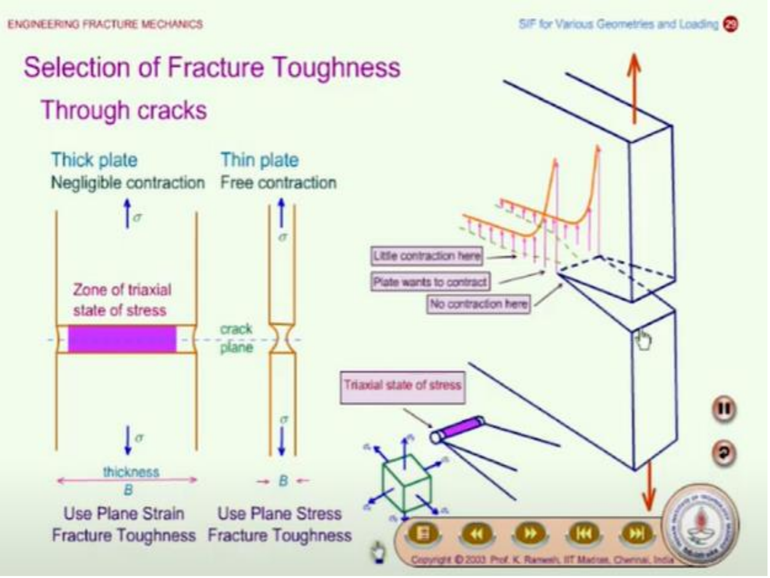
Mechanical Behavior of Materials
Prof. S. Sankaran
Department of Metallurgical and Materials Engineering
Indian Institute of Technology - Madras

Module No # 12
Lecture No # 55
Fracture Mechanics – VII

Hello I am professor S Sankaran in the department of metallurgical and material engineering.

(Refer Slide Time: 00:17)



Hello everyone welcome to this lecture in the last class we have looked at the displacement field from the for mode 1 and then we compared that solutions with the stress field mode 1 derivation and then we realized that you know for stress field we had the you know the near field solution. But for a displacement field, it is a bounded solutions this is what we have seen. So, now the our attention is to get into the plastic zone size at a crack tip and we are going to look at that but before that I just want to cover a small toπc which is part of the yesterday's stress intensity factor for different geometries and then we will discuss this that is selection of fracture toughness through cracks. So, you will have this specimen which is having an edge crack like this and how do we select the fracture terms? That is something we will see just before we go into plastic zone size and calculation.

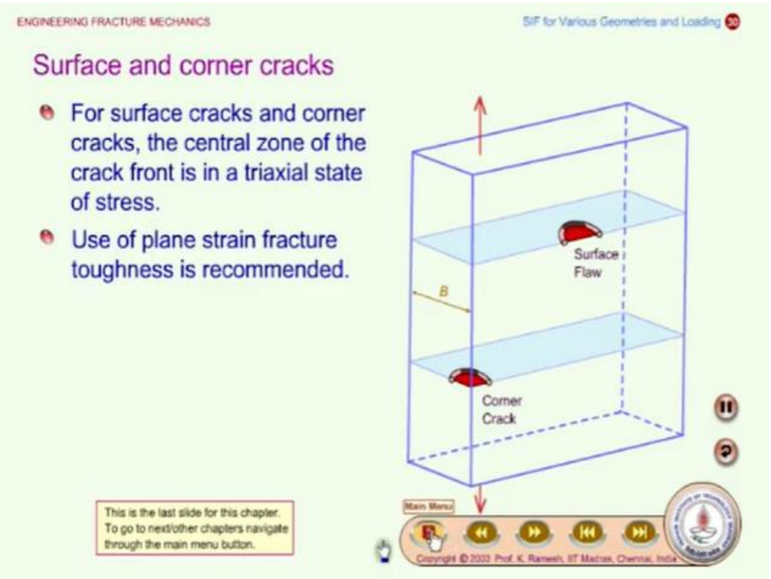
I will play this animation and what you see here is the at this crack tip, the stresses are very high so this is what we have to remember. So, what happens what is the influence of this very important idea? That is why I want to just emphasize this point before we get into plastic zone size calculation. So, because of this high stress at the crack tip what are the consequence? the consequences are the following.

It immediately close to this tip there is no contraction in this direction but exactly in this zone plastic, I mean the plate wants to contract. In this zone the plate will try to contract and what about the region ahead of this crack tip? A little contraction will take place so what exactly is happening in this region why the contract takes place. So, you just imagine that you take a cylindrical material from this zone and then try to put it here what we need to understand is?

When the cracked tip is very sharp $\sigma_x = \sigma_y$. if the crack tip is blunt then at the phase of the crack it is $\sigma_x = 0$. This is all we know so but the at this zone strain may be will be 0 but not this stress. Because we are talking about the plane strain condition so in the when we wrote a stress strain matrix in the beginning, in the for a plane strain condition the strain is 2X2 matrix but not the stress is still 3X3 matrix so that means what we can try to understand from here is? This particular point is subjected to triaxial state of stress in a plane strain condition so we have to look at thick plate verses thin plate. So thick plate where negligible contraction and you can see that the zone of a triaxial state of stress which is interior of the specimen except the surface layers, few lengths of surface layer and for the thin plate it is a free contraction will be there and then we should use a plane strain fracture toughness for the thin plate and we should use a plane stress fracture toughness for the thin plate, excuse me.

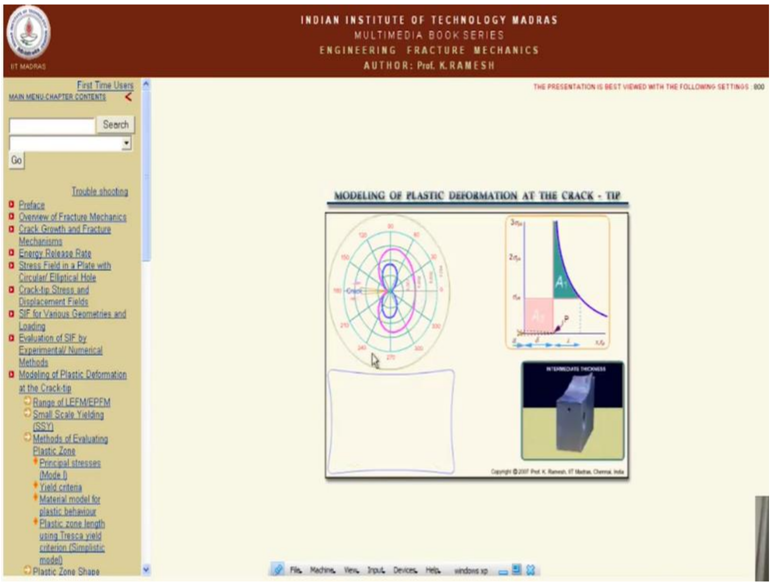
And use plane strain fracture toughness for the thickness plane so this is something we have to understand because and this is quite at the back drop of this understanding is the crack tip and the thick plate undergoes triaxial to the state of stress.

