

**Mechanical Behaviour of Materials**  
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**Lecture – 30**  
**Strengthening Mechanisms in Crystalline Materials - III**

Hello, I am Professor S. Sankaran in the Department of Metallurgical and Materials Engineering. Welcome back to the lecture on the solid solution strengthening mechanisms, which we have discussed in the last class. We are going to continue that concept now.

(Refer Slide Time: 00:32)

**Solid-Solution Strengthening**

- The magnitude of the size-effect interaction energy,  $U_D^E$  between an edge dislocation and a spherically symmetric distortion is estimated.

$$U_D^E = \frac{4(1+\nu)Gbr^3\epsilon_b \sin\theta}{3(1-\nu)R}$$

Where R is the distance between the dislocation core and the solute atom and  $\theta$  is the angle between the slip direction and a line connecting the dislocation core and the solute atom.

- The parameter  $\epsilon_b$  is the measure of the relative size difference between the solute and solvent atoms. In a binary alloy it can be estimated as the fractional change in lattice parameter per unit concentration of solute atom, i.e.,

$$\epsilon_b = \frac{1}{a} \frac{da}{dc}$$

- The modulus interaction energies as


$$U_G^S = \frac{G\epsilon_b^2 b^2 r^3}{6\pi R^2}$$


and

$$U_G^E = \frac{U_G^S}{(1-\nu)}$$

where  $U_G^S$  and  $U_G^E$  are the modulus interaction energies for screw and edge dislocations and  $\epsilon_b^+$  is defined by

$$\epsilon_b^+ = \frac{\epsilon_b}{\left(1 + \frac{1}{2}|\epsilon_b|\right)}$$





If you look at the magnitude of size effect interaction energy that is  $U_D^E$  between the edge dislocation and a spherically symmetric distortion is estimated. Which is given by this expression

$$U_D^E = \frac{4(1+\nu)Gbr^3\epsilon_b \sin\theta}{3(1-\nu)R}$$

Where R is the distance between the dislocation core and the solute atom and  $\theta$  is the angle between the slip direction and a line connecting the dislocation core and the solute atom. You see we are now looking at the semi quantitative expression for the size effect.

We were talking about distortion and dilation. When you talk about substitutional solid solution, whether it is interstitial or substitutional the size effect is given by this kind of an expression which directly related to parameters like R distance between the dislocation core and the solute atoms and  $\theta$  the orientation with respect to the slip direction and line

connecting the dislocation core and solute atom. You have to recall if cannot just ignore the concept which we have learned earlier, if you look at the dislocation dynamics or motion.

We have seen that the dislocation core has to be at an angle with respect to its other object whether it could be a solute or whether it could be another dislocation, the orientation is important. So, that is why it has directly related to and an inverse related to the  $R$ . The parameter  $\epsilon b$  is a measure of relative size difference between the solute and solvent atoms. For example, in a binary alloy it can be estimated as the fractional change in the lattice parameter per unit concentration of solute atom.

This is very important point. So, this is directly related to  $\epsilon b$  which is a measure of relative size difference. How it is measured?  $\epsilon_b = \frac{1}{a} \frac{da}{dc}$ , that is fractional change in the lattice parameter per unit concentration of the solute atom. The modulus interaction energy as  $U_G^S$ ,

here it is  $U_D$  this is the size of it. Here it is  $U_G$  which is given by  $U_G^S = \frac{G \epsilon'_G b^2 r^3}{6 \pi r^2}$  and it

can be rewritten for the edge dislocation. This is for a screw dislocation  $U_G^S$  means this is for screw and this is for edge earlier we have seen.

So, edge again you know that the only difference is the elastic constant term. So, edge dislocation  $1 / (1 - \nu)$  is a constant term which is going to come throughout the discussion

that we have already discussed, where  $U_G^S$  and  $U_G^E$  are the modulus interaction energy for

screw and edge dislocation and  $\epsilon'_G$  is defined by  $\epsilon'_G = \frac{\epsilon_G}{(1 + \frac{1}{2} |\epsilon_G|)}$

$$U_G^E = \frac{U_G^S}{(1 - \nu)}$$

So, this is how the interaction energies are derived semi quantitatively. So, all that you have to know about this expression is the parameters. What parameters are important, how they are related with the energy? These are all the concept you have to catch rather than identifying how these equations are derived that is not important here, we have to just get the parameters, which is related to the core idea.

