Mechanical Behaviour of Materials
Prof. S. Sankaran
Department of Metallurgical and Materials Engineering
Indian Institute of Technology, Madras

Lecture - 04
Strength of Materials – a short overview part - II

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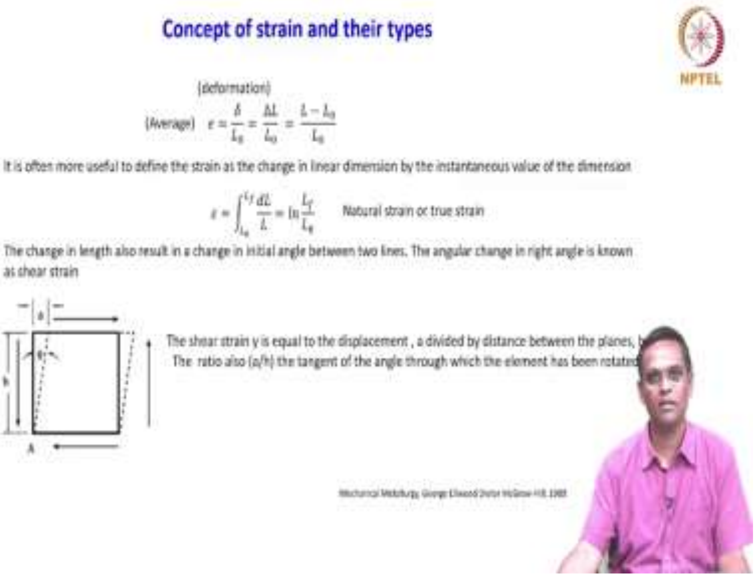

Hello, I am Professor S. Sankaran in the department of metallurgical and materials engineering. So, the concept of strain, how strain is defined is linear average strain that just like just now, we have seen that. So, it is always useful to define the strain as a change in linear dimension by the instantaneous value of the dimension rather than this average strain which is expressed as

$$\varepsilon = \int_{L_0}^{L_f} \frac{dL}{L} = \ln \frac{L_f}{L_0}$$

So, this is natural strain or true strain. So, the change in length also result in change in initial angle between the two lines. So, the angular change in the right angle is known as shear strain. So, the linear dimension what we have looked at for, as a deformation can also involve this change in angle. For example, if you take this square and these two dotted lines make some angle with respect to this face.

So, this shear is measured by the displacement a/h which is nothing but a tangent which is nothing but a tangent. The shear strain γ is equal to the displacement a divided by the distance between the planes h. The ratio (a / h) the tangent of the angle through which the element has been rotated. So, the message is the strain need not just be a linear displacement, it can also involve shear or change in angles which is a shear strain. So, this is again a convention. So, which is shear strain is denoted by gamma is equal to a / h is equal to tan theta.

$$\gamma = \frac{a}{h} = tan\theta = \theta$$

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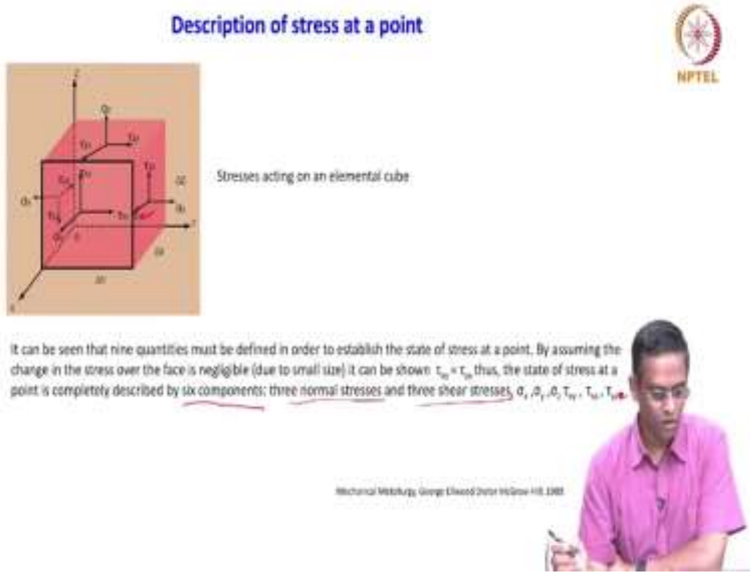