(Refer Slide Time: 05:04)



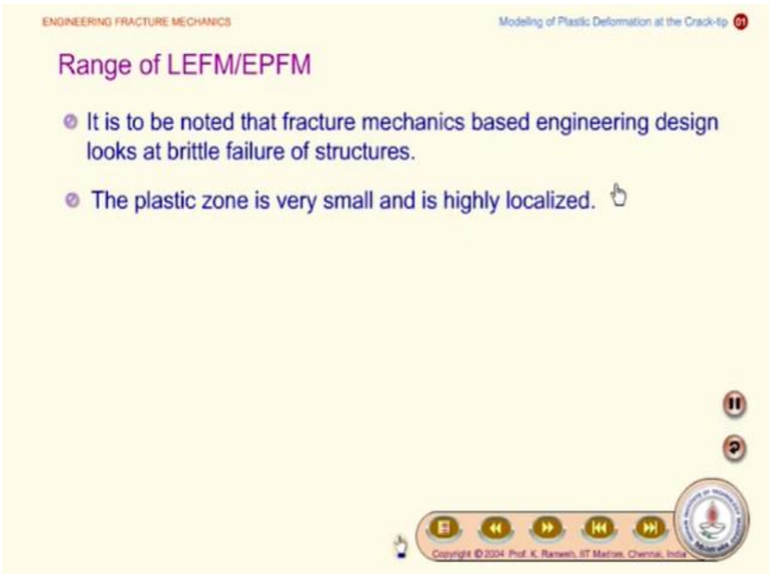
And then one practical relevance of this discussion is this surface and corner cracks so you just look at this animation where you have the plate of this dimension with thickness b and then you have a corner crack and then surface crack. So, the surface cracks and corner crack, the central zone of the crack front is in a triaxial state of stress and use a plane strain fracture toughness, I mean use of plane strain fracture toughness is recommended for the analysis of this type of crack. So, this is a most important point which we need to understand before we get into the plastic zone analysis.

(Refer Slide Time: 05:52)



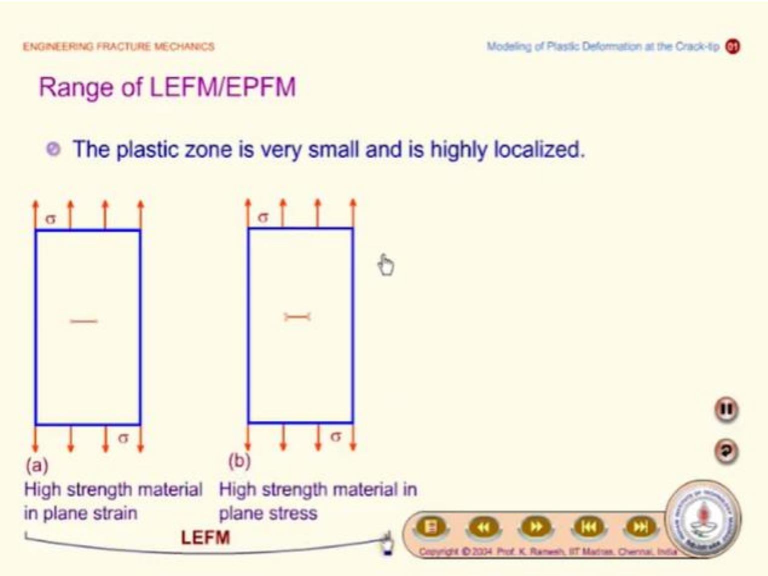
Now what I will do? I will turn your attention to estimation of plastic deformation and we will find this the chapter modeling of plastic deformation in the crack tip. So, these are all the animation we are going to consider.

(Refer Slide Time: 06:06)



So if you look at what we already seen in the introduction slide, we have talked about range of LEFM and EPFM it is to be noted that fracture mechanics based engineering design looks at brittle failure of structure this is primary focus on fracture mechanics. The plastic zone is very small and is highly localized so this is assumption by which all the brittle material are handed by fracture mechanics that is the important point.

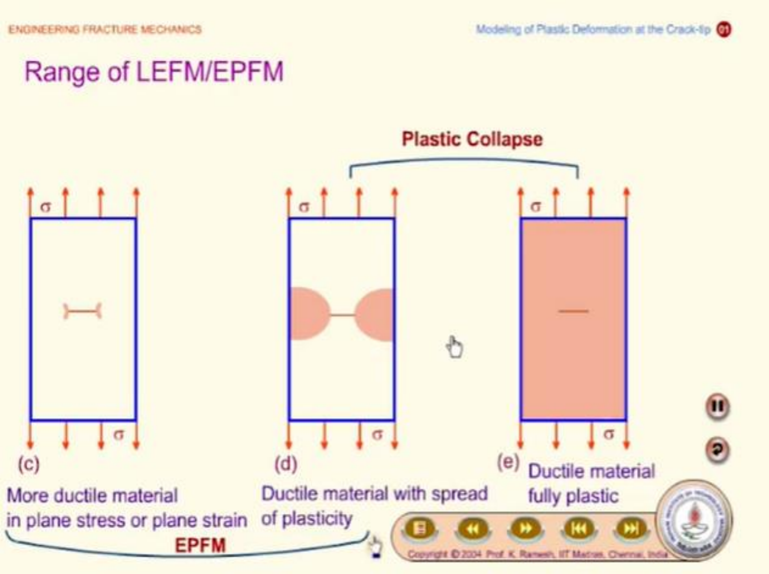
(Refer Slide Time: 06:47)



But when we say the plastic zone is very small what kind of plastic zone we are looking at, so this is some schematic which talks about where the high strength material in a plane strain condition and then high strength material in a plane stress condition so you can see that the plastic zone which is shown here is very very localized. But one very important to note is the plastic zone in the plane strain is much smaller as compared to a plane stress situation.

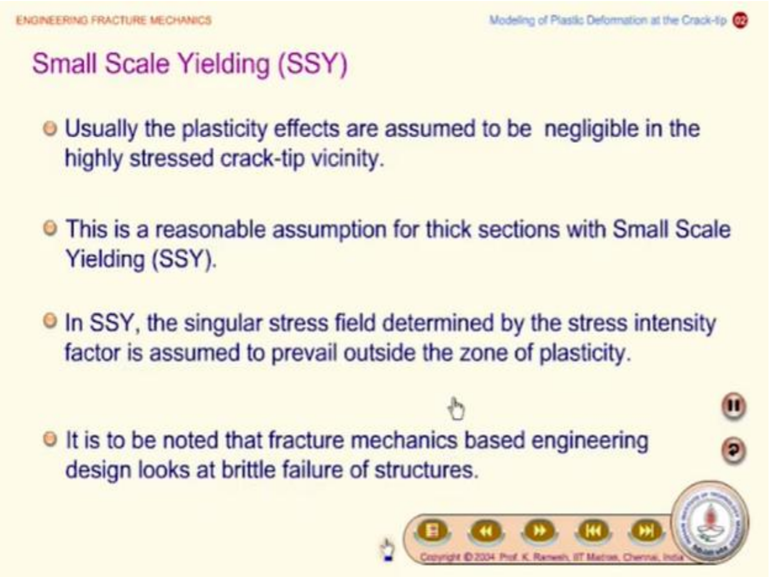
So, these are the two conditions where the LEFM comfortably handles to estimate the stress intensity factor and so on right.

(Refer Slide Time: 06:47)



But the moment you go to very large plastic zone size for more ductile material in a plane stress or plane strain or you have the ductile material with spread of plasticity only with EPFM can handle this and of course elastic, when plastic collapse where ductile material fully plastic then these are not applicable whatever the idea which is discussed in earlier is not applicable to this. But up to this, these theories are calculations are valid.

