Mechanical Behavior of Materials
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Lecture No 51
Fracture Mechanics - III

(Video Time: 00:18)

Hello, I am Professor S. Sankaran in the Department of Metallurgical And Materials Engineering. Hello, everyone welcome to this lecture and today we are going to turn our attention to a new topic in the fracture mechanics called energy release rate. So, this parameter is a very important parameter if you recall what I have shown in the table as various parameters like *G*, *K* and you know *J* and COD so on and this form comes as a first fracture mechanics parameter.

So, in fracture mechanics this is a, you know very important parameter due to its energy approach. Most of the problems in fracture mechanics are dealt with energy approach without much of mathematical complications. And also conceptually there are a lot of you know ideas have been developed using this energetically straight. So, we will begin this chapter by quoting the Inglis solution if you recall these during the historical development I just mentioned.

Inglis solution gave as the first clue about the severity of the crack. So, quickly we will review what is this? Then we will go get into the energy late chapter. So what you are seeing in this slide is I just enlarge this. So, this is an elliptical hole in an infinite plate and what Inglis has shown is that what you are seeing here is as the $\sigma_{max} = (1 + 2a/b)\sigma_o$ that is what he has shown.

And this is the σ_{yy} and this is σ_x. you can see that is a variation here. So, this is an elliptical hole which is of the central elliptical hole of the length 2*a*. So, this is *a* and this is *b* that is what we have seen. So, what is the importance of this Inglis solution? For *a* = *b* (a circular hole) the moment *a* is become *b* then it becomes circular, the maximum value of σ_{yy} is becomes 3 times the far field stress that is $3\sigma_o$ which agrees with the results given by Kirsch.

So, Kirsch has shown this already. But the moment it becomes you know ellipse then it becomes like that. The maximum value of σ_{yy} increases without bound as $b/a \rightarrow 0$, that means the semi-minor axis b becomes smaller and smaller related to a, then what happens it becomes like a sharp line a sharp crack. So, ellipse becomes a crack in that case and one requires a newer approaches to handle such problem.

So, why do we say that because the moment b tends to 0 in that equation the stress becomes infinite like the multiplication factor of σ_o becomes infinity. So, what does it mean? That could be 2, 3 interpretation you can think of, one is the stresses are very high even with the small applied load things can become pieces within 2 pieces. So, such things never happen. So, that is one possibility of thinking the next thing is a stress can be very high it can exceed maximum it reaches to a yield stress of a material that is close to reality.

So, this Inglis solution gave a very good alertness about the stress concentration and the crack tip but it is not practically applicable that is why it says a newer approach is mandatory. So, with this we will go to the next discussion on this energetically straight. So, we will start again with the Inglis solution for the continuity and then we will carry our discussion further. So, this is this is the same figure what I have just shown that is from the different page of the book.

So, I just moved from to the actual chapter here. So, this is for an infinite plate with an elliptical hole, Inglis solution for a maximum stress is there I wrote as σ_{max} here it is $\sigma_{yy,}$ that is in this there that schematic I showed because that profile stress profile was nicely shown in that. So,

$$\sigma_{yy}^{\max} = \sigma\left(1 + \frac{2a}{b}\right)$$

when $b \rightarrow 0$, no material can sustain this stress.

So, that is the kind of you know it an impractical solution this analysis gives, that is why a newer approach is required. Thus, even for a small applied load σ_{yy} the above equation tells that σ_{yy}^{max} will be very large and it would exceed the ultimate strength of the material, which is not the case anyway. So, what can we do? So, this is one dilemma. So, what happens the equation suggests that even for a small load the crack may grow and break the component into pieces and that creates a dilemma that is called the Griffith’s dilemma.

However, this is contrary to our observations. Griffith argued that some other mechanisms must be in operation which helps our sorry which helps solid to sustain solid forms. So, what is that? Similar to surface tension of liquid, all surfaces of solid are associated with surface energies or free energies. So, what we have to appreciate here is the Griffith's first thought process to bring in surface energy to fracture mechanics to explain this you know the crack propagation is the important step.

We will see how it helps. These energies develop because atoms close to the surface behave differently from an atom at the interior of the solid. The interior atom is attached or repulsed by the neighbouring atoms more or less uniformly in all the directions. So, these fundamental ideas the people who are dealing with materials they all know right. The atomi at surface experiences a different kind of equilibrium as compared to one exhibited interior.