So, having introduced the notations of the stress and strain we will now focus our idea on description of stress at a point. So, how do we describe the stress at a point? So, this is a cube and we just got you know the length Δx, Δy and Δz is that the height and you have seen you are seeing that there are so, many components of the stresses denoted in this cube. So, x, y, z is a Cartesian coordinate.

And I say just mentioned in the previous slides in on a total stress always you know it is convenient to resolve intoa total stress into normal stress and the shear stress. So, that is how that is what it is shown in each of this phase. For example, you have this σ_y and σ_x and σ_z. they are all normal stresses that is they are perpendicular to the plane. This is y plane and this is an x plane and this is z plane they are normal to this direction, this direction and direction. So, that is they are called normal stress.

But if you look at the shear stresses, there are two shear stresses on each plane, like I just mentioned before two shear stresses and why there are two subscripts in shear stresses. But in the case of normal stresses only one subscript is given it is understood that this normal stress act perpendicular to this plane. So, σ_y means a normal stress acting perpendicular to this plane y here.

On the other hand, if you have two shear stresses, you need two subscripts because we need to know first of all, where this shear stress lies. So, that means, the first subscript denotes it lies in the y plane and the second subscript shows that it is which direction. For example, if you take this τ_{yz} is that the shear stress is in the y plane that is first one and the direction is x. So, similarly, you take this is in z plane but the direction is y. You take this shear stress it is in z plane.

So, first subscript is z that it is in this x direction, so, that zx. So, you need two subscripts to denote and describe a shear stress, this is a mistake it should be x. So, for describing a normal stress, you can denote it by σ_x or σ_y and σ_z and so, on. So, this is about the description about normal stress and shear stress. But we are talking about description of stress at a point.

So, what this diagram suggests is to describe a stress at a point you need almost a nine components of stress that is what it describes, it gives you a clue. If you are interested in stress at a point, you need a minimum of nine components. So, in this geometry, we have taken a cube as volume element. So, nine quantities must be defined in order to establish the state of stress at the point by assuming this change in the stress over the face is negligible.

Due to small size it can be shown that $\tau_{xy} = \tau_{yx}$ that is the state of stress at a point is completely disturbed by six components. So, if you assume this, identities, then the nine becomes six components to describe the stress out of which three are normal stresses and three are shear stresses. Which are σ_x, σ_y, σ_z, τ_{xy}, τ_{xz} and τ_{yz} something? So that is how it is described. So, now, we are now slowly concentrating on description.

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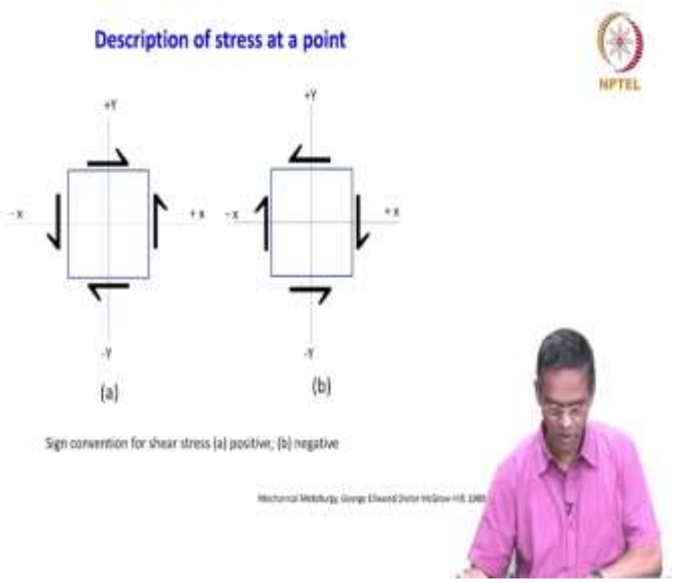

