

Mechanical Behaviour of Materials
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Lecture - 33
Mechanical Testing - I

Hello, I am Professor S. Sankaran in the Department of Metallurgical and Materials Engineering. Hello everyone welcome to this lecture on mechanical testing now, we have been seen and know all the physical metallurgical aspects of materials in the context of mechanical behavior in the previous classes and from this point onwards we will discuss about mechanical testing which is a very important aspect in this field of mechanical engineering as well as metallurgical and materials engineering.

And as we talked in the introduction section, I will not get into the specific method of testing all these mechanical tests which we can always find it from the standard ASTM standards or any other standards which is universally accepted. Rather we would like to look at the interpretation of the test outcome. For example, we are talking about the spectrum of materials which are going to respond to the mechanical force this is all about this course.

So, we would look at for example, particular the state of testing for example tensile when the materials subjected to tensile testing then how it responds. So, we will look at these kind of aspects. So, similarly, we will go through a gamut of testing like tensile impact factor and most popular one among this test is simple tensile test and then from this tensile test, people try to derive as much information as possible.

And then we will spend a lot more time on this tensile testing and a lot of information a wealth of information is available in the variety of core materials and when people take away quite a bit of information about the simple tensile tests. So, we will spend more time on that, but even before tensile test, there is much more simpler test is possible in terms of mechanical behavior, if you want to assess a straightforward, very quick assessment of mechanical behavior is a hardness test.

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The Hardness Test



Scratch Hardness

- Hardness is not a fundamental property of a material and depending on the context of measurement, it may mean resistance to **scratching, abrasion, cutting and penetration**
- For example, scratch hardness may not be of any significance to the structural engineer, but it is of paramount importance to the mineralogist and geologist
- In the Mohs scale of scratch hardness, hardness is rated by a group of 10 common minerals arranged in an arbitrary scale of progressively increasing hardness from 1 to 10. The minerals are arranged in the following order:
 1. Talc ($\text{Mg}_3(\text{Si}_2\text{O}_5)_2(\text{OH})_2$)
 2. Gypsum ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$)
 3. Calcite (CaCO_3)
 4. Fluorite (CaF_2)
 5. Apatite ($[\text{CaF}(\text{Ca}_4(\text{PO}_4)_3)]$)
 6. Feldspar
 7. Quartz (SiO_2)
 8. Topaz ($[\text{OH}(\text{F})_2\text{Al}_2\text{SiO}_4]$)
 9. Corundum (Al_2O_3)
 10. Diamond

- Here, the order 1-10 represents the Mohs hardness numbers
- In using the Mohs scale, the hardness number is found scratching the minerals progressively by the test material until the hardest material that the test material will scratch, is found
- Hardness of the test material then lies between the two standards that it will just scratch and the one that it fails to scratch
- For example, tungsten carbide lies between 9 and 10, fingernail is between 2 and 3, lead is at 1, copper is between 3 and 4, glass is between 5 and 6 and hardened steel is between 6 and 7

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So, we will start with a very simple test called hardness test and even in the hardness test, there are a variety of hardness tests are possible because with respect to various kinds of materials, so, those days there are particular type of hardness tests were specifically meant for metals and other hard materials, but then when you talk about today, different kinds of materials to ceramics composites, polymers. So, visco elastic material, non-crystalline material, and so on so forth. So, the hardness test also has a spectrum of methods we can just come across so, let us start with something called a scratch hardness the very basic hardness testing method and before even getting the scratch hardness let us see what is hardness is not a fundamental property it is not like a young's modulus or shear modulus, Poisson's ratio and so on.

However, it is very important in the context of engineering applications. So, we will see how it is. So, hardness is not a fundamental property of the material and depending upon the context of measurement, it may mean resistance to scratching, abrasion, cutting and penetration. So, these are all the attributes here how whether it is resistant to scratch resistance to abrasion or resistance to cutting or resistance to penetration.

For example, scratch hardness may not be of any significance to structural engineer. But it is of paramount importance to mineralogist and geologist. This is quite obvious, like a structural engineer will not get enough information about the material through scratch hardness, because for a structural engineer the strength and toughness there are the primary important properties.

