

Mechanical Behaviour of Materials
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Lecture – 36
Mechanical Testing - IV


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Slip Character and Strain-Hardening Coefficients for Several Metals

Metal	Stacking Fault Energy (mJ/m ²)	Strain-Hardening Coefficient	Slip Character
Stainless steel	< 10	~0.45	Planar
Cu	~90	~0.3	Planar/wavy
Al	~250	~0.15	Wavy

- The direct connection between strain hardening and dislocation mobility makes it possible to relate strain-hardening coefficients with stacking fault energy values as shown in Table.
- When the stacking fault energy is low, cross-slip is restricted so that barriers to dislocation movement remain effective to higher stress levels than in material of higher stacking fault energy.
- That is to say, the low stacking-fault-energy material strain hardens to a greater extent. Note that the strain-hardening coefficient increases with decreasing stacking fault energy while the slip character changes from a wavy to a planar mode.


NPTEL



Deformation and Fracture Mechanics of Engineering Materials by Richard W. Astenberg, John Wiley & Sons, 2002 11

Hello I am Professor S. Sankaran in the Department of Metallurgical and Materials Engineering. Hello everyone welcome to this lecture again, let us continue our discussion on the tensile test and we are looking at the results of typical tensile test and then we are looking at basically the interpretation of the results which is basic outcome of simple tensile tests. So, in the last class, we have looked at the engineering stress strain curve and various parameters one can derive from that outcome of the results.

So, we also looked at some derived parameters like to stress, strain, curve and what is the usefulness of this curve and how it is related to engineering stress strain parameters. So, we will continue our discussion on that and what we are going to discuss today is how this slip character and the strain hardening coefficients for several metals are compare when you compare these parameters how do they appear. So for example, if you look at this table where they stacking fault energy and strain hardening coefficient and slip characters they are all summarized for typically for stainless steel copper and aluminium. What you have to notice here, is the parameter stacking fault energy we have already seen this, this is not new to us we have a sufficient background to understand this, what is energy which is expressed in the milli joules per meter square(mJ/m²).

So, this stacking fault energy plays an important role in determining the deformation characteristics of polycrystalline material, this is a fundamental property or characteristic of each material. So, if you take the stacking fault energy by just looking at the values you will be able to predict some kind of dislocation activity. So, the stainless steel typically which is having the stacking fault energy less than 10 mJ/m^2 .

As got a strain hardening coefficient of 0.45 and then the slip character is named as planar, so we will try to see what it is. Then if you go to the next metal copper which is having a significantly higher stacking fault energy has gone the strain hardening coefficient of 0.3 and slip characteristic described as planar/wavy. Finally, for aluminium this stacking fault energy is very high around 250 mJ/m^2 , for that the strain hardening coefficient is 0.15 and slip characteristic this designated as wavy.

So, now we have to just look at the individual you know deformation characteristics of each of these metals. So, the table basically summarizes the material having two extreme cases of stacking fault energy of course, the copper is having some middle range.

So, how this is going to determine the deformation characteristic of these materials? So, now we can relate these things with the strain hardening coefficient.

So, what is the strain hardening coefficient tells us, from the previous discussion what do we understand, the more the strain hardening value that means it will the material we will have huge ability to strain hardening ability will be very high. I mean that is the index of strain hardening coefficient. So, the material which is having very small stacking fault energy you can see that the strain hardening coefficient value is higher.

So, we have very nice trend here, lowest the stacking fault energy, highest strain hardening coefficient, highest strain stacking fault energy lowest strain fault energy and these two are characterized as two extreme slip character one is planar other is wavy. You can relate all of them very interesting table and very important table as well. So, if the material having very low stacking fault energy it is going to strain harden significantly, but the slip characteristic is planar that means the dislocations will try to move only in a planar fashion, not in the three dimensional moments which is not going to climb or cross slip.

So, easily they are all going to just pile up the glide will be this they are going to glide on a single plane and that is going to be a primary deformation more that is all we have to look at here. For a copper which is having the middle range of a stacking fault energy which will strain harden but not to the extent of stainless steel but at the same time it will be better than the aluminium, it only going to go to strain hardening.

