Mechanical Behaviour of Materials
Prof. S. Sankaran
Department of Metallurgical and Materials Engineering
Indian Institute of Technology – Madras

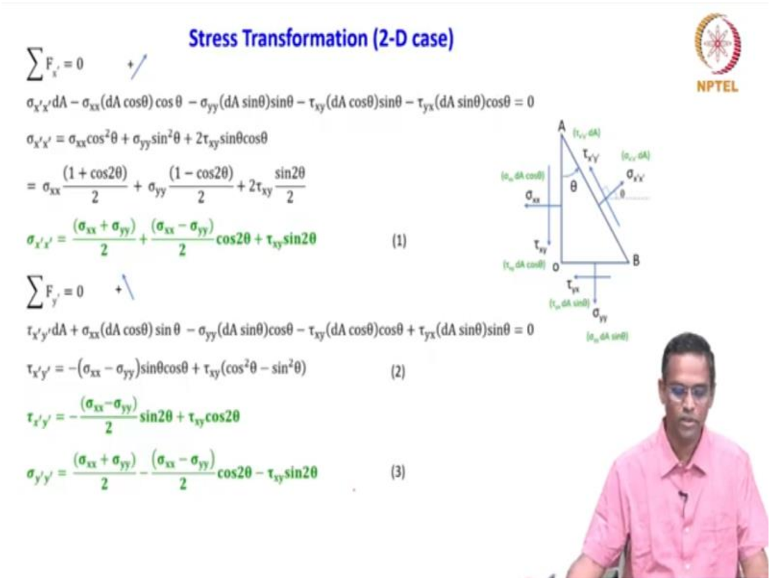
Lecture-05
Strength of Materials - A short overview part - III

Hello, I am Professor Sankaran in the department of metallurgical and materials engineering. Hello, let us continue over a discussion on state of stress in two dimensions, what we have just looked at so far is we tried to kind of derive an expression for getting the principal stresses in plane stress conditions are in two dimensions on an oblique plane that means, we are trying to find an equation which will help us to find the principal stresses as a function of any orientation that is all we are trying to do. I will just quickly review what we have seen yesterday and then we would like to continue further.

(Refer Slide Time: 01:05)

So, if you look at what we have just seen yesterday is like, quickly I will just go through we have taken this kind of geometry to describe this and then we have taken a wedge out of this square member and then we looked at the forces which is normal to this oblique plane, then we try to do the force balancing because we assume the strength of materials assumes that this is the member is in the equilibrium. And then we try to calculate the force equilibrium from each site, I mean for a normal stress as well as the shear stress.

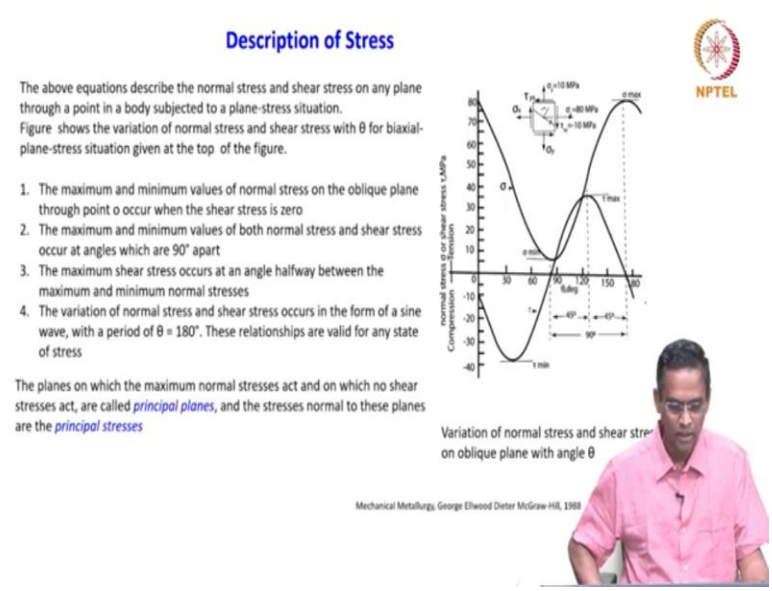
(Refer Slide Time: 01:53)



And then from there we calculated the expression for normal stress and shear stress in a general manner. So, this is what we have just done yesterday. So, these two equations, I would say 3 equations are very basic equations, once we have this equation ready, then as you can see that we can do a lot of manipulation to get different quantities for example, we are all interested or especially in the theory of yielding and so on or metal forming you know the stress where it is maximum.

