Mechanical Behaviour of Materials
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Lecture - 35
Mechanical Testing - III

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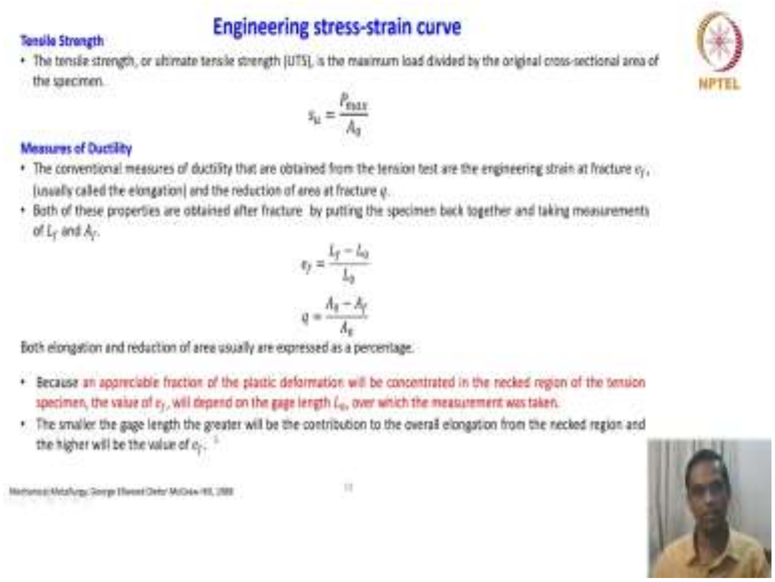

Hello I am Professor S. Sankaran in the Department of Metallurgical and Materials Engineering. So, we will now go to another parameter called tensile strength that is the tensile strength or ultimate tensile strength UTS the maximum load divided by the original cross sectional of the specimen which is called also called tensile stress is a maximum load maximum strength which is given by S_u, S is engineering stress u is ultimate which is P_{max} / A_0.

And that is about the strength and we will talk about measure of ductility, what are the measure of ductility the conventional measures of ductility that are obtained from the tension test or the engineering strain at factor that is e_f, usually called the elongation and the reduction of area at fracture that is q. So, elongation and reduction in the area are the two measures of ductility in our material that is actually obtained from this simple tensile test.

Both of these properties are obtained after a fracture by putting the specimen back to there and taking measurements of L_f and A_f. L f is the final length and I mean length after a fracture, A_f is the area after the fracture. So, which is given by the formula is this $e_f = (L_f - L_0)/ L_0$. which is elongations and reduction area q is initial area that is (A_0 - A_f)/ A_0, both elimination reduction area and usually expressed as percentages.

Because an appreciable fraction of plastic deformation will be concentrated in that necked region of the tension specimen, the value of e_f will depend on the gage length L_0 over which the measurement was taken, very important. Just because necking takes place we will see the importance of making and what it you know how does it determine some of the basic properties or how does it capture the basic properties of material we are going to see it.

But we are now looking at the ductility parameters. And considering elongation as the ductility parameter is an issue, issue is what. Since the plastic deformation will be constant in the necked region, the elongation will depend upon the gage length L_0, over which it has been does measurement, I mean over which it is being measured, the smaller the gage length the greater will be that contribution to the overall elongation from the necked region. And the higher will be the value of elongation. So, there is a bias here depending upon what gage length you are choosing so it is always important.

