Mechanical Behavior of Materials
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Lecture No 52
Fracture Mechanics - IV

(Video Time: 00:18)

Hello, I am professor S. Sankaran in the Department of Metallurgical and Materials Engineering. How it was verified. So, Griffith carried out a series of experiments on a glass tubes and the spherical vessels subjected to internal pressure, p. These were pre-cracked and then annealed to eliminate residual stresses, if any and estimated $\sigma\sqrt{a}$ as 0.25-0.28 MPa √m.

So, this is one set of value, he considered that the surface tension of the glass is a linear function of temperature and extrapolated surface tension values of the glass fibers between 1110 ^{o}C and 745 ^{o}C to room temperature and he considered E for a glass is about 62 GPa and surface energy of glass γ_{glass} is 0.54 N/m. This analysis gives 0.15 MPa √m. So, this is the right hand side of this expression.

So, what we have seen in the previous slide is the left hand side of the expression. Though this value is not exactly the same, but there exists a correlation. Correlation between what? What is given that is surface energy and the crack length. So, the stress required for the incremental crack growth has been established by this equation and also verified by Griffith by some experiments. Though the values are not exactly the same but there is a good correlation that exists, that means the idea is good to carry it further.

So, we can now give the definition of the energy release rate which is G_1 that means mode one. It is a crack driving force for a mode one which is equal to

$$G_I = \frac{\partial U_a}{\partial (2a)} = \frac{\pi \sigma^2 a}{E}$$

Energy release rate is energy released per unit extension of crack front per unit thickness of the body per crack tip. So, this is also very important idea. We are looking at you know energy I mean energy per unit thickness of the body per crack tip.

So, central crack means you have a two tips. So, this energy is per tip, this is very important ok. The unit is Joules/m^2 or N/m also called as crack driving force per unit extension and rate has no reference to time in this expression which normally we when we see anything we call it by rate which involves a time but here there is no time reference is there. So, it is a misnomer.

And we also have another parameter in fracture mechanics called resistance which is given by

$$R = \frac{\partial U_s}{\partial (2a)} = 2\gamma_s$$

So, this is again an inherent property of a material, we have to remember the resistance R is an inherent property of any material which is related to surface energy. So, that is how you have to remember now. If you recall we have already seen that you know in the earlier classes we derived an expression for the theoretical strength of the material.

We looked at some expression for theoretical strengths that is what we have just found out and the theoretical strength is always you know very high as compared to the experimental values and then we came to an understanding that the dislocations are responsible for this kind of a lower sigma value or yield strength value as compared to theoretical. This is what we have seen in the earlier lectures.

And we concluded it is because of dislocation in this analysis also Griffith have assumed that that flaw whatever the dislocation we have just assumed in that theoretical evaluation, here Griffith has assumed that as a flaw size basically in the brittle material. So, he assumed that factor that the dislocation factor here has a flaw here which is nothing but a crack length in the brittle solid.

So, why I am saying that here because where we looked at the; inherent resistance of the material and which is also related to the flaw inside the material. So, here also you will have a characteristic resistance of each material which is related to a surface energy that is how we will look at it. So, G in terms of change in potential energy this is another important way of representing the G in the fracture mechanics.

So, better to keep a note on that, consider the case of an incremental increase in the crack area that is ΔA. To cause this crack growth, ΔW_{ext} external is done by the external forces and the strain energy within the body of the component changes by ΔU. Then the available energy, $G_1 \Delta A$, provides the energy balance as a follows. So,

$$G_1 \Delta A = \Delta W_{ext} - \Delta U.$$

So, this is a one of the, you know basic expressions you know given by the strength of materials concept as well as in fracture mechanics.

So, we are trying to look at the $\Delta W_{ext} - \Delta U$ is also considered as a potential energy. So, to try to write G in terms of potential energy this is considered. So, dividing the above equation by ΔA and taking the limit $\Delta A \rightarrow 0$ this is very important step and then you get an expression

$$G_{\mathrm{I}} = -\frac{d}{dA}\left(U - W_{ext}\right)$$

$\Pi = U - W_{ext}$ is commonly known as potential energy. This is the definition given in the strength of materials. So, you just write that expression

$$G_{\mathrm{I}} = -\frac{d\Pi}{dA}$$

this is in terms of potential energy. This equation is quite powerful to evaluate the energy release rate of a system. G_1 is always positive. So, remember this quantity is always positive G_1 is positive for a crack studied for probable growth. So, what is ΔA? ΔA is nothing but $B\,\Delta a$, where B is thickness of the specimen, Δa is incremental crack length.

