


**Mechanical Behaviour of Materials**  
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**Lecture - 38**  
**Mechanical Testing - VI**


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**Yield-Point Phenomenon and Strain Aging**

- The serrated character of curve C in Fig. sometimes observed in plain carbon steel alloys and certain aluminum alloys (e.g., Al-Mg alloys) can also be explained in terms of dislocation-solute atom interactions
- Curve C is inhomogeneous in nature but does not have the distinctive horizontal segment that characterizes the upper and lower yield point phenomenon seen in A.
- Bursts of mechanical twin formation and growth are one possible cause of the serrated behavior seen in curve C
- Dislocation motion is the dominant (or only) plasticity mechanism active under normal loading conditions.
- In these cases, the appearance of curve is known as *Portevin – Le Chatelier effect*, and is evidence of *dynamic strain aging*
- As long as the diffusion rate for the solute atoms is equal to or slightly greater than the rate of plastic deformation, dislocations will alternately break free from solute atmospheres and then be repinned, producing serrated curve C
- If the strain rate and test temperature were outside the critical range, homogeneous dislocation flow would take place since solute atmosphere formation would no longer be favored; accordingly, the stress-strain curve would be smooth (curve D)
- Dynamic strain aging can cause poor surface quality and reduced ductility associated with the formation of undesirable surface marks like Lüders bands, so it is preferable to avoid it through careful alloy selection and choice of processing conditions when possible

Deformation and Fracture Mechanics of Engineering Materials by Richards W. Hertzberg, John Wiley & Sons, 2012 37  
Mechanical Metallurgy, George E. Dieter McGraw-Hill, 1988



Hello I am Professor S. Sankaran in the Department of Metallurgical and Materials Engineering. So now we come to the serrated character of the curve C in that previous slide. The serrated character of the curve C in figure sometimes observed in the plain carbon steel and alloys and certain aluminium alloys for example, aluminium - magnesium alloys can also be explained in terms of dislocation solute atom interactions that is curve C.

Curve C is inhomogeneous in nature but does not have the distinctive horizontal segment that characterizes the upper and lower yield point phenomenon seen in A. Bursts of mechanical twin formation and growth are one possible cause of the serrated behaviours in curve C. Dislocation motion is the dominant or the only plasticity mechanism active under normal loading conditions. In these cases the appearance of curve C is known as Portevin - Le Chatelier effect and is evidence of dynamic strain aging. So, this particular serrated flow of materials or serrated flow execution by the stress strain curve is a phenomenon called dynamic strain aging. So, we are discussing two things here either it could be a solute dislocation interaction pinning, unpinning and then again it comes out of the atmosphere and relaxes and then repin then also this kind of serrated flow comes or it could be a bursts of pin formation product nucleation and growth this also will cause the serrated behaviour. As long as the diffusion rate for solute atoms equal to or slightly greater than the rate of plastic

deformation, dislocations will alternatively break free from the solute atmospheres and then be repined just I said before as long as this can happen pinning, repining So, the serrated flow will keep on occurring till the end of the fracture producing the serrated curve C. If the strain rate and the test temperature were outside the critical range, homogeneous dislocation flow would take place since solute atmosphere formation would no longer be favoured, accordingly the stress strain curve would be smooth. Dynamic strain aging can cause poor surface quality and reduced ductility, associated with the formation of undesirable surface marks like Luders band. So, it is preferable to avoid it through careful alloy selection and choice of processing condition when possible. So, like we discussed in the case of Luders band, these dynamic strain aging also wants to leave poor surface quality. So, we need to select the appropriate processing or alloy design or other conditions to overcome this.

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**Effect of Strain rate on Flow Properties**

- The rate at which strain is applied to a specimen can have an important influence on the flow stress. Strain rate is defined as  $\dot{\epsilon} = d\epsilon/dt$ , and is conventionally expressed in units of  $s^{-1}$ , i.e., "per second."
- The spectrum of available strain rates is given in Table.
- Figure shows that increasing strain rate increases flow stress.
- Moreover, the strain-rate dependence of strength increases with increasing temperature.
- The yield stress and flow stress at lower plastic strains are more dependent on strain rate than the tensile strength.