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### Solid-Solution Strengthening

- The parameter  $\epsilon_G$  of Eq. is analogous to  $\epsilon_b$ . That is,  $\epsilon_G$  represents the fractional change in shear modulus per unit solute concentration;


$$\epsilon_G = \frac{1}{G} \frac{dG}{dc}$$

- In contrast to the size-effect interaction energy, the modulus interaction energy can be either positive or negative depending on the sign of  $\epsilon_G$ .
- However, **Fleischer** has shown that, at least for some substitutional solid-solution strengtheners, this energy and the resulting shear stress correlate with a parameter,  $\epsilon_s$ , defined as


$$\epsilon_s = |\epsilon'_G - \beta \epsilon_b|$$


- In Eqs above,  $\epsilon_b$  is always taken as positive as is the parameter  $\beta$ . For "soft" atoms  $\epsilon'_G$  is negative and thus the size and modulus effects reinforce each other.
- For "hard" atoms,  $\epsilon'_G$  is positive and the opposite situation holds. The parameter  $\beta$  in Eq. is an empirical one related to the relative importance of screw and edge dislocations during plastic flow and, therefore, also depends on the relative importance of the size effect.
- Solid-solution strengthening can also be discussed in terms of concepts introduced earlier. If  $L'$  is the effective obstacle spacing, the increase in flow stress associated with the solute atoms is

$$\tau = \frac{F_{max}}{bL'}$$



Mechanical Behavior of Materials, Thomas H. Courtney, Pearson Press Inc., 2005





The parameter  $\epsilon_G$  of the equation is analogous to  $\epsilon_b$  that is  $\epsilon_G$  represents the fractional change in the shear modulus per unit solute concentration. Where earlier we have seen  $b$  is there, there it is the size difference, here it is a shear modulus per unit concentration, that is  $\epsilon_G = \frac{1}{G} \frac{dG}{dc}$ . In contrast to the size effect interaction energy, the modulus interaction energy can be either positive or negative depending on this sign of  $\epsilon_G$ , not size it is the sign of  $\epsilon_G$ .

However, Fleischer has shown that at least for some substitutional solid solution strengthens this energy and the resulting shear stress correlate with the parameter  $\epsilon_s$ , which is defined as  $\epsilon_s = |\epsilon'_G - \beta \epsilon_b|$ . In this above equation,  $\epsilon_b$  is always taken as positive as the parameter  $\beta$ .

For soft atoms  $\epsilon'_G$  is negative and thus the size and modulus effects reinforce each other.

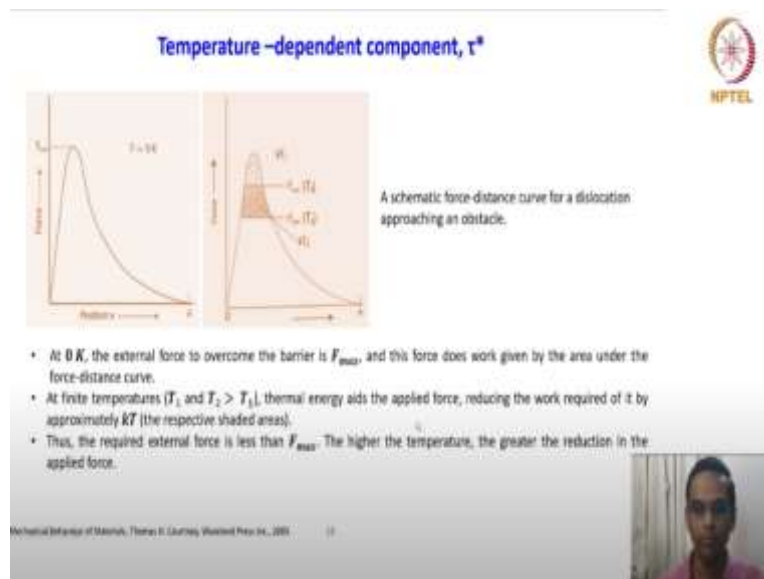
You can verify this with the plot which we are shown in the beginning of the solid solution class, where we have shown this size effect and modulus effect reinforced for the soft. For a hard atoms  $\epsilon'_G$  is positive and the positive situation holds the parameter  $\beta$  in the equation is an empirical one related to the relative importance of screw and edge dislocations during plastic flow and therefore, also depends on the relative importance of this size effect.

