

**Mechanical Behaviour of Materials**  
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**Lecture - 28**  
**Strengthening Mechanism in Crystalline Materials – I**

Hello I am Professor S. Sankaran in the Department of Metallurgical and Materials Engineering. Hello everyone, welcome to the lecture today and we would like to start the new topic on strengthening mechanisms in crystalline materials. So, when you talk about strengthening mechanisms, if you look back what all the background that is required to get the grip on this subject is like again I as I mentioned the earlier sections some idea about the crystallography and dislocations and stress fields and so on.

So, when you see strengthening mechanisms you are going to strengthen, we are going to talk about strengthening the crystalline lattice. So, there are several ways by which you can do this process. So, we would like to see one by one and before we really getting into this topic, suppose if you consider the fundamental ideas that is required to grasp before we get into this is like stress fields in the lattice.

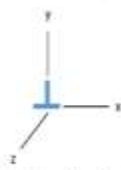
Suppose, if you talk about solid solution strengthening and then the stress field around the solid solutions that is one primary concern. The other thing is we just talked about a lot as dislocation and these stress field and strain field around the dislocation also has an important role to play, in fact it has got a primary role to play because we are going to talk about dislocation mediated activities are like deformation we talked about dislocation mediated plasticity. Here also when we involve plastic deformation to incorporate the strength.

Then again same idea will bring in but then we stopped talking about solid solution strengthening that is a first one to look at it, then you talk about two things as the addition of solid solution I mean additional foreign item to the matrix then what are all the changes it brings in and what kind of stress field surrounding the solute atom with respect to the solvent atom or solvent matrix so, these are all the concerns.

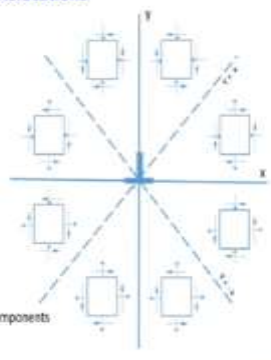
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### Stress Fields around Dislocations

- The dilational and shear stresses and the signs of these stresses are illustrated in Figure.
- There are no shear stresses acting in the direction parallel to the dislocation line.
- There is, however, a third dilational stress, normal to the plane of the figure.



The sense of the shear ( $\tau_{xy}$ ) and dilational ( $\sigma_{xx}, \sigma_{yy}$ ) stress components of an edge dislocation.



Mechanical Behaviour of Materials, Thomas & Courtney, Newland Press Inc., 2000

So, we will recollect what we have already seen in this dislocation we will be we have this idea about what kind of stress field around the dislocation. So, we have seen that it is quite complicated but we have the idea about the extra plane, the state of stresses, hydrostatic compression and below it is hydrostatic tension and along this you know, Y axis it is completely hydrostatic there is no shear stresses in this on the other hand if you look at the X axis, there is no hydrostatic stresses but only a shear stresses in both sides. So, these are all we already seen.

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### Forces on Dislocations

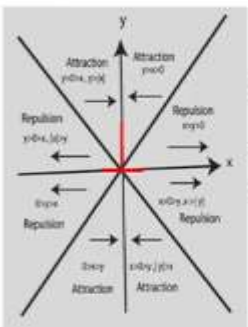


Figure shows the regions of attraction and repulsion.

- The stress,  $\tau_{xy}$ , is zero at  $x = 0$ ,  $x = y$ , and  $x = \infty$ .
- Between  $x = 0$  and  $x = y$ ,  $\tau_{xy}$  is negative, indicating that the stress field would cause attraction of edge dislocations of the same sign to each other.
- Therefore, edge dislocations of the same sign tend to line up one above the other, as shown in Figure before.
- For  $x$  greater than  $y$ , the stress,  $\tau_{xy}$ , is positive indicating that the stress field would tend to repel another edge dislocation of the same sign.

Mechanical Behaviour of Materials, W.J. Haddad, Cambridge University Press, 2010

And we also looked at the stress field in this form if you recall which clearly demonstrates the whether it is attraction or repulsion in the fourth quadrant with respect to the coordinates of x and y. So, this is very important to recollect and depending upon this, what kind of stress field is there in this dislocations and also what kind of stress fields exist in the solute

environment, the interaction energy is going to be decided. So, the whole solid solution strengthening mechanisms that revolves around these two fundamental aspects.