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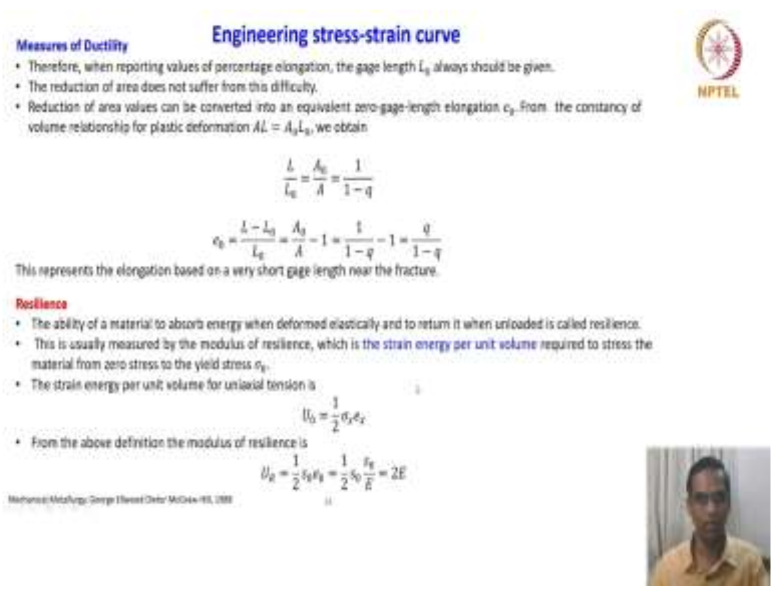

Therefore when we are reporting values of percentage elongation, the gage length L_0 always should be given, the reporting this without mentioning the gage length the percentage elongation will have a biased value we need to remember this so the reduction in the area does not suffer from this difficulty. So, you can say that reduction area is a better measure of that ductility than the percentage elongation because of this I mean the gage length dependence.

Reduction of area values can be converted into an equivalent to the zero gage length elongation from the constancy of volume relationship for plastic deformation that is $AL = A_0 L_0$ we obtain. We can play around with his relations from the constancy of volume relationship.

$$\frac{L}{L_0} = \frac{A_0}{A} = \frac{1}{1-q}$$

$$e_0 = \frac{L - L_0}{L_0} = \frac{A_0}{A} - 1 = \frac{1}{1-q} - 1 = \frac{q}{1-q}$$

This represents the elongation based on a very short gage length near the fracture please you have to note that. The other fundamental property which can be assessed by simple tensile test is resilience. What is resilience? The ability of material to absorb energy when deformed elastically and to return it when unloaded is called resilience.

This is usually measured by the modulus of resilience which is the strain energy per unit volume required to stress the material from 0 stress to the yield stress σ_0, the strain energy per unit volume for uniaxial tension is given by $U_0 = 1/2 (\sigma_x e_x)$, so stress into strain. From the above definition the modulus of resilience is U_R can be written like this.

$$U_R = \frac{1}{2} S_0 e_0 = \frac{1}{2} S_0 \frac{s_0}{E} = 2E$$

So, here what is this suddenly s_0 is coming, so s_0 this nothing but the proof stress what is proof stress if the material is not showing an end point we take something called a proof stress we will demonstrate it now how do we take proof stress ? The proof stress this is considered yield stress. So, s engineering stress s_0 is a proof stress.

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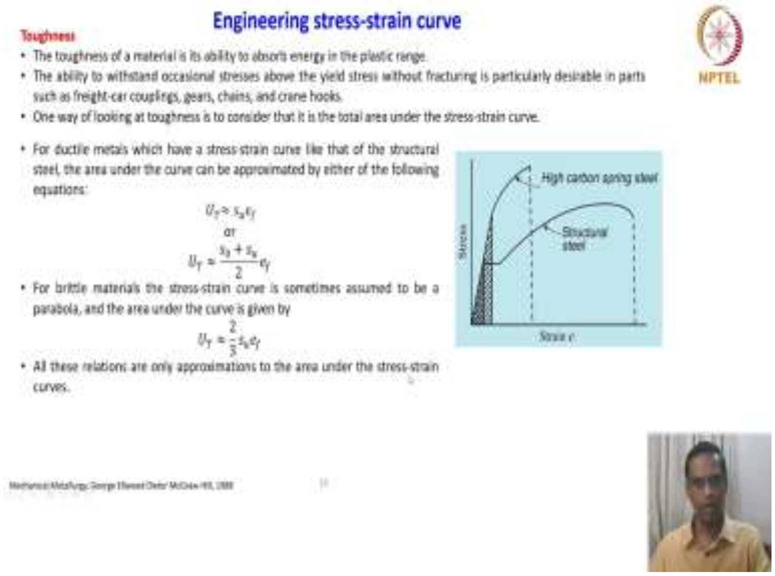