So, that is why this comes as an explanation. But, an atom of the free surface has no surrounding atoms on one side, thus requires a different kind of equilibrium. In fact atoms on the free surface and the ones below have readjust to form an equilibrium thereby developing a strain in the material close to the free surface. This is one way of interpreting a surface energy an elemental way of explaining what happens in this free surface. Such deformation requires energy and it is known as surface energy.

So, very, very a simple definition for a surface energy you can think of. But, Griffith realized that a crack in a body will not extend unless energy is released in the process to overcome the energy needed for forming two new surfaces; one below and one above the crack plane. So, this is very important point, what he realized is the energy required to create new two surfaces, see he is not just thinking about crack alone.

He is just bringing nicely the surface energy into a crack problem saying that to the crack itself, if you look at the geometry it contains two new surfaces. So, to create such a two new surfaces, it requires some energy. So, that is that is that is an idea how he interprets. The surface energy of the material depends on material properties. However, its magnitude is rather small, of the order of 1 J/m^2.

So, what are the surface energy we are aware of? If you look at for the common materials you can see that you know for a copper it is 0.98 surface energy is measured I mean denoted as γ_s which is which has got units of J/m^2. And the other parameter you are seeing is a γ_p, γ_p is the energy required to cause plastic deformation near the cracked surface. So, this parameter we will use it little later but nevertheless it is related to surface energy that is why it is shown here.

So, we will keep it after the Griffiths analysis we will get into the Irwin's analysis of the crack problem, we will use this γ_p. For example, if you look at the surface range of mild steel it is 1.2 and if you look at the γ_p it is significantly high. So, it is a ductile material. So, the energy required to cause a plastic deformation near the crack surface is very, very high. So, for Al alloy it is 0.6 and the γ_p is 4000.

Interestingly Griffith was working on a glass. So, you see that the surface energy is 2.3 it is significantly high as compared to the other metals common metals and then you have the interesting value for ice and the diamond is the highest surface energy. So, what we now do is we are going to review some of our old ideas I mean whatever we have already learned We are going to look at the elastic strain energy.

Energy stored in an elastic body when the load is supplied. Strain energy stored in a linear speed see we are going to talk now about strain energy and we are also going to involve a surface energy to explain the crack problems. So, all this you know some small, small definitions will come in very handy and also the associated mathematical simple expressions to explain the energy of this you know crack formation and so on. So, that is why we are going to look at it.

So, these concepts you know but we will just try to understand it for the completeness. So, this is a very nice animation for a simple spring pulled in one direction the load p is applied in this direction and then you see that load versus displacement plot and what you are seeing is the strain energy of this action is given by ½Pδ.

$$U = \frac{1}{2} P\delta$$

So, why it is ½$P\delta$ because the load is gradually applied, if it is not gradually applied it is a $P\delta$. So, then it becomes like this since gradually applied it becomes ½$P\delta$. So, you have to remember that. So, we can look at the strain energy stored in a body when only normal stress is acting. So, this is a 3-dimensional cube kind of material and which is being pulled in this x-direction. So, the σ_x is a normal direction I mean normal stress and how to write this change in strain energy it is dU is equal to half here again it is gradually applied.

$$\begin{aligned} dU &= \frac{1}{2}(\sigma_x \, dy \, dz)\varepsilon_x \, dx \\ &= \frac{1}{2}\sigma_x \varepsilon_x \, dx \, dy \, dz \\ &= \frac{1}{2}\sigma_x \varepsilon_x \, dV \end{aligned}$$

σ_x is a normal stress and dy and dz is the area on which this is acting and this is a displacement. So, what you get is ½ $\sigma_x \varepsilon_x dV$. So, this is a volume. So, strain energy change per unit volume that is how you will get finally for all this small small basic definition. And now we will look at the strain energy stored in a body when only a pure shear stress is acting. So this also you know is very familiar to you this one. So, what is that expression for strain energy that this is nothing but

$$\begin{aligned} dU &= \frac{1}{2}(\tau_{xy} \, dx \, dz)\gamma_{xy} \, dy \\ &= \frac{1}{2}\tau_{xy} \gamma_{xy} \, dx \, dy \, dz \\ &= \frac{1}{2}\tau_{xy} \gamma_{xy} \, dV \end{aligned}$$

this is the area on which the shear stress acts and this is a displacement shear strain. So, you get ½ $\tau_{xy} \gamma_{xy} dV$. So, this is the strain energy stored in the material due to shear stress. Strain energy stored in a member when all the stress components are acting both normal and shear stress then the expression becomes like this.