So, to summarize, what is the we have idea as far as the shear stresses and the normal stresses around these dislocations are concerned, we know that the screw dislocation is primary associated with the shear stresses, but the edge dislocation has got both shear and normal stresses. In fact you have one shear stress component and then three normal components edge dislocations and that also hydrostatic in nature and as far as a screw dislocations concern it is mostly shear component.

The other important idea we have to remember is at the moment we talked about the shear components it is related to distortion of the bond, but the hydrostatic or normal stresses as we talk about it is also related to dilation. So, we have to remember these two aspects. So, in the case of screw dislocations, we can talk about only a distortion that is a distortion energy associated with this. On the other hand if you go to edge dislocation you have both dilation as well as distortion.

So, it is with this background if you approach the solid solution strengthening then it is easy to understand and then move forward. So, the first and foremost topic is general description of strengthening, we will first look at the general strengthening aspects.

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**General Description of Strengthening**

- The strength of a crystalline material is increased by obstacles that restrict dislocation motion within it.

The stress required to overcome the obstacles depends on the effective spacing ( $L$ ) between the obstacles along the dislocation line and the angle ( $\theta_c$ ) to which the dislocation bends before it breaks through them.

- The stress required to produce continued motion of the dislocation through the obstacle array is the macroscopic yield or flow stress.

Mechanical Behaviour of Materials, Thomas H. Courtney, Wiley-Interscience, 2005

The strength of a crystalline material is increased by obstacles that restrict dislocation motion within it. So, to stop the dislocation motion we have seen several forms but we looked at when friction stress that is no exists in the lattice a perfect lattice is called the parallel stress we have seen that. So that also will have an influence here but here we are talking about an

external obstacle and a foreign object which is going to stop the dislocation motion in the parent matrix, this is what we are talking about.

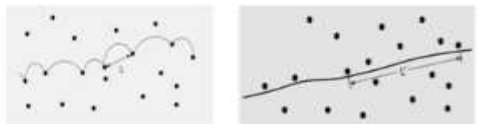
So, let us look at the schematic here, that is shown here what you are seeing here is a matrix where you have all the dispersion kind of obstacle, we are not defining this and this is the dislocation motion direction. So, and what you are seeing is the dislocation line which is trying to pass through this obstacle and then what you are seeing here, is these particles are trying to stop that is why it is the dislocation line is getting bent here right over here.

And the angle between these two lines tension we call it and it is  $\phi_c$  and then the distance between the two obstacles are  $L'$ . So, each particle in the matrix is considered as a obstacle. So, what we are really talking about is the stress required to overcome the obstacles depends on the effective spacing  $L'$  between the obstacles along the dislocation line and the angle  $\phi_c$  to which the dislocation bends before it breaks through. It is a kind of extrusion between these two obstacles.

This dislocation line is trying to come out or extrude themselves between these two obstacles that is what we are trying to show. The stress required to produce continued motion of the dislocation through the obstacle array is the macroscopic yield or flow stress. So, this is one way of you know a holistic view what is that we talked about as a macroscopic yield or flow stress which is nothing but the stress required to produce a continued motion of dislocation through the obstacle, array of obstacles, it could be any second phase particle or it could be defects, it could be boundary etcetera, so that is the idea.



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**General Description of Strengthening**



- The obstacle "strength" essentially determines this flow stress. "Strong" obstacles resist dislocation penetration.
- This is reflected by  $\phi_c$ , approaching zero. The spacing between obstacles is also important.
- For strong obstacles, the effective spacing is the mean spacing between obstacles on the slip plane ( $L$ ). If, on the other hand, the obstacles are weak,  $\phi_c$  is very large, approaching  $180^\circ$ , and this means that the dislocation line is much straighter for the situation of weak obstacles.
- As a corollary, the "effective" obstacle spacing,  $L'$ , is much greater than  $L$ .
- The stress required to overcome the obstacles depends on the effective spacing ( $L'$ ) between the obstacles along the dislocation line and the angle ( $\phi_c$ ) to which the dislocation bends before it breaks through them.

Mechanical Behaviour of Materials, Thomas F. Courtney, Wiley-Interscience Inc., 2003



Now we can have this obstacle in the different forms, the obstacle strength essentially determines the flow stress. So, we are talking about here, the second phase particle in the matrix but then whether the second phase particle is hard or soft enough as compared to matrix there is always a relative comparison of the hard or soft nature of the second phase particle with respect to the matrix that is why term strength comes into picture. So, strong obstacles resist dislocation penetration.