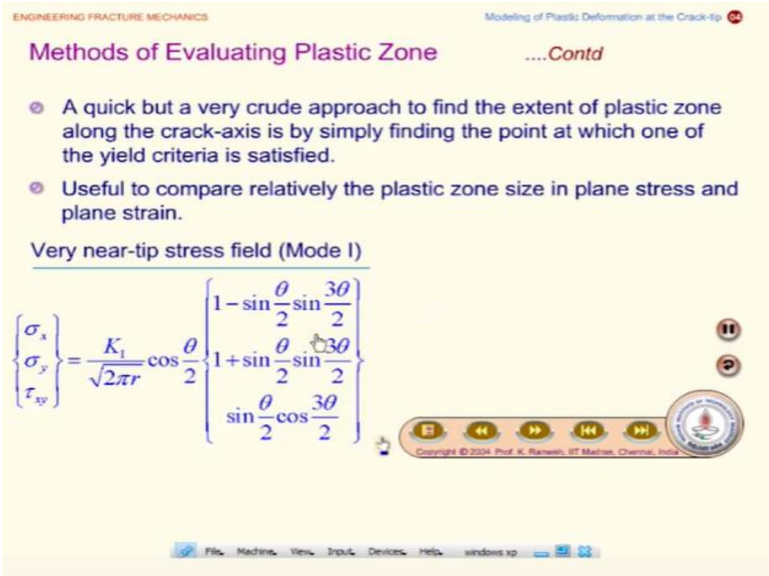
(Refer Slide Time: 08:09)



So in this context we also mentioned in the production something called small scale yielding what is small scale yielding? Usually, the plasticity effects are assumed to be negligible in the highly stresses crack tip vicinity and this is a reasonable assumption for a thick sections with small scale yielding is called SSY. In SSY the singular stress field determined by this stress intensity factor is assumed to prevail outside the zone of plasticity.

You see, the region which is very close to the cracked up is called singular stress field you know it is called singular stress field which is determined the stress intensity factor. It is to be noted that fracture mechanics-based engineering design looks at brittle failure of structures this is what we are SSY is suppose to look at it.

(Refer Slide Time: 09:14)

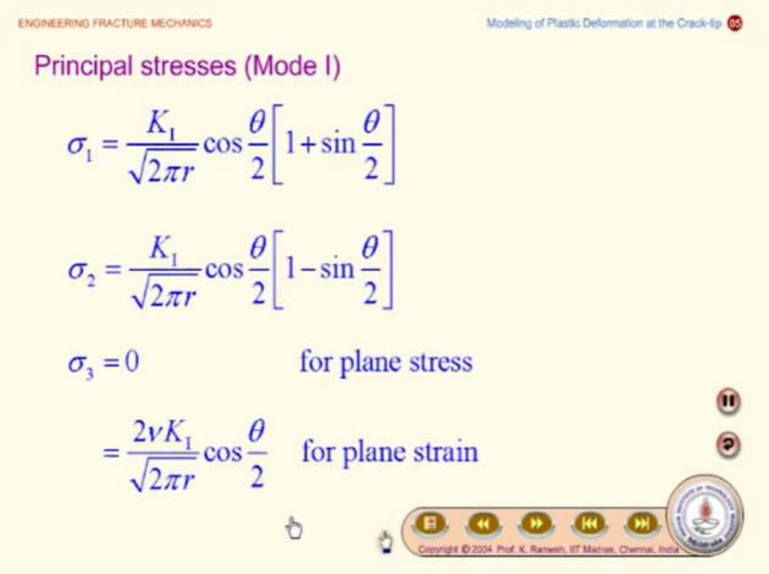


So, what are the methods available for evaluating plastic zone it is very quiet difficult to give a proper description of plastic zone shape and size. But whatever we are going to look at right now a very simplistic and crude way of looking at it let us see one by one. In all the models to simplify the analysis usually the material is assumed to be elastic-perfectly plastic. In the presence of plastic zone at the crack tip the stiffness of the component decreases this study we have seen that is the compliance increases. To incorporate the effect of plasticity in the fracture analysis the crack is mathematically modeled to be longer than the actual length. So, this is one way of addressing the plastic zone size so you include the plastic zone region also as the crack length. So, integrate that region and then calculate as the a so are a +Δ, something like that so you integrate the plastic zone a head of the crack tip that is one way of looking at it.

So now what are the other points, a quick but a very crude approach to find the extent of plastic zone along the crack axis is by simply finding the point at which one of the yield criteria is satisfied. So, we have seen at least yield criteria we have seen one is von mises and another is tresca yield criterion. So, one of this yield criteria we can use and try to get the you know yield stress and then proceed with the plastic zone size calculation.

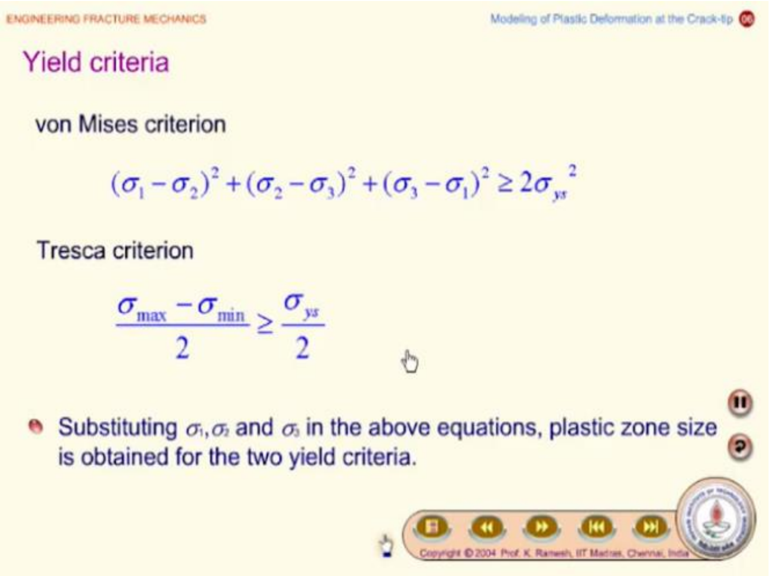
This kind of method is useful to compare relatively the plastic zone size in a plane stress and a plane strain condition. So where is the starting point? so we know that the very near tip stress yield under mode 1 is given by this stress field equation that is we know right.

(Refer Slide Time: 11:32)



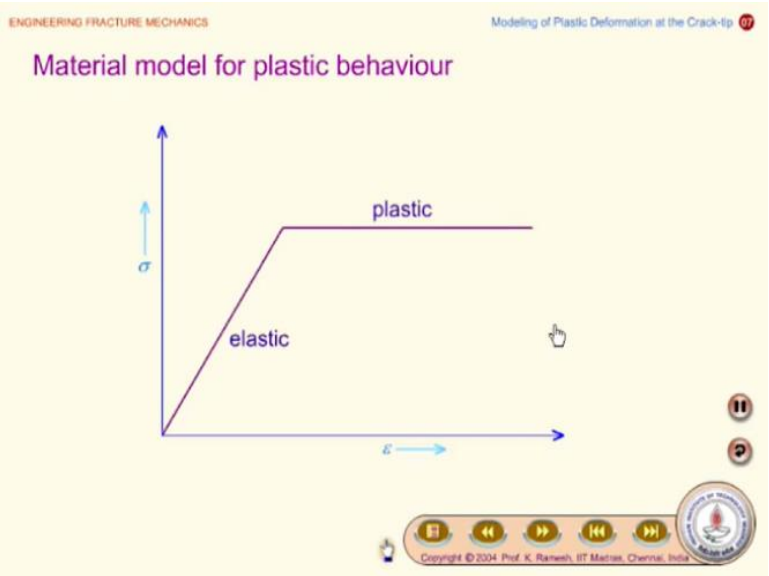
So from this we can take the principal stress for mode 1, $\sigma_1 = \frac{K_1}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left[1 + sin\frac{\theta}{2}\right]$,so this is you can derive it from the stress equation. So similarly, for σ_2, $\sigma_2 = \frac{K_1}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left[1 - sin\frac{\theta}{2}\right]$. And σ_3 is 0, for the plane stress and the plane strain again we have the expression. So, you should remember this for a plane state the stress matrix or stress tensor is 3X3 matrix. So, σ_3 is $\sigma_3 = \frac{2\upsilon K_1}{\sqrt{2\pi r}} cos\frac{\theta}{2}$. So, this something which you have to take it from the stress field equations.