And we should also know the description of sign convention. So, how for example, if you look at this diagram a, the shear stresses are shown. So and this type of notation are termed as a positive shear stress. So, you can you can understand this diagram it is like this. So, you have the element like this and the shear stress act on this. So, it comes like this. So, it is it is forcing like this and this this element will force this. So, this way it is cycle. So, it is counterbalance in both directions right. So, like that, so, it is in the positive side, so, it is in the positive axis. So, similarly, here it is the negative. So, shear stress these notations are useful when you try to do some calculations or describe the state of stress and then try to calculate a transformation of stress all these things are useful. So, these are some of the basics we need to know.

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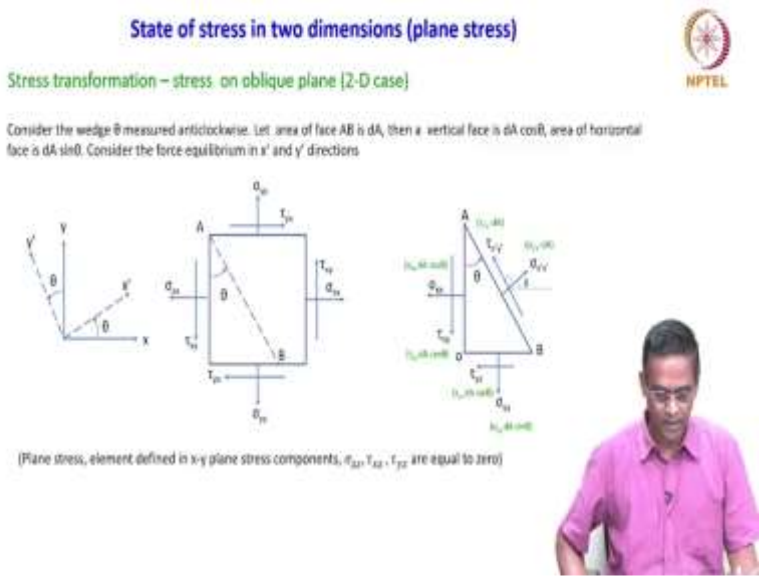

So, what we are now going to do is we are interested in finding without a stress at a point we are going to describe in a three dimension, but before we go into three dimension, we will first look at them in two dimensions or plane stress, this is also useful because later we will describe that most of the complicated problems which is being solved with the two dimensional assumption then it is much more simpler to solve that is called plane stress problem or plane train problem.

But, in this case, before we go to three dimensional description of stress are looking at your stress tensor, we will start with a state of stress in two dimensions and before we go there, we are also going to consider the stress transformation that means, the stress on oblique plane, it is not that we are going to describe stress at a fixed coordinates or fixed position. So, we will also like to look at the stress at a point and it influence you know, suppose if it changes the orientation from one coordinates to other. How it will have the effect the effect of orientation also has to be accounted for that is why we take these stress transformation. So, if you take a stress on an oblique plane, any arbitrary oriented object plane and then how it will work out to be including the orientation effect, so, that is what it is going to be described in this. So, before I read, look at this let us take this kind of geometry.

Where you have this square which is subjected to biaxial loading that is why we say that it is a plane stress. So, $\sigma_{xx,}$ σ_{yy} and τ_{xy} and τ_{yx} and $\tau_{xy,}$ τ_{yx} here, so, this is again a positive shear stress just I showed before and which is there in the normal Cartesian coordinate x and y this is the reference coordinate coordinates and suppose, if you tilt this whole configuration into some other angle, so, then that angle can be θ.