For example, principal planes where the stress which is acting we are characterizing as principal stress and the maximum shear stress which plane we have to identify. So, these are all the some of the benefits of looking at all this equation in detail.

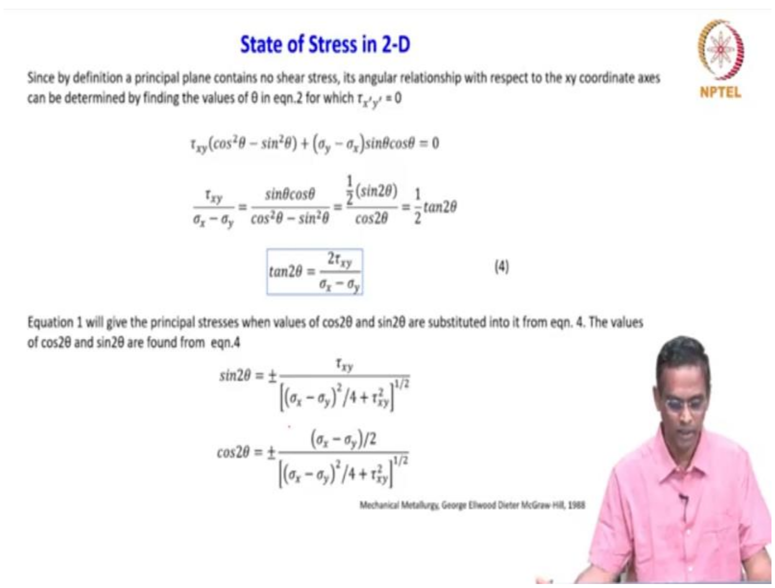
(Refer Slide Time: 02:59)



So, this we will see one by one then again we just discussed about these four points about the relation between the normal stress and shear stress and the maximum and minimum values of normal stress on an oblique plane through point O occur when the shear stress is 0 just be looked at all this points. And the maximum and minimum values of both normal stress and sheer stress occur at an angle which is 90˚ apart.

So, this also we have just witnessed yesterday, the maximum shear stress occurs at an angle half way between maximum and minimum normal stress there was a typo error yesterday here, it was maximum and minimum normal stress. So, that is what also is showing in this image. And finally, the variation of normal stress and shear stress occurs in the form of a sine wave. So, we have seen witnessing that as usual. So, we discussed that.

(Refer Slide Time: 04:08)

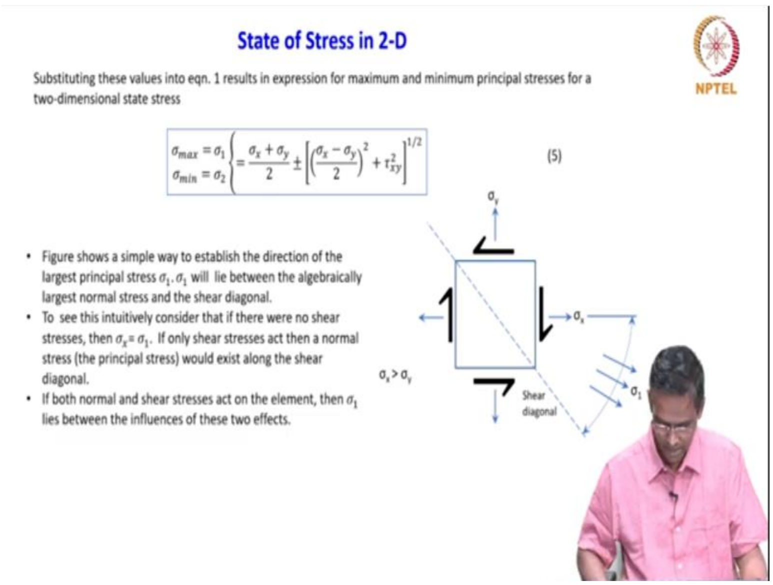


And we were trying to calculate the principal planes from these equations. No shear stress that means the principal stress wherever the principal stresses acting on a principal plane, there is no shear stress and its angular relationship with respect to x y coordinate axes can be found out from this shear stress equation. That means, I will just go back. So, this equation we are talking about. So, this and then we are taking this relation and then we are trying to manipulate that and then if you rearrange these equations.

And then you get this tan two θ is equal to 2 τ_{xy} / σ_x - σ_y which gives an idea of you know where the principal plane could be and I just mentioned yesterday that you can use this on this right angle triangle in this it is2θ. So, then you can find out the hypotenuse values using this relation by as per the Pythagoras theorem, then you can see that $\sin 2\theta$, $\cos\theta$ value can be readily found out from this simple step.