So, it gives a volume therefore the above equation can be written as

$$G_{\mathrm{I}} = -\frac{1}{B}\frac{d\Pi}{da}$$

we can write this expression for a constant load and constant displacement. So, we have known this expression before. So, you just put that

$$\Delta\Pi = -\frac{1}{2}P\,dv = -\Delta U$$

for a constant displacement again we know this expression already which is ΔU. we can write it like this

$$G_I = \frac{1}{B}\left|\frac{dU}{da}\right|$$

and this is for the constant load from the energy.

These are the two energy release rate expression for constant load and constant displacement. So, this we have already seen. So, just for the, you know recollecting the conceptual idea, how we looked at. So, this is just for recollecting you know why we got this expression. So, you can look at it again. So, you get the strain energy,

$$\Delta U = \frac{1}{2}P_1\,dv$$

and the external work done is

$$\Delta W_{ext} = P_1\,dv$$

So, the $\Delta\Pi = -\Delta U$.

So, you can get it like this and similarly for a constant displacement again, you will see this good idea to recall this very nice and important concepts of calculating the strain energy. We have all already all this point we have already seen just to apply the final expression and you get the G_1 in terms of ΔU under the constant displacement. So, you get here as

$$\Delta U = \frac{1}{2}v_1\,\mathrm{d}P$$

and

$$\Delta\Pi = \frac{1}{2}v\,\mathrm{d}P = \Delta U$$

which is nothing but ΔU. So, this is already known.

You see we will also look at this energy release rate by compliance approach. Compliance why do we do that? we because in fracture mechanics, keeping the mathematics and not only mathematics, but also in experiments keep it simple people have recommended compliance approach will be better off. So, just look just to have an idea about what is this. Stiffness of the component decreases with increasing the crack length this we have seen.

Compliance (C) is the inverse of stiffness (k) this also we know. In fracture mechanics it is easier to deal with compliance. For a general case of loading, load displacement relation is given by $P = k\,v$, where P is applied load, k is stiffness and v is a displacement. And the above equation can be written as

$$\boldsymbol{v = \frac{1}{k} P}$$

Or

$$\boldsymbol{v = C\,P}\ .$$

So, this is a simple compliance equation and then we will now use this equation to write this immediately state.

The problem is solved for a two extreme cases: one is constant load in which the displacement of the load point increases as the crack grows and the another is constant displacement where the load at the load point decreases as the crack grows. These two already we have looked at even in the experiment. But in compliance approach, we will write it like this
delta phi is equal to delta U minus delta W external.

So, we can rewrite this expressions like this and we know this now sorry it is me, let me replace this it is going very fast so. Yeah, for $\Delta\Pi$ we have this expression and which is, this is what we are to write and v can be written like this.

$$\begin{aligned} \Delta\Pi &= \Delta U - \Delta W_{\text{ext}} \\ &= \frac{1}{2} P dv - P dv \\ &= -\frac{1}{2} P dv \end{aligned} \qquad \begin{aligned} v &= CP \\ dv &= C\cancelto{0}{dP} + PdC \\ dv &= PdC \end{aligned}$$

So, we have to replace dv by this expression. So, since it is a constant load. So, this term becomes zero, then $dv = PdC$ and then this is becoming like this.

So,

$$\Delta\Pi = -\frac{1}{2} P\, P\, dC$$
$$= -\frac{1}{2} P^2 dC$$

that is a first idea. So, the G_1 is written as.

$$G_{\mathrm{I}} = \underset{\Delta A \to 0}{Lt} \frac{\Delta\Pi}{\Delta A}$$
$$\Delta A = B da$$

$$G_{\mathrm{I}} = \frac{P^2}{2B} \frac{dC}{da}$$

So, this is a expression for strain energy release rate in terms of compliance. So, for the constant displacement also you should get the same expression because we have seen that the energy available for the crack extension is the same. So, you get this expression again a very simple substitution again.

So, there it is a *dv* here it is a *dP*. So, for a constant displacement $dv = 0$, this becomes zero that means you will get interdependent relation here, then it becomes, yeah $CdP = -PdC$ and then you can substitute this *dP* into this and the $v = CP$, then you get the same expression

$$G_{\mathrm{I}} = \frac{P^2}{2B} \frac{dC}{da}$$

So, this is again reinforcing that experiments like whether it is a constant load or constant displacement, the energy available for the crack extension to grow is the same. So that is very nicely put forward by this compliance concept as well.