So, obviously it is going to have or it is going to display both the slip characters, namely planar and wavy. So, this is something very important you have to keep in mind and if you look at the aluminium which is having very high stacking fault energy and then you can delete all this technical energy values with the dislocation dynamics and mechanics, which we are already familiar with.

But when the stacking fault energy is very high in this context where we are talking about mechanical testing, especially a tensile test, the material is, you know having the low strain hardening coefficient does not going to strain harden much as compared to the stainless steel or copper and the slip character is going to be wavy. So, what does wavy slip mean it is going to the dislocation mobility will be in three dimensional.

I mean the three dimensional mobility is facilitated by the high stacking fault energy, it can just quickly move up and down you can easily cross slip and climb all that but it will not be very climb and all it is primarily in high temperature mechanisms. But here we are talking about easy cross slip and then you can move on to a different slip plane which is available at a different height. So, that is all we have seen.


So, when the deformation takes place with such an ease so, the substructure which is going to form is there are going to appear as a wavy nature that is why it is called wavy slip nature. So, all the dislocation walls, bundles, dislocation cells, sub structures, all of them will appear in a wavy form that means it is not an organized form on the other hand the same dislocation substructures will appear in a planar fashion. So, it will not be like a random waviness will not be there. So, it will be all planar segments.

So that is just looking at the dislocation substructures you will be able to characterize the deformation behaviour, in a first look if so, we can just understand this and then of course, depending upon the interest you can go to any level of analysis and investigation. So, the strain hardening coefficients which you obtained from a simple tensile test to tell us a quite a bit of a deformation characteristics, if you connect that with the stacking fault energy that is the message I want to bring here. The direct connection between the strain hardening and dislocation mobility makes it possible to relate strain hardening coefficients with stacking fault energy values as shown in that table. So, when the stacking fault energy is low, the cross slip is restricted so that barriers to dislocation movements remain effective to highest stress levels than in the material of higher stacking fault energy. That is to say, the low stacking fault energy material strain hardens to a greater extent. Note that the strain hardening

coefficient increases with decreasing stacking fault energy while slip character changes from wavy to a planar mode.

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Instability in tension



- Necking or localized deformation begins at maximum load, where the increase in stress due to decrease in the cross-sectional area of the specimen becomes greater than the increase in the load-carrying ability of the metal due to strain hardening.
- This condition of instability leading to localized deformation is defined by the condition $dP = 0$.

- From the constancy-of-volume relationship,

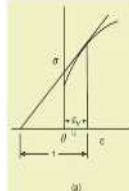
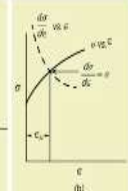
$$\frac{dL}{L} = -\frac{dA}{A} = d\epsilon$$

and from the instability condition

$$\frac{dA}{A} = -\frac{d\sigma}{\sigma}$$

so that at a point of tensile instability

$$\frac{d\sigma}{d\epsilon} = \sigma$$

Mechanical Metallurgy, George E. Dieter, Jr. McGraw-Hill, 1988
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So that is one aspect of the tensile test, another important aspect is instability in tension. So, normally we look at in a tensile test the specimen is subjected to making beyond certain a load, so that is called instability in tension. So, let us learn a little more on this necking or localized deformation begins at maximum load where the increase in stress due to decrease in cross sectional area of the specimen becomes greater than the increase in the load carrying ability of the metal due to strain hardening.

The moment the cross section of the specimen becomes smaller and it becomes a problem there because the load carrying ability of material is affected that is what we have seen. So, that is what the strain hardening coefficient also will indicate this condition of instability leading to localized deformation is defined by the condition $dP = 0$. The changing load at the maximum that is $dP = 0$.

So, we can just relate this use this condition of instability using this relation we will be able to derive few more equations $P = \sigma A$ and $dP = \sigma dA + A d\sigma = 0$ and from the volume constancy relationship you can write it like this, $dL / L = - dA / A = d\epsilon$ and from this instability condition if you apply this instability condition here $dP = 0$, $-dA / A = d\sigma / \sigma$.