And then what is the use of finding this if you can find this 2 $\sin\theta$ and $\cos 2\theta$ then we can plug these values into the gentle expression which we have derived for $\sigma_{x'}$ and $\sigma_{y'}$ for finding out the principal stresses on an oblique plane. So, that is what we have seen yesterday.

(Refer Slide Time: 06:09)

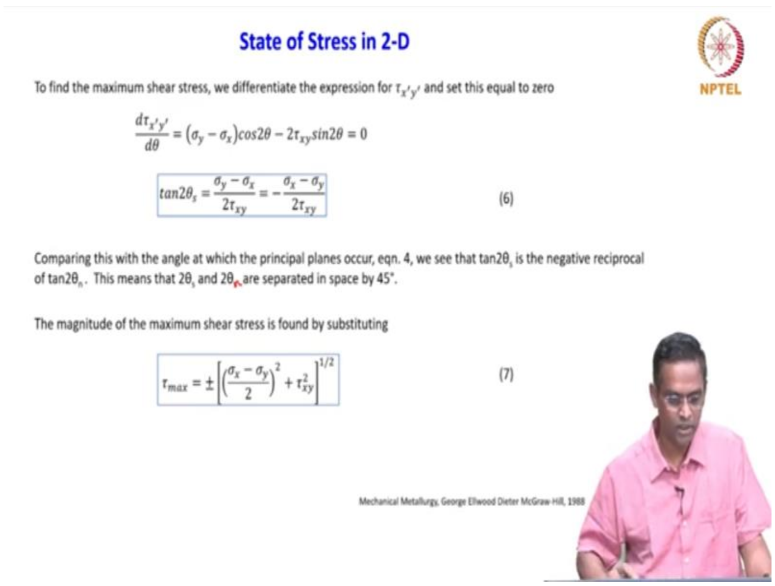


And this is the final expression substituting this sin and cos values in equation 1 results in the expression for maximum and minimum principal stresses for a two dimensional state of stress. So, this equation gives that and also we stopped somewhere in this discussion what we are interested is to find out the plane of normal you know the orientation of a normal stress or a principal stress or normal stress which contains both shear and normal component, how to find the plane orientation that is what we were discussing.

So, for that to we take this diagram, where it shows the biaxial stress σ_x and σ_y and then this is diagonal shear stress and as per the convention this σ_x is always greater than σ_y that is even if it is 3 dimensions that σ_2 will be greater than σ_3. So, that is a convention. So, the figure shows a simple way to establish the direction of largest principles for σ_1. σ_1 will lie between the algebraically largest normal stress and the shear diagonal.

To see this intuitively considered that if there were no shear stress then $\sigma_x = \sigma_1$. So, if there is no shear stress then we are talking about this direction this direction. So, and if it is completely it is only shear stress then it is this direction normal to the stress along the shear diagonal if both normal and shear stress act on this element then σ_1 lies between the influences of these two somewhere here. So, this is 1 nice way of looking at where the σ_1 will be.

(Refer Slide Time: 08:27)



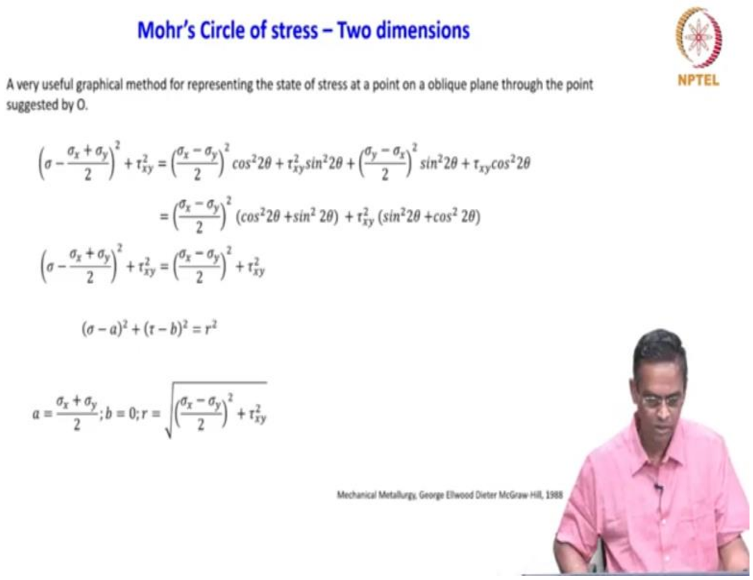
And we are interested in maximum shear stress. So, especially, you know in theory of yielding σ_{max} is τ_{max} I would say is very important parameter where you know the yielding takes place and we will discuss that in detail in our yielding theory and so, on. To find out the maximum shear stress, we differentiate the expression of τ x prime, y prime what we have the same equation what we have been using and set this equal to 0.