On the other hand, people like the largest geologist will find it very interesting data from out of this kind of scratch hardness. So, this is a very fundamental hardness. So, let us see what it is in the Mohs scale of scratch hardness. So, the Mohs the scratch hardness is measured in Mohs scale hardness is rated by a group of 10 common minerals arranged in an arbitrary scale of progressively increasing hardness from 1 to 10.

The minerals are arranged in the following order. What are those minerals? These are all the minerals from a talc, gypsum, calcite, fluorite, apatite, feldspar, quartz, topaz, corundum and diamond. So, here the order is 1 to 10 represent the most hardness numbers in using the Mohs scale the hardness number is found scratching the minerals progressively by the test material, the material in which we are interested to find the hardness.

We are going to scratch with the testing material until the hardest material, that the test material base scratch which is formed. So, what does it mean I want to find the hardness of something like that some material x so, I take this x material and make a scratch on all these minerals from 1 to 10? So, that it will be a material on which my material the x will make a scratch on them is followed.

The hardness of the test material then lies between the 2 standards that it will just scratch and one that it fails to scratch. So, obviously, some of the materials will try to scratch some material it will not scratch. So, it will be in between these 2 numbers. So that is the measure of hardness. For example, tungsten carbide lies between 9 and 10, fingernail is between 2 and 3, lead is at 1, copper is between 3 and 4 glass is between 5 and 6 and hardened steel is between 6 and 7.

So, it is quite interesting just by comparison of scratch mark on the various minerals you try to assess the hardness of the material. This is one way of assessing the strength in a very crude way but then it is very useful for some people.

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The Rebound Hardness



- The **Shore scleroscope** (a portable instrument) determines the rebound hardness by measuring the rebound height of a standard weight (hammer) dropped from a fixed height onto a surface.
- It utilizes a small diamond tipped hammer (weight 1/16 pound) with rounded tip, which is allowed to free fall inside a glass tube from a height of 10 in. onto the test surface.
- The height of rebound is measured against a graduated scale inside the tube. The scale is arbitrary and graduated to 140 divisions, and it is based on a rebound height of 100 divisions from hardened high carbon steel.
- Accurate vertical alignment of the tube is necessary for accurate results.
- The hardness of large structural parts can be conveniently measured using the instrument.
- The scleroscope measures the **dynamic hardness** and the method is also known as the **elastic rebound method**.
- The dynamic hardness is a measure of the elastic energy stored in the body due to impact by the indenter.
- Therefore, materials like rubber with low elastic modulus usually give rebound hardness numbers higher than that of steel.

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The next type of harnesses rebound harness; What is this rebound harness? The shore telescope a portable instrument determines the rebound hardness by measuring the rebound height of a standard weight typically hammer dropped from a fixed height onto a surface. So, it is called also called the shore hardness though the equipment name is shore telescope it utilizes a small diamond tip to hammer (weight 1 / 16 pound).

With a rounded tip which is allowed to free fall inside a glass tube from a height of 10 in. onto the test surface. So, basically, you have this tip which is enclosed in a tube and then you try to drop it on the surface of interest. So, the height of rebound is measured against the graduated scale inside the tube. The scale is arbitrary and graduated to 140 divisions and it is based on the rebound height of 100 divisions from the hardened high carbon steel.

So, it is the kind of relative rebound height that decides the or that gives the kind of idea about the hardness of the material or indirectly a strength accurate vertical alignment of the tube is necessary for accurate results the hardness of large structural parts can be conveniently measured using this instrument. This scleroscope measures the dynamic hardness and the method is also known as elastic rebound method.

Because the rebound takes place because of elasticity so it is called elastic rebound method. The dynamic hardness is a measure of elastic energy stored in the body due to impact by the indenter. So, it is a measure of elastic energy as well stored energy as well. For example, if you take a big polymeric sheet, very big polymer sheet typically used a variety of applications.