So that at a point of tensile instability we reach an equation $d\sigma / d\epsilon = \sigma$, this is very useful and very important relation. Now we can look at all this in a graphical way. So, how to find the point of making at the maximum load there are two ways. So, the one is this is the σ versus ϵ plot where you look at the point and then draw it sub tangent of unity draw from the maximum load draw a sub tangent of unity that will indicate the maximum load of necking, the point at which it is going to neck.

Or the other way to find out is find out the intersection where, the $d\sigma / d\epsilon = \sigma$.

So, that is what we perform here. So, either this or this both of them will find out the point of necking at maximum load. So, this is very important thing and you can also check in your simple tensile test you can plot these two things work hardening rate versus strain and stress versus strain plot you will be able to see that the maximum beauteous will be the lowest value of $d\sigma / d\epsilon$ will be matching this so you will see that.

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Instability in tension

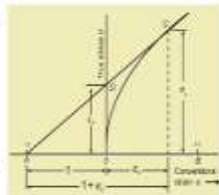
- The necking criterion can be expressed more explicitly if engineering strain is used.


$$\frac{d\sigma}{d\epsilon} = \frac{d\sigma}{de} \frac{de}{d\epsilon} = \frac{d\sigma}{de} \frac{dL/L_0}{dL/L} = \frac{d\sigma}{de} \frac{L}{L_0} = \frac{d\sigma}{de} (1 + e) = \sigma$$


$$\frac{d\sigma}{de} = \frac{\sigma}{1 + e}$$

- This equation permits an interesting geometrical construction called **Considere's construction** for the determination of the point of maximum load.
- The stress-strain curve is plotted in terms of true stress against conventional linear strain.
- Let point A represent a negative strain of 1.0. A line drawn from point A which is tangent to the stress-strain curve will establish the point of maximum load, for according to Eq. the slope at this point is $\sigma/(1 + e)$.
- we obtain a simple relationship for the strain at which necking occurs. This strain is the true uniform strain e_u .

$$e_u = n$$







So, another interesting way of analyzing this making phenomenon is let us go through that the necking criterion can be expressed more explicitly if engineering strain is used $d\sigma / d\epsilon$ can be written like this, where $d\sigma / de$ into $de / d\epsilon$. So, engineering a strain we know that we can express in terms of dL / L_0 and again the 2 strain is dL / L . So, we can manipulate this

$$\frac{d\sigma}{d\epsilon} = \frac{d\sigma}{de} \frac{de}{d\epsilon} = \frac{d\sigma}{de} \frac{dL/L_0}{dL/L} = \frac{d\sigma}{de} \frac{L}{L_0} = \frac{d\sigma}{de} (1 + e) = \sigma$$

$$\frac{d\sigma}{de} = \frac{\sigma}{1 + e} \frac{d\sigma}{de} = \frac{\sigma}{1 + e}$$

So, what is the usefulness of this you can this equation permits an interesting geometrical construction called considere's construction for the determination of the point of maximum load.

Here again we are going to do the same exercise like what we have seen in the previous slide here it is called considere's construction or considere's condition we can say that. So, what you have to see here is this is a σ versus ϵ plot so, you try to you first take a negative access point A up to the unity, - 1 and from there you try to draw tangent to this curve and that point C that will give you the point of necking at maximum load that is what this equation says.

So, if you simply look at this slope that is $d\sigma / d\epsilon$ which is nothing but $\sigma / (1 + e)$, that is what is shown here. So, the slope is $\sigma / (1 + e)$, so this is what is shown here. So, this stress strain curve plotted it in terms of the true stress against conventional linear strain. This is a kind of relation between the true stress versus conventional strain that is also the way of looking at this.

So, this is the description is given here let point A represent a negative strain of 1.0. A line drawn from the point A which is tangent to the stress strain curve will establish a point of maximum load according to this equation. So, another important relation we can get it, we can obtain a simple relationship for a strain at which necking occurs, this time is through uniform strain.

If you recall I just mentioned in the previous lecture that we just looked at discussing the state hardening exponents when expression something closed to this $d\sigma / d\epsilon = n(\sigma / \epsilon)$, that expression we have seen. So, if you recall that expression and then compare these two expressions this expression and that expression then you can relate this $\epsilon_u = n$ that means the uniform strain to the maximum load which is equal to n this is also very important relation.