So, now we are talking about the other events like the dislocation not only gets stopped, but it also will stop them to penetrate the particles or penetrate obstacles. So, this is reflected by  $\phi_c$  approaching 0. So, what is  $\phi_c$  approaching 0? You can see that these dislocation lines they are all trying to extrude between these obstacle particles then the moment you know it is the penetration is completely resisted, then you see that the  $\phi_c$  is almost becoming 0. So, that is idea so the  $\phi_c$  is 0. The spacing between obstacles is also important.


So, in the strengthening mechanisms especially of this type that obstacles dispersion strengthening or second phase particle in the matrix, the spacing between the obstacles is a quite important parameter. If you look at this image this is also showing some second phase particle in the matrix but the situation here is quite different. What is the difference here? Here the dislocation line is not curve like this that means. The obstacles are pinning this and then resisting the motion and then the dislocation line is trying to extrude between them, but here the dislocation line is simply travelled through, penetrate through, cut through all this so that is the difference. For strong obstacles the effective spacing is the mean spacing between obstacles on the slip plane. If on the other hand the obstacles are weak  $\phi_c$  is very large so this is a difference.

So, we are talking about  $\phi_c$  approaching 0 here but here it is the line is just simply going to move like this. So, there is no I mean significant influence of  $\phi_c$  here  $\phi_c$  almost equal to  $180^\circ$ . So, the angle between this dislocation line and obstacle will be almost  $180^\circ$ . that means, the dislocation line is much straighter for situation of weak obstacles. So, this is what it is penetration means? The dislocation is cutting through, penetrating through. As a corollary the effective obstacle spacing  $L'$  is much greater than  $L$ . So, very important point so here in the previous case when the obstacle is completely resisting the dislocation line to penetrate then the effective spacing is different that is  $L$  here, but on the theory it is not like that the dislocation is simply going to cut through.

For example if you take this particular line and what you are seeing here is one particle citing here, one particle citing here which is measured as the  $L'$  but at the same time you can also see the other particle citing here other particles, so it is an average of these particles distances that is going to be  $L'$ , not just the distance between the two particles. So, there is a difference between hard and weak obstacles are the dislocation motion this is very important.

The stress required to overcome the obstacles depends on the effective spacing  $L'$  between the obstacles along the dislocation line and the angle  $\phi_c$  to which the dislocation bends before it breaks through them, this is important.


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### General Description of Strengthening

- The stress required to bend the dislocation to the angle  $\phi_c$  is calculated in a manner analogous to that used to determine the stress required to operate a Frank Read source.
- An appropriate line tension balance (taking the dislocation line tension as  $Gb/2$ ) at the critical angle shows that the necessary shear stress for continued dislocation motion is
 
$$\tau \equiv \frac{Gb}{L'} \cos \frac{\phi_c}{2}$$
 where  $L'$  is the effective spacing.
- As noted, for strong obstacles  $\phi_c \equiv 0$  and  $L' \equiv L$ . Thus, the maximum strength associated with a given obstacle array is:
 
$$\tau_{max} = \frac{Gb}{L}$$
- With decreasing obstacle strength,  $\phi_c$  increases. An overestimate of the operative stress for strong obstacles is obtained with  $L' = L$ , i.e.,
 
$$\tau(\text{strong obstacles}) \equiv \frac{Gb}{L} \cos \frac{\phi_c}{2}$$
- Friedel has considered the case of weak obstacles and has shown that as  $\phi_c \rightarrow \pi$ ,  $L'$  becomes approximately equal to  $\frac{L}{(\cos(\frac{\phi_c}{2}))^{3/2}}$ . Thus, the approximate strength provided by weak obstacles is given by
 
$$\tau(\text{weak obstacles}) \equiv \frac{Gb}{L} \left( \cos \frac{\phi_c}{2} \right)^{3/2}$$

Mechanical Behaviour of Materials, Thomas H. Courtney, Pearson Press Inc., 2003



So, the stress required to bend the dislocation to the angle  $\phi_c$  is calculated in a manner analogous to that used to determine the stress required to operate a Frank Read Source. You see now we know what is Frank Read source? We have already seen this and how did we calculate the stress required to generate the dislocation multiplication. So, you have to recall that, so we introduced a term called line tension.