Range of strain rate	Condition or type test
$10^{-8}$ to $10^{-5} s^{-1}$	Creep tests at constant load or stress
$10^{-5}$ to $10^{-1} s^{-1}$	"Static" tension tests with hydraulic or screw-driven machines
$10^{-1}$ to $10^2 s^{-1}$	Dynamic tension or compression tests
$10^2$ to $10^4 s^{-1}$	High-speed testing using impact bars (must consider wave propagation effects)
$10^4$ to $10^8 s^{-1}$	Hypervelocity impact using gas guns or explosively driven projectiles (shock-wave propagation)

Flow stress at  $\epsilon = 0.002$  versus strain rate for 6063-T6 aluminum alloy

Mechanical Metallurgy, George E. Dieter, McGraw-Hill, 1988

That is about the yield point phenomenon and strain aging as you can just briefly we looked at. Now we go to effect of strain rate on flow properties. Suppose, how the strain rate is going to get affected. The rate at which the strain is applied to a specimen can have an important influence of the flow stress. The strain rate is defined as  $\dot{\epsilon}$  which is  $d\epsilon / dt$  and it is conventionally expressed in units of per second.

The spectrum of available strain rate is given in this table. So what are the typical strain rates we come across the  $10^{-8}$  to  $10^{-5} s^{-1}$  typically used in creep tested constant load or constant stress.  $10^{-5}$  to  $10^{-1} s^{-1}$ , a static tension tests with hydraulic or screw driven machines.  $10^{-1}$  to  $10^2 s^{-1}$  which is a dynamic tension or compression test.

and  $10^2$  to  $10^4$  high speed testing using impact bars must consider wave propagation effects because we are dealing with very, very high strain rate.  $10^4$  to  $10^8$  is hypervelocity impact

using gas guns or explosively driven projectiles, shockwave propagations. So, these are the typical strain rate ranges and the corresponding test is given which is very important you should know this.

And most of the tension tests are done with the initial strain rate you have to understand that it is not you can also do the constant strain rate experiments, but it is conventionally or normally it is not done in every laboratory it is always considered the initial strain rate that you are doing. So, this is a flow stress at the strain of 0.002 versus strain rate for 6063 aluminum alloy can see that at various temperatures stress versus strain rate.

You can see that how the strain rate at this temperature I mean, for a given temperature to the increased strain rate the stress increases. The temperature is low at the  $\sigma$  versus that also we know but this is what it is shown here. The yield stress and flow stress at lower plastic strains are more dependent on strain rate than the tensile strength. So, this is what it is the yield strength and flow stress at lower plastic strains.

High rates of strain cause the yield point to appear in tests on low carbon steel that do not show yield point under ordinary rates of loading this is another interesting point of that means, your rate of loading is so high. So, this is what we just talked about in the previous slide that the diffusion rate versus the plastic flow rate which is dominate whichever is faster will suffer as the other.


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### Effect of Strain rate on Flow Properties

- A general relationship between flow stress and strain rate, at constant strain and temperature is
$$\sigma = C(\dot{\epsilon})^m \big|_{\epsilon, T}$$

where  $m$  is known as the **strain-rate sensitivity**. The exponent  $m$  can be obtained from the slope of a plot of  $\log \sigma$  vs.  $\log \dot{\epsilon}$ .
- However, a more sensitive way is a rate-change test in which  $m$  is determined by measuring the change in flow stress brought about by a change in  $\dot{\epsilon}$  at a constant  $\epsilon$  and  $T$ .

$$m = \left( \frac{\partial \ln \sigma}{\partial \ln \dot{\epsilon}} \right)_{\epsilon, T} = \frac{\dot{\epsilon}}{\sigma} \left( \frac{\partial \sigma}{\partial \dot{\epsilon}} \right)_{\epsilon, T} = \frac{\Delta \log \sigma}{\Delta \log \dot{\epsilon}} = \frac{\log \sigma_2 - \log \sigma_1}{\log \dot{\epsilon}_2 - \log \dot{\epsilon}_1} = \frac{\log(\sigma_2/\sigma_1)}{\log(\dot{\epsilon}_2/\dot{\epsilon}_1)}$$

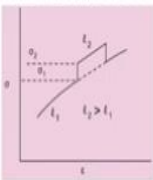


- Strain-rate sensitivity of metals is quite low ( $< 0.1$ ) at room temperature but  $m$  increases with temperature, especially at temperatures above half of the absolute melting point.
- In hot-working conditions  $m$  values of 0.1 to 0.2 are common.
- The above equation is not the best description of the strain-rate dependence of flow stress for steels.
- For these materials, a semilogarithmic relationship between flow stress and strain rate appears to hold.

$$\sigma = k_1 + k_2 \ln \frac{\dot{\epsilon}}{\dot{\epsilon}_0}$$


where  $k_1, k_2$ , and  $\dot{\epsilon}_0$  are constants.