Solid solution strengthening can also be discussed in terms of concepts introduced earlier. What is that concept? If  $L'$  is the effective obstacle spacing the increase in the flow stress associated with the solute atoms is  $\tau = F_{max} / b L'$ . This expression, we have seen earlier we

have seen, this is a very general and generic expression and whatever we have seen that the semi quantitative expressions shown here can also be that one way of looking at it what people have reported.

And the other way of looking at it is keeping this basic equation and then we can address the above problem in a similar manner that is the idea.

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Now, another important parameter we are going to discuss that is  $\sigma$ . So far, we have discussed the dislocation motion and in the presence of some obstacle. How the dislocation overcomes the obstacle and that determines the flow stress. So, if you recall in the earlier stage, when we talked about the flow properties, we also talked about the temperature dependent component of stress.

If you recall there is a plot where we showed that temperature dependent component why we discussed in the strain rate and temperature effect of flow properties, that plot we have seen. So, we will now see, since in the context of all the solute atoms being an obstacle to the dislocation motion, how this stress that  $F$  applied or  $\tau$  applied stress can be aided by the temperature component.

To have some idea about that we will have this slide. So, look at this plot this is a force versus position plot and  $F_{max}$  is shown here this is a hypothetical situation where  $T$  is  $0$  Kelvin, because we are interested in finding out the temperature effect on the force requirement, how the temperature is going to aid. So, at  $0$  Kelvin the external force to

overcome the barrier is  $F_{\text{Max}}$  and this force does work given by the area under the force distance step. So, basically this area under the curve is the total work.

Suppose if we raise the temperature and that is what shown here. The force position curve is shown for 2 different temperatures one is  $T_1$  and another is at  $T_2$ . So, the  $T_1$  the  $F_{\text{max}} = kT_1$  at  $T_2$  the  $F_{\text{applied}}$  is the work is equal to  $kT_2$ . So, how does this help? At finding temperatures where  $T_1$  and  $T_2$ , where  $T_2$  is greater than  $T_1$ . I think that is a typographical error in this. This supposed to be  $T_2$  and this supposed to be  $T_1$  because otherwise this would not hold could any way does not matter.

$T_2$  is greater than  $T_1$  terminal energy aids the applied force reducing the work required by approximately  $kT$  times. Thus, the required external forces less than  $f_{\text{max}}$ . For example, if you take at  $F_{T_1}$  this much of work is getting reduced or the force is reduced for the  $T_2$  this much of work is getting reduced. So, that is what it is. Actually, that is no problem in this plot. Because that means  $T_2$  takes more, I mean it gives more addition aid in terms of work done. So, it is correct only.

So, the  $T_2$  gives the work equivalent to  $kT_2$  and this is the work given by this region is  $kT_1$ . So, the greatest reduction in the applied force. So, this amount of energy is getting reduced in this y axis that is the idea.


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### Temperature-dependent component, $\tau^*$

- At a finite temperature, thermal vibrations can produce an energy on the order of  $kT$  to help the obstacle overcome the barrier.
- Thus, the work done by the external stress is reduced by  $kT$  and this is reflected in a reduction in the applied stress ( $\tau_{\text{app}} = F_{\text{app}}/bL'$ ) required for bypassing the obstacle.
- When the temperature is high enough ( $T_c$ ) that  $kT$  is equal to the external work required at 0K, the obstacle presents no further barrier to glide; thermal energy supplies the necessary work for overcoming barriers.
- If the force-distance profile of is known, the temperature dependence of the flow stress can be determined; for certain defects  $\tau^*$  can be expressed as

$$\left(\frac{\tau}{\tau_0'}\right)^{1/2} = 1 - \left(\frac{T}{T_c}\right)^{1/2}$$

where  $\tau_0'$  is the thermal component of the yield stress at 0 K and  $T_c$  is the above-mentioned critical temperature. The relationship between  $\tau$  and  $T$  predicted by Eq. has been verified experimentally for several materials.