So, an appropriate line tension balance taking the dislocation line tension as  $Gb^2 / 2$ , this is what we have seen if you go back and see this is what we have shown already and the critical angle shows that the necessary shear stress for the continuous dislocation motion is  $\tau$  approximately equal to  $Gb$  divided by  $L'$  times  $\cos \phi_c / 2$ , where  $L'$  is the effective spacing. See, what you have to remember now any expression we are showing in this at least these domains they are all semi qualitative.

$$\tau = \frac{Gb}{L'} \cos \frac{\phi_c}{2}$$

It is semi quantitative, not qualitative, it is mostly qualitative but when you try to do a quantitative it is not 100% it is a semi quantitative expressions mostly because the complexity of the problem you are seeing that how complex it is. So, most of the expressions what you are going to see in this particular section is semi qualitative in nature. As noted for strong obstacles  $\phi_c = 0$  and  $L'$  is  $L$  that is the maximum strength associated with the given obstacle array is  $\tau_{\max} = Gb / L$ .

With a decreasing obstacle strength,  $\phi_c$  increases that means the particles become weak this is what you have to understand and overestimate of the operative stress for a strong obstacles is obtained when  $L' = L$ , then  $\tau$  for a strong obstacles is equal to  $Gb / L$  times  $\cos(\phi_c / 2)$ .

$$\tau = \frac{Gb}{L'} \left( \cos \frac{\phi_c}{2} \right)^{\frac{3}{2}}$$

Friedel has considered the case of weak obstacles and has shown that  $\phi_c$  tends to  $\pi$ ,

$$L' = L / ((\cos \phi_c / 2)^{1/2})$$

Thus, the approximate strength provided by the weak up cycle is given by

$$\tau (\text{weak obstacles}) = Gb / L \text{ times } (\cos (\phi_c / 2))^{\frac{3}{2}}.$$

So, these 2 expressions give some idea about the stress required to surpass the obstacle whether it is when it is weak or strong or you can say hard or soft some semi quantitative idea.


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### Work Hardening

- Dislocation interactions during easy glide of single-crystal plasticity provide an example of **soft obstacles**.
- Stage I hardening can be considered to arise from the stress field interaction of dislocations moving on parallel slip planes.
- Dislocation intersections **produce hard obstacles**.
- This is manifested by the high work-hardening rates of polycrystals and single crystals during Stage II deformation.
- These jogs are "hard" obstacles, and they are circumvented by dislocation extrusion between them in a manner analogous to operation of a Frank-Read source.
- The flow stress can be described in terms of the approximate Eq. mentioned above, that is, assuming all dislocations represent obstacles, then, if the average dislocation density is  $\rho$ , the obstacle spacing is obtained using the relationship:  $L^2 \rho = \text{constant}$ . The shear flow stress is then given by
 
$$\tau = \tau_0 + \alpha G b \sqrt{\rho}$$

where  $\tau_0$  is the intrinsic strength of a material having a dislocation density low enough so that dislocation interactions are inconsequential, and the empirical constant  $\alpha$  represents the correction factor necessitated by the approximate applicability of shear stress equation.
- For BCC metals,  $\alpha$  is 0.4 and for FCC metals  $\alpha$  is 0.2.

Mechanical Behaviour of Materials, Thomas H. Courtney, Wiley-Interscience, 2000





So, the next the mechanism which we are going to talk about is work hardening, the term is quite familiar to us now, we have already looked at some of the crucial mechanisms which is involved in plastic deformation of single crystals. So, we are talking about poly crystal now. So, what is work hardening? So, dislocation interactions during easy glide of single crystal plasticity provide an example of soft obstacles.

In a single crystal deformation we talked about quite a lot of dislocation interaction and reactions they are all considered soft obstacles. So, in stage 1 hardening can be considered to raise from the stress field interaction of dislocations moving on parallel slip planes and dislocation intersections produce hard obstacles. So, this also we have seen what kind of intersections will produce hard obstacles the only example is jog, jog vacancies and so on they are all hard obstacles is all we have seen.

This is manifested by the high work-hardening rates of a polycrystals and the single crystal during stage two deformation. So, this also we know as we move from stage one to stage two to stage three, the obstacle generated even within the dislocation reactions, they are quite high. The jogs are hard obstacles and they are circumvented by dislocation extrusion between them in a manner analogous to operation of the Frank Read source.