Strain-rate change test to determine strain-rate sensitivity



Mechanical Metallurgy, George E. Dieter McGraw-Hill, 1988

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So, we just look at the some of the constitutive relations a general relationship between the flow stress and the strain rate at constant strain and temperature is given by  $\sigma = C(\dot{\epsilon})^m$  at constant strain and temperature where,  $m$  is known as strain rate sensitivity very important parameter is useful in the high temperature deformation. The exponent  $m$  can be obtained

from the slope of a plot of  $\log \sigma$  versus  $\log \dot{\epsilon}$  similar to Holloman equation here we get it is  $m$ . However, a more sensitive way the rate change test in which  $m$  is determined by measuring the change in flow stress brought by about a change in  $\dot{\epsilon}$  at constant  $\epsilon$  and  $T$

which is given by 
$$m = \left( \frac{\partial \sigma}{\partial \dot{\epsilon}} \right)_{\epsilon, T} = \frac{\dot{\epsilon}}{\sigma} \left( \frac{\partial \sigma}{\partial \dot{\epsilon}} \right)_{\epsilon, T} = \frac{\Delta \log \sigma}{\Delta \log \dot{\epsilon}} = \frac{\log \sigma_2 - \log \sigma_1}{\log \dot{\epsilon}_2 - \log \dot{\epsilon}_1} = \frac{\log (\sigma_2 / \sigma_1)}{\log (\dot{\epsilon}_2 / \dot{\epsilon}_1)}$$


Suppose, if you do it at two different strain rates then we can be written like this, the final expression is this which is very use to that is what is written here, this is a more sensitive way of determining the log. So that means  $m = \log (\sigma^2 / \sigma^1) / \log (\dot{\epsilon}^2 / \dot{\epsilon}^1)$ . Strain rate sensitivity of metals is quite low less than 0.1 at room temperature but  $m$  increases with temperature very important point it is important only at high temperature deformation as I mentioned.

Especially at temperatures above half of the absolute melting point  $0.5 T_m$  above  $0.5 T_m$  it is more significant. So, this is what is shown in a schematic here plot  $\sigma$  versus  $\dot{\epsilon}$ , test conducted at two different strain rates. So strain rate change test or it is also called strain rate jump test to determine strain rate sensitivity. In hot-working conditions,  $m$  values of 0.1 to 0.2 are common values. The above equation is not the best description of strain rate dependence of the flow stress for steels. For these materials; is semilogarithmic relationship between flow stress and the strain rate appears to hold. So, that means for materials like steels this is relation course better than the other equation that is  $\sigma = k_1 + k_2 \log (\dot{\epsilon} / \dot{\epsilon}_0)$  where  $k_1$ ,  $k_2$  and  $\dot{\epsilon}_0$  are constants. So this is again another empirical relation we can use.

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### Effect of Strain rate on Flow Properties

- Strain-rate sensitivity is a good indicator of changes in deformation behavior and measurements of  $m$  provide a key link between dislocation concepts of plastic deformation and the more macroscopic measurements made in the tension test.
- It is known that the velocity of dislocation motion was very strongly dependent on stress according to  $v = A\sigma^m$
- Moreover, strain rate is related to velocity of mobile dislocations by



$$\dot{\epsilon} = \rho b v$$

From previous Eqs.

$$\frac{1}{m} = \frac{\partial \ln \dot{\epsilon}}{\partial \ln \sigma} = \frac{\partial \ln v}{\partial \ln \sigma} + \frac{\partial \ln \rho}{\partial \ln \sigma}$$

and from above Eq.


$$\frac{\partial \ln v}{\partial \ln \sigma} = m'$$

Therefore,

$$m' = \frac{1}{m} - \frac{\partial \ln \rho}{\partial \ln \sigma}$$

Mechanical Metallurgy, George E. Dieter McGraw Hill, 1988

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Strain rate sensitivity is a good indicator of changes in deformation behaviour and the measurements of  $m$  provide a key link between dislocation concepts of plastic information

and their more macroscopic measurements made in the tension test very important idea we will see. It is known that the velocity of dislocation motion was very strongly dependent on the stress according to this relation  $v = A \sigma^{m'}$ .