So, now we will move to the important concept, I will not go into some other examples now. I will now move on to, ok, we will now get into some new topic involving energy release rate necessary and sufficient conditions for fracture. What is this? $G_1 = R_1$ is a necessary condition. what is R_1? It is a resistance of, for any given material which is inherent related to surface energy this is what we are seeing.

So, this is the first condition for the crack to grow that is $G_1 = R_1$. It is a necessary condition for a brittle material equation 1 is also a sufficient condition. So in the mathematical formulation, we have these kind of conditions one is necessary condition another is sufficient condition and in brittle solids, this equation itself serves as both necessary and sufficient condition for the crack growth.

We will see how it is demonstrated in the coming slides. For instability occurs when

$$\frac{\partial G}{\partial a} = \frac{\partial R}{\partial a}$$

this is a called sufficient condition. it is not just you know $G_1 = R_1$, their derivative also should be equal then only the fracture instability occurs. So, this is called a sufficient condition. Facture of high strength ductile material is possible only when both the conditions are satisfied.

So, this is very, very important concept or idea for a, we are going to talk about a stable crack growth and R concept. So, for that this particular idea is very, very important. So, the graphical representation of condition of fracture for a brittle material is shown in this slide. So, critical stress or a crack length can be determined by this expression which we already derived. And for a given stress fracture occurs when crack length a reaches a_c. This is what the relaxation analogy has taught us.

So, you will go back to that plot again we recall that plot as I mentioned we will look at this again. So, the relaxation analogy has given us a concept of critical crack length where you can see that. And now we will extend this idea to a graphical representation again of a condition of fracture for a brittle material. How are we going to actually extend that idea, the critical crack length idea. We are going to now see whether these increase or the crack growth, whether it obeys both you know necessary condition and sufficient condition and so on.

So, what is plotted here is energy versus a crack length, this is an actual initial crack length and this is an incremental crack growth as the crack growth happens. So, what is shown here is energy in the y axis and this line is $\gamma_{critical}$ which is R the resistance. The resistance is plotted here for this which is very constant for a given material. So, you have this cracked initial flaw in the material of length a_1 and then you try to you know try to grow.

So, what does that mean, I mean when the crack length is a_1 and then they apply the stresses of this much that means no crack initiation will take place because it is much below the resistance level. So, we need to reach $G_1 = R_{1,}$ unless it reaches this point the crack will not grow. So, that is why, then only it will satisfy the necessary condition. So, that is what is shown here. So, the now the stress is increased to the next level. Even now the σ is less than R.

So, the fracture does not occur in this region and the moment you reaches this point that is σ_{c1} that is σ critical. Fracture initiates from here, that means G_1 is becoming equal to R_1 the crack initiates from this point and what you are seeing here is, this is for the constant load and this is for the fixed grips. So, what is the difference between these two? So, for this the crack to grow the energy comes from the external load. But, for this the energy comes within the system the strain energy within the system.

So, that you have to remember. The another important point to keep in mind is you see that as the crack length grows as the length grows the energy available for the crack to move is significantly increasing. That means it will propel the crack growth that means the velocity the crack growth velocity will be significantly increased. So, as it grows, that is also another important point to keep in mind.

Under fixed grips, energy release rate decreases as the crack length advances. So, for a given crack length a, fracture occurs when σ reaches σ_c. Longer crack requires less σ for a fracture. So, this is what is shown here. So, for a_1 this kind of a stress for a_2 then again it becomes $\sigma_{c,}$ I mean σ_{c2} and this is σ_{c1} it is what is shown here. So, this is for a fixed grip and this is for a constant load.

For both constant load and fixed grips, the condition for the onset of crack growth is the same. At the point of crack instability, the energy release rate is same in both cases. So, this is also true. Point of instability also the energy release rate is there. So, we are now talking about at the initiation point alone, we are not talking anything beyond that. At what stage, the crack would initiate. So, that is the necessary condition we are discussing.

In fixed grips, since external load does no work, the energy release rate decreases as the crack advances. So, this is quite obvious. The energy comes within the system. So, it comes down. For further discussion, let us consider the constant load case the increase of energy availability leads to increase in the crack velocity. I just mentioned in the previous slide, because as the crack length advances the energy available is also significantly increasing.

So, then that creates or that facilitates the crack velocity and beyond a certain velocity crack branching can occur. You see in the introduction of fracture mechanics slide, I just showed you that in a shadow graph, a crack will accelerate and then it goes and stops beyond the speed and then it try to branches. So, that is also explained by this energy concept you will see now. I will just recall that plot just another two slides.