Mechanical Behaviour of Materials, Thomas R. Courtney, Westland Press, 2020 79

So, at finite temperature thermal vibration can produce an energy on the order of  $kT$  to help the obstacle overcome the barrier. Thus, the work done by the external stresses reduced by  $kT$  and this is reflected in the reduction in the applied stress, ( $\tau_{\text{applied}} = F_{\text{applied}}/bL'$ ) required to bypass internal struggle. So, what happens is the; whatever the empirical relation shows in

terms of either it could be energy or it could be force or stress that if higher temperature is they are going to bring down that total stress that is the idea.

So, at high temperature all these obstacles can be overcome by this dislocation much more easily. When the temperature is high enough that is  $T_c$  that is  $kT$  is equal to the external work required at 0K. The obstacles present no further barrier to glide. Thermal energy supplies the necessary work for overcoming these barriers. So, that is an idea. If the force-distance profile of the curve which is shown is known.

The temperature dependence of the flow stress can be determined for certain defects  $\tau^*$  can be expressed as  $(\frac{\tau}{\tau_0})^{1/2} = 1 - (\frac{T}{T_c})^{1/2}$ . So, this is a relation between the temperature

difference on the flow stress. So, if we have an idea about the force distance profile of any particular situation or system then this kind of relation is established. For example, where  $\tau^*$  is the thermal component of the yield stress at 0K and  $T_c$  is the above-mentioned critical temperature.

What is the above-mentioned critical temperature? The temperature at which the  $kT$  is equal to the external work that is the critical temperature. The relationship between  $\tau^*$  and  $T$  predicted by this equation has been verified experimentally for several materials. So, what is that we have shown? What we have shown is the temperature dependent component of  $\tau$  that is a stress can be significantly reduced, if the temperature I mean if the obstacle is facing the dislocation at higher temperatures.

We will see this term in the high temperature deformation also the temperature dependent component of shear stress. Now, we move on to the other strengthening mechanism - particle hardening. So, we have now seen a boundary strengthening, solid solution strengthening, work hardening and this is particle hardening. This is also sometimes called a precipitation hardening, dispersion hardening and so on. But then we give a very gentle description like particle hardening.

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**Particle Hardening**

- Three types of interphase boundaries (IPBs).
- In (a), a coherent or ordered IPB exists between  $\alpha$  and  $\beta$  phases. The atoms match up, one to one, along such a boundary. Owing to different lattice parameters of the phases, a coherency strain energy is associated with this type of boundary.
- In (b) a fully disordered IPB is shown. Here, there are no coherency strains. A dislocation can penetrate an ordered IPB, but not a disordered one.
- (c) An intermediate type of IPB (a partially ordered one). Here, coherency strains are partially relieved by the periodic introduction of dislocations along the boundary.

Mechanical Behavior of Materials, Thomas H. Courtney, Wiley Interscience, 2002

What are the important points we need to understand before we get into this topic? What we are seeing here is interface boundary;  $\alpha$  is one crystalline material or the lattice which is described like this. The  $\beta$  is other crystalline lattice and then you have this interface boundary and what is that you are seeing? When you see this kind of boundary how will you describe them? They are all coherent boundaries. three types of interface boundaries we are going to see and this is the first coherent boundary.

A coherent or ordered interface boundary exists between  $\alpha$  and  $\beta$  phase. The atoms match up. What is match up? Basically, each one atom is directly connected to the other external atoms without any defect. One to one along such a boundary. Owing to different lattice parameters of the phases. Yet, coherency strain energy is associated with this type of boundary. You see this inter atomic distance and this is not exactly the same.

There is a term difference, but nevertheless these two phases are in coherent with each other that means, these bonds are getting stretched a little bit across the interface. So, that is that stretching that of the bond extra strain which is experienced by these bonds, we have an energy that is called a coherency strain energy. And if you look at the second type of interface boundary it looks like this and what is that we are seeing here?

It is not the same like what we have seen here definitely it is quite different. How do we compare this fully disordered interface boundary? Here there are no coherency strains. A dislocation can penetrate an ordered interface boundary, but not disordered one. So, this is where we have to pay our attention. So, when you have a coherent interface boundary the




dislocation motion is easy, it can glide through any glide plane without any problem across the grain interface boundaries.