(Refer Slide Time: 12:35)



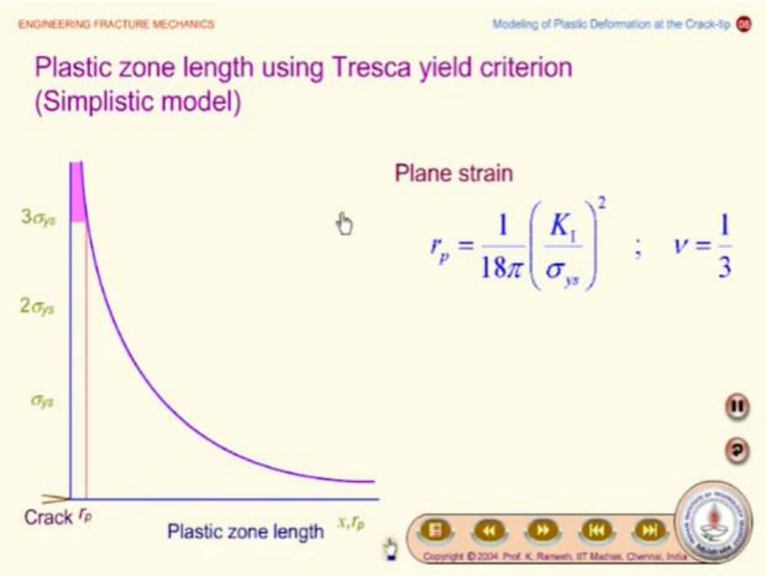
Now we have yield criteria first one is criteria this we all know σ_1 like this we all know$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2\ \sigma_{ys}^2$. So, this is von mises and tresca slightly different it is $\frac{\sigma_{max} - \sigma_{min}}{2} \geq \frac{\sigma_{ys}}{2}$, Substituting σ_1 σ_2 and σ_3 in the both equation plastic zone size is obtained for two yield criteria. So, both of them can be utilized but we will try to use one of them and then try to find this plastic zone.

(refer Slide Time: 13:22)



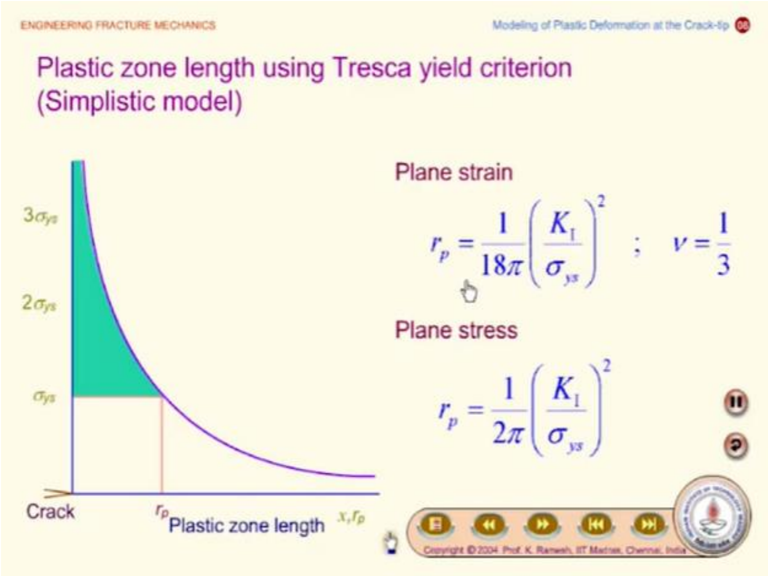
So, what is the material model we are considering a material model to be elastic perfectly plastic so that is how the materials is supposed to behave as per the, this model is concerned.

(refer Slide Time: 13:37)



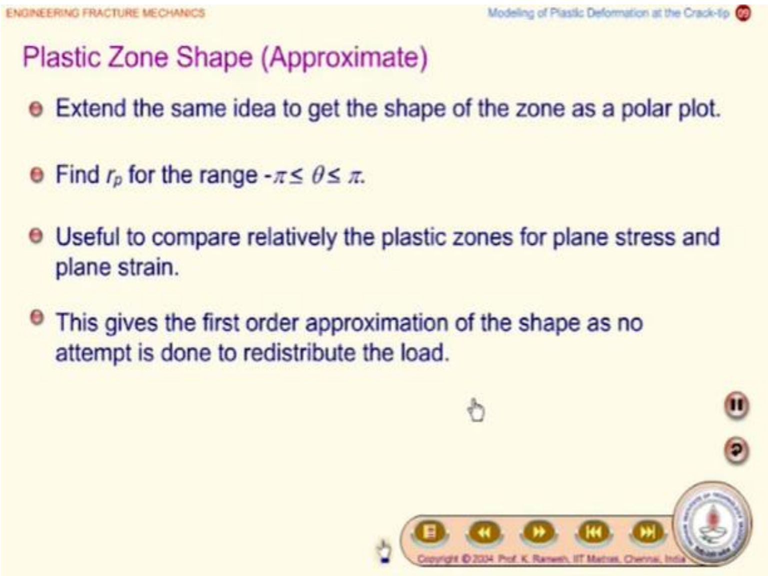
So if we try to calculate the plastic zone length using Tresca yield criterion, which is simplistic model you see this σ versus r plot. So, the σ_y is going to vary like this so for a plane strain if we use the Tresca criteria we can simply mark this r_p from here and then you can take it as the 3 σ_{ys}. So how do we take this? Because in a plane strain condition the σ_3 we can use it from the elastic you know the 3-dimensional Hooks law which is you know σ_3 =μ times σ_1 and σ_2 and then this can be substituted directly in this Tesca yield criterion then you see that the yield stress is three times the σ_{ys} that is what the Tresca yield criteria predicts. So, you can just simply mark this and that is considered as r_P so this is the plastic zone size according to this if you assume python ratio as 1/3, so that we have to remember. So then you get is $r_p = \frac{1}{18\pi}\left(\frac{K_1}{\sigma_{ys}}\right)^2 \; ; \; \upsilon = \frac{1}{3}$, So this is a plastic zone size r_p under the plane strain condition under the Tresca yield criteria.

(Refer Slide Time: 15:14)



And then if you what will you get for a the plane stress condition σ_3 is 0 so the σ is σ_{ys} so this is given by this r_p. So the plane stress condition the plastic zone $r_p = \frac{1}{2\pi}\left(\frac{K_1}{\sigma_{ys}}\right)^2$. So, you can see that you know the plastic zone size in r_p is much larger than the plastic zone size in the plane strain condition. So, these are all very crude and a simplistic value but nevertheless it give an idea of the magnitude of the r_p in a plane strain conditions versus plane stress condition. So that is the merit of this kind of analysis.

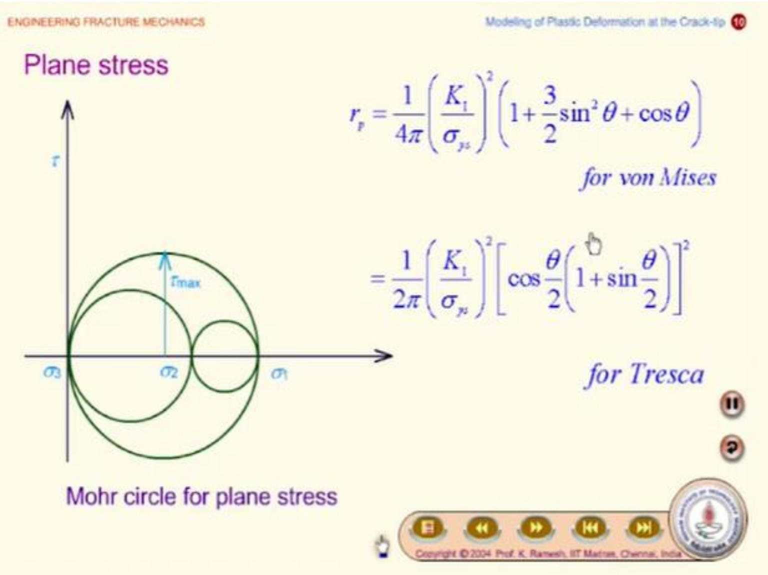
(Refer Slide Time: 16:13)



So how do we understand the shape it is very quite difficult that is why it is written as approximate. Extend the same idea to get the shape of the zone as the polar plot so we that we can represent r_p in the range of π, I mean θ can be expressed in between -π to +π and useful compare relatively the plastic zones for the plane stress and plane strain. And this gives the first order approximation of the shape as no attempt is done to redistribute the load.