So, this is what we get and this you can just rewrite in this form $\tan 2\theta_s = \sigma_y - \sigma_x / 2\tau_{xy}$ and this can be rewritten like this for our convenience, because we have another expression for tan 2θ this before we have just seen where it was giving some angular relationship for the principal planes. So, that tan2θ was describing the angular relationship for the normal stress and this is for a shear stress.

So, we can now compare these two . Comparing this with the angle at which the principal planes occur that is equation 4. We see that $\tan 2\theta_s$ that is tan2 θ be representing the maximum shear is negative reciprocal of $\tan 2\theta_n$. So, this is a negative reciprocal of this means that t$2\theta_s$ and2 θ_n are separated in space by 45 degrees. So, you see that by looking at this expressions which is also a gives a very nice idea about the orientation between these two planes.

We have also physically seen that you know how these two are separated but these equations also prove that the orientation between these two stresses. So, that is what is it this is a permanent correction I will make so, it will be n supposed to be read as n. So, what is the maximum shear stress the magnitude, the magnitude is given by τ_{max} is equal to plus or minus $(\sigma_x - \sigma_y / 2)^2 + \tau_{xy}$ all squared to the power half.

(Refer Slide Time: 11:two two)



So, this expression is important and we will now move on to something a new way of representing the stress versus strain. So, very useful graphical method for representing a state of stress or the point on an oblique plane through your point suggested by Mohr is a scientist suggested this methodology, so, it is called Mohr's circle. So, what does it convey, but before we go into the details of what this Mohr's circle means.

What we will do is we will take I have just written an expression here how do we got this expression I hope you all will have the familiarity with this terms what I have done is you have the two dimensional principal stress expressions and shear stress expressions for an arbitrary orientation. So, what we have done is we have taken the 2θ terms on 1 side and the non 2 θ the other side added up, squared and added up. So, that is what it is.

So, if you look at that two equations that shear stress expression as well as the normal stress expression, I have combined and separated the 2θ terms right hand side and then non 2θ to eliminate the 2θ we are doing this manipulation and to get some expression what we what we get. So, that you can readily do we have not changed anything, we have just rearranged those two equations and squared them and added up. So, you can just verify that.

So, this can be written like this, rewritten like this, simple rearrangement and finally, you get this

$$(\sigma - \frac{\sigma_x + \sigma_y}{2})^2 + \tau^2 xy = \frac{\left(\sigma_x - \sigma_y\right)^2}{4} + \tau^2 xy$$

. So, this equation is in the form

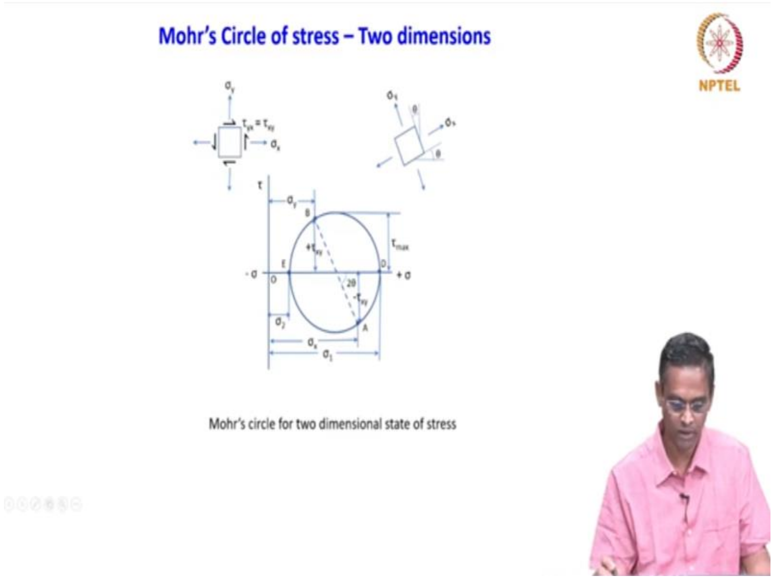
$$(\sigma - a)^2 + (\tau - b)^2 = r^2$$

So, this equation is very familiar equation of circle..

$$a = \frac{\sigma_x + \sigma_y}{2}; b = 0; r = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau^2 xy}$$

So, this is this is exactly the equation of Mohr's circle so, this is representation this way of representing this state of stress in two dimensions is called Mohr's circle.

