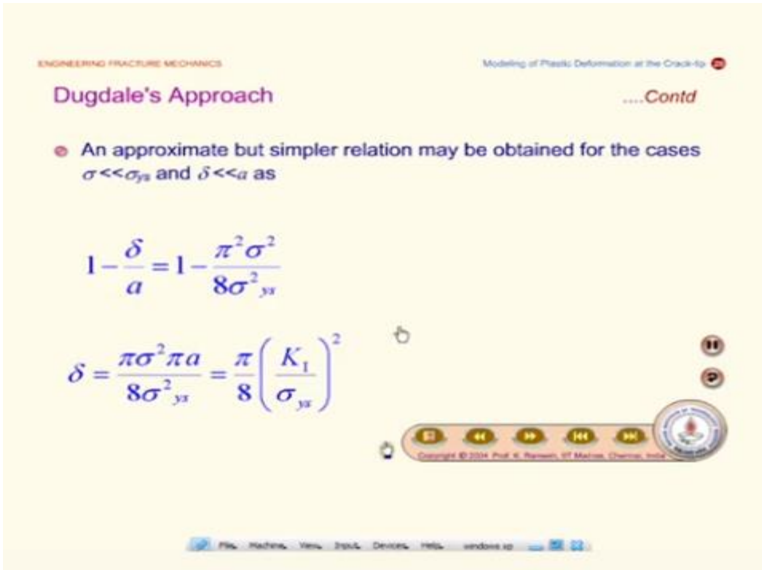


An approximate but simpler relation maybe obtained for the cases where $\sigma$ is far less than $\sigma_{ys}$. So, this is something we have to remember, see the $\sigma$ when you consider this then always the $\sigma$ which is appearing this kind of equation is much smaller as compared to the yield stress of the material. So, and this assumption is valid because it will give the a good approximate estimate of this plastic zone. So, then again you consider that $\delta$ is far less than a and then you can rewrite that expression like that $1 - \delta/a = 1 - (\pi^2 \sigma^2/8 \sigma_{ys}^2)$. So, then we get an expression for a $\delta$ like this which is $\frac{\pi}{8}\left(\frac{K_1}{\sigma_{ys}}\right)^2$.

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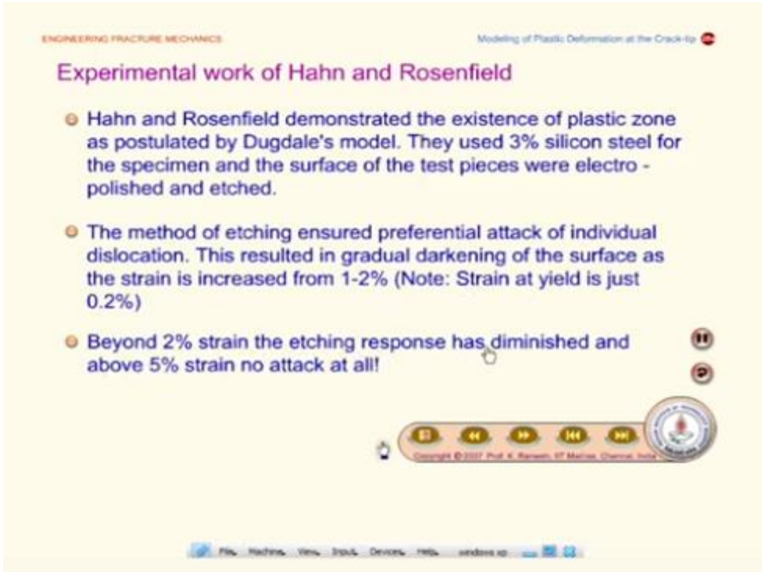


So now we can use this and then try to estimate through the experiments. Hahn and Rosenfield demonstrates the existence of plastic zone as postulated by the dugdale model. They use 3% silicon steel for the specimen and surface of the test pieces were electro polished and etched. So the method of etching ensured preferential attack of individual dislocation is resultant in gradual darkening of the surface as the strain increase from 1 or 2%. Beyond 2% strain the etching response has diminished and; above 5% strain no attack and all. So this is some kind of verification done by Hahn and Rosenfield about Dugdale model.