$$dU = \frac{1}{2}\left[\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz}\right]dV$$

$$U = \frac{1}{2}\int_V \left[\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz}\right]dV$$

This is a one equation which explains all the stresses and then we can now substitute these strain components in terms of stress components then we can use this generalized Hook's law.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$
$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right]$$
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

And this generalized Hook's law also you all familiar we have spent enough time on this just to we are recollecting all of them. So, the strain expressed in terms of stress. So, it gives a 3-dimensional picture and is also called the generalized Hook's law. So, we will be using that and again the strain energy in terms of I mean shear strain and in terms of stress components we can write it like this

$$U = \frac{1}{2}\int_V \left[\frac{1}{E}\left(\sigma_x^2 + \sigma_y^2 + \sigma_z^2\right) - \frac{2\nu}{E}\left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x\right) + \frac{1}{G}\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]dV$$

and I am not going to spend much time on this because all these things are very simple equations you already know that. And our aim is to just to get into the some kind of what kind of energy strain energy formulation appears. What kind of expressions you get that is an idea. So, if you look at the strain energy in terms of applied load *P* for an axially loaded member.

$$dU = \frac{1}{2}\left(\frac{P}{A}dy\,dz\right)\frac{P}{AE}dx$$
$$= \frac{1}{2}\frac{P^2}{A^2E}dA\,dx$$
$$U = \frac{1}{2}\int_V \frac{P^2}{A^2E}dA\,dx$$
$$U = \frac{1}{2}\int_0^L \frac{P^2}{AE}dx$$

So, like this see all these expressions are applicable for the slender members in an applied mechanics or strength of materials language. So, slender members means what the cross sectional area is much much smaller than the other dimension the longitudinal dimensions or so.

So, these expressions are you know used to describe the energy terms or you know strain energy stored in a small bodies. So, that is how you should look at it. So, in this expression what is that you are getting we are getting the strain energy is half into integral over 0 to l and p square by AE into dx. So, this is all a very simple thing and you what you have to remember is you see that what kind of you know formulation is finally we get for the strain energy is $U = \frac{1}{2}\sigma^2/E$.

So, we will remember this. So, this is one expression we are going to keep on using it as long as we are using this strain energy concept. And this is for the torsion member subjected to torsion and the strain energy is calculated by this you know shear formula.

$$I_P = \int_A r^2 dA$$

$$U = \int_V \frac{M_t^2 r^2}{2GI_P^2}\, dA\, dz$$
$$= \int_0^L \frac{M_t^2}{2GI_P}\, dz$$

$$U = \int_0^L \frac{M_t^2}{2GI_P} dz$$

This kind of expression we have seen it in the torsion test and where M_t is the moment of torsion and I_p is the polar inertia moment of inertia which is all given by the strength of materials books and small, small equations.

And then you get this final expression like this and we can also recall our flexural stress this is a flexural formula again you know given by the strength of material concepts.

$$\frac{M_b}{I_z} = -\frac{\sigma_x}{y} = \frac{E}{\rho}$$

$$\sigma_x = -\frac{M_b y}{I_z}$$

$$\varepsilon_x = -\frac{M_b y}{E I_z}$$

And what you see here is σ_x is equal to bending moment of bending moment into y given by I_z. So, this also we have used actually. So, if you write for the strain energy expression then again you get this simple formula.

$$dU = \frac{1}{2}\sigma_x \varepsilon_x \, dV$$

$$= \frac{1}{2}\left(-\frac{M_b y}{I_z}\right)\left(-\frac{M_b y}{EI_z}\right) dV$$

$$\textbf{since } I_z = \int_A y^2 \, dA$$

$$U = \int_V \frac{M_b^2 y^2}{2EI_z^2} \, dA \, dx$$

$$= \int_0^L \frac{M_b^2}{2EI_z} \, dx$$

So, this all these small small formulas will be handy if we can recall and then we can apply to this slender members that is our idea. So, now what we will do is we will look at the energy release rate *G* preliminaries. This is a parameter defined by energy approach to characterize the crack tip and energy approach has the advantage that one does not have to account for a large stresses that are developed in the vicinity of the crack.