So, the toughness, so next parameter is toughness, the toughness of the material is the ability to absorb energy in the plastic range. So, we were talking about resilience which is in an elastic range. So, this is particularly the plastic range. The ability to withstand occasional stresses above the yield stress without factoring is particularly desirable, in parts such as freight car couplings, gears, chains, and crane hooks. So, these are some of the typical applications where the components will be subjected to the occasional stresses which are above the yield stress.

One way of looking at toughness is to consider that it is the total area under the stress strain curve. So, you see here the typical stress strain plot is shown one for structural steel and another for high carbon spring steel. You can see both of them the shaded area shows about resilience and the total area under this is toughness so. For ductile metals which have a stress strain curve like that of the structural steel, the area under the curve can be approximated by either of the following equations.

So, we can use this approximate equation U_T approximate = $s_u . e_f$ or $U_T = (s_0 + s_u) / 2\ e_f$. For brittle materials the stress strain curve is sometimes assumed to be a parabola, and the area under the curve is given by U_T approximately = 2 / 3 ($s_u e_f$). All these relations are only approximations to the area under the stress strain curves. So, these are very preliminary and basic equations are the property one can derive from a simple tensile test.

This is what we are trying to show here we are talking about strength we are relating the yield strength or tensile strength or fracture strength. And we are also talking about ductility which can be measured by the percentage elongation or reduction in the area you are talking about resiliency we are taking about toughness and so on.

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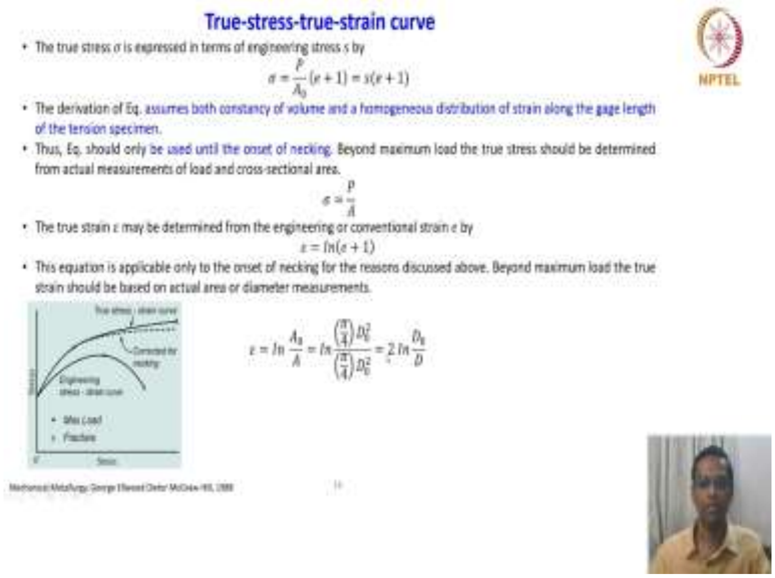

So, now we will talk about a true stress true strain curve so why do we need this in the first place? What is the definition of a true stress? What is the definition of engineering stress? Engineering stress is load divided by what is an area of cross section. On the other hand true stress is load divided by the instantaneous area of cross section that means the true stress exhibits a complete characteristic of a deformation behavior of a material.

Rather than the engineering stress that is how you have to continue. So, the true stress σ is expressed in terms of engineering stress from the length from the volume constancy of plastic deformation we can derive this very simple but it can be given the true stress can be given like engineering stress times (engineering stress strain+1). The derivation of equation assumes both constants of volume and homogeneous distribution of strain along the gage length of the tension specimen. So, this is very important so where does this homogeneous distribution takes place in the stress strain curve up to maximum load only up to maximum load, the homogeneous distribution of strain happens then up to this point this can be used. So, thus the equation should be use until the concept of necking so once the necking starts taking place it is no longer a homogeneous distribution. So, these relationships will not hold good.