Moreover, strain rate is related to velocity of mobile dislocation by this expression this we have already seen  $\dot{\epsilon} = \rho b v$ . So, from this previous equations we can relate or rewrite this

$1/m = (\partial \ln \dot{\epsilon} / \partial \ln \sigma) = (\partial \ln v / \partial \ln \sigma) + (\partial \ln \rho / \partial \ln \sigma)$ . So, you can use this relation and then apply this relation then you get  $\partial \ln n / \partial \ln \sigma = m'$

Or you can simply write  $m' = 1/m - (\partial \ln \rho / \partial \ln \sigma)$ .

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### Effect of Strain rate on Flow Properties


- Thus, if there is no change in the mobile dislocation density with increasing stress,  $m' = 1/m$ .
- This hardly is a reasonable assumption. However, if  $1/m$  is plotted as a function of strain, the curve extrapolated to zero gives a value close to  $m'$  determined from etch-pit measurements of dislocation velocity.
- While strain-rate sensitivity may be quite low in metals at room temperature, in other materials it may be quite appreciable. The extreme case is a Newtonian viscous solid, where the flow stress is described by  $\sigma = \eta \dot{\epsilon}$

and by comparison with Eq. , we find that  $m = 1$ .

- High strain-rate sensitivity is a characteristic of superplastic metals and alloys.
- **Superplasticity** refers to extreme extensibility with elongations usually between 100 and 1,000 percent.
- Superplastic metals have a grain size or inter phase spacing of the order of  $1 \mu\text{m}$ . Testing at high temperature and low strain rates accentuates superplastic behavior.
- The large elongations result from the suppression of necking in these materials with high values of  $m$ .
- An extreme case is hot glass ( $m = 1$ ) which can be drawn from the melt into glass fibers without the fibers necking down.

Mechanical Metallurgy, George E. Dieter, McGraw-Hill, 1988





Thus if there is no change in the mobile dislocation density with increasing stress  $m' = 1/m$ , this hardly is a reasonable assumption. However, if  $1/m$  is plotted as a function of strain the curve extrapolated to 0 gives a value close to  $m'$ , determined from etch-pit measurements of dislocation its dislocation velocity. While strain rate sensitivity may be quite low in metals at room temperature in other materials, it may be quite appreciable.

The extreme cases of Newtonian viscous solid where the flow stresses described by  $\sigma = \eta \dot{\epsilon}$ . So we are now trying to relate this strain rate sensitivity with different temperatures as the temperature goes up. The extreme case of strain rate sensitivity can be which will describe it Newtonian viscous solid which will obey this flow equation like this. And by comparison with the previous equation effect of strain rate sensitivity we find that  $m = 1$ .

The high strain rate sensitivity is a characteristic of superplastic metals and alloys, we are slowly taking this idea of strain rate sensitivity at moderate temperature to very high temperatures and now we are trying to explain the new phenomenon called superplastic metals or superplastic behaviour. So high the material exhibit any material which exhibit high strain rate sensitivity is a characteristic of superplastic metal and alloys.

Superplasticity refers to extreme extensibility with elongations usually between 100 and 1000% of elongation. We will see this little more detail, we will look at closely the phenomenon when we look at the high temperature definition the next chapter we will see, but we are just introducing this concept because we are talking about effect of strain rate on flow properties there also this idea comes up because we are bringing  $m$  value that is strain rate sensitivity and then high strain rate sensitivity is the characteristic of superplasticity.

Superplastic metals have a grain size or inter phase spacing of the order of 1 micron testing at high temperature and low strain rates accentuates superplastic behaviour. The large elongation result from the suppression of necking in these materials with the high values of  $m$ . So, this sentence is very important sentence we have to wait a little bit of attention. The large elongation results from the suppression of necking. So, we just looked at the necking phenomenon elaborately.

So we have to suppress the making that means, this material should have high values of  $m$ . So, high values of  $m$  also not gives a indirect measure of resistance to make it so, this point you have to keep in mind any material exhibits a high value of  $m$  that means, this material has the highest resistance to making then it will promote superplastic behaviour that means higher percentage of elongation.

An extreme case is hot glass where  $m = 1$  which is almost like you know a viscous solid which can be drawn from the melt into glass fibers without the fibers necking down. So, this is one classical example where you can see the strain rate sensitivity is very high.