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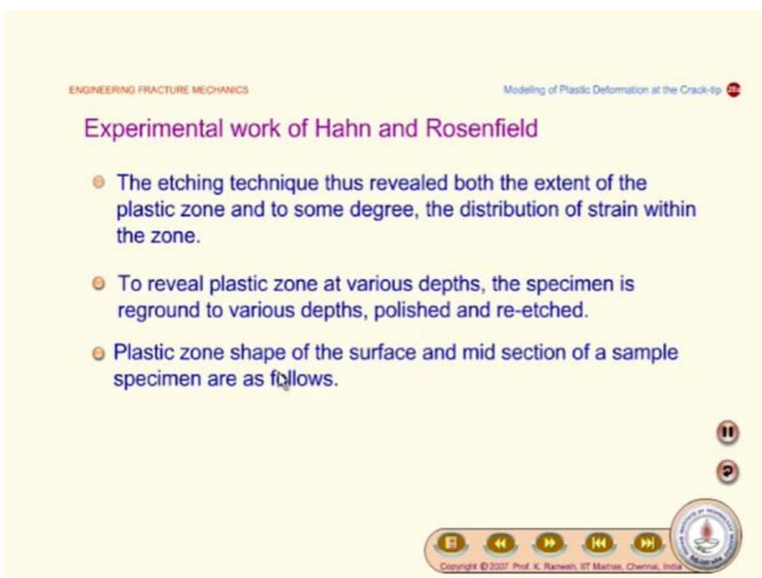


So, the etching technique does reveal both extent of plastic zone and to some degree of distribution of strain within the zone. So, the redistribution of the load is also being witnessed by the experiments to reveal the plastic zone at various depths the specimen is reground to various depths polished and reached plastics zone shape of surface and midsection of sample specimen are as follows.

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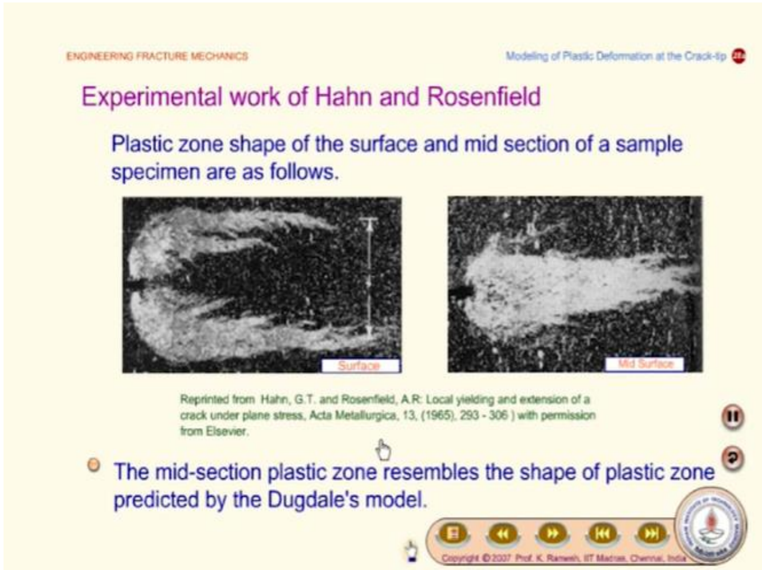


So what is obtained by these Hahn and Rosenfield is give here so this is very important. What is seen here is the surface what you see is a surface and this is the side view. So this is very interesting to note that whatever you know is what are the you know model proposed by the Dugdale it is exactly resembling let us say the mid-section plastic zone resembles the shape of the plastic zone predicted by that dugdale model that is very important.