Beyond maximum load that true stress should be determined from actual measurements of load cross sectional area that is σ = P / A, the true strain ε may be determined from the engineering or conventional strain e by ε = ln(e + 1). So, what is shown in this scheme schematic here is the engineering stress strain curve together with a true stress strain curve.

The maximum load which is shown is correspondingly it is here in the true stress strain curve. So, this is a fracture and this is the maximum load and you can see that since the true stresses defined by the load divided by the instantaneous area as the necking takes place as the diameter the cross sectional area keeps on coming down, so that it is just keep on increasing the fraction, of course there is a dotted line which is suppose if you try to predict this behavior by some of the semi empirical relations, there is also correction for necking behavior it is also suggested.

So, we will see one by one. what is it, so these equations is applicable only to the onset of necking for the reasons discussed above, beyond maximum load the true strain should be based on the actual area or diameter measurements. So, which is given by this $\varepsilon = \ln A_0 / A$ which can be related to this finally can be measured, what is measured is the diameter that is $\varepsilon = 2 \ln(D_0 / D)$.

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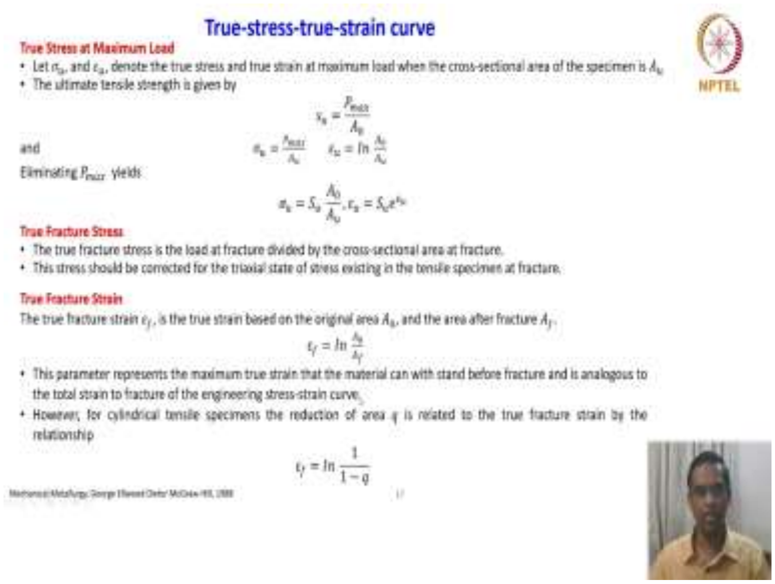

So, what is true stress at maximum load this is again very simple substitutions but then we need to know this because all these assumptions are valid up to the maximum load. So, you take this expression s_u σ_u ε_u with respect to the maximum load only. We can also I mean rewrite this expression like this $\sigma_u = s_u (A_0 / A_u)$ or $\varepsilon_u = S_u e^{\varepsilon u}$, this is another relations you can derive from this simple equations.

True factor stress again is the load at fracture divided by the cross sectional area of the fracture. This trust should be corrected for a triaxial state of stress existing in the tensile specific fracture. So, what we are trying to say is you simply take this value we have to take care of the other correcting factors like you know correction for making correction for triaxial state of stress and so on. True fracture stress again true fracture strain which is given by ε_f true strain at fracture is equal to ln (A_0 / A_f). This parameter represents the maximum true strain that the material can withstand before the fracture and it is analogous to the total strain to fracture of engineering stress strain curve. However for cylindrical tensile stress means the reduction of area q is related to the true fracture strain by the relationship $\varepsilon_f = \ln(1 / 1 - q)$. So, I am going a little fast because there are simple substitutions but you have to know this all these parameters so that you will not misinterpret when it comes to interpretation.