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**Effect of Strain rate on Flow Properties**

From the definition of true strain

$$\dot{\epsilon} = \frac{1}{L} \frac{dL}{dt} = - \frac{1}{A} \frac{dA}{dt}$$

and combining Eqns.

$$-\frac{dA}{dt} = A \dot{\epsilon} = A^{1-1/m} \left( \frac{P}{C} \right)^{1/m}$$

and

$$-\frac{dA}{dt} = \left( \frac{P}{C} \right)^{1/m} \left( \frac{1}{A^{(1-m)/m}} \right)$$

- Equation states that so long as m is < 1 the smaller the cross-sectional area, the more rapidly the area is reduced.
- Figure shows how the area decrease varies with m.
- When m = 1 the deformation is Newtonian viscous and dA/dt is independent of A and any incipient neck is simply preserved during elongation and does not propagate inward.
- As m approaches unity, the rate of growth of incipient necks is drastically reduced.

Mechanical Metallurgy, George E. Dieter McGraw Hill, 1988

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So now we try to relate that with some basic equations  $P / A$  is  $\sigma$  which is can be related to this  $C (\epsilon)^m$  and we can rewrite this equation for the  $\dot{\epsilon}$  and we can also connect to the other definition of strain rate and then combine these two and get one a general relation for describing the strain rate sensitivity So, it is also will give another idea the equation states that so long as m is less than 1 the smaller the cross sectional area the more rapidly the area is reduced.

And this figure shows what how the area decrease varies with m, so this is  $-(dA / dt)$  and this is area so, how the m value changed, for m = 1 this is almost constant here, there is no this area reduction is not a issue here. But on the other hand, if you have m = 3 / 4, m = 1 / 2, m = 1 / 4 then you can see the influence of this area reduction. When m = 1 the deformation is Newtonian viscous and  $(dA/dt)$  is independent of A and any incipient neck is simply preserved during elongation and does not propagate inward.

So, all this curves indirectly indicates the resistance to how the material resist to necking this is what you get out of this at the end. As m approaches unity the rate of growth of incipient necks is drastically reduced.

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### Effect of temperature on flow properties



- The stress-strain curve and the flow and fracture properties derived from the tension test are strongly dependent on the temperature at which the test was conducted.
- In general, strength decreases, and ductility increases as the test temperature is increased.
- However, structural changes such as precipitation, strainaging, or recrystallization may occur in certain temperature ranges to alter this general behavior.
- Thermally activated processes assist deformation and reduce strength at elevated temperatures.
- The best way to compare the mechanical properties of different materials at various temperatures is in terms of the ratio of the test temperature to the melting point, expressed in degrees kelvin.
- This ratio is often referred to as the *homologous temperature*.
- When comparing the flow stress of two materials at an equivalent homologous temperature, it is advisable to correct for the effect of temperature on elastic modulus by comparing ratios of  $\frac{\sigma}{E}$  rather than simple ratios of flow stress.

Mechanical Metallurgy, George E. Dieter, McGraw Hill, 1988

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Now we will pay our attention to temperature so far we just looked at the strain rates and now we look at the effect of temperature. The stress-strain curve and the flow and the fracture properties derived from the tension tests are strongly different on the temperature at which the test was conducted. In general strength decreases and ductility increases as the test temperature is increased. However structural changes such as precipitation, strainaging or recrystallization may occur in certain temperature ranges to alter this general behaviour.

So, you have to be careful in making this I mean making this statement you have to see that the temperature does not influence any other phase transformation reaction or precipitation strainaging, just before we have seen simple you know the pinning and unpinning of dislocation causes strain aging behaviour and then recrystallization grain growth all these things you have to keep in mind. Thermally activated processes assist deformation and reduce strength at elevated temperatures.

The best way to compare the mechanical properties of different materials at various temperatures is in terms of the ratio of the test temperature to the melting point expressed in degrees kelvin. The ratio is often referred to as the homologous temperature see this is one way of normalization you do just to keep I know reduce the influence of melting point or suppress the influence of melting point you refer to this parameter or homologous temperature.

And then study the compared to mechanical properties of different materials this also we will see it as we study this high temperature deformation behaviour in detail we will use only this



parameters that time also you will appreciate this much better. When comparing the flow stress of two materials at an equivalent homologous temperature, it is advisable to correct for the effect of temperature on elastic modulus by comparing the ratio of  $\sigma / E$  rather than simple ratio of flow stress.

So, this is basically a sum of the instructions or information when we take too much of information from the simple tensile test. So, what are the aspects to look at that is how these points are given here.