(Refer Slide Time: 14:41)



So, we will see what is the small circle looks like. So, you can look at the member which is subjected to biaxial tension σ_x and σ_y and then this is the shear stress acting for this situation leap it can be an arbitrary orientation to be rotated with respect to θ like this. So, for this situation the Mohr's circle is drawn here. So, what is involved in this Mohr's circle drawing, so, the x axis is principal, or normal stress and the y axis is shear stress.

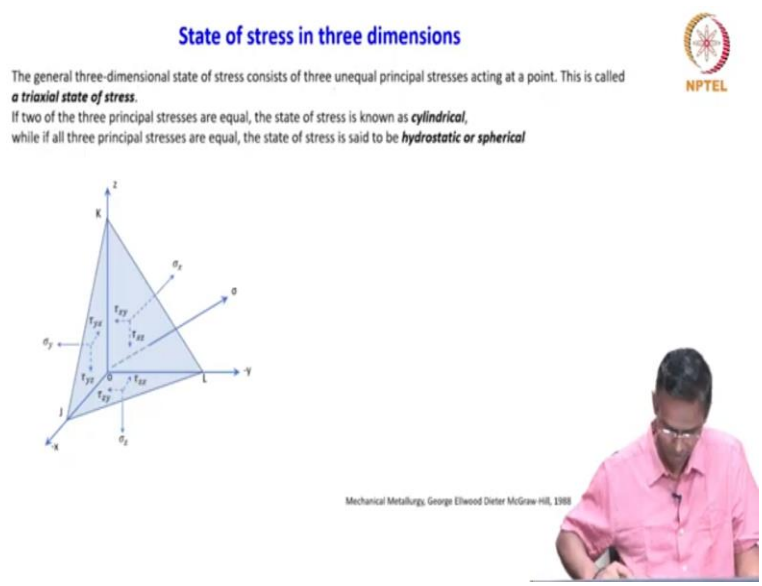
So, that is the axis labeling. This is positive and this is negative and this is positive and this is negative and suppose, you take the any σ x which is normal to this plane if you can just mark it like this positive and negative A B, if you if you mark it like this the intersection is considered as the center of the circle and then what you see here is D and E where the shear stress is 0 that means, are these two points it is pure principal stress it is there is no shear stress and the σ_x that is where we are interested.

So, you can see that this is point σ_x and the corresponding value here is this is τ. So, what you are seeing here is $+\tau_{xy}$ and this is $-\tau_{xy}$. So, for example, if you like I just mentioned yesterday, so, this shear stress will drive this direction and this shear stress will drive this direction. So, this is positive direction. So, the positive shear is here put here and then the negative is here and the maximum the σ_1 which is algebraically largest a normal stress is σ_1 and what we are interested in plotting this σ_x.

So, the angle between this σ_1 and σ_x is 2θ in Mohr's circle it is always at the θ is plotted as two θ not just θ. So, it is always 2θ and then you can see that the angle between this σ_1 and σ_x is 2θ and this is a σ_2 σ_x and this one is σ_x and this one is σ_y and this one is σ_2 and this is σ_1 and what is the maximum is here that is τ_{max}. But we are interested in looking at this particular normal stress, which is on arbitrary θ then this diagram is useful, because it can take any angle.

So, at any angle it will plot here then you will see the corresponding normal stress and the shear stress here. So, here it is negative shear stress it is positive shear stress. So, depending upon the rotation, which sign we are looking at so, we can easily relate this very nicely you can represent the normal and shear stress for a given orientation. So, that is the usefulness of this state of stress. So, we can also look at the equations which we have derived, but this will give much more visual benefit. So, grasping the idea of how to see this shear stress and normal stress as a function of orientation for all the orientation you can see, so, this is for two dimensions.

(Refer Slide Time: 19:30)



And we now will slowly move to the state of stress in the three dimension. So, far we just looked at biaxial. So, biaxial when we say biaxial, σ_1 and σ_2 are supposed to be equal in magnitude biaxial. And, in general, the three dimensional state of stress consists of three unequal principal stresses acting out in point this is called a triaxial state of stress. So, you can have unequal principal stresses even in biaxial you can just check whether it is equal in magnitude or unequal in magnitude as state of stress.