So, the x will become x' and y will become y', then what will happen to this geometry this also will get rotated. So, in order to describe this state of stress, let us consider the wedge this. So, we are going to cut a wedge which is denoted the dotted line here measured anti-clockwise. This is measured anti-clockwise let because this is what the rotation is so, it should be anti-clockwise let area of face AB is dA.

So, AB, this is area line is dA then the vertical faces dA $\cos\theta$, so, we are going to consider this AB is an oblique plane. So, if you look at this projection, it is this vertical, is this is a vertical face. This is dA $\cos\theta$ and an area of the horizontal this portion is dA$\sin\theta$. So, this is this is an assumptions. So, now, we will describe them just the wedge portion itself.

So, that is just a cut and then put it here for convenience. So, you can look at the coordinates much more simpler here. So, what you are seeing here is this is a new phase and $\sigma_{x'x'}$, that is a double x I mean prime is a normal stress to this oblique plane and this is a shear stress, $\tau_{x'y'}$, acting on this and rest all the same only thing is what I have just given here is we are interested in look at the force equilibrium in x′ and y′ direction. So, we are looking at this direction and this direction of the rotation. So, we are going to look at the force equilibrium in this new workstation. And for that we are getting ready with this description like this. So, this is what we have described is dA cos θ is the area of the vertical portion and dA sin θ is the horizontal area of the horizontal OB, and OA.

So, the σ_x, this direction multiplied by this area will give the force, that is what it is. So, that is very easy to calculate the force, if you just write it like this, it is easy. So, this is a normal stress and multiplied by area will do the force corresponding to this. Shear stress multiplied by the area give the corresponding force shear force, normal force shear force, shear force and then normal force this direction and this direction.

But we are interested in this rotated an oblique plane and we are now trying to calculate the force equilibrium based on this particular plane. So how are we going to do it, so that is what you want to look at. So there is no confusion here, do not get confused, you can look at this geometry, once again for your convenience, I have put all of them in a different colour. So that it directly gives the force. So when you say that the object is in a static equilibrium, then that net force is 0? So that is what we are going to do.

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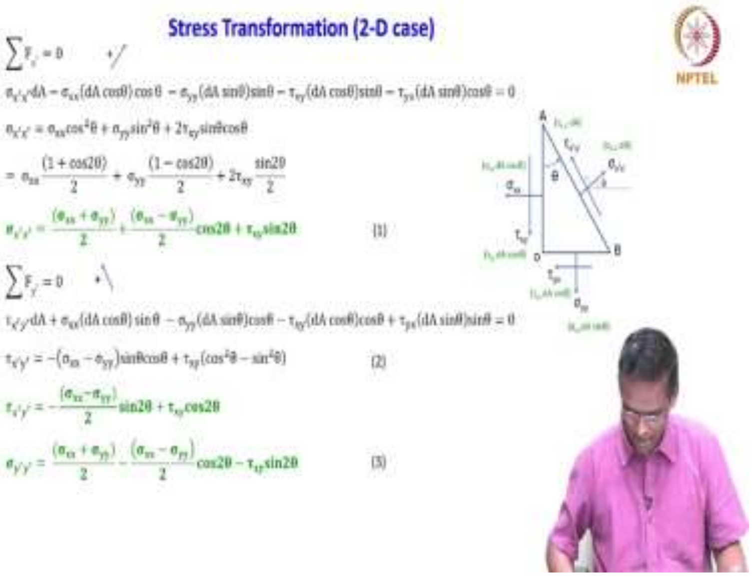

So, again, I will bring that geometry for your convenience. So, we are going to look at this total for net force, we are going to assume that is 0 and then we are going to write that what are all the total force pertaining to this direction we are going to look at this and this is a convention, sign convention what we have just talked. So, now, let us look at this equation

$\sigma_{x'x'}$ da is a force which is acting on this that is this is the normal force I said.