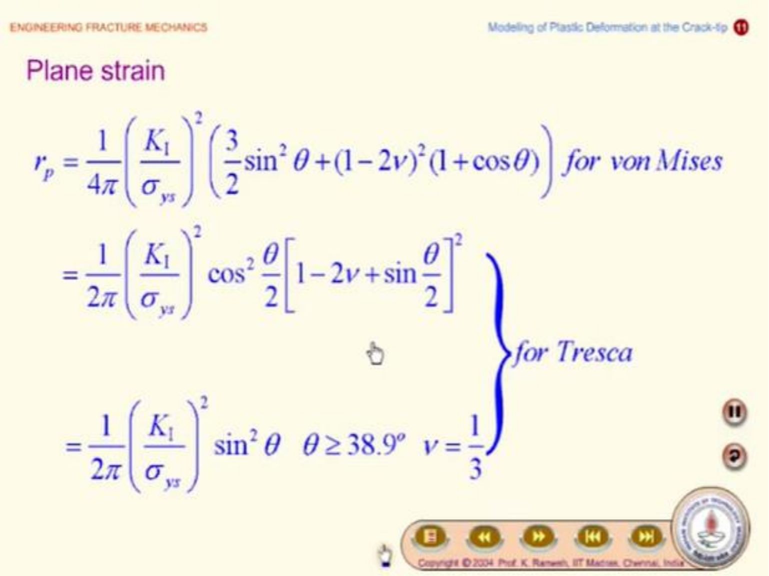
So in this simplistic assumption it is considered a first order approximation because there is no redistribution of the load is done which we will do it in later stage but let us see how this shape is obtained from this.

(Refer Slide Time: 17:06)



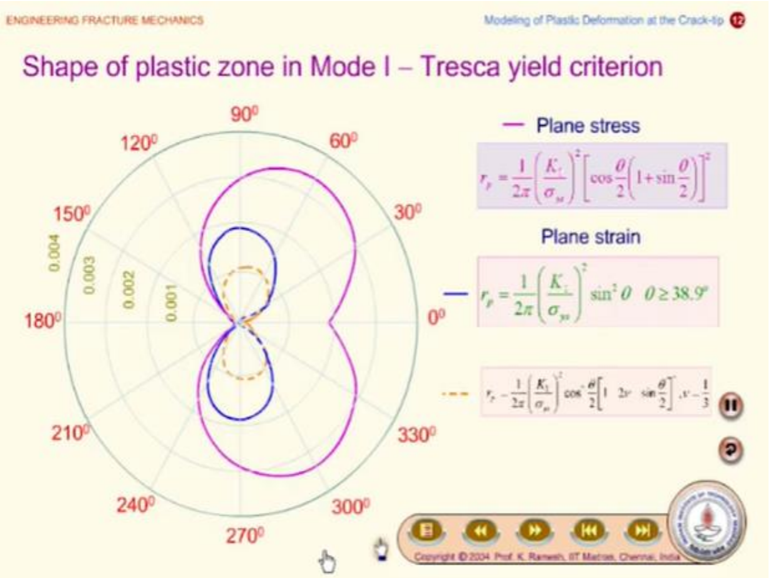
So we know that you know for a plane stress condition we can use this you know Mohr circle and r_p when is written as r, θ. Then it will be of this form that is $r_p = \frac{1}{4\pi}\left(\frac{K_1}{\sigma_{ys}}\right)^2 \left(1 + \frac{3}{2} sin^2\theta + cos\theta\right)$, when we use von mises criteria. And then you get this expression $r_p = \frac{1}{2\pi}\left(\frac{K_1}{\sigma_{ys}}\right)^2 \left[cos\frac{\theta}{2}\left(1 + sin\frac{\theta}{2}\right)\right]^2$, when we use tresca criterion.

(Refer Slide Time: 17:54)



So, we can use this expression to plot for applying plain strain condition this is again this formula is used and if you use a Tresca yield criterion then this is the formula.

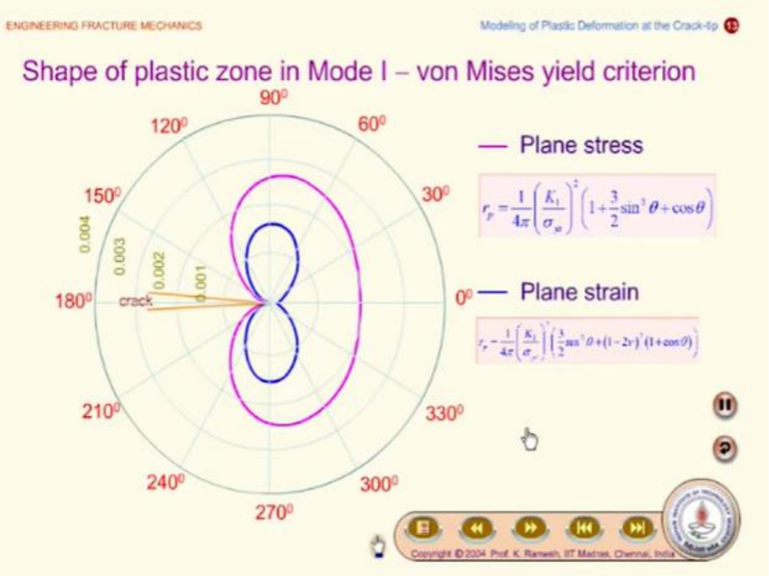
(Refer Slide Time: 18:20)



So we can try to use these expression to plot by giving a different values of theta and first we will see that Tresca yield criterion in the plane stress equation if you use this equation. Then you get this kind of a shape so this is a very crude method of doing it but it gives some approximate shape and for the plane strain this equation is used and then used this kind of small size as compared to the previous one.

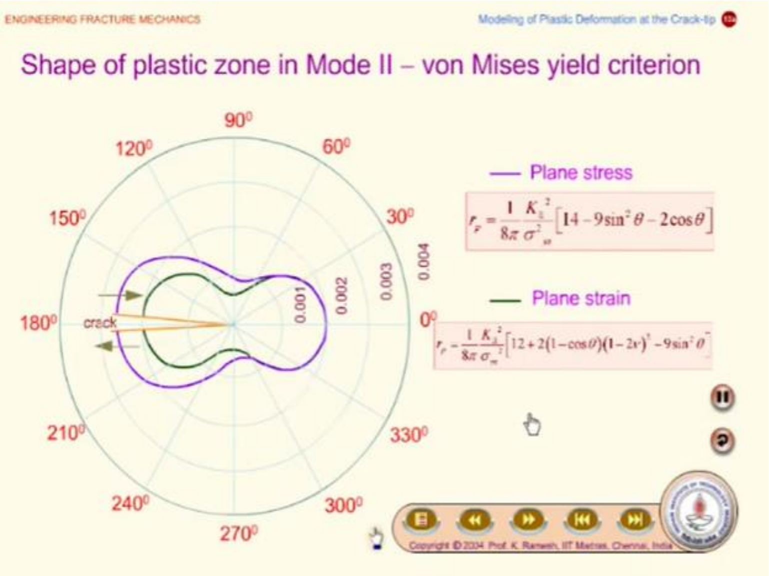
So, that it gives some idea about the you know these two are compared the plan stress or plane strain conditions are compared.