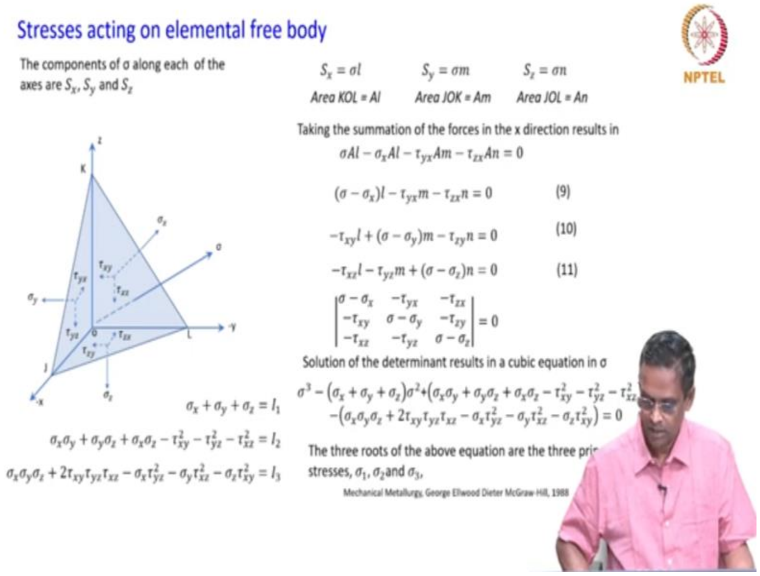
But in general this is a three dimension state of stress can be an unequal principal stresses and if the two of the three principles should sorry equal then the state of stress is known as cylindrical. So, even in the triaxial state of stress if out of three if the two are equal in magnitude, the third is smaller than the other two then it is called cylindrical and if the all the three principal stresses are equal then the state of stress is said to be hydrostatic or spherical.

So, these are the terms which are very important and very basic and you have to keep in mind while describing the state of stress especially in three dimensions. So, we know now triaxial state of stress, cylindrical state of stress, and hydrostatic state of stress or spherical state of stress how it will be. So, now, we are just going to describe how to get these equations for the state of stress in three dimensions it is simply an extension of what we have just looked at for two dimensions.

So, I am not going to spend more time on this but just for a completion, I wanted you to look at it because you should not ignore this and we are going to take up this same kind of Cartesian coordinate what we have done in two dimension. So, you just take this J, K, L plane which is a plane which is just cut from the perfect cube you know kind of 1,1, 1 plane and then all the and then here is the σ a normal stress which is coming out of this plane.

And then we can just look at all these you know these normal stresses are marked σ_x and σ_y with the two shear stress so, this is what we have just seen yesterday, we need to resolve this total stress into normal stress and shear stress when while are we describing the state of stress in three dimensions this is what we just looked at yesterday. So, like that we have just marked all this shear stress components as well as the normal stress components.

(Refer Slide Time: 23:02)



So, what we are now going to do is just very briefly describe the components of σ. So, the competence of σ along each of the axes are S_x, S_y and S_z. So, in a two dimensional if you go back and look at this geometry what we looked at it we took a component normal stress component, shear stress component on each direction there it was just a line, we assume that there is a line because we just took a projection of oblique plane here it will be a since it is a 3 dimensional it will be a plane.

So, we will consider the plane. So, these are the components the so, the S_x is σ_l, S_y is called σ_m and S_z is that is equal to σ_n. So, l, m, n are the direction cosines of σ which makes with the x y z coordinates that is one thing you have to remember. So, the components of the normal stress is to multiply with the direction cosine and as I just said the two d plane what are the two d planes here we are to think of K, O, L.

This K, O, L is that is the area of the triangle is A and this is belong to x direction. So, it is multiplied by l. So, area J, O, K this plane that is associated with y, so, A m and then area J, O, L An so, this is a similar way what we have taken. So, now, we will take the summation of the forces again the assumption is same this body is assumed to be an equilibrium. So, we take the summation of forces on a particular direction and equated to 0.