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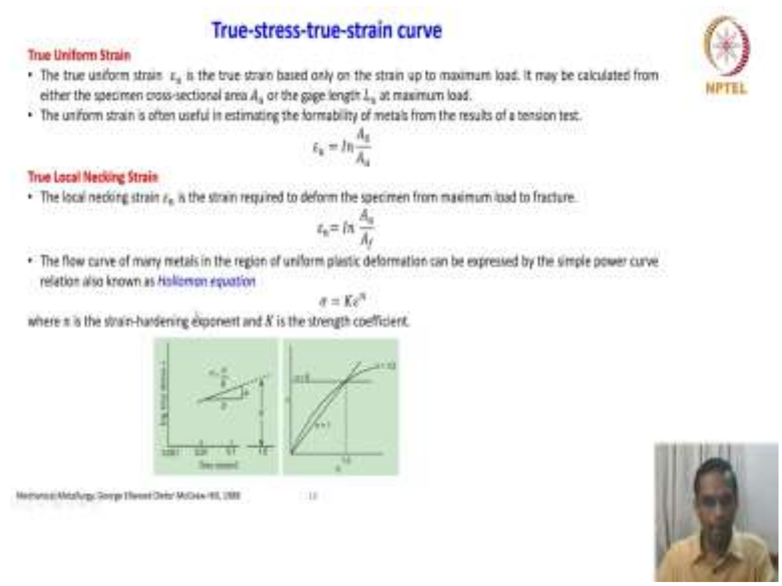

True uniform strain very important parameter what is that the true uniform strain ε_u is the true strain based on the strain up to the maximum load it may be calculated from either specimen cross sectional area A_u or the gage length at the maximum load. The uniform strain is often useful in estimating the formability of metals from the results of a tension test. So, you please see that by just measuring the uniform strain we have some idea about it formability.

Formability is a again an important parameters in the metal forming applications. So, one can get some idea by sharing this $\varepsilon_u = \ln (A_0 / A_u)$. True local necking strain what is that? The local making strain ε_n, is the strain required to deform the specimen from maximum load to fracture. So, $\varepsilon_n = \ln (A_u / A_f)$, the flow of many metals in the region of uniform plastic deformation can be expressed by a simple power calculation also known as Holloman equation.

Which is equal to $\sigma = K \varepsilon^n$ so this is quite popular in empirical relation known as Holloman equation which is valid up to maximum load in a stress strain curve where n is the strain hardening exponent, K is the strength coefficient very widely used most popular and very simple equation so widely used and it is not just one expression like this for approximating stress strain behavior.

There are several of them so we will see couples of them which is most widely used and in research people if you go and look at the literature there are n number of empirical relation are done to predict this stress strain behavior of variety of materials. So, we can take any empirical relations according to the material which you are working are looking for information and then try to estimate all these parameters one by one couple of them or we will show it in this course.

So, next things what is shown here is how to estimate this strain hardening exponent n so and the K so basically it is a log of true stress versus true strain. So, you get that a straight line there log of true stress true strain you will get a straight line this and the slope will give you the n = a / b is n and this is the K, for different what is the significance of n. If n = 0 the stress strain curve will look like this.

So, what is the meaning of a stress strain curve appearing like this, it is perfectly plastic and what is n = 1, n = 1 is a linear relationship perfectly elastic curve and n =1/2, it has material undergoes kind of strain hardening. So, something like that so these plots give the, at least captures the idea of what is the meaning of the n strain hardening export is very important parameter. As I said by measuring n you can relate that to formability of metals and strain hardening ability and so on so forth we are going to see them one by one.