And this area is AB a AB plane is dA. So this is a force that is positive, because we are looking at this direction. At - sigma xx dA cosθ that means that as I said that the projection of this is vertical and this force is negative because you are looking at them this direction, so negative sign. σ_{xx} dA cosθ stress into area is a force, but then this additional cos θ that comes to because of this orientation, we have to take care.

So, σ_x dA cosθ, cosθ minus this σ_y this component which is a negative again because it is this orientation $-\sigma_{yy}$ dA sineθ and we are looking at this direction sine theta and this is again what is x y is again negative $-\tau_{xy}$ dA cos θ sinθ and this is $-\tau_{yx}$ dA sinθ cosθ. So, if you if you if you just write this one line equation by looking at this diagram then everything will be clear.

$$\sigma_{x'x'} dA - \sigma_{xx} (dA cos\theta) cos\theta - \sigma_{yy} (dA sin\theta) sin\theta - \tau_{xy}(dA cos\theta) sin\theta - \tau_{yx}(dA sin\theta) cos\theta = 0$$

You do not have to worry about it first time just for your own practice to gain a confidence you do it have your one and then look at this results, then it is easy. Otherwise, it will be, may look complicated, but look at the geometry first, what are the descriptors look at them carefully and then try to write of your own then it is easy it since I have just typed it here it may look very simple also, but when you sit and write you may get confused.

So, you try to practice this then it is easy. So, once you have written this, it is straightforward after that everything is very simple steps. So it is basically rearrangement

$$\sigma_{x'x'} = \sigma_{xx} cos^2 \theta + \sigma_{yy} sin^2 \theta + 2\tau_{xy} sin\theta cos\theta$$

then we have removed DA which is a common both sides.

So, this now, as simple parametric identity you can just bring in.

So cos^2 θ can be replaced with this (1 + cos 2θ / 2) and sin^2θ can be replaced by (1 - cos 2 θ/ 2) and sinθcosθ is replaced by a (sin ^{2}θ /2). So, now we have the equation.

So, this is one first equation of normal stress with arbitrary orientation.

$$\sigma_{x'x'} = \frac{(\sigma_{xx} + \sigma_{yy})}{2} + \frac{(\sigma_{xx} + \sigma_{yy})}{2} cos2\theta + \tau_{xy} sin2\theta$$

So, if you that what does it mean that means, if you have if you want to find out the normal stress in a public plane, if you know that theta plugin the θ values, you will get the normal stress value. So, now, similarly, we are interested in a shear stress. So, we will look at in the y direction that means $\sigma_{y'}$ direction that is $\tau_{x' y'}$ here. So, similarly, you just look at all the components here.

So, here $\tau_{x' y'} dA$. This is the force the shear force in this oblique plane, so, that is positive and now you see, plus σ_{xx} (dA cosθ)sinθ, because we are looking at this component this direction, which is this component, so, that is positive here and then minus rest all remaining same here because this component which is affecting or coming to this direction $-\sigma_{yy}$ dA sinθ cosθ and - τ_{xy} dA cosθcosθ rest all the same.

Though this one and this one is same orientation, but the τ_{yx} dA sinθand sinθ, that we just again changing the direction similar to an armistice, so, $\tau_{x'x'} dA - \sigma_{xx}\,(dA\cos\theta)\sin\theta \;-\; \sigma_{yy}\,(dA\sin\theta)\cos\theta - \tau_{xy}(dA\cos\theta)\cos\theta - \tau_{yx}(dA\sin\theta)\cos\theta = 0$

this is another equation you have to just look at this geometry and then then it is easy. So, similar to this, this is got one equation, then we can replace that earlier in that with the common practice you remove and then you rewrite this and final expression can be

$$\tau_{x'x'} = -\frac{(\sigma_{xx} - \sigma_{yy})}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'y'} = \frac{(\sigma_{xx} + \sigma_{yy})}{2} - \frac{(\sigma_{xx} - \sigma_{yy})}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

So, this is another shear stress equation similar to this normal stress if you plug in this 2 θ value in an arbitrary plane or member you will get the shear stress and similar to $\sigma_{x'}$ you can also do the same thing for σ_{yy}. It will also similar expression you will get.