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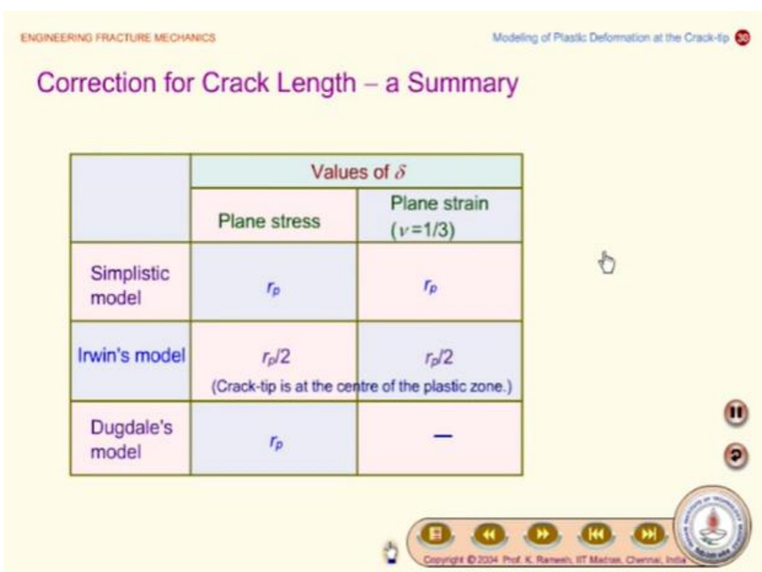

So now let us look at the summary of the plastic zone lengths for a simplistic model you have for a plane stress and a plane strain model is this we have already seen. And Irwin model is given by these expressions and then Dugdale model is only for a plane stress situation on the plane strains situation just to recall what al the plastic zone side we are able to appreciate. And what about the correction of crack length that is also a simplistic model gives considers the $r_p$ in both cases plain stresses and plan strain.

Irwin model gives you know $r_p/2$ they crack tip at the center of the plastic zone in both the cases it is approximately $r_p/2$ and the Dugdale model again considers the as an $r_p$.

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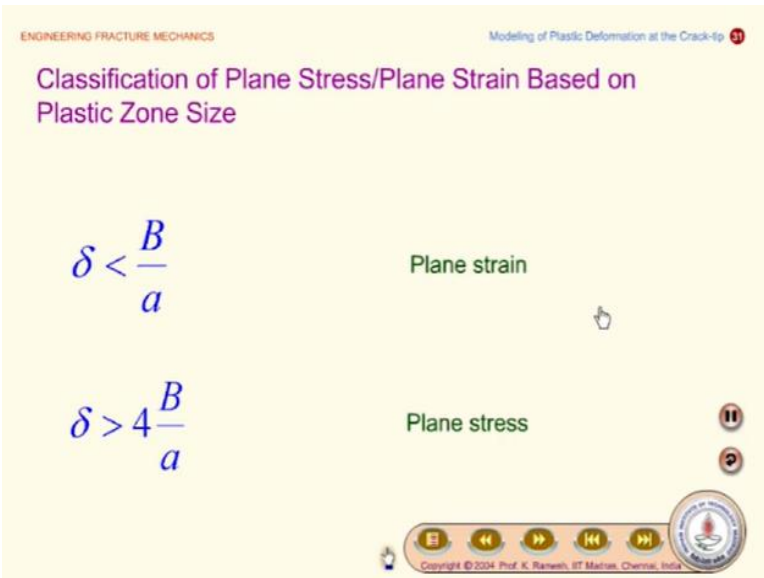


So now what we will do is, we will try to classify them classification of the plain stress and plain strain based on the plastic zone side. How, do we classify based on the classic zone size if $\delta$ is less than B/a then that is belong to plain strain condition if the $\delta$ is greater than 4 B/a, is the plane stress conditions. So this is one important idea to keep in mind.

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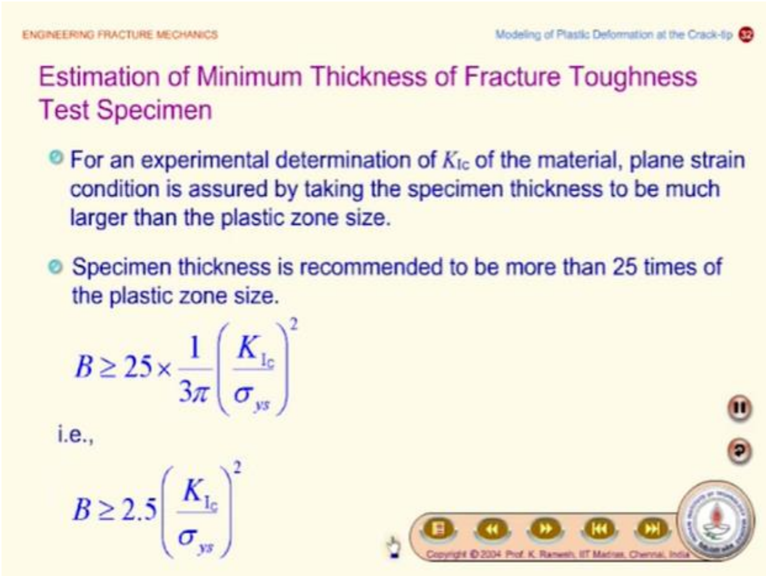