People even use those thick polymeric sheets in the foundations which as a shock absorber, big foundations there. The hardness of this material is crucial it has got a specific shore hardness a commercially it is told to shore hardness So, this is all very important so, all the

polymeric materials semi-crystalline material, rubbery material they use these kinds of hardness therefore, materials like rubber with the low elastic modulus usually give rebound hardness numbers higher than that of a steel that is quite obvious. So, any low elastic modulus materials how to assess the hardness is by the rebound elastic rebound method.

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Hardness tests

| Test | Indenter | Side view | Top view | Load | Formula for hardness number |
|------------|---|-----------|----------|---------------------------------------|--|
| Brinell | 10 mm sphere of steel or tungsten carbide | | | $P = 3000 \text{ kg}$ | $BHN = \frac{P}{\pi D (D - \sqrt{D^2 - d^2})}$ |
| Vickers | Flattened pyramid | | | $P = 0.05 \text{ to } 100 \text{ kg}$ | $VHN = \frac{1.854 P}{d^2}$ |
| Rockwell | Flattened diamond | | | $P = 0.05 \text{ to } 150 \text{ kg}$ | $RHN = \frac{100}{1 - \frac{d}{2}}$ |
| Rockwell C | Diamond cone | | | $P = 10, 30, 50, 100, 150 \text{ kg}$ | $RHC = \frac{100}{1 - \frac{d}{2}}$ |
| Rockwell B | 1/16 inch diameter steel sphere | | | $P = 10, 30, 50, 100, 150 \text{ kg}$ | $RHB = \frac{100}{1 - \frac{d}{2}}$ |
| Rockwell A | 1/16 inch diameter steel sphere | | | $P = 10, 30, 50, 100, 150 \text{ kg}$ | $RHA = \frac{100}{1 - \frac{d}{2}}$ |

The Properties and Properties of Materials, John Wiley, 2001
Mechanical Behavior of Materials by Meyers and Chawla, Cambridge University Press, 2009
Callister's Materials Science and Engineering, Material Science, R. Balakrishna, John Wiley India (P) Ltd., 2007

Macroindentation impressions

- The hardness test measures the resistance of a material to an indenter or cutting tool.
- The indenter is usually a ball, pyramid or cone made of a material much harder than that being tested – for example, hardened steel, sintered tungsten carbide, or diamond.
- An empirical hardness number may be calculated from the results of such tests by knowledge of the load applied and cross-sectional area or depth of the impression.

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Let us now come to the other primary method of hardness testing, look at this table it is a huge table it is quite popular table which is given most of the fundamental first year level of textbooks. It is given because it is very fundamental property and then you have Brinell hardness, Vickers hardness, Knoop hardness and Rockwell hardness. Rockwell hardness of course, it is measured at a different different scale depending upon the indenter the geometry and the load.

So, you see that what is common in all the hardness testing method each hardness test is going to use a particular type of indenter, in this case it is about 10 mm sphere of steel or a tungsten carbide ball and it is going to look like from the side view like this. So, that capital D is the diameter of the wall on the sphere and small d the distance or the diameter of the impression it is going to make on the surface.

So, this is the top view of the impression after indentation mark and this is the impression made because of the load which has been given by this and then the formula for hardness number. So, what is this formula BHN? Brinell hardness number. So, like that Vickers hardness it is using a diamond pyramid indenter which has got the geometry like this from the side view the angle between these two angular faces about 136° and then it makes an impression of a square of course, it is you can see that this is since a taper.

So, it will make a taper inside it will make an impression inside and then the diagonals are measured with d_1 and d_2 if it is very different but if it is ideally it is expected to be same and this is the formula for calculating this and Knoop hardness which uses a diamond pyramid indenter again. So, it makes an impression like this and the Rockwell which is measured at different scale also uses a different type of indenter.

Diamond cone indenter is like this and it makes impression like surface circular impression and also it is 1 / 16-inch diameter, steel sphere or 1 / 18 diameter, steel sphere also will make ball will also make a impression like this on the material and then depending upon the load with which you are making an interpretation the hardness is measured in the respective different, different scales.