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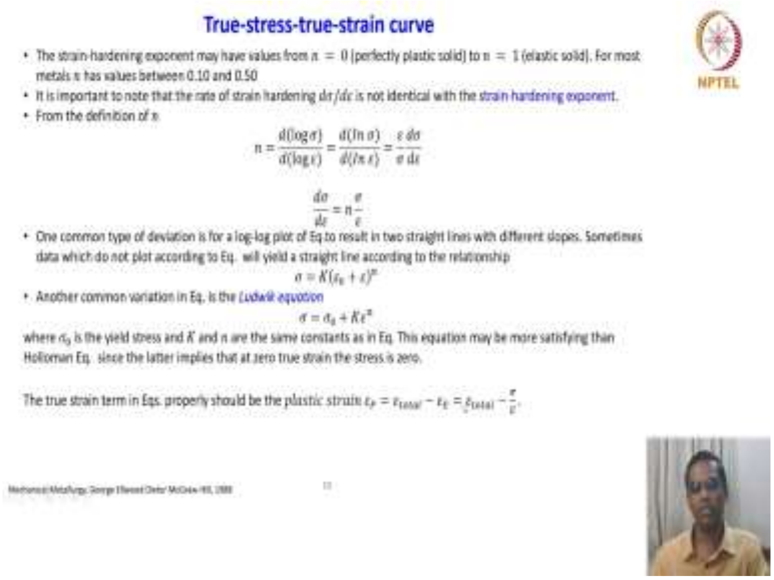

The strain hardening exponent may have values from n = 0 perfectly plastic solid, n = 1 plastic solid for most metals and has values between 0.10 and 0.50. It is important to note that the rate of strain hardening dσ / d ε is not identical with the strain hardening exponent. From the definition of n because we are doing you know a plot of log σ versus log ε, then we find the n as a slope.

So, this can be written like this a natural, $n = \frac{d(\log\sigma)}{d(\log\varepsilon)} = \frac{d(ln\sigma)}{d(ln\varepsilon)} = \frac{\varepsilon}{\sigma}\frac{d\sigma}{d\varepsilon}$

that is what is written here the strain hardening rate dσ/dε is not identical with the strain hardening exponent. So, the are very different this relation we will use it later to use this relation to show that we will be able to get the uniform strain ε_u can be directly related to n.

So, again n be related to uniform strength ε_u so later we will show that and this relation we will recall that time. One common type of deviation is for log-log plot of the above equation Holloman equation to result in two straight lines with the different slopes. Sometimes data which do not plot according to the above equation will yield a straight line according to the relationship which is given here. So, sometimes the Holloman equation is modified in this fashion that is $\sigma = K(\varepsilon_0+\varepsilon)^n$. Another common equation is called the Ludwik equation like which is reads like this $\sigma = \sigma_0 + K\varepsilon^n$, where σ_0 is the yield stress and K and n are the same constant as the Holloman equation this equation may be more satisfying than the Holloman equation.

Since the later implies that at 0 to strain the stresses 0. So, it assumes the friction stress at least so 0 true strain, I mean 0 true strain the stresses is 0 here. So that is what is the true strain terms in the above equations both the equations properly should be the plastic strain actually which is ε p

$= \varepsilon_{total} - \varepsilon_{elastic} = \varepsilon_{total} - \sigma / E$. So, we are since we are talking about the strain hardening ability the more appropriate strain is plastic strain.

So, we are now looked at two models that is Holloman equation and Ludwik equation these are the two basic models we looked at it. We will look at some more models which will describe this stress strain behaviour.