So, what is shown in this plot, this is å that is m/s crack growth velocity versus K_1 stress intensity factor. So, the curve is like going in a shape of a γ that is why it is called a γ curve. You should not confuse this with the surface energy. So, what happens is, this is for the Homolite 100 material and what is being estimated here is the start of branching the crack, branching is starting from here and then it goes up to this point.

So, the crack branching is experimentally observed and like I just mentioned in the previous slides, this is shown by Canto. So, the shadow graph clearly shows that the crack branching is indeed possible, because the crack cannot grow or crack cannot travel beyond the Rayleigh wave speed and then once it reaches a critical speed, it try to branches out to dissipate the energy, kinetic energy.

So, how do we support this argument with the energy release rate concept that is what we are going to see. So, energy release rate has given some idea by simple model, a schematic, these are all schematic but then what you have to appreciate is the conceptual you know understanding comes better by this kind of schematic, this important idea. So, we will now look at the simplified model of crack branching based on energy approach that crack branching without considering the kinetic energy.

Please understand that this is very important and also we have to remember that this is an infinite plate and it is the central crack is growing without any boundary that is also very important. So, it is kind of a fictitious experiment just to appreciate the energy release rate concept. So, now let us see what happens. So, now the crack of various, I mean crack length a, this is a critical crack length that is why it is called a_c and this is increments in the crack length and plotted against a resistance R. So, the initial crack goes like that and then you just watch as it just crosses the energy R to 2R then the crack branching takes place and when it reaches the 3R then further crack branches is taking place and this is happening at the a constant load.

So, this one contributes to the kinetic energy of crack and the blue one, the symbolism note that the height of this line doubles and triples. Decides the number of crack branches. So, it just give the visual idea or visual impact how it happens. So, I will play this schematic once again. So, it is nicely shown that the crack attains a particular velocity, then it try to dissipate the energy like this into branches. So, 2R, 3R branches so, that the bottom line is the energy release rate is able to give some conceptual explanation of crack branching that is how we should look at it.

So the next idea is Irwin-Orowan extension of Griffith's analysis. So, I just mentioned Irwin extended the analysis of Griffith by introducing γ_p. So, this is what we are going to see how he has used the same idea of Griffith and then still he explained the energy concepts for the high strength alloys. Griffith looked at only for brittle solids, but Irwin extended that idea to high strength and ductile alloys.

In brittle materials, advancing cracks require small energies of the order of surface energies, and therefore, once a crack starts advancing, it runs through the body easily causing catastrophic failure. In most engineering materials metals and plastic etc., energy much larger than the surface energy is required to grow a crack. That is quite obvious therefore besides surface energy of the solids, some other mechanisms are operating which involve large amounts of energy.

So, this is what Irwin started arguing and Inglis showed that the stresses of the crack tips are quite large; so large that they cause anelastic deformation that is plastic deformation is referred here, this is not anelasticity which is a time dependent deformation. We are not talking here, we are essentially talking about plastic deformation in front of the crack tip. The anelastic or plastic deformation, such as plastic flowing metals, is mostly irreversible and if the stresses are released the body will not attain its original configuration near the crack tip.

So, we are now talking about the plastic zone formation under the crack tip. The energy that causes the plastic behaviour is eventually converted into heat energy and is lost to the surroundings. See most of the energy or applied stress involved in a deformation, it goes as a heat most of the energy goes as a heat only very small amount of energy is being used to actually deform the material, especially in a plastic deformation.

So, now Irwin has modified this idea and then he said that in metals, plastic deformation in the vicinity of the crack tip is caused mainly by motion and generation of dislocations, rotation of grains and grain boundaries, formation of voids, etc., all these things we have enough sufficient background to appreciate. And if the material is of low yield stress, the size of the plastic zone is large. So, for example mild steel. So, you know the size of the plastic zone near the crack tip would be very high.

A large plastic zone means that a large amount of energy is required to advance the crack tip, that means the plastic zone size is going to now decide the energy required for the track advancement. That is the modification by the Irwin. So, the overall surface energy can be written as $\gamma = \gamma_s + \gamma_p$. All the other expressions are same, exactly the same similar expressions we can use but instead of just s it should be $\gamma_s + \gamma_p$. So, γ_s surface energy posseessed by surface even it has not been subjected to any plastic deformation.

γ_p surface energy caused by the plastic deformation near the cracked surfaces. So, that is how we have to remember this idea. Now we will go to another important concept the resistance to crack growth in high strength alloys in a plain strain condition we are going to see. Again same plot energy versus change in crack length and this is thectrack length is plotted in this side and growth is plotted this side and in high strength ductile materials, resistance to crack growth does not remain constant but increases as the crack goes.