(Refer Slide Time: 19:09)



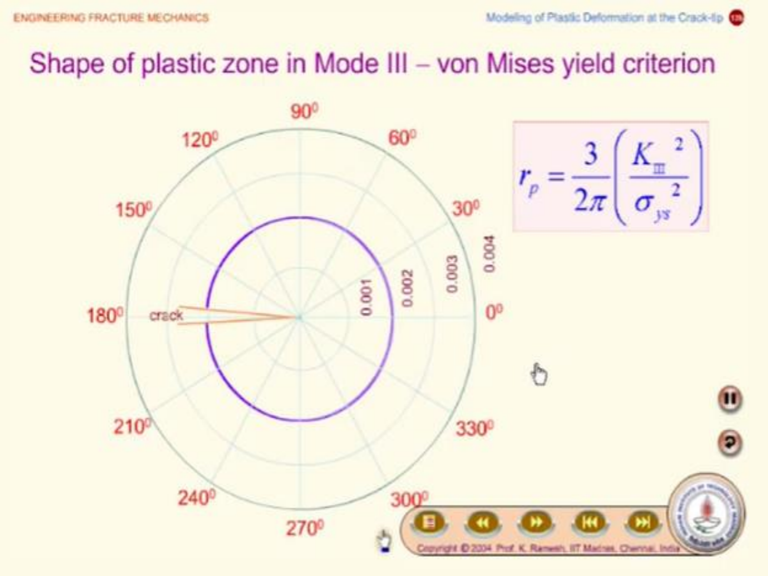
So this is for the von mises criteria so similar plot is obtained when we use this expression. So this gives some visual idea about how the plastic zone at the character base is predicted using this criteria.

(Refer Slide Time: 19:40)



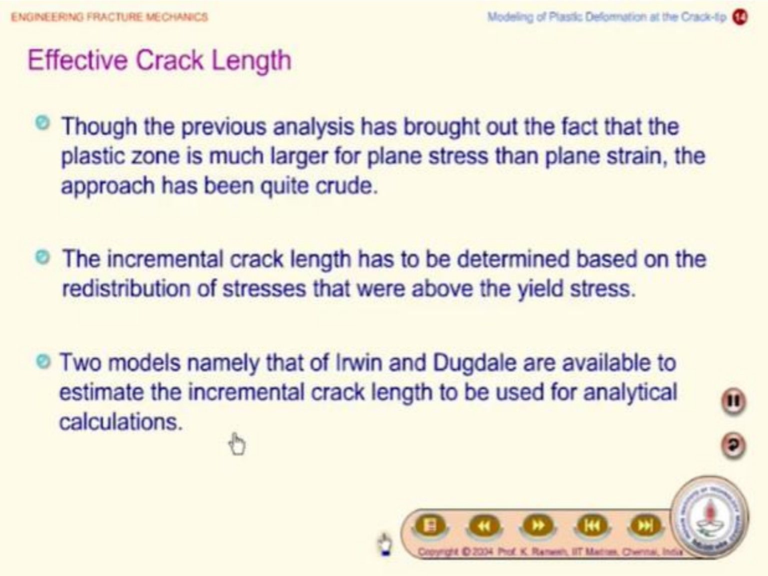
So this is for just to comparison for more to we have not looked at the stress intensity factor for mode 2. But nevertheless, you can just refer the Professor Ramesh Ghosh or any other textbooks just for completion but the interest of time I am getting into almost. But nevertheless it is all same but you can look at the plastic zones size for the mode 2.

(Refer Slide Time: 20:12)



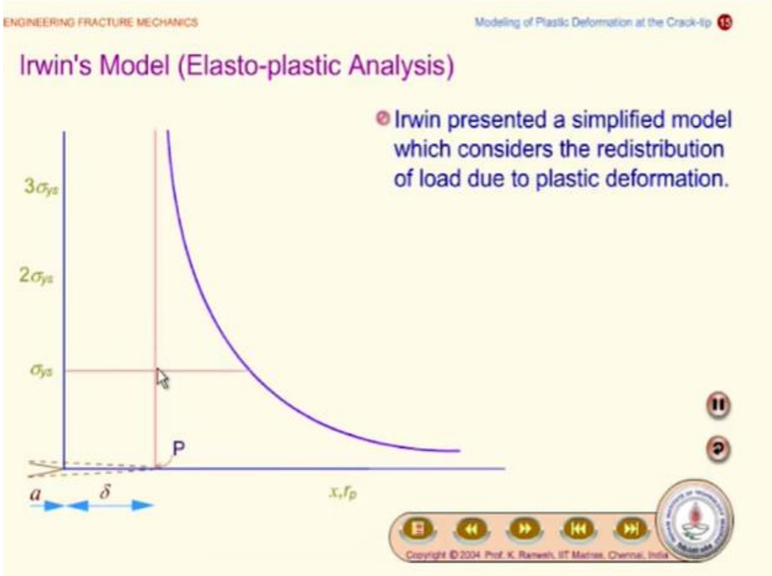
As well as we will see for mode 3 so which is quite illustrative to look at the plastic zone size which is given by this two yield criterions. Even though it is approximate but it is some idea.

(Refer Slide Time: 20:35)



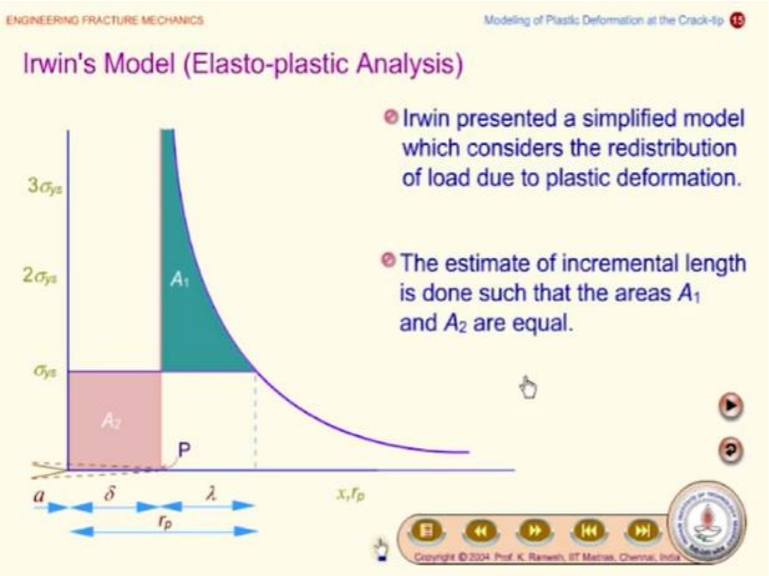
So now the problem is to calculate the effective crack length how do we assess the effective crack length. Though the previous analysis has brought out the fact that the plastic zone is much larger for a plain stress than the plane strain, they approach has been quite crude. So, what do we do? They incremental crack length has to be determined based on the redistribution of stresses that were above the yield stress. So, this is something you have to understand so all the previous models do not consider the redistribution of stresses and twp models namely that of Irwin and Dugdale are available to estimate the incremental crack length to be used for analytical calculations. So very important models for plastic zone size calculations we will see one by one.

(Refer Slide Time: 21:29)



The first one is Irwin model which is also considered as elastoplastic analysis. Irwin presented a simplified model which considers the redistribution of load due to plastic deformation. So how the redistribution of model is done just look at this plot, the same plot σ versus r $_p$. And if you look at this where the yield stress is considered.