So how do we know? estimation of minimum thickness of the fracture toughness displacements. So if you want to this is going to be huge challenge now what is the minimum thickness for estimating the fracture toughness the displacement. For an experimental determination of $k_{1c}$ the material plane strain condition is assured by taking the specimen thickness to be much larger then the plastic zone size this is first point.

Specimen thicknesses is recommended to be more than 25 time of plastic zone size that means B should be $B \geq 25 \times \frac{1}{3\pi}\left(\frac{K_1}{\sigma_{ys}}\right)^2$, that is $B \geq 2.5\left(\frac{K_{1c}}{\sigma_{ys}}\right)^2$. So, in order to you know take the valid

$K_{1c}$ test this is the first primary conditions sample thickness has to be of this order. We will see, when we do that I mean fracture toughness testing we will cut all this condition much more closely.

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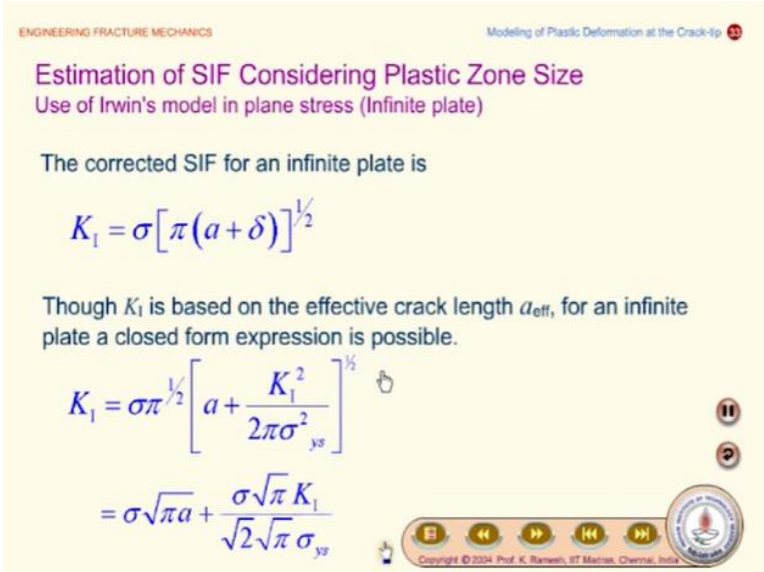


So how do we estimate the stress incentive factor considering the plastic zone size using the Irwin model in especially in the plane stress condition. The corrected stress intensity factor of infinite plate is given by this $K_1$ is equal to $\sigma[\pi(a+\delta)^{1/2}]$ thought $K_1$ is based on the effective crack length $a_{effective}$, for an infinite plate, a closed form expression is possible as given by this models $\boldsymbol{K_1 = \sigma\pi^{\frac{1}{2}}\left[a+\frac{K_1{}^2}{2\pi\sigma_{ys}^2}\right]^{1/2}}$, You will get $\sigma\sqrt{\pi}\ a$ +[ $(\sigma\sqrt{\pi}\ K_1)/\sqrt{2}\sqrt{\pi}\ \sigma_{ys}$]. So this is one way of getting the $K_1$, with the plastic zone so finally we will get this $\boldsymbol{K_1 = \frac{\sigma\sqrt{\pi a}}{\left[1-\frac{1}{2}\left(\sigma/\sigma_{ys}\right)^2\right]^{1/2}}}$. So this is one very important expression for stress intensity factor.