So, what is the usefulness of the strain hardening coefficient can also be good that is equivalent to uniform strain. So, the uniform strain is very important in the case of a material which is being subjected to metal forming operations. So, the measure of ductility and formability, some idea it gives so these relations are very important and useful in that context.

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Instability in tension

The true fracture strength is the true stress at fracture:

$$\sigma_f = \frac{P_f}{A_f}$$

Where the final cross-sectional area following fracture A_f is determined from measurements of averaged minimum diameter of the failed halves of the specimen put together. Similarly, the fracture ductility is the true strain at the final fracture:

$$\epsilon_f = \ln \frac{A_0}{A_f} = 2 \ln \frac{d_0}{d_f} = \ln \frac{1}{1 - RA}$$

where the reduction in area is given by

$$RA = \frac{A_0 - A_f}{A_0} = 1 - e^{-\epsilon_f}$$

At fracture,


$$\sigma_f = K_1 (\epsilon_f)^n \quad \text{or} \quad K_1 = \frac{\sigma_f}{\epsilon_f^n}$$


Then from previous Eqs.

$$\epsilon_p = \left(\frac{\sigma}{K_1} \right)^{1/n} = \left(\frac{\sigma \epsilon_f^n}{\sigma_f} \right)^{1/n} = \epsilon_f \left(\frac{\sigma}{\sigma_f} \right)^{1/n}$$

Since $\epsilon_u = \frac{\sigma}{E}$, the total strain may now be expressed as

$$\epsilon = \epsilon_e + \epsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K_1} \right)^{1/n} = \frac{\sigma}{E} + \epsilon_f \left(\frac{\sigma}{\sigma_f} \right)^{1/n}$$





Dislocations and Mechanical Behavior of Materials, M.N. Shetty, TM Learning Pvt. Ltd., 2022

So, one final description about this instability in tension which is also related to the different relationship we are looking at it, the true fracture strength is the true stress at fracture. So, beyond the yielding point, sorry beyond the maximum load, we just mentioned in the previous lectures that all these relationships are not going to be effective. So, after the maximum load, what all we can derive and combining these two strains that is the in tension of this slide.

So, we are talking about the true fracture strain that means the true stress at fracture so, $\sigma_f = P_f / A_f$, where the final cross sectional area following factor A_f is determined from the measurement of average minimum diameter of failed halves of the specimen put together. Similarly, the fracture ductility is a true strain at the final fracture, so this also can be measured and then we will see what? What we can do with this expression.

So, the reduction in the area can be written like this $(1 - e^{-\epsilon_f})$ and at fracture we can simply substitute this values, I mean this is a simple Hollomon type of equation where you are now looking at only at the fracture. So, you it is not σ , σ_f it is not K it is $K_1(\epsilon_f)^n$ or you can write that expression for K like this and then substitute this into the plastic strain ϵ_p is $(\sigma / K_1)^{1/n}$ and then this can be rewritten like this.

We simply substitute this value of K_1 here, so then we get finally $\epsilon_p = \epsilon_f (\sigma / \sigma_f)^{1/n}$. So, we can now write, bring the another relation this is engineering strain which is equal to σ / E , the total strain we can bringing so we now know that ϵ_p and the ϵ_e . So, total strain = $\epsilon_e + \epsilon_p$ which is nothing but this expression $\sigma / E + (\sigma / K_1)^{1/n}$, which is nothing but

$$\epsilon = \sigma / E + \epsilon_f (\sigma / \sigma_f)^{1/n}.$$

So, this kind of just small manipulation gives us to visualize how to express the total strain in the tensile test results, so, some useful relations you can relate.

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Instability in tension

- During tensile deformation, strain is periodically localized at "weak links" along the sample length.
- For strains less than ϵ_{Eu} , the work hardening in these areas strengthens the material enough relative to the material outside of them so that the instability is removed.
- The work-hardening rate decreases with strain.
- At ϵ_E , the fractional decrease in cross-sectional area at a "weak link" exactly compensates the increase in flow strength due to work hardening there.
- Thus, a permanent instability is formed at a strain infinitesimally greater than ϵ_{Eu} and this leads to neck formation.
- Further deformation is localized in the instability region and the neck becomes more pronounced.