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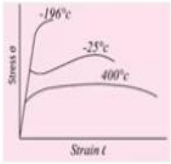
### Effect of temperature on flow properties

- The temperature dependence of flow stress at constant strain and strain rate generally can be represented by
$$\sigma = C_2 e^{\frac{Q}{RT}} \dot{\epsilon}^n$$


where  $Q$  = an activation energy for plastic flow,  $J \text{ mol}^{-1}$ ,  $R$  = universal gas constant,  $8.314 \text{ J mol}^{-1} \text{ K}^{-1}$   
 $T$  = testing temperature,  $K$

- If this expression is obeyed, a plot of  $\ln \sigma$  versus  $1/T$  will give a straight line with a slope  $Q/R$ .
- The value of flow stress depends on the dislocation structure existing at the time the flow stress is measured.
- Dislocation structure will change with temperature, strain rate, and strain.
- One way to separate these effects is to evaluate  $Q$  by a temperature change test.
- The stress-strain curve is conducted at constant strain rate, and at the desired value of plastic strain the temperature is changed from  $T_2$  to  $T_1$ . After the temperature comes to equilibrium, the activation energy is given by





$$Q = R \ln \left( \frac{\sigma_1}{\sigma_2} \right) \frac{T_1 T_2}{T_2 - T_1}$$



Chemical Metallurgy, George Ellwood Dieter McGraw Hill, 1988

So, the temperature dependence of flow stress at constant strain and strain rate generally can be expressed by  $\sigma = C_2 e^{(Q / RT)}$  at the constant strain where  $Q$  is an activation energy for plastic flow in joules/ mole,  $R$  is universal gas constant,  $T$  testing temperature in kelvin. If this expression is obeyed the plot of  $\ln \sigma$  versus  $1 / T$  will give a straight line with a slope of  $Q / R$ .

So, what is the point the value of flow stress depends on the dislocation structure existing at the time the flow stress is measured. Dislocation structure will change with temperature strain rate and strain. One way to separate these effects is to evaluate  $Q$  by a temperature change test. So, if you conduct the deformation at different, different temperatures and then evaluate this we can find out the  $Q$  the activation energy without the influence of the other dislocation other parameters like dislocation substructures and so, on.

The stress strain curve is conducted at constant strain rate and the desired value of plastic strain the temperature is changed from  $T_2$  to  $T_1$ . After the temperature comes to equilibrium

the activation energy is given by  $Q = R \ln(\sigma_1 / \sigma_2) \times T_1 T_2 / (T_2 - T_1)$ . So, this is very important relation find out the activation energy for the plastic flow activation energy for what? Activation for plastic flow under the influence of high temperature.

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### Constitutive equations


- Constitutive equations describe the relations between stress and strain in terms of the variables of strain rate and temperature. The simple power-law relationship (Holloman) and its variants are elementary forms of a constitutive equation.
- An early concept in developing constituent relationships was the idea that the flow stress depended only on the instantaneous values of strain, strain rate, and temperature.
$$f(\sigma, \epsilon, \dot{\epsilon}, T) = 0$$
- This is directly analogous to the concept that a thermodynamic system, in equilibrium, can be expressed by the state variables pressure, volume, and temperature.
- A much simpler but useful relation describes the combined temperature and strain-rate dependence of flow stress.
$$\sigma = f(Z) = f(\dot{\epsilon} e^{\Delta H/RT}) l_{\epsilon}$$


where  $\Delta H$  is an activation energy,  $J \text{ mol}^{-1}$ , that is related to the activation energy  $Q$ .  $Q = m\Delta H$ , where  $m$  is the strain-rate sensitivity.

- The quantity  $Z$  is called the **Zener-Hollomon parameter**. It may also be referred to as a **temperature-modified strain rate**.
$$Z = \dot{\epsilon} e^{\Delta H/RT}$$
- A relationship which is particularly useful for correlating stress, temperature, and strain rate under hot-working conditions was proposed by Sellars and Tegart.
$$\dot{\epsilon} = A(\sin \alpha \sigma)^{n'} e^{-Q/RT}$$

where  $A$ ,  $\alpha$ , and  $n'$  are experimentally determined constants and  $Q$  is an activation energy.

Mechanical Metallurgy, George E. Dieter McGraw-Hill, 1988





So we will look at some of the constitutive equations which are very useful or important for the further discussion in this subject. So, what are constitutive equations, constitutive equation describes the relationship between stress and strain in terms of variables of strain rate and temperature. The simple power law relationship like Holloman and its variants are elementary forms of constitutive equations.