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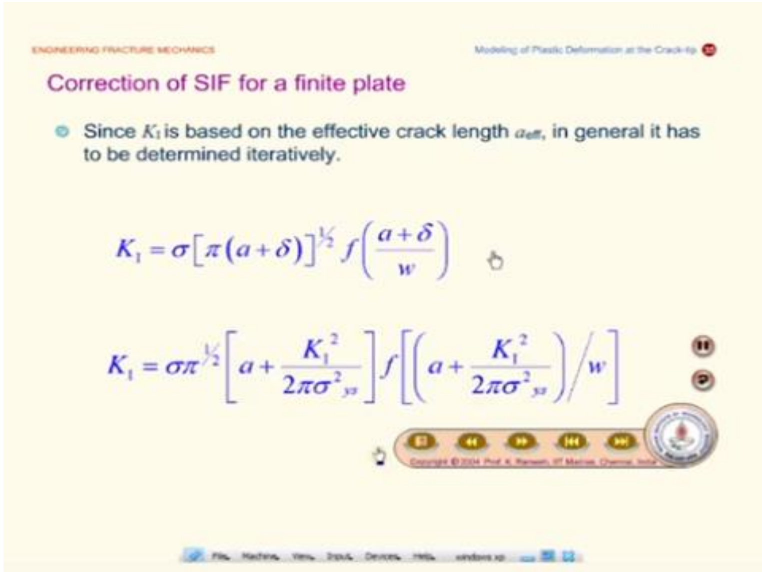


Corrective in the stress incentive called a finite plate so we know that now so far we have looked at finite plate and we will now come to finite plate. Since $K_1$ is based on effective track length a effective in general it has to be determined iteratively. And $K_1 = \sigma\,[\pi(a + \delta)]^{1/2} f(a + \delta/w)$. So, this we have already seen you are not new to this it is a function of a instead of a by w it is a + δ by w. So now you get the final expression like this $\boldsymbol{K_1 = \sigma\pi^{\frac{1}{2}}\left[a + \frac{K_1{}^2}{2\pi\sigma_{ys}^2}\right] f\left[\frac{\left(a+\frac{K_1{}^2}{2\pi\sigma_{ys}^2}\right)}{w}\right]}$. So this completes you know the stress intensity factor for a finite plate.

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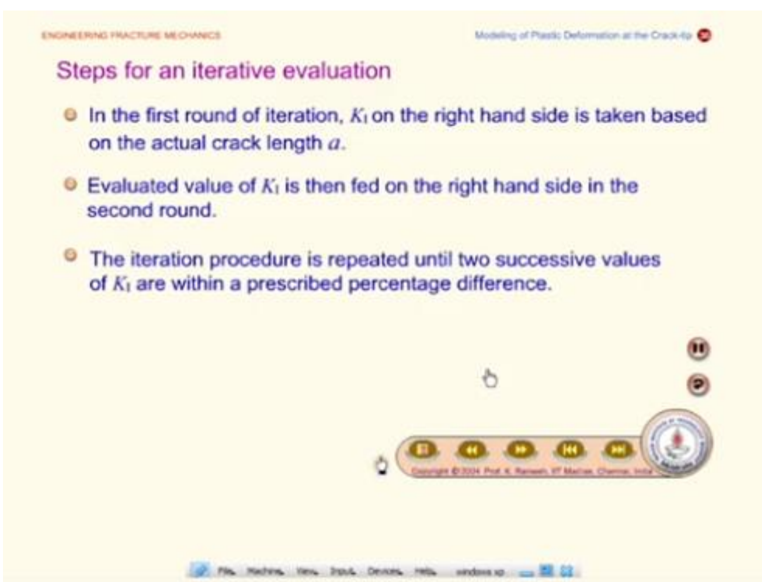

So we will solve some numerical examples later then you will get the impression of the plastic zone are the influence of plastic zone on the stress intensity factor. So now there are some iterative procedures to evaluate the $K_1$ so I am not going to spend time on this.