So, these three equations are the fundamental equations from where we can just build up to find out all type of stresses and orientation effects we can correlate. So, this equation this particular type of derivation is quite useful if you practice by yourself.

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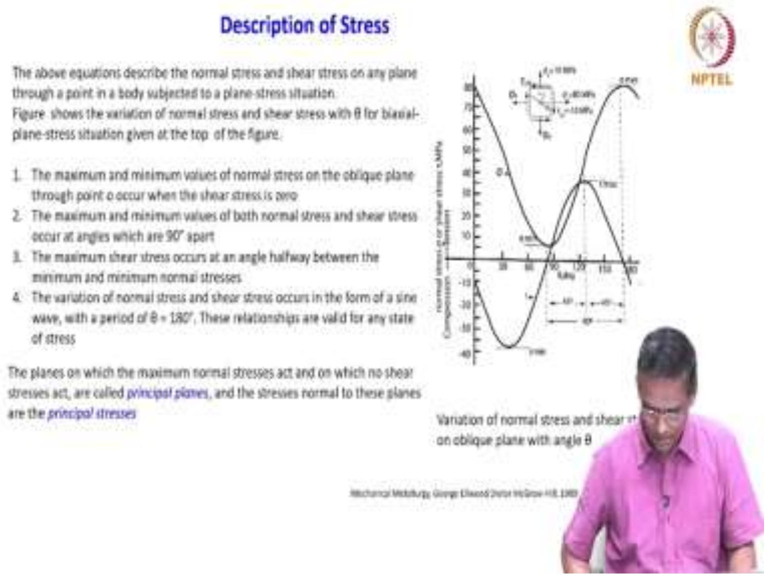

And we will now just look at some description about this. So, the above equations describe the normal stress and the shear stress on any plane through a point in your body subjected to plane stress to situation. So, it is a stress transformation in 2D or description of stress at a point in an oblique plane with no with the 2 dimension whatever you call it, but it is a plane stress situation. So, we are talking about σ_x and σ_y plane the situation.

So, it is interesting to note this graph and how this for example, if you look at this similar geometry what I have drawn, suppose, if you assume this biaxial situation, there it is here σ_x and σ_y has got some specific values. So, in order to appreciate what this graph is showing. So, the σ_x is equal to 80 MPa and σ_y is equal to 10 MPa and shear stress is -10 MPa. So, it is plotted on this graph. This is an normal stress or shear stress, τ versus the orientation.

So, all that we have now looked at is how this you know, stress in a 2D will have an effect of θ that is orientation or transformation, whatever we call it, so, this gives them an clear demonstration how it looks like. So, what is that we are seeing here. So, the figure shows the variation of normal stress and shear stress with θ for a biaxial plane situation given at the top of the figure. So, this is a figure we are referring to. The maximum and minimum values of normal stress on the oblique plane to the point O occurs when the shear stress is 0.

So, what does it mean how what are we seeing here, the shear stress is 0 is here. So, this is where the maximum and the minimum values of normal stress on an oblique plane. So, the maximum and minimum it happens here. So, this is 0 and this is 0, that is what it says maximum and minimum normal stresses on oblique plane throw point O will occur when shear stresses 0 for shear stress 0 here and here. So, the maximum, minimum values of both normal stress and justice occur at an angle which is which are 90 degrees apart.

So, the maximum and minimum values of both normal and shear stress, so, normal and shear stress maximum and minimum, maximum, minimum, maximum, minimum the interval is 90˚ that is what it says. So, the maximum shear stress occurs at an angle halfway between the minimum normal stresses. So, this is what it is halfway between the angles the maximum shear stress of B between the minimum of normal stresses.