Energy method avoids analysis close to the crack tip. Energy approach can only be used for conservative systems where there is no energy dissipation. Energy release rate is primarily developed only for brittle materials. So, these are all the points are important to keep in mind because the energy release rate is going to give a conceptual idea and it is not going to be very accurate that is why we are saying that it avoids analysis close to the crack tip.

Because in crack tip stress and displacement fields are very complex we will see in the later section how the expressions are derived. And then how it is useful then again we will come back and relate those stress and strain displacement fields and try to relate to the energy that is a, that is what we will do at the end. But then right now we will just look at energy release state as a concept used which is used only for the conservative systems without energy dissipation.

So, that is the idea. So, what happens when what are the changes in the component when the crack advances, very important idea. So, that when the crack advances, the stiffness of the component decreases this is very common sense. If the crack length is keep on increasing obviously the stiffness of the component will decrease and the strain energy in the component decreases or increases.

So, this statement is we will now qualify for this by looking at small simple experiments which we are going to show in the next few slides. Strain energy in a component may decrease or may increase. So, we will look at them in detail the points of the components at which the external loads are applied may or may not move. So, we are going to look at the strain energy aspect for a constant load as well as a fixed grip.

So, we will see work is being done on the component by these forces if the points move. Energy is being consumed to create two new surfaces. So, this is one an important step or a conceptual step put forward by Griffith by I mean approaching the crack problem by you know try to define or try to get an expression for an energy release rate. So, we will see one by one. So, what is that we are seeing here in this slide we are going to see a strain energy as a function of load and displacement.

Like we said in the previous slide but the first experiment is at constant load. So what is that you are seeing this is a double cantilever specimen which is one lever is fixed on the grips and then which is having the crack length a and there is a rope over a pulley and then the load P is applied here which is a constant load. And this for this experiment this load versus displacement plot is shown here for the crack length a.

You should ask why are we interested in conducting these strain energy experiments for a constant load and constant displacement. Because in fracture mechanics make the mathematics very simple and these kind of approaches are done we will we will just see how. So, there is another test we are comparing here with a different crack length this is a similar specimen but what you see here is the crack length is $a + da$ and that is why the load also has slightly come down from this level. So, you can see that.

So, the crack length is $a + da$. So, for this again you will see that load versus displacement is going to be below this curve. So, this is for the crack length a and this is for crack length $a + da$. So, now what we will do is now we are trying to slowly increase the load from P_1, P to P_1 then you can see that already the crack is growing and then as the track grows you can just watch this P_1 the dotted line is the original direction but then the moment the crack length increases the P_1 tries to come here I will just play this.

So, now the crack length becomes $a + da$ the P_1 has just started from here and then reach this curve. Then it will follow this curve because the crack length is becomes $a + da$. So, what is that we are trying to understand from here. So, now we can look at the strain energy for these two systems. The first case that is I mean the crack length the initial crack length which was there in $a + da$ the strain energy is given by this triangle and the earlier crack length was a. So, the strain energy is given by this triangle. So, that is what we are going to now see.

So, strain energy is given by ΔU that is this triangle was minus this triangle. So, you can write it like this

$$\Delta U = \frac{1}{2} P_1 (v_2 - v_1)$$
.

So, v_2 this is this triangle minus this triangle. So, you can see that this triangle is much bigger triangle than this. So, obviously, this will become; so, this is what is this triangle this is a truncated triangle is the change in strain energy between these two events.

The change in free energy is shown in this triangle and what is the net result? The net result is the strain energy of the system increases. So, as the crack length grows in this cantilever beam experiments the strain energy increases. So, this is what it is given, but what is the external work done? External work done because we are applying directional load constant load. So, the external work done is given by this red rectangle because the moment you start you know changing the load from P to P_1.

Then this line moves and jumps to this another line that load displacement plot. So, this area under this curve is called external coordinates. So, we play the animation once again just for the clarity. Yeah, this is load versus displacement data for this and your second experiment it is $a + da$ and now you see that load is coming down then it transfers to the next one. Then this is the strain energy for the second experiment.