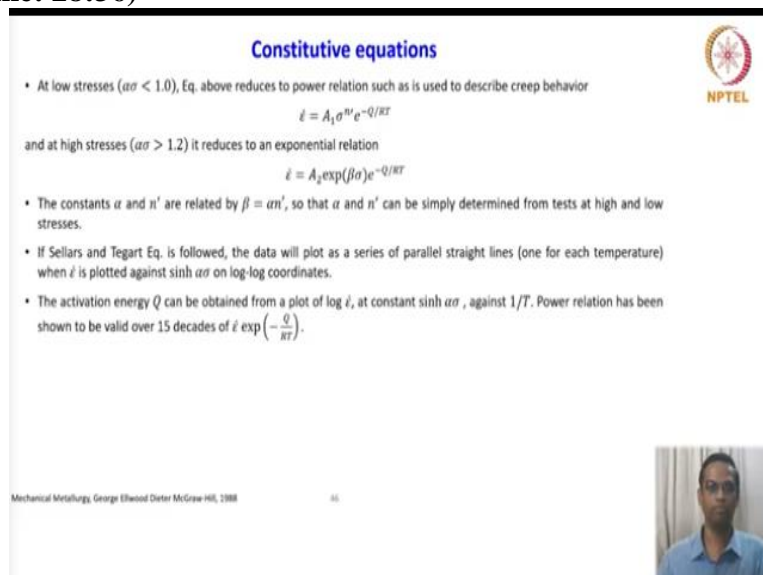
An early concept in developing constituent relationship was the idea that the flows stress dependent only on the instantaneous values of strain, strain rate and temperature. So, this is a function which is depend on  $f(\sigma, \epsilon, \dot{\epsilon}, T) = 0$ . This is directly analogous to the concept that a thermodynamic system in equilibrium can be expressed by the state variables pressure, volume and temperature.

So, we will try to derive and constitute equation looking at all these aspects. So, it was bit complex and then some of the assumptions are being made and then I am not getting into the details, but I will give some simpler results reported in the literature. A much simpler but useful relation describes the combined temperature and strain rate dependence of the flow stress  $\sigma = f(Z)$ , which is  $f$  as a function of strain rate times  $e^{(\Delta H / RT)}$  at constant strain.

Where  $\Delta H$  is an activation energy in joule per mole, that is related to activation energy  $Q$ ,  $Q = m \Delta H$ , where  $m$  is the strain rate sensitivity so these parameters you would have seen already. The quantity  $Z$  is called Zener-Holloman parameter. It may also be referred as temperature modified strain rate  $Z = \dot{\epsilon} e^{(\Delta H / RT)}$ , see this Zener-Holloman parameter is also very important parameter which describes the high temperature deformation or the temperature influence on flow properties of materials.

These are all very key constitutive equation that is why I just want to bring it so the Holloman equation is described by this. A relationship which is particularly useful for correlating stress temperature and strain rate under the hot working conditions was proposed by originally Sellars and Tegart so, which is leads like this  $\dot{\epsilon} = A \sinh \alpha \sigma^{n'} e^{-Q/RT}$ , where  $A$ ,  $\alpha$  and  $n'$  experimentally determined constants and  $Q$  is the activation energy.

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**Constitutive equations**

- At low stresses ( $\alpha\sigma < 1.0$ ), Eq. above reduces to power relation such as is used to describe creep behavior
 
$$\dot{\epsilon} = A_1 \sigma^{n'} e^{-Q/RT}$$
- and at high stresses ( $\alpha\sigma > 1.2$ ) it reduces to an exponential relation
 
$$\dot{\epsilon} = A_2 \exp(\beta\sigma) e^{-Q/RT}$$
- The constants  $\alpha$  and  $n'$  are related by  $\beta = \alpha n'$ , so that  $\alpha$  and  $n'$  can be simply determined from tests at high and low stresses.
- If Sellars and Tegart Eq. is followed, the data will plot as a series of parallel straight lines (one for each temperature) when  $\dot{\epsilon}$  is plotted against  $\sinh \alpha\sigma$  on log-log coordinates.
- The activation energy  $Q$  can be obtained from a plot of  $\log \dot{\epsilon}$ , at constant  $\sinh \alpha\sigma$ , against  $1/T$ . Power relation has been shown to be valid over 15 decades of  $\dot{\epsilon} \exp\left(-\frac{Q}{RT}\right)$ .