So, what is that we need to know this is a very simple test or primary test every industry or anybody who is interested in knowing the mechanical behavior of material or any material they directly indirectly assess the strength they do not have the direct access to measure the strength. So, this is one way of connecting the strength to this hardness measurement. So, we need to under understand what is the physics behind it and then how to interpret the results. This is what we need to know.

So, it makes this kind of impression what I have shown here is a Brinell is a one which makes a very huge impression a spherical impression. Rockwell-C is much smaller spherical impression and then there is a superficial Rockwell, which is again which is small. The Vickers is very small a pyramid impression will be going to make. So, the hardness test measures the resistance of a material to an indenter or cutting tool.

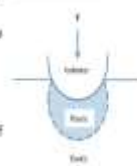
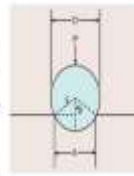
The indenter is usually a ball, pyramid or a cone made up of a much harder than the being tested. For example, hardened steel, sintered tungsten carbide, diamond these are all listed here. So, you can see this an empirical hardness number may be calculated from the results of such tests by knowledge of load applied and cross-sectional area or depth of impression. So, these are all empirical relation based on the knowledge of load applied that is P is a load divided by the cross-sectional area of the impression. So, these are all empirical relation which gives some idea about the material hardness.

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Indentation Mechanics

- It is found that by applying yield criteria, the condition for plasticity is first reached at a point below the centre of the circle of contact at a distance equal to approximately one fourth the diameter of the circle of contact.
- When the indenter is pressed, shear takes place on countless number of slip planes of maximum shear stress near the indenter.
- An amount of material gets displaced due to the decrease in volume of the elastic material.
- The elastic region surrounding the plastic zone then puts a constraint on the plastic zone from further deformation.
- As a result, the mean pressure $p_m (= 4P/\pi a^2)$ exerted by the indenter far exceeds the flow stress σ_0 of the material. This is expressed as

$$p_m = \frac{4P}{\pi a^2} = C \sigma_0$$
 where C is the constraint factor.
- Based on the elastoplastic analysis, the constraint factor C is found to vary between 2.6 if maximum shear stress theory is used and 3.0 if von Mises criterion is used.
- These values apply to materials that do not work harden.
- For work hardening materials, σ_0 in Eq. must be considered as the average flow stress for and average flow strain in the plastic zone.



Mechanical Behaviour of Materials, Thomas H. Courtney, Elsevier Press Inc., 2005
 Constitutive and Mechanical Behaviour of Materials, R.N. Thirum, PHI Learning Pvt. Ltd., 2012



What is the mechanics? Let us understand the mechanics. So, this is the let us consider this spherical impression out of this Brinell hardness tester. So, as I just mentioned that this is capital D of the diameter of the ball indenter and this small d is the diameter of the impression it makes on the surface here. So, how do we connect this small d with the capital D. So, it makes, suppose you just extend the surface line here. And then and write a, I mean write after that, you just mark out perpendicular to this then it makes a angle just find out the angle with which it is just expanding on this. So, the angle is angle between the perpendicular and this horizontal line is ϕ . So, the diameter of the impression D can be the $d \sin \phi$ is equal to capital D. So, depending upon the depth of the impression the depth of the segmentation which goes into this and this can be a $\sin \phi$ of d can be related to the capital D.

So, that is how we can find out and normally what happens is indenter makes an impression which is plastic a plastic impression and then the surrounding material is plastic and then because of this it leaves a plastic zone surrounding by the elastic region there is something which you know, we need to understand especially in the state of stress now that we know mechanics little bit. So, we need to understand the state of stress behind the indenter. So, what is that so, it is found that by applying yield criteria, the condition for plasticity is first reached at a point below the center of the circle of the contact at a distance equal to approximately one fourth diameter of the circle of contact. When the indenter is pressed, shear takes place on countless number of slip planes of maximum shear stress near the indenter. So, this is an indenter as it just penetrates the material it generates countless number of slip planes. Especially the maximum shear stress near the indenter that especially the just below the indenter it is going to create a lot of slip planes, that means, so, we are talking