And this is the strain energy for the first experiment the change in strain energy is this one what is the truncated triangle and which is given by ½ $P_1 dV$ that means the strain energy of the system increases the work done is $P_1 dV$. So, now we will look at the same experiments for the constant displacement. So, what is constant displacement means? It is the specimen is kept under the fixed grips.

So, the fixed drips the crack length is a and if you look at the load versus displacement plot something like this and then again another second specimen with the track length of $a + da$ again kept that fixed grip and there is no external load here please understand it is a fixed grip. So, the energy required to move the crack comes within the system that is strain energy that we will discuss now. So, what is that we are seeing again the same thing the load the stiffness of the second specimen which is having a higher crack length obviously will have the lower stiffness that is why it is like this.

And now you see what happens as the crack length I mean If the increase the load if the crack started moving here then if the crack length becomes $a + da$. Similar to the previous experiment it is just coming down the load is coming down because the crack length is increasing the load is coming down. So, you can see that it reaches a P_2 and then we can write similar expressions. So, what one important point you have to remember is as the crack advances, no external work is done in this case because they are all kept under the fixed grid on the system because the external load is not allowed to move.

So, that is one important. So, that means external work done $\Delta W_{ext} = 0$ but stain energy if you look at ΔU that is change in strain energy by this the difference between the area of this these two triangles. $\Delta U = ½v_1 dP$. So, this is a if you look at the change in strain energy is much smaller truncated triangle compared to the previous one. So, in this case for a fixed grip experiment or constant load displacement experiment the strain energy of the system decreases.

So, in one case it was increases for a constant load the other case at constant displacement it is decreases. So, what is that we are going to say now for a general loading that means both constant load and constant displacement you can write it like this in the limit when $\Delta P \rightarrow 0$ that means this will become 0 or this will become $\Delta v \rightarrow 0$ this will become 0. If this becomes 0 and this will become 0 then what happens?

The quantum of energy whether the constant load or displacement the quantum of energy available for a crack extension is the same very important point to remember. Whether we use constant load or constant displacement the energy available for the crack extension is the same. So, that is what is saying here the first case in it strain energy increases and in the other case it decreases.

In practice any general load displacement behavior can be thought of as discrete steps of constant load-displacement. So, you can see that this is this is like you know it is a constant load-displacement constant load constant displacement kind of small small steps are conceived as a general loading. So, you know general load you just conceived like that. So, what happens is strain energy in the presence of a crack could be arrived at based on dimensional analysis or a relaxation analogy, or actual calculation based on the crack face displacements.

But it requires knowledge of stress and displacement fields. So, like I mentioned in the beginning of this lecture the right now we are not looking at the stress field at the crack tip or strain field at the tip. We are looking at the trying to explain the energy you know arrive at an expression for energy release rate based on the energy concept. So, we will look at it how do we do that. For a central crack that means a crack with the two tips the strain energy is given by U_a.

U means a plate which is a component having already a crack of length *a*. So,

$$U_a = \frac{\pi\sigma^2 a^2}{E}$$

So, this is an expression for the strain energy of a plate containing a crack. This result is for a crack in an infinite panel of unit thickness. So, this is also important. So, right now we can assume that all this strain energy calculation is for the unit thickness. Otherwise there is another term will come to this expression.

What we will now do is we have to now see how do we derive this. How this analogy is relaxation analogy is envisaged. So, consider this schematic this is a plate of thickness b and it which is being stretched. So, we can think of you know when I say the plate you may think that is a solid plate and especially we are talking about a brittle materials only. Just for our you know understanding we can just think of an elastic strip which is being stretched and then you know sealed here in a stretched condition.

You can think of that situation which is also true for the you know brittle solid that is the actual thing we are discussing but to understand the, you know what we are referring here we can think of some elastic material here and then now introduce a crack at this center. So, what happens this is a crack and then crack is you know appearing as an ellipse here elliptical hole. And what is this two triangle which is shown which is the, you see this is a strained region it could be of any dimension but for simple schematic to just get an idea, we are taking this triangle because the mathematics is simple.

So, you assume that this is height is I mean 2*a* times λ, λ is proportionality constant and the crack is 2, crack length is 2*a*. Note the area on which the strain energy is released. So, what we are seeing is the two new surfaces are created here and for that creation of the energy if the energy comes from this, this strain energy region from this place. Because, there is no external load applied here because it is a fixed grips right, it is already fixed the spin must be fixed and then we introduce a crack here.