This is minimum and should be a maximum normal stresses. So, this is halfway. So, variation of normal stress and shear stress occurs in the form of a sine wave with a period of θ is equal to 180 degree, these relationships are valid for any state of stress. So, this is the sine wave function. So, similarly, you will have a sine wave function of this. So, the another classification is the planes on which the maximum normal stress act and on which no shear stress act are called principal planes.

And the stress is normal to these planes are the principal stresses. So, before we get into the much more detailed definition of this. So, let us get into the nomenclature of what is principal plane and principal stresses. So, this is the starting point. A plane on which maximum normal stress act and no shear stress are called principal planes something like this, this situation shear stress is 0 here and the normal stress will have a maximum.

So, this will happen at particular planes the loose planes are called principal planes and normal to these planes they are called principal stresses. So, principal stresses means there is no shear 0 shear stress. That is the bottom line here we have to remember always.

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So, since by definition a principal plane contains no shear stress its angular relationship with respect to the xy coordinate axis can be determined by finding the values of θ in equation 2 for which $\tau_{x'y'}$ is equal to 0. So, we have now derived some I mean equation which describes the shear stress on a plane stress situation for an oblique plane or arbitrary θ.

So, if in order to find out the angular relationship if that equation is being made into 0 then we will be having some relations we can develop some relation. What is that? This is the relation we have taken from the previous slides the if you look at the shear stress expression, this is the same shear stress expression then we are trying to make it equal to 0 and then try to rearrange that.

So, if you do.

$$\frac{\tau_{xy}}{(\sigma_x \; - \sigma_y)} = \frac{sin\theta\, cos\theta}{cos^2\theta - \; sin^2\theta} = \frac{1/2(sin2\theta\;)}{con2\theta} = \frac{1}{2}\; tan2\theta$$

$$tan2\theta \;= \frac{2\,\tau_{xy}}{\sigma_x \; - \sigma_y}$$

. So, this is an angular relationship we are talking about with respect to these stresses.

What is that I am trying to show here equation 1 what is equation 1 each equation 1 is σ_{xx} what we have derived in this 2D case if you plug in all the values of cos 2θsin 2θ. then the values it will give the principal stresses the equation will do the principal stresses that is σ1 and σ 2. So, how do we find this the value of this. So, we are seeing this. So, what is this if we can take from this simulation, we can simply we can also do this.

Suppose you assume this as a 2θ. So, from this relation you will be able to find out what is this using Pythagoras theorem. So, you because of using that relation, so, you will be able to find sin 2θ. So, from this equation you put it this and then this is tan 2θ is to τ x y / sigma x - sigma y will give you the simple Pythagoras relation.

$$sin2\theta = \pm \frac{\tau_{xy}}{[\frac{(\sigma_x - \sigma_y)^2}{4} + \tau^2 xy]^{1/2}}$$

$$cos2\theta = \pm \frac{\frac{\sigma_x - \sigma_y}{2}}{[\frac{(\sigma_x - \sigma_y)^2}{4} + \tau^2 xy]^{1/2}}$$

So, this will be your hypotenuse, so, it comes like this. So, similar equation for cos 2 theta just I am just giving you.

So, that you will be able to appreciate from this relationship you will be easily able to derive this using simple mathematics and still appreciate the physical concept they are trying to visualize how do we find this principal stresses and biaxial situation.