about this region just to below this. So, you have from surface to this maximum depth, there is a huge distance. So, that means all these depths, then the slip is not going to be uniform. So, you should expect some non-uniform plastic deformation. So, what is that? That is what we are going to see, an amount of material gets displaced due to the decrease in the volume of elastic material. The elastic region surrounding the plastic zone then puts a constraint on the plastic zone from the further deformation. You see this. This particular elastic region surrounding this plastic zone is going to constrain this plastic deformation to further deform. so, that is something we have to understand. So, what is so, what happens as a result the mean pressure (P_m)

$$P_m = \frac{4P}{\pi d^2}$$



This is a load divided by the circular area exerted by the indenter far exceeds the flow stress sigma of the material. So, it crosses this mean pressure crosses the yield strength of the material. So, this can be written like this.

$$P_m = \frac{4P}{\pi d^2} = C\sigma_0$$

where C, is the constraint factor based on the elastoplastic analysis, the constraint factor C is found to vary between 2.6 if maximum shear stress theory is used and it is 3 if von Mises criterion is used. So, you know now, all this basically criterium and the material flow and all that, so, you should be able to relate all those things here even in the indentation mechanics. So, these values apply to materials that do not work harden, it is a very important point if the material is susceptible to work hardening, then these measurements, I mean these assumptions will not hold good. For work hardening material sigma not in the equation must be considered as the average flow stress for and average flow strain in the plastic zone. So, it is not independent, but it is an average as a whole.

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Indentation Mechanics

Limits of plastic and elastic deformation around an indentation
 (a) Riding type, common in hard or work hardened materials
 (b) Sinking type, common in soft or annealed materials

- One of the principal errors that occurs when measuring hardness is in measuring the diameter of the indentation which gets altered due to the **elastic recovery** of the material
- The **riding type** of indentation is observed in hard or cold worked materials where no more strain hardening is possible. As the indenter is pressed, the material simply tends to pile up around the indenter
- This results in an overestimate of the indentation diameter, leading to lower hardness determination
- In soft and well-annealed materials, a **sinking type** of indentation results owing to a high rate of strain hardening accompanied by elastic relaxation around the indenter
- In this case, the measured indentation diameter will be smaller than the actual measurement, resulting in an erroneously high hardness estimate
- In the process of indentation, the material immediately below the indenter is plastically deformed and the **elastic material surrounding the plastic zone hinders** further deformation
- As a result, a **strain gradient** manifests itself within the volume below the indenter, the strain being greatest near the indenter surface and least at the elastoplastic interface

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So, now, let us understand a little more about this mechanics. So, the material is undergone some plastic deformation under the indenter, but what is shown here there are two possibilities, limits of plastic and elastic deformation around the indentation a rid type common in hard or work hardened material if the material is work hardening, I mean the material undergoes work hardening during this hardness testing then we can create a bridge like this.

On the other hand, a sinking type a common in softer annealing materials if it is too soft material is being tested for hardness using this spherical indenter but it can also produce a sinking edge surrounding the indentation. So, one of the principal errors that occurs when measuring the hardness is in measuring the diameter of the indentation, which gets altered due to elastic recovery of the material.

That riding type of indentation is observed in hot or cold worked materials where no more strain hardening is possible. As indenter is pressed, the material simply tends to pile up around the indenter. This results an over estimate of indentation diameter leading to lower hardness determination. In a soft or well-annealed material, a sinking type of indentation results owing to your high rate of strength hardening, accompanied by elastic relaxation around the indentation or indenter.