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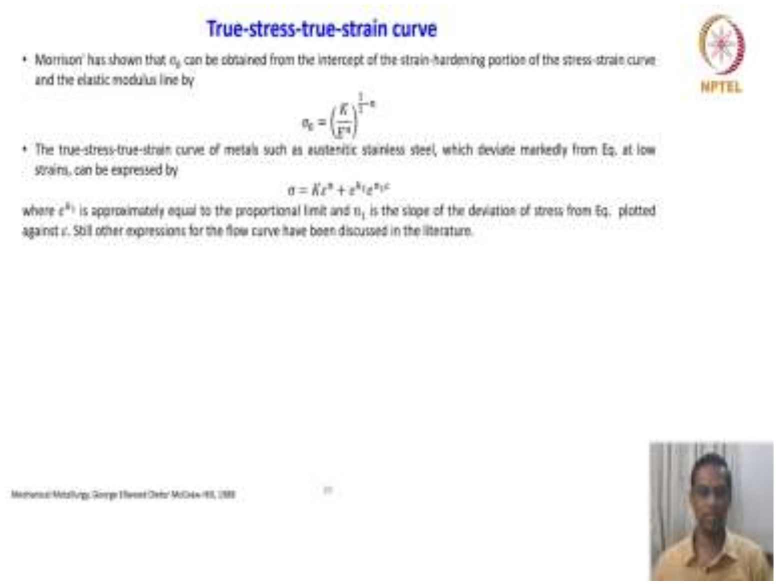

Morrison has shown that σ_0 can be obtained from the intercept of the strain hardening portion of this stress strain curve and the elastic modulus line by this relations $\sigma_0 = (K / E^n)^{(1/n-1)}$. The true stress true strain curve of metals such as austenitic stainless steel which deviates markedly from the equations at low strains can be expressed this type of equation.

$\boldsymbol{\sigma = K\varepsilon^n + e^{k_1} e^{\varepsilon\, n_1}}$,where e to the k_1 is approximately equal to the proportional limit and the

n_1 is the slope of deviation stress from the equation plotted against ε, still other expressions for the flow curve has been discussed in the literature. So, like I said there are like this several expressions are discussed to predict the stress strain curves of metals and materials. These are all I just brought this here just to give you an idea how people try to interpret this results.

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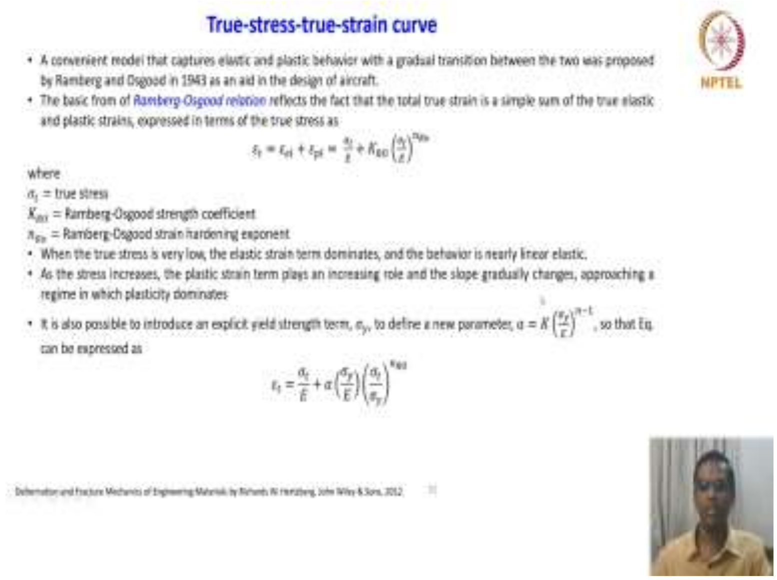

A convenient model that captures elastic and plastic behavior with the gradual transition from the two was proposed by Ramberg and Osgood in 1943 as an aid in the design of the aircraft. I brought one important model apart from these two Ludwik and Holloman, I wanted to present this model because it captures both elastic and plastic behavior together and it is also quite popularity mechanics community very popular model.

The basic form of Ramberg and Osgood relation reflects the fact that the totals true strain is a simple sum of true elastic and plastic strains expressed in terms of true stress as $\varepsilon_t = \varepsilon_{elastic} + \varepsilon_{plastic}$, which can be written as σ_t that is true stress divided by E, do not confuse this with the σ_t this is σ_t means σ true, true stress plus K_{RO} times σ_t / E to the power n_{RO}.