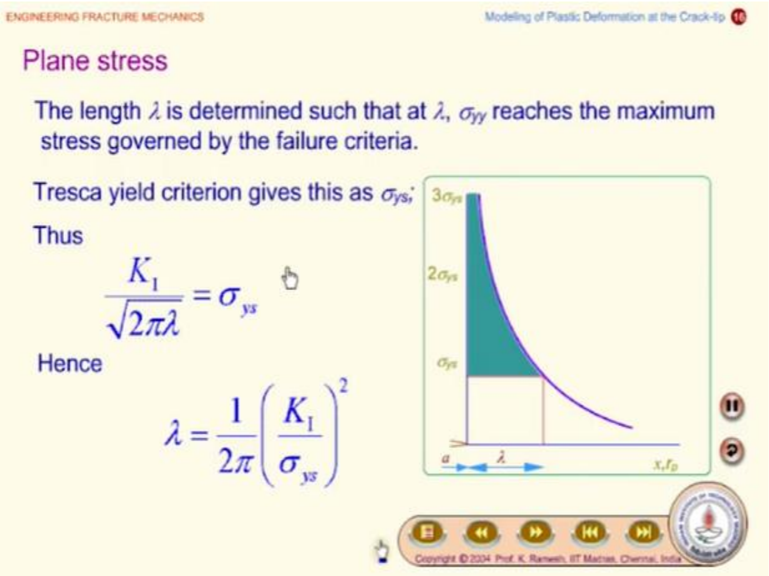
(Refer Slide Time: 22:05)



I will just, I want to replay this because I want to stop this first and then so I will just stop here then this is the, you know the load which is above the yield point which is not being considered in the previous analysis. So, this is the load which we are trying to redistribute and then try to recalculate the plastic zone length, so what happens? So that is how it is done so what is the methodology are been followed this kind of analysis which is shown in this.

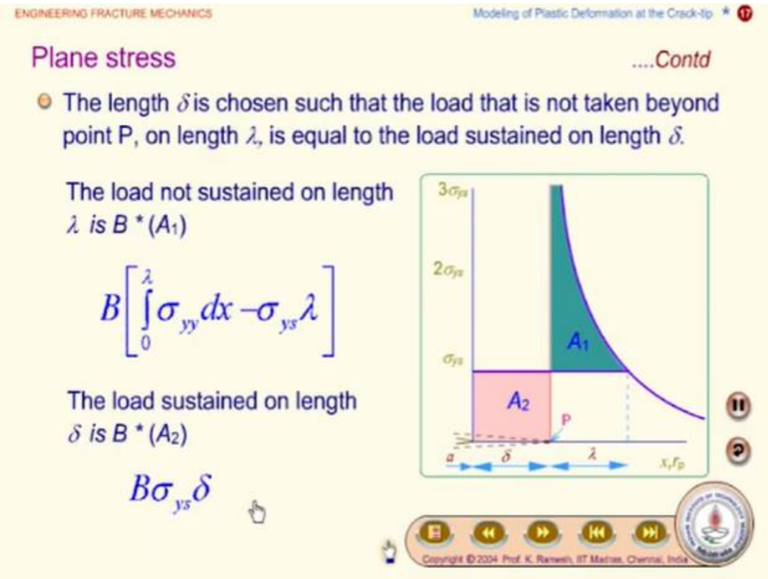
So the σ_y variation is just moved and what is shown here is he assumed that the estimate of incremental length is done such that the areas a $_1$ and a $_2$ are equal. So this is already done the one which is ignored previously is now considered and make sure that this area and this area are equal. So that is the physics, basically so this is crack length a and this is δ and from here to here it is λ. So the total $\delta + \lambda$ is r $_p$, here so that is the assumption by the Irwin's model

(Refer Slide Time: 23:42)



So now we have to do the mathematics the length λ is determined such that at λ σ $_y$ by reaches the maximum stress governed by the failure criteria. So Tresca yield criteria gives as σ_{ys} and we know that now $k_1/2\pi\ \lambda = \sigma_{ys}$. This is according to Tresca yield criteria hence we can write expression for λ from this $\lambda = \frac{1}{2\pi}\left(\frac{K_1}{\sigma_{ys}}\right)^2$. So, this is one step and then from there we can just move to calculate the δ.

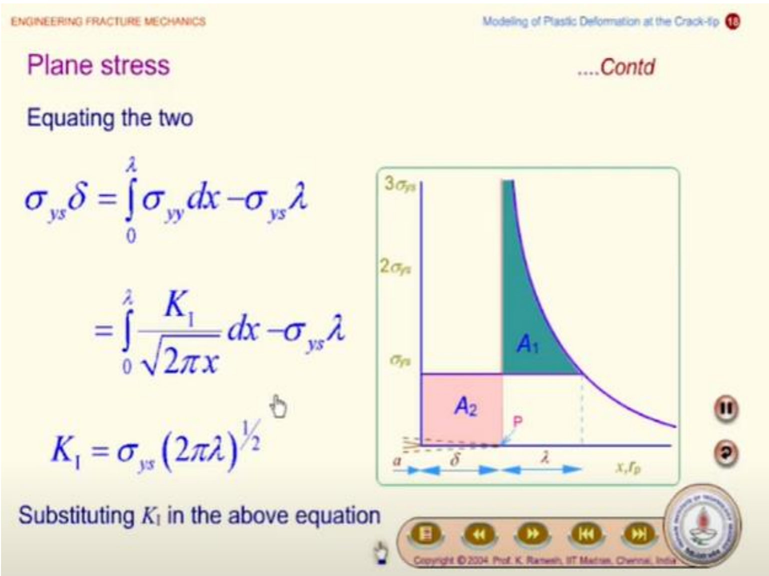
(Refer Slide Time: 24:27)



The length δ is chosen such that the load that is not taken beyond point p on the length λ is equal to the load sustained on the δ. So, this is what, it is so this one and this one. The load not sustained on length λ what is that? That is nothing but b* A_1, A_1 is the area under this curve B is the thickness of the specimen so you get this for the full load so B$\left[\int_{\delta}^{\lambda} \sigma_{xx} dx - \sigma_{yy} dy \right]$. So, here is a catch whether it has to be limits which are given here is

confusing if you write it like this. But it is not δ to λ it is 0 to λ. that is why it is we have to be careful. We are interested to integrate this value of the so it is should be o to λ not δ to λ. So you integral to find the integral value of this the load sustained on the length is B into A_2. And B σ_{ys} δ this is from this so this δ this is σ_{ys}.

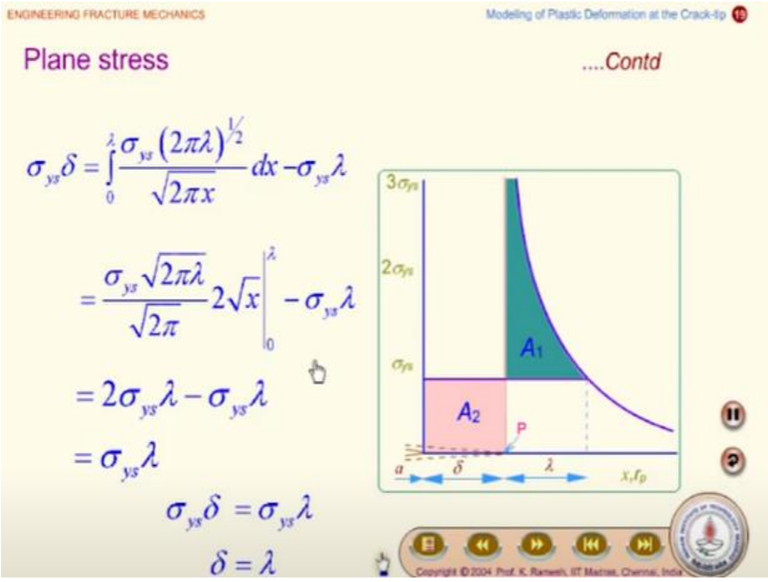
(Refer Slide Time: 26:16)



So B into δ $_{ys}$ is the load so now we have to equalize this two and then what we get. That is what you will see so this animation keep coming to give us a reference so, that we will not miss the any mathematical step. So $\sigma_{ys}\delta = \int_0^{\lambda} \frac{K_1}{\sqrt{2\pi x}} dx - \sigma_{ys}\lambda$, which can be given by this.