But here the dislocation cannot simply glide through. So, the interface is disordered. What is the third interface boundary? What does it say? An intermediate type of interface boundary a partially ordered one. Here the coherency strains are partially relieved by the periodic introduction of dislocation along the boundary. You see it is exactly similar to the first one, but then you have some extra half plane is introduced here.

So, that means, it is going to destroy this coherency but partially at locally and it is also going to relieve that strain. That strain energy comes because of this stretching of this bond. That stretching is getting relieved here. So, that is why introduction of these kind of partial, I mean edge dislocations, if you assume that along this interface, if you keep on introducing the dislocation like this, it is going to reduce that energy.

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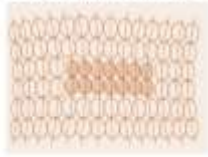


**Deforming Particles**

- The nature of deformable particles varies significantly from one material system to another.

**1. Coherency hardening**


- Precipitate coherency hardening is analogous to the size effect in solid solution hardening.
- A precipitate ordinarily has an atomic volume different than the matrix from which it forms. If the precipitate is coherent, this leads to an internal lattice strain.



- Schemata of a coherent precipitate (shaded atoms) in a matrix. Here, the precipitate has lattice parameter less than that of matrix.
- The atomic "mismatch" across the IPB leads to an internal stress field that interacts with moving dislocations.

- The associated stress field interacts with dislocations, either attracting or repelling them; either situation, of course, results in an increase in yield strength.

Mechanical Behavior of Materials, Thomas H. Courtney, Wiley, 2005, 11



First, we will discuss about deforming particles. We are going to talk about particle hardening and in the first cases deforming particles. The nature of deformable particles varies significantly from one material system to another, which one is characteristic. In deforming particles, we will first discuss about coherency hardening. What is coherency hardening? Precipitate coherency hardening is analogous to the size effect in solid solution hardening.

In the previous slide we have seen an interface boundary with the coherency coherent interface boundary we have seen. Suppose if the precipitate is forming a coherency I means



the precipitate is enabling this coherency hardening then that should have a boundary which is seen in the previous slide. That means, the interface boundary should be coherent with the matrix. The hardening comes from the idea similar to what is solid solution strengthening that is what we are saying. A precipitate orderly has an atomic volume difference than the matrix from which it forms

If the precipitate is coherent this leads to an internal lattice strain. There are two things one precipitate evolves from the matrix that is called precipitation and when we introduce a particle externally that is we will talk about dispersion probability later we will see, but here we are talking about a precipitation which comes out of the matrix. So, look at this schematic which nicely shows that idea.

It is a schematics of a coherent precipitate shaded atoms these are all coherent precipitant in a matrix here the precipitate has a lattice parameter less than that of the matrix. It is coherent, but the lattice parameters are small. The atomic match across the interface boundary leads to an external stress field that interacts with the moving dislocation. So, because of this small size, we know now, if the solute atom is small and then what kind of a stress fields it will produce. We have just seen in the solid solutions strengthening.

So, that similar stress fields will interact with the dislocation. Then we know what happens also. The associated stress field interacts with the dislocation either attracting or repulsing them either situation of course the result is increased in the yield strain. This also we have seen, whether it attracts or repels the net result is increase in the flow stress. In approximate expression for the increase in the results shear stress during early-stage precipitation.

Remember, we are talking about early-stage precipitation because it is deformable particles. If you recall, we were talking about dislocations with a straight line, which is going to just cut through them penetrate through them such kind of a situation that can happen only in the early stage of precipitation because that time the size will be extremely small. Of course, the other parameter also has to follow the trend since the modulus should be less and hardness should be less and so on.

But then if it is a shareable or deformable particle this is the expression, semi quantitative expression which says that  $\tau_{coh} \cong 7|\epsilon_{coh}|^{\frac{3}{2}} G \left(\frac{rf}{b}\right)^{1/2}$

where r is the precipitate radius, f is the volume fraction and the parameter  $\epsilon$  is analogous to the size parameter of solid solution strengthening. So, we know this is  $\epsilon$  is a size difference we have seen that. So, this is an empirical relation for the coherency hardening which is analogous to solid solution strengthening.