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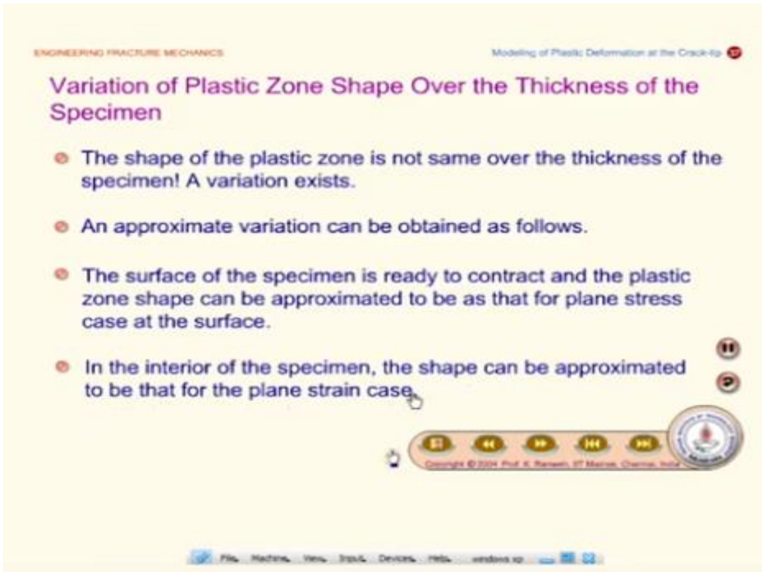


So, we will take it up later. So, now what is important point to note here is variation of plastic zone shape over the thickness of the specimen how do we visualize this shape. The shape of the plastic zone is not same over the thickness over the specimen a variation exists and approximately variation can be obtained as follows the surface of the specimen is ready to contract, and the plastic zone shape can be approximated to be as that for a plane stress case at the surface. In the interior of the specimen the shape can be approximated to be that for the plane strain case this is what we have seen in the selection of fracture toughness slide.

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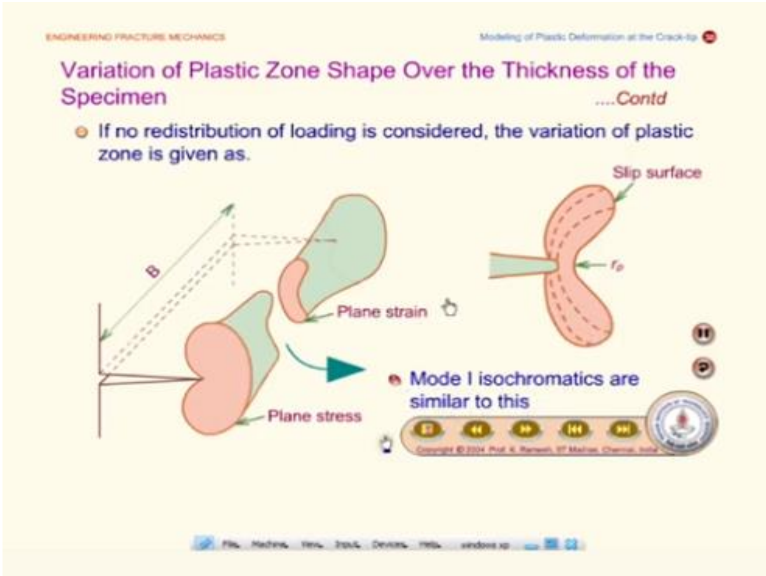


If no re-distribution of loading is considered the variation of the plastic zone is given as this schematic. So this is very interesting schematic you see that this specimen thickness is shown here as a B. And this is cracked up this is at the surface which is a plane stress condition and this is inside this thickness, which is the plane strain condition you can see that this is magnified view is given here. These are all slip surfaces and this is called you know butterfly plastic zone this also mode 1, isochromatics are similar to this.

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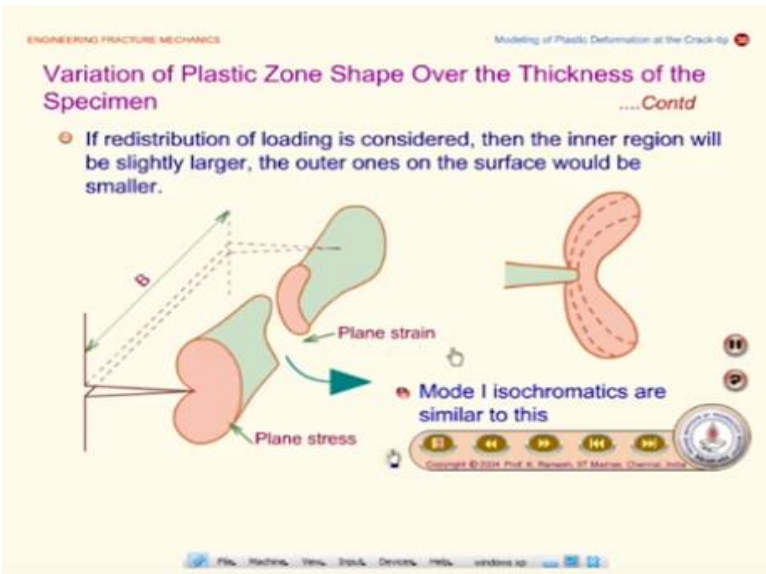