$\epsilon < \epsilon_{Eu}$ $\epsilon < \epsilon_{Eu}$ $\epsilon = \epsilon_{Eu}$ $\epsilon > \epsilon_{Eu}$
 (a) (b) (c) (d)

Notional Behavior of Materials, Thomas H. Courtney, Wiley-Interscience, Inc., 2005

One final description about this instability in tension what you are seeing in this schematic is this is a typical dog bone density specimen which is subjected to potential tensile force here, the strain is less than ϵ_{Eu} . So, the capital E_u is the maximum load engineering strain that is called E is engineering and u is the ultimate. So, this notation is given in this reference so I have not changed that reference but it is nothing but the engineering strain at the maximum load, so that is what you are looking at.

So, when this strain is less than that maximum load, nothing happens and then as the deformation proceeds, what happens? There is no necking here. So, what is the physics we are trying to explain here you see. During tensile deformation strain is periodically localized to weak links. So, when you say weak links here it means, the diameter in the gauge length is much smaller as compared to the shoulder portion. So, that is what we are seeing that.

And for strains less than ϵ_{Eu} , that means, the maximum strain that is the strain of the maximum load if it is less than maximum load the work hardening in these areas strengthens the material enough relative to the material outside of them, so that the instability is removed. So, basically when this cross section is being pulled in tension the immediately the necking does in form, the necking form is much later stage. So, what happens during that time before necking after plastic deformation commences.


So, in between these two times, what is the physics that is what we are trying to see so that means what happens? So, the rest of the whole area is getting strengthened so that is why there is almost like you know, it is equalizing the whole specimen through the uniform strain or something like that. Then the work hardening rate decreases with strain. At the maximum load the strain at maximum load the fractional decrease in the cross sectional area at a weak link exactly compensated increase in the flow strength due to the work hardening there.

So, it is something like what we have just seen $d\sigma / d\varepsilon = \sigma$ this is what it is, the moment that condition reaches that is actually this stage the necking begins. Thus a permanent instability is formed at strain infinitesimally greater than ε_{Eu} , that is strengthens the maximum and this leads to neck formation. So, we are not saying anything new here, but then these visual pictures of making what happens to the strain before the making takes place, but then after the plastic deformation commences.

So, in between how the strain distribution occurs within the specimen that aspect is being brought in this schematic so nicely so that is why I just wanted to share that.


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Definition of and relationships between true and engineering stress and strain



Parameter	Fundamental definition	Prior to necking	Following necking
Engineering stress (σ_E)	$\sigma_E = \frac{F}{A_0}$	$\sigma_E = \frac{F}{A_0}$	$\sigma_E = \frac{F}{A_0}$
True Stress (σ_T)	$\sigma_T = \frac{F}{A_i}$	$\sigma_T = \frac{F}{A_i} = \sigma_E (1 + \varepsilon_E)$	$\sigma_T = \frac{F}{A_{neck}}$
Engineering Strain (ε_E)	$\varepsilon_E = \frac{\delta l}{l_0}$	$\varepsilon_E = \frac{\delta l}{l_0}$	$\varepsilon_E = \frac{\delta l}{l_0}$
True Strain (ε_T)	$\varepsilon_T = \ln \frac{A_0}{A_{min}}$	$\varepsilon_T = \ln \frac{l_i}{l_0} = \ln \frac{A_0}{A_i} = \ln(1 + \varepsilon_E)$	$\varepsilon_T = \ln \frac{A_0}{A_{neck}}$

Behavior of Materials, Thomas H. Courtney, Wiley Press Inc., 2003



So, whatever we have just seen this table summarizes the whole result it will be very handy if you just look at them in one table and we have the fundamental definition of engineering stress, true stress, engineering strain, true strain and then this relationship just I mean they appear as it is prior to necking or with some equivalent relations, but after necking how these relationships is converted or it has to be looked at because all these relations are valid till the maximum load.

Prior to necking after necking we can use only these kinds of relation. We cannot use the same relation. So, that is one advantage we can just refer this table and then try to understand, which are the relations are valid before necking and after necking.