And then the new two new surfaces were created and for creating that creating that two surfaces new surfaces the energy comes from this nearby region that is. So, we should assume at this point this is simple schematic. So, we are trying to make some analogy. So, if you take U_a

$$U_a = (\textbf{Volume of Triangles}) \times \left(\frac{\sigma^2}{2E}\right)$$

So, that is how we are going to calculate the U_a.

This expression we have already seen 2 times, this because it is a two surfaces and then the thickness of the plate is B. So, you get this expression like

$$U_a = \frac{2\lambda a^2 B\sigma^2}{E}$$

and $\lambda = \pi/2$ for thin plates, this also we will correlate in the later derivation how we justify this. So, all this you know the, I mean I would say the symbols are quite you know unique for fracture mechanics.

So, you have to keep that in mind B is for thickness λ is you know it is proportionality constant here it is not normally we use that for you know wavelength and so on. So, you have to be little alert to get into this kind of symbolism. So, we get this you know strain energy

$$U_a = \frac{\pi a^2 B\sigma^2}{E}$$

So, now what we will do? Now we look at variation of surface energy and the strain energy in the fixed grips for a constant displacement.

Because that what we have looked at is one side we are trying to take the other ideas into our discussion. So, that is why we are starting this if λ_s is the surface energy per unit area of one surface, the surface energy required for a model of thickness B which is equal to

$$U_s = 2(2a)B\gamma_s = 4a\,B\,\gamma_s$$

So, we are bringing the surface energy. So, we are trying to do energy balance.

So, for that what is that we are trying to do here, energy versus crack length plot we are creating and this equation is nothing but a straight line equation. So, the energy is you know increasing with the crack length. So, it goes like this. So, this is a surface energy for a fixed grip and from the relaxation analogy we got this expression like this which is

$$U_a = \frac{\pi a^2 B \sigma^2}{E}$$

And then what is that I mean this is a straight line and what is this curve shows this is kind of a this is a a^2. So, that means a parabola a second degree curve. So, the net is something like this. So, the net our total is $U_T = U_S + U_a$ which gives a curve like this that means it gives a maxima it gives the maximum. So, what does it mean? It demarcates this curve into two segments and which is very very important conceptually.

Why it is important because it gives or it indicates a new parameter called critical crack length it is a_c, incremental crack growth by crack growth mechanism and catastrophic crack growth by fracture mechanism. So, what you have to understand here is when we talk about energy release rate we are talking about energy for the crack to grow incrementally that is Δa what is the energy required for the crack to grow incrementally.

So, that is what it is called incremental crank growth by a crack growth mechanism. So, we have looked at several crack growth mechanisms but it start any one of the mechanisms can involve in growing the crack to a_c. So, it gives a very important idea that you know critical crack length the idea of critical crack length is provided by this relaxation analogy concept basically this I mean energy release rate concept.

So, catastrophic crack growth by fracture occurs after this critical crack length. So, very very important idea to keep in mind, we will bring this we will keep this idea and then we will bring it again as we just progress in this discussion. So, what we are interested in is the incremental change in the crack length to occur. The incremental energy requirement should be satisfied we are trying to do energy balance and this all the analysis is meant for brittle material. So,

$$\frac{\partial U_a}{\partial a} = \frac{\partial \text{ (Surface Energy)}}{\partial a}$$

that means what?

Here the change in energy as a function of crack length is equal to change in surface energy, it is a factor. So, this is the kind of energy balance we are trying to do and then if you can rewrite those equations expressions what we have got we can write it like this

$$\frac{\pi\sigma^2 2a}{E} = 4\gamma_s$$

and if you differentiate this equation with respect to 2*a* and then you get this expression.
Why we just differentiate with 2 here because 2*a* is the crack length. So, the final expression is

$$\frac{\pi\sigma^2 a}{E} = 2\gamma_s$$

and then you can rewrite this expression like this

$$\sigma\sqrt{a} = \sqrt{\frac{2E\gamma_s}{\pi}}$$

very very important equation as far as the energy release rate is concerned. And most of the textbooks also write this equation like σ, the a will be inside the I mean they will be right hand side inside the square root. So, you can we can verify this expressions in the subsequent discussion how it was verified. **(Video Ends: 43:51)**