And if the redistribution is calculated and then again this plotted like this schematically though it does not give much difference between the re-distribution but nevertheless it gives the idea, I mean original idea into the same where the plastic zone size will be bigger on the surface that is a plane stress condition and in interior will be a little larger. And which will represent the plain strain condition.

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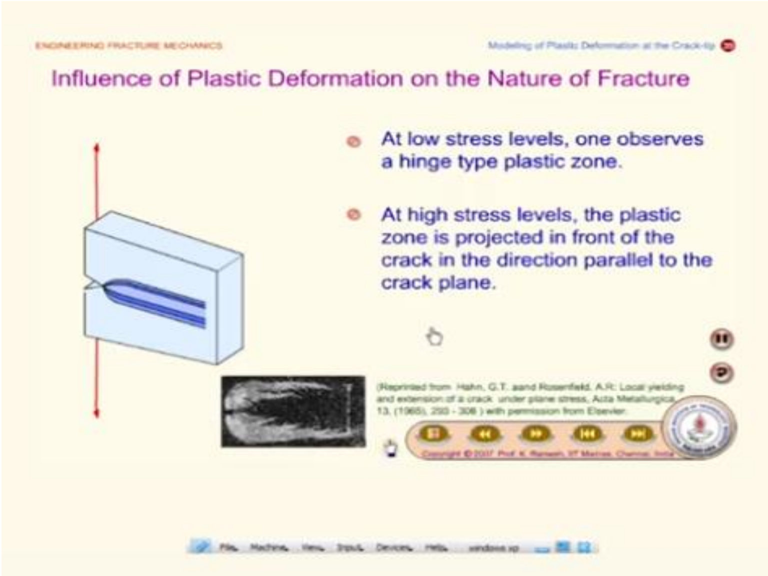


So now another important point is influence of plastic deformation on the nature of the fracture. At low stress levels one observes a hinge type of plastic zone, at high stress level the plastic zone is projected in front of the crack in the direction parallel to the crack plate. So, look at this animation so this is the kind of you know plastic zone region is demonstrated. But it is also you know experimentally observed by you know, Hein and Rosenfield which we have already just seen in the previous slide. So we will now get the details how we can understand this.

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So slip plane in the plain strain you can see that you know these are maximum shear plane of maximum shear and then the crack tip region is shown here how the slip takes place. So slip takes place like this it is sliding in each other each the region is sliding over each other in that process that crack extends by tearing that is what shown here. In this schematic and then what happens when slightly interior you know the plastic zone it is quite different.

Now you see that you know the plane of maximum shear test is something similar to what we have already seen it does not know. It is these types of you know, 45° planes which is not in the same plane of the stress axis but you see that you know how we 45° plane moves in this zone. That is how the plastic zone develops so this some nice schematic which gives the visual impact of how do we crack stresses or cracked tip plastic zone evolves. So, this is slip planes in the plane stress again the plane of maximum shear and this particularly we have already seen this, in one of the introduction slide. So, what you are seeing here is the plastic zone you know from the crack tip it goes like this the crack grows in the plastic zone and the finally it fails by the shear lip, this is something which we have to remember. So, what I will do is I am just rushing through because I just want to give you an overall flavour now.

But my intention is not to you know dump too much of information but the same time my concern is the materials students should have also ideas to in their mind when they compare you know fracture mechanics problem. Then it is you know easy for you to you know understand the general literature and then you can be you know understand all the languages taught by the full community, that is my intention. So, I will stop here and then we will continue in the next lecture in another topic in fracture mechanics, thank you.

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