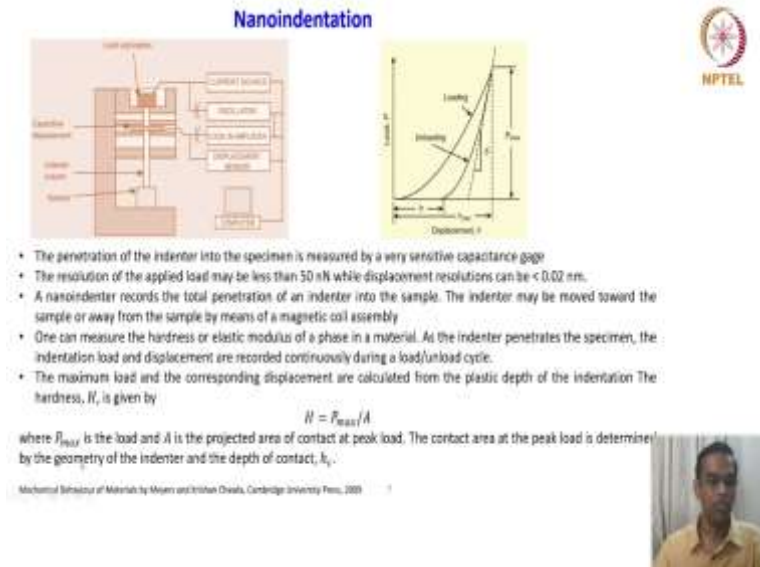
In this case, the measured indentation diameter will be smaller than the actual measurement resulting in an erroneously high hardness estimate. So, what it emphasizes here is irrespective of any hardness testing method, what this illustration demonstrate is the impression which creates the indentation impression has to be made perfectly otherwise it will have positive and negative influence.

So, the geometry of the impression is so crucial in taking the results of any type of hardness. That is the main idea. In the process of indentation, the material immediately below the indenter is plastically deformed and the elastic material surrounding the plastic zone hinders the further deformation. So, this again we have shown in the schematic. As a result, a strain gradient manifests itself within the volume below the indenter, the strain being greatest, near the indenter surface and the least at the elastoplastic interface. Very, very important.

So, the strain will be very high at the bottom and it will be small in the elastoplastic interface like this here. So, indentation geometry clearly or an indentation mechanics clearly shows that there is a strain gradient exists in every impression. So, this is the primary information we have to keep in mind.

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Nanoindentation



- The penetration of the indenter into the specimen is measured by a very sensitive capacitance gage
- The resolution of the applied load may be less than 50 nN while displacement resolutions can be < 0.02 nm.
- A nanoindenter records the total penetration of an indenter into the sample. The indenter may be moved toward the sample or away from the sample by means of a magnetic coil assembly
- One can measure the hardness or elastic modulus of a phase in a material. As the indenter penetrates the specimen, the indentation load and displacement are recorded continuously during a load/unload cycle.
- The maximum load and the corresponding displacement are calculated from the plastic depth of the indentation. The hardness, H , is given by

$$H = F_{\max} / A$$

where F_{\max} is the load and A is the projected area of contact at peak load. The contact area at the peak load is determined by the geometry of the indenter and the depth of contact, h_0 .

Mechanical Behaviour of Materials by Meyers and Krishan Choudhary, Cambridge University Press, 2009

So, what are the other hardness test testing, sophisticated hardness testing. So, this is nanoindentation this this particular tool again is quite popular because it uses very very small loads and then you can find out the hardness for the even a small segment very sub-micron features you can go and test the hardness in the microstructure. So, that is what it is let us see, what it is. The penetration of the indenter into a specimen is measured by very sensitive capacitance gauge are very sensitive capacitance gauge the resolution of the applied load may be less than 50 Nn, note that number 15 nano Newtons while displacement resolution can be as small as less than 0.02nm. So, you see that you will be able to measure the hardness such a small dimension in the microstructure.

So, this is the advantage of this. So, what is this? This is a load displacement plot. So, we will see one by one how to understand this curve. A Nanoindenter records the total penetration of an indenter into the sample the indenter maybe move toward the sample or away from the sample by means of magnetic coil assembly. So, one can measure the hardness or elastic modulus of a face in a material as indenter penetrates the specimen, the indentation load and displacement are recorded continuously during a load unload cycle. So, what is plotted in this diagram is a loading and unloading cycle. So, you see that the load loading increases it reaches a maximum that is a p_{\max} and then unloading takes place. So, that means that the net penetration is h that this h what is the maximum penetration depth is h_{\max} .