So, it is not a flat R like we have used in our previous schematic in a high-strength alloys, it grows little bit. So, that is why it is just shown in a little curved phase. So, R increases like this for a high strength alloys, the increase in the resistance is small for the plane strain case. So, now that you have to, from now onwards in fact every concept in fracture mechanics we will look at for a plane stress condition as well as plane strain.

Even in the beginning, we have just defined these two stress states, if you recall the introduction to this course we have derived a stress tensor for a plane stress and plain strain conditions. So, in fracture mechanics, every concept is explained in terms of plane stress and plane strain condition. So, this is for resistance to crack growth in high-strength alloys it is like this. So, now let me play this. So, the moment it reaches the G_1 reaches R_1 then the crack will initiate, right then you will see that, that is for the constant you know load type of thing.

So, this is how the energy versus crack growth will appear in a plain strain condition, but in a plain stress condition the idea is quite different. So, the R curve is not that shallow but here is quite steep in a plane stress condition. Increase in resistance is very steep. So the initial crack length now what happens is yeah now the crack length a_1 with this σ_1 does not start the fracture and the moment the stress reaches the G_2, it becomes critical and then yeah. So, now I will stop here and then try to understand this.

Stable fracture phenomenon is observed. What is stable fracture phenomenon? So, you see this is a crack length and it initial crack length with the different stresses the moment it reaches this point G = R, then fracture initiates then after that the crack started growing growing like if you look at this the Δa is here. So, at this point if you remove the load the crack will not grow. The crack will remain in the system without growing that is why it is called a stable crack growth. We are talking about a stable fracture is here.

So, after that it reaches critical G_c then what happens, then the catastrophic fracture continues from here. Fracture initiates at σ_2, stable still just below G_c, beyond G_c it is unstable. So, below G_c, at any point if you remove the load, the structure will not collapse or crack will not grow that is why it is called a stable fracture after G_c it becomes unstable fracture. So, crack extends from a_1 to $a_1 + \Delta a_1$ by stable fracture. At G_c, $G_1 = R_1$ and

$$\frac{\partial G_I}{\partial a} = \frac{\partial R_I}{\partial a}.$$

So, here you see it is very interesting, we are not only satisfying the necessary condition but also a sufficient condition. So, which explains the stable crack growth behaviour in the plane stress conditions of high-strength alloys. So, here the energy release rate concept is very useful in realizing the stable crack growth phenomena very, very important idea.

σ_c is not fracture strength at crack length $a_1 + \Delta a_1$ but for a_1 so, this something you know this is always a critical crack length but we should not consider Δa_1. So, a_1 remains always a critical crack length but this is, you should not add this that is the caution here. This is a serious error normally reported in the fracture mechanics calculations in this interpretation and so on.

You see for a plane stress condition if the G varies linearly with the crack length, the condition

$$\frac{\partial G_I}{\partial a} = \frac{\partial R_I}{\partial a}$$

Leads to a contradiction. Because, if you keep on adding different different crack length when you look at this idea a_1 reaches a critical value then as for a different different crack length it goes and meets at the different different position and this is not going to be true because the G should be the same for any crack length, that is why this kind of analysis is going to have a problem.

So, then what is the remedy, what is the remedy? Remedy is we have to consider something called *a/w*. So, what is *a/w*? See in this previous experiments we consider the crack in the infinite plate and there is no you know boundary. So, the moment you bring in that boundary that is width of the specimen, then that is being corrected. So, in this case, you can see that for a finite panels G_1 lines are curved.

In the infinite panel the line goes and you know make an intercept or make a tangent at a different point but for a finite panels then you bring a parameter called β which is function of *a/w* then G is not just a straight line, it is a curved line. So, for example if you take a_3, for a crack a_3 if the G is G_a here, it meets the G_a here for it crack length a_4, the G is G_b here. So, G and G_b are almost very close.

So, this takes care of that problem you bring in a β which is a function of *a/w*. So, this leads to approximately the same G_c for all the crack lengths and this is in agreement with the critical fracture toughness K_C obtained based on stress criteria. So, what we have understood from this today's lecture is, energy release rate is a significant you know made a significant improvement in understanding the crack problems.

And it also explained you know the concept of critical crack length and it explains the stable drag growth and it paved the way you know for the further understanding of the fracture mechanics problems. So, without getting into much of a mathematics, simply using this concept of energy, a lot of understanding has come from the complicated crack problems. So, with this we will you know kind of finish this discussion on energy release rate.

In the next class we will start discussing about the stress fields and the displacement fields. I will stop here. We will continue in the next class, thank you.

(Video Ends: 41:23)