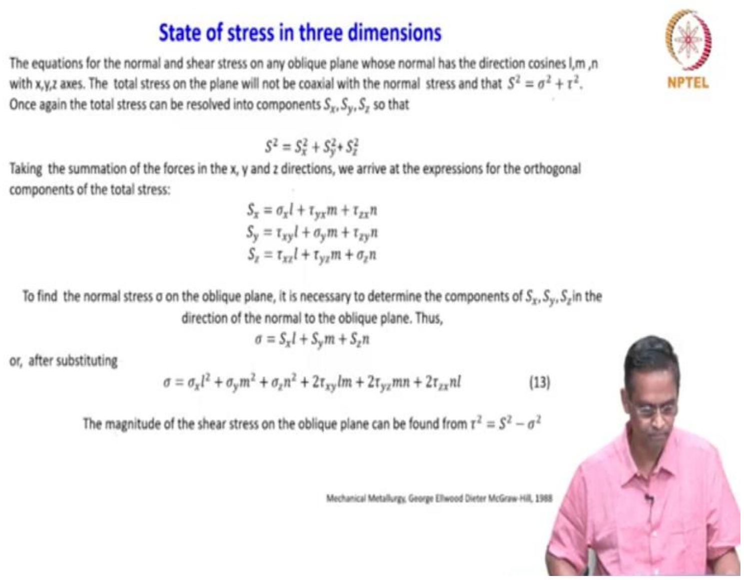
Then we are interested in the σ_a normal stress to this plane K, J, L and that area is A and we are interested in L. So, we are interested in x direction so, that is why it is σ Al that means, a stress into area so, it gives a force. So, same exactly similar to what we have done in the two dimension. So, here also you just look at it without any problem. So, if you sum it in this direction, then σ Al - σ_x Al - this is x component here and minus τ_{yx} Am - τ_{zx} An and so, you look at this x component here.

So, they are all summed up here and then equal to 0 then it is a next is a very simple step rearrange them and then you get the expression like this and similarly, you can do it for y direction and z direction. So, these are three homogeneous linear equation and the non-trivial solution is only can be found from the finding the determinant value so, you can just put them into this determinant and what you can see is the solution of the determined results in that cubic equation in σ.

So, this is how the cubic they if you expand this dataset, you will get this expression. So, this all very simple operation, but what you can now observe is there is a combination of stress associated with each of these σ. So, what it is that means, the three roots of the above equation are three principal stresses σ_1, σ_2 and σ_3. So, this combination of stresses associated with this coefficient of the σ they are not going to change even if you change the coordinate system.

Any different coordinate system and that is why they are called invariants, stress invariants So, you should know what is stress invariant, so, these are the combination of stress coefficients of σ the first one invariant is $\sigma_x + \sigma_y$ and σ_z that is I_1 and then $I_2$is written like this, this is I_3. So, these are the 3 invariants of this cubic equation the solution of this cubic equation. So, we will just refer this invariants when we look at the when we apply this concept into deformation much more than you will get the grip of what are we talking about.

(Refer Slide Time: 28:2 6)



So, let me number these and what we are now seeing here, so, far is a normal stress three principal stresses in three dimensions. Similarly, we can also look at the normal and shear stress on any oblique plane whose normal has a direction cosines l, m and n with respect to x,y and z axis. So, the one thing we have to remember here is the total stress on the plane will not be coaxial with a normal status and so, this condition that exists there.

So, once again we try to resolve these components in order to find this normal shear stress this this exercise we have done already, so, it is not three dimensions we will have 1 more extra term here. So, the resolved component is

$$s^2 = s_x^2 + s_y^2 + s_z^2$$

and taking summation of forces in all these three directions, we arrived expression for orthogonal competence of the total justice. So, this can be expanded like this.

$$s_x = \sigma_x \, l + \tau_{yx} m + \tau_{zx} \, n$$

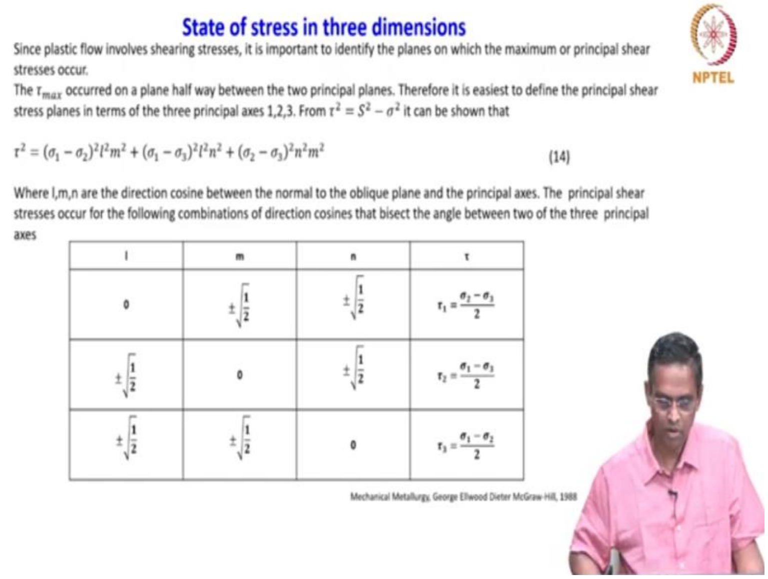
$$s_y = \tau_{yx} l + \sigma_y \, m + \tau_{zy} \, n$$

$$s_z = \tau_{xz} l + \tau_{yz} m + \sigma_z \, n$$

So, we are trying to write to resolve this, to find the normal stress σ on oblique plane it is necessary to determine the competence of S_x, S_y, S_z in the direction of normal to the oblique plane thus we write σ is equal to $S_x l + S_y m + S_z n$. So, you plug in all these values here and then you get this expression. So, again the magnitude of the shear stress on oblique plane can be formed from