So, you can you can also just you know take some other parameters from this curve the S stands for the stiffness. The stiffness can be measured by you know changing load this basically a slope that means what dP / dH which will give you the stiffness of the face which you are interested. So, what is important in this hardness tester. This hardness tester, uses a very small load and it can sense a very small displacements in sub-micron scale.

So, we will be able to understand the strength of very tiny you know sub-micron features or sub-micron entities in the microstructure you can measure the mechanical strength so, that is the idea. The maximum load and the corresponding displacement are calculated from the plastic depth of indentation the hardness H is given by $H = P_{\max} / A$. It is a load by area where P_{\max} is a load and A is the projected area of the contact and a peak load.

The contact area of the peak load is determined by the geometry of indenter and the depth of contact h_c . so, again here up here also the impression of the indenter is quite crucial. See as such it is very sensitive to the load and displacement. So, it is for more crucial to measure the exact impression.

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Nanoindentation



- Assuming that the indenter does not deform significantly, we can write $A = f(h_c)$
- The form of function f must be established experimentally. The area A can be calculated by means of the following expression:

$$A = a + bh_i^{1/2} + ch_i + dh_i^{3/2} + 24.56h_i^2$$

where h_i is the plastic depth of the indentation and $a, b, c,$ and d are adjustable coefficients

For a perfect tip, $a = b = c = d = 0$, and the only coefficient is 24.56

- The stiffness, S , can be obtained from the load, P vs. penetration depth, h by the following expression relating the reduced modulus, E_r , the contact area A and the stiffness, S :

$$S = dP/dh = \left(\frac{2}{\sqrt{\pi}}\right) E_r \sqrt{A}$$

- The reduced modulus E_r of indenter-sample combination takes into account the fact that elastic deformation under load occurs in the sample as well as in the indenter
- The reduced modulus is given by

$$E_r = \frac{(1 - \nu_i^2)/E_i}{1 + \frac{(1 - \nu_s^2)/E_s}{(1 - \nu_i^2)/E_i}}$$

where E_i and E_s are the Young's moduli, and ν_i and ν_s are the Poisson's ratio of the indenter and sample, respectively.

- The initial unloading slope gives us the reduced modulus provided one can measure the contact area at the peak load

Mechanical Behaviour of Materials by Meyers and Chawla, Cambridge University Press, 2009



Let us see what are the available methods, assuming that the indenter does not deform significantly we can write a $A = f(h_c)$ that is the form of function f must be established experimentally. The area A can be calculated by means of the following expression

$$A = a + bh_i^{1/2} + ch_i + dh_i^{3/2} + 24.56h_i^2.$$

So, this is kind of an empirical calculation where h_i is the plastic depth of indentation and a, b, c and d are the other adjustable coefficients.

Why we need this adjustable coefficients, for a perfect tip $a = b = c = d = 0$ and the only coefficient is 24.56. So, that means if you are making a perfect impression with this indentation then all these coefficients will become 0, these are all adjustable coefficients for the different non perfect impression. So, that is how we understand the stiffness S can be obtained from the load P versus penetration depth h by the following expression relating the reduced modulus E_r , why it is called reduced modulus.

The contact area and stiffness S can be given like this $S = dP / dh$ we have seen in the previous plot is a slope which is, $(2 / \sqrt{\pi}) E_r \sqrt{A}$. Reduced modulus E_r of the indenter sample combination takes into account the fact that elastic deformation under load occurs in the sample as well as in the indenter. So that is why it is reduced modulus.

So, the reduced modulus is given by $E_r = \frac{(1 - \nu_i^2)}{E_i} + \frac{(1 - \nu_s^2)}{E_s}$

where E_i, E_s are the Young's moduli and ν_i and ν_s are Poisson's which should not $\nu \nu_s$

ν_i or the Poisson's ratio of indenter and sample respectively. The initial unloading slope gives us the reduced modulus provided one can measure the contact area of the peak load.