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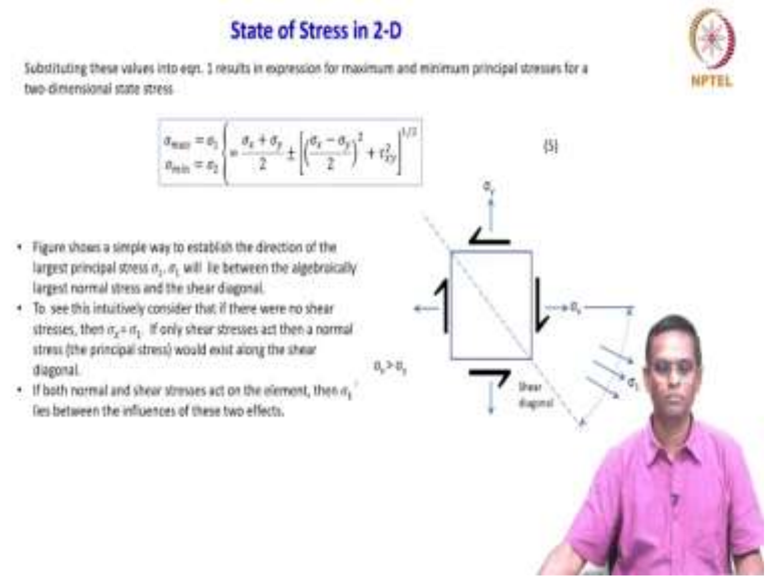

So, again from using this after substituting this you will this is the final expression you will get from this the original equation that So, this is the expression for maximum and minimum principal stresses for a D dimensional state of stress.

$$\sigma_{\max,}, \sigma_{\min,} = \sigma_1 , \sigma_2 \{= \frac{\sigma_x + \sigma_y}{2} \pm [\frac{(\sigma_x - \sigma_y)^2}{4} + \tau^2 xy]^{1/2}$$

So, what you see here is once you get into the grip of getting the common equation it is very easy to plug in the values of theta and then arrive at some expression for principal stress. If we look the equation look very complicated, but if you just sit and work and work it out, you will not find it difficult. The most important point is you will get the physical picture, you should keep that physical you know the dimensions everything in mind and then it will do it will be very easy to grasp this.

What we can do is now, this what is this figure? this figure shows shear stresses, this is a negative shear stress and this is a shear diagonal and the σ_x is a normal stress and σ_y again a normal stress. Suppose, if you want to get an idea of how to establish the σ_1, how do we find the big σ_1? This figure shows a simple way to establish the direction of a largest principles just σ_1 and σ_1 will lie between algebraically largest normal stress and shear diagonal.

You see, when we describe this sigma and in the state of stress in a three dimension again we will come to this point σ_1 is the largest principle stress and then σ_2 and then σ_3 and the maximum shear stress will occur at 45 here 45 degree and then when you when the σ_1 has 0 shear stress then it becomes σ_x normal stress that is principal stress if there is 0 shear stress it is principal stress and then it is a pure shear here to see this intuitively considered that if there is no shear stresses.

Then σ_x becomes σ_1, that is what I am saying suppose, if you say σ_x does not have a shear stress, then it becomes σ_1 if only shear stress act on the normal stress that is principal stress would exist along the shear if the principal stress itself a pure shear, then it is only on the diagonal. So, if both normal stress and shear stress act on the element then σ_1 lies between the influences of these 2 effects.

So, this is a very nice way of looking at it. If both normal and shear stress act on an element, so, it has to it has to live somewhere in between. So, because it has got both the components whether normal it will be a normal stress here at as well as your copyright somewhere if it is not pure shear it is not pure principal stresses but somewhere in between. If the both normal and shear stress act on element, then sigma 1 lies between the influences of these 2 effects. So, that is the effect of I wanted to show in this image.

So, here again σ_x is greater than sigma y like you know σ_1 is greater than σ_2 and σ_3 something like that. So, this image clearly shows the how to visualize the σ_1 or how to visualize the sigma when a body is subjected to both normal stress and shear stress. I think we will stop here and we will continue the discussion on this stresses and their derivation in the next class.

So, what I would like to tell you before I wind up. So, these geometries and the equations unless you practice of your own it will be difficult. So, you just go through the slides because in a PPT it may appear so simple. So, but I am not saying that it is difficult or something elementary only, but the point is you have to practice by yourself after going through this you try to practice yourself then it is easy. Thank you.