$\tau^2 = S^2 - \sigma^{2,}$ which comes from this. So, these are all simple ways of looking at how we can visualize this normal and shear stress and then we can also work out a little more detail to get this expression.

(Refer Slide Time: 30:38)

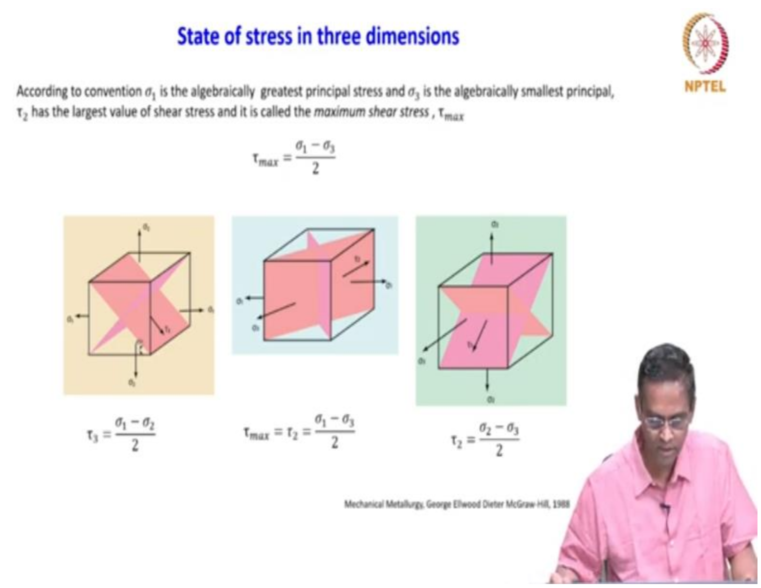


Let me finish that and why are we looking at it because, you know, plasticity we will be using the shear stresses quite often and it is also important to identify the planes on which the maximum and principal and shear stress occurs. So, that is another idea. So, we know that already this aspect we know this that is τ_{max} occurred on a plane halfway between the two principal planes. Therefore, it is easiest to define a principal shear stress in terms of 3 principal axes 1,2 , 3.

And we can show that this the previous equations from there we can just we can also show that the τ^2 is equal to, in the form of l, m, n though direction coefficients for to look at the principal planes we can use this terms and there is another way since, they are all direction cosines between normal to the oblique plane and the principal axis the principal shear stresses occur for the following combinations of direction cosines that bisect the angle between the two of the three principal axes. So, this is very interesting.

So, you can just specifically find out what combination of this direction cosine will give what type of shear stress combinations. So, this is what it is. So, if you can plug in this value of l, m, n then you get this kind of a 3*3matrix where you can see I know what type of shear stress or what type of shear plane will be operating in if this is called the direction cosines. So, you will get τ_1, $\tau_2$and τ_3.

(Refer Slide Time: 32 :49)



So, this you can just put it as in the form of graphs we can put and I will just first show this and so, this is the pictorial representation of this two planes right. So, according to the convention this we have already discussed σ 1 is the algebraically greatest principal stress and σ_3 valuably smallest, so, τ_2 has the largest value of shear stress and this is called maximum shear stress τ_{max}. So, this is this we know this is an important and we know this is a maximum τ max is equal to

$\sigma_1 - \sigma_3$.

So, if you look at the table also before τ_2 is a maximum. So, these are the three known set of principal planes actually, you can see that if you look at this diagram one thing is very clear for a given pair of normal stresses, there are a pair of you know principal shear planes which know bisects the normal stress here, here also it bisects. So, you can also see that the similar things are happening here for the τ max and then this is a τ_2.

So, what we are now trying to convey through this all this derivation, we are not only physically able to see how it is the principal plane or principal plane where the principal stress acts, principal plane where the principal shear stress acts and how they are connected angularly these are all the things we have witnessed with this kind of exercise. it may it may look little boring or tiresome but if you look at the end of and what you can just grasp out of all this derivation then it is not a bad idea to go through this. So, this is about the state of stress in three dimensions.