Because we know how the value of λ and σ yy is given by this equation. I mean yield criterion so we can substitute this and then we get the expression for K 1 which is equal to $\sigma_{ys}(2\pi\lambda)^{1/2}$. Substituting K_1 with above equation again.

(Refer Slide Time: 27:19)

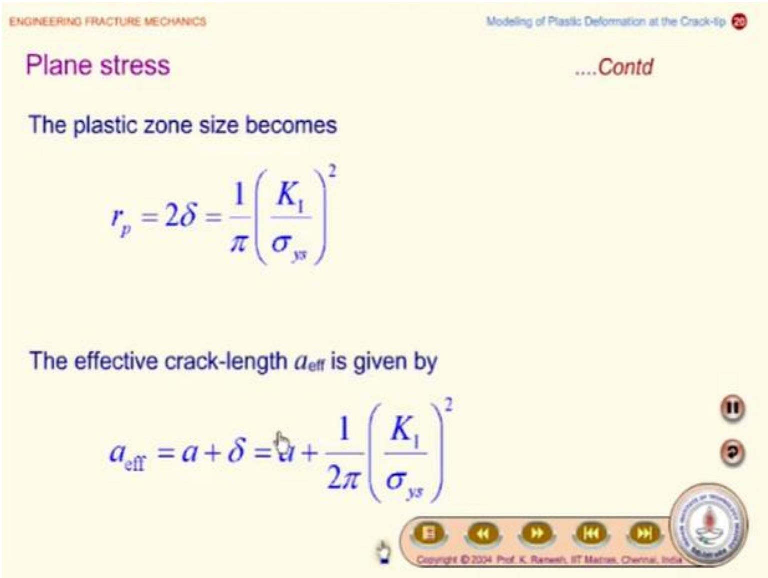


We will get expression to solve δ and λ of course so $\sigma_{ys}\delta = \int_0^{\lambda} \frac{(2\pi\lambda)^{1/2}}{\sqrt{2\pi x}} dx - \sigma_{ys}\lambda$, Then

you can just rewrite this in this form then what you get is 2 σ_{ys} λ – σ_{ys} λ this is straight forward. So it becomes σ_{ys} λ and σ_{ys} δ = σ_{ys} λ so δ = λ.

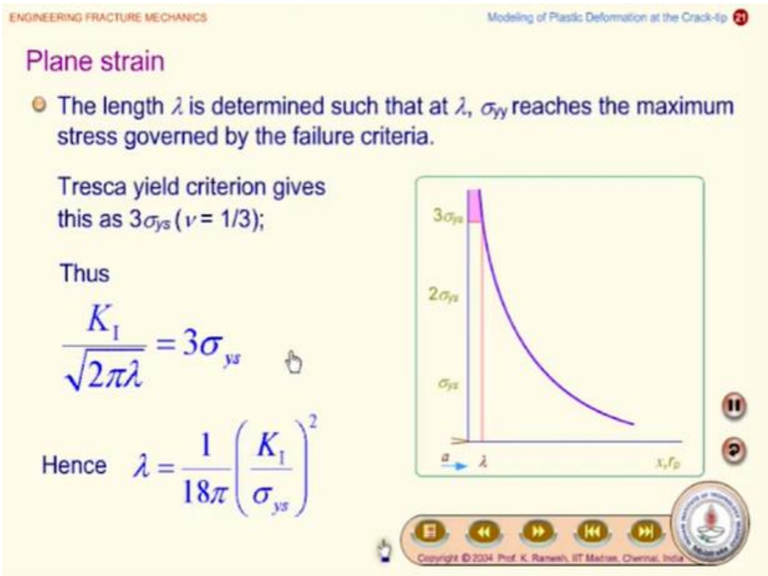
So this is a very interesting result we are getting so what is the plastic zone size we will get after this.

(Refer Slide Time: 28:13)



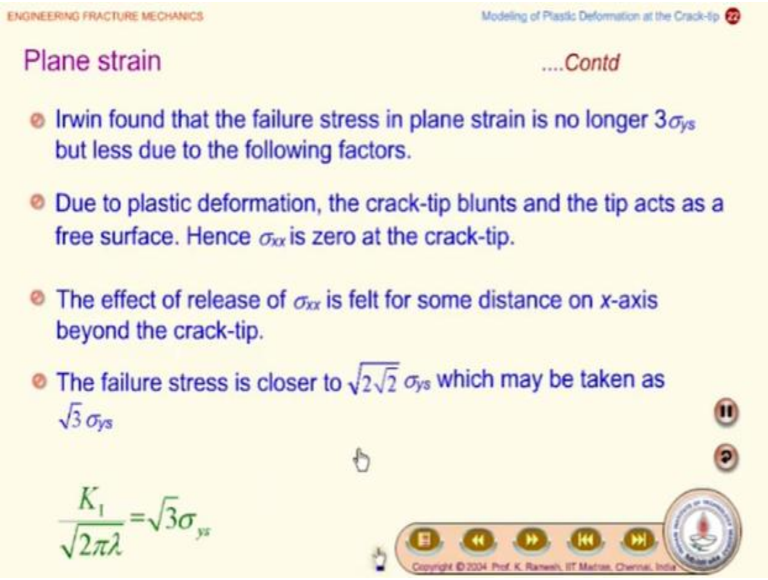
So the plastic zone size becomes $r_p = 2\delta$ which is equal to $1/\pi(k_1/\sigma_{ys})^2$. So the effective crack length a effective is given by a + δ which is nothing but $(a + 1/2\pi)(k_1/\sigma_{ys})^2$. So this is very you know a very useful information because this Irwin's considers the redistribution of the load. So the effective crack length you know which is very important in calculating the stress intensity fact has been arrived by first approximation.

(Refer Slide Time: 28:55)



I mean, the first model given by Irwin the length λ is determined such that at λ σ_{yy}, which is the maximum stress governed by the failure criteria. The Tresca yield criteria, give this has 2 σ_{ys} like this we have seen earlier. So we can work it out like this $k_1/\sqrt{2\pi\lambda} = 3$ and this is already known. We have seen this already.

(Refer Slide Time: 29:30)



Irwin found that the failure stress in the plane stress is no longer 3 σ_{ys} but less due to the following factors. Basically, it is correcting this simplistic model due to plastic deformation the crack tip bends and the tip acts as the free surface hence σ_{xx} is 0 at the character. So this is this point we have already seen when the crack tip is you know blunt by plastic zone then σ_{xx} is 0.

The effect of release of σ_{xs} is felt for some distance on an x axis beyond the character this also we have just seen before, while we were looking you know selection of fracture toughness. I showed that in the schematic the σ_{xx} is 0 at that you know blunt region and then after certain region it is not 0 something like that. The failure stress is closure to square root of $\sqrt{2}\sqrt{2}$ σ_{ys} which may be taken as $\sqrt{3}$ σ_{xx}.

Instead of 3 σ_{ys} into square root of 3 σ, I mean square s we can take it like that as proposed by the Irwin corrections for the simplistic model