Mechanical Metallurgy, George E. Dieter McGraw-Hill, 1988

At low stresses  $\alpha \sigma$  is less than 1 the above equation reduces to the power relations such as is used to describe the creep behaviour. So, normally this kind of expression is used in creep behaviour, we will see them later, but I am just introducing here to make sure that how the temperature influence of flow behaviour is addressed. So,  $\dot{\epsilon} = A_1 \sigma^{n'} e^{-Q/RT}$

And high stresses where  $\alpha \sigma$  is greater than 1.2 it reduces to an exponential relation like this  $\dot{\epsilon} = A_2 \exp(\beta\sigma) e^{-Q/RT}$ . The constant  $\alpha$  and  $n'$  are related by  $\beta = \alpha n'$  so that  $\alpha$  and  $n'$  can be simply determined from the test at any high and low stresses. So, if you follow this Sellars and Tegart equation shown in the previous slides followed the data will plot as a series of parallel straight lines, one for each temperature.

When  $\dot{\epsilon}$  is plotted against the sinh hyperbolic  $\alpha\sigma$  on log-log coordinates. The activation energy  $Q$  can be obtained from plot of  $\log \dot{\epsilon}$  at constant sinh hyperbolic  $\alpha\sigma$  against  $1/T$ . Power relation has been shown the value over 15 decades of  $\dot{\epsilon} \exp^{(-Q/RT)}$ . So these constitutive relations have really work for calculating the stress temperature strain rate. So very important relations, so we will use some of these relations and take it forward to describe high temperature behaviour like creep and stress ruptures and so on.

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### Effect of temperature on flow properties


**Stress-Strain Curves**

- The flow stress of metals increases with increasing strain rate and decreasing temperature, because thermally activated dislocation motion is inhibited
- The **Johnson-Cook equation**


$$\sigma = (\sigma_0 + K\epsilon^n) \left( 1 + C \ln \frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right) \left[ 1 - \left( \frac{T - T_r}{T_m - T_r} \right)^m \right]$$

is widely used in large-scale deformation codes

- The three groups of terms in parentheses represent work-hardening, strain rate, and thermal effects, respectively
- The constants  $K$ ,  $n$ ,  $C$ , and  $m$  are material parameters, and  $T_r$  is the reference temperature,  $T_m$  the melting point, and to the reference strain rate
- The basic idea is to develop one equation that represents the mechanical response of a material from 0K to 0.5  $T_m$  and from very low strain rates ( $\sim 10^{-5} \text{ s}^{-1}$ ) to very high strain rates ( $\sim 10^5 \text{ s}^{-1}$ )



Deformation and Fracture Mechanics of Engineering Materials by Richards W. Hertzberg, John Wiley & Sons, 2012 47



So let us look at finally, the effect of temperature stress strain curves the flow stress of metal increases with increase in strain rate and decrease in temperature which is already known as and this is because of thermally activated dislocation motion is inhibited and there is a famous equation called Johnson-Cook equation which reads like this

$$\sigma = (\sigma_0 + K\epsilon^n) \left( 1 + C \ln \frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right) \left[ 1 - \left( \frac{T - T_r}{T_m - T_r} \right)^m \right]$$

So, what is the equation tells so we looked at so, far the flow behaviour independently like initially we looked at simple room temperature deformation and then we looked at the strain rate effect and then we looked at the temperature effect so everything is combined in 1 equation so that is why this equation is quite popular and useful. So, which is widely used with the large scale deformation codes.

In fact it has been used in the large deformation code. The three groups of terms in parentheses represent work hardening, strain rate and thermal effects respectively. The constants  $K$ ,  $n$ ,  $C$  and  $m$  are material parameters and  $T_r$  is the reference temperature  $T_m$  is a

melting point and to the reference strain rate. The basic idea is to develop one equation that represents the mechanical response of the material from 0 kelvin to  $0.5 T_m$ .

And from very low strain rates such as  $10^{-5} \text{ s}^{-1}$  to very high strain rate,  $10^{-5} \text{ s}^{-1}$ . So, this equation is powerful in that sense it try to represent the material behaviour with all this constraints, low strain rate to high strain rate, low temperature too high temperature and then involving work hardening all of them. So, this equation is also very useful to describe the material behaviour. So, I will stop here and we will continue our discussion on mechanical testing in the next lecture. Thank you.