Where σ_t is true stress K_{RO} is Ramberg Osgood strength coefficient n_{RO} Ramberg Osgood strength hardening exponent. When the true stress is very low the elastic strain terms dominates and the behavior is nearly linear elastic. At the stress increases the plastic strain term plays an increasing role and the slope gradually changes approaching a regime in which plasticity dominates.

So, how do we write this equation, it is also possible to introduce an explicit yield strength term σ_y to define a new parameter alpha which is equal to $K (\sigma_y / E)^{n-1}$. So that the whole equation expressed as $\varepsilon_t = \sigma_t / E + \alpha(\sigma_y / E)(\sigma_t / \sigma_y) n_{RO}$, this is understandable when alpha is written like this and you can write like this one times this one. So, this situation is like this.

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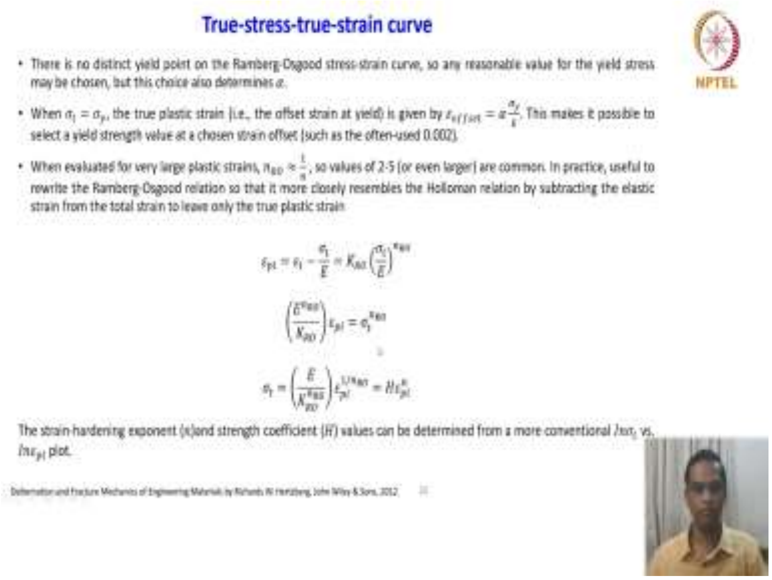

So, what is the benefit, there is no distinct yield point on Ramberg Osgood stress strain curve, so any reasonable value for the yield stress maybe chosen, but this choice also determines α. When σ_t that is true stresses is equal to σ_y, yield stress that true plastic strain that is offset strain at the yield is given by $\varepsilon_{offset} = \alpha (\sigma_y / E)$, this makes it possible to select the yield strength value at a chosen strain offset (such as often used 0.0020.

I just mentioned in the very first introduction of engineering stress strain curve, there is an offset yield and yield stress that is called also called true stress. So, this is what it is so the strain axis you mark this 0.002 value and from there you draw a straight line wherever it intersects the curve that is 0.2% offset yield strength or small true stress. When evaluated for very large plastic strains n_{RO} which is approximately equal to 1 / n.

So, the values of 2 to 5 or even larger are common in practice useful to rewrite the Ramberg Osgood relation. So that more closely resembles the Holloman relation by subtracting the elastic strain from the total string to leave only the true plastic strain. So, we can write it like this $\varepsilon_{plastic} = \varepsilon_t - \sigma_t / E = K_{RO} (\sigma_t / E)^{n_{RO}}$ which can be rewritten like this.

And finally you can substitute into this form $\boldsymbol{\sigma_t = \left(\frac{E}{K_{RO}^{n_{RO}}}\right) \varepsilon_{plastic}^{1/n_{RO}} = H \varepsilon_{pl}^{n}}$. The strain hardening exponent n and the strength coefficient H values can be determined from the more conventional ln ε_t versus ln$\varepsilon_{plastic}$ .So, this is one way of approximating the stress strain curve again. So, we will continue this discussion in how to look at this you know a total strain in a stress strain curve and from there we will move on. So, I will stop here now I will see you in the next class thank you.