So, Frank Read source also, how it produces we have seen that the dislocation line is getting extruded between two obstacles and then they joined together and then they form a loop and then continuous something like that. The flow stress can be described in terms of approximate equation mentioned above that is assuming all the dislocation relationship that is  $L^2 \cdot \rho = \text{constant}$



The shear flow stress is then given by

$$\tau = \tau_0 + \alpha G b (\rho)^{1/2}$$

So, this is one important flow stress equation in terms of dislocation density, we know what is  $\tau_0$  is fictional stress we have already seen that. So, where  $\tau_0$  is the intrinsic strength of a material having dislocation density low enough so, that the dislocation interactions are inconsequential and the empirical constant  $\alpha$  represent the correction factor necessitated by the approximate applicability of shear stress equation. For a BCC metal  $\alpha$  is 0.4 and FCC material  $\alpha$  is 0.2.

So, this particular hardening mechanism is purely phased on dislocation, dislocation interactions. Since, we have already discussed quite a bit about the dislocation interactions and also plasticity of single crystal and polycrystal little bit so, the concept here is the same. So, that is why I just want to briefly mention here and this equation gives you an idea how the dislocation density controls the shear flow stress.

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**Boundary Strengthening**

- Grain boundaries are particularly effective obstacles to dislocation motion, a crystallographic factors do not permit the passage of a dislocation from one grain to an adjacent one through a grain boundary.
- The stress concentration due to the dislocation pileup in the active grain increases with the number of dislocations that come up against the impenetrable boundary.

Microyielding in a grain (Grain 1) favorably oriented for slip may precede macroscopic yielding.

- Macroscopic flow requires dislocation activity in all grains (e.g., Grain 2), and this may be induced by the internal stress caused by the dislocation pileup at the boundary in Grain 1.
- This stress may cause dislocation emission from the boundary or may activate a dislocation source (at point r) in Grain 2.
- The magnitude of the stress concentration depends on the number of dislocations in the pileup, and increases with the grain diameter,  $d$ .

Mechanical Behavior of Materials, Thomas N. Courtney, Woodhead Press Inc., 2005.

The next strengthening mechanisms which is important is a boundary strengthening, see we are now talking about obstacles for the dislocation motion. And then we initially we said that the obstacle could be dislocation itself or it could be a second phase particle and then we have also seen that whether second phase particle can be hard or soft. But now, we are talking about a boundary, the boundary is a very strong obstacle for a dislocation motion.

So, that we are going to see that grain boundaries are particularly effective obstacles to dislocation motion crystallographic factors do not permit the passage of a dislocation from

one grain to adjacent one through a grain boundary, it is not easy because you know that in a polycrystalline material each grain is oriented in a particular manner they are not the same and that is why we have the boundary compatibility issue and all we have seen while talking about plastic deformation.

The stress concentration due to the dislocation pileup in the active grain increases with number of dislocation that comes up against the impenetrable boundary. So, let us read this sentence again very important the stress concentration due to dislocation pileup, we know the word dislocation pileup what does it mean? Dislocation get pileup against the one obstacle, we also seen in this term how the dislocation get pileup against the obstacle in the dislocation mechanics. So, this can happen within the grain active grain.

What do you mean by active grain? Active grain means this is the one grain which is favourably oriented that means, this grain has got maximum shear factor and orientations are favourable and the slip activity is going on that information proceeds that is what it means active rate increases with the number of dislocation that come up against the impenetrable boundary. Impenetrable boundary that means, because these next to grain need not be favourably oriented or it may not be experiencing the similar critical resolved shear stress to take over or to continue the plastic deformation which is active in one  $x$  or  $y$  grain. It can simply pass it there is a problem there is a boundary that is why it is called impenetrable that means the dislocations cannot simply pass through it will get stop. So, that is what it is shown in the schematic you can see that the grain 1 with diameter  $d$ .

And imagine that this dislocation pileup is happening in this grain 1 that means the slip systems in this grain are active or multiple slips are happening whatever it is, at least you can see that the dislocation is getting pileup against this boundary. And this is a second grain and then you have some source is there where the dislocation there is a potential for the dislocation multiplication which is there from the boundary with the distance  $r$ .

But then to activate this dislocation multiplication and this stress has to assist the dislocation pileup from here and the boundary stress concentration which is built up here will try to assist that are trying to activate that source so that is something we are trying to see. so, the microyielding in grain 1, what is microyielding that means, though it is a polycrystalline deformation. Since, only selective grains are going to start the plastic deformation because of its orientation and slip factor and so on. So, the yielding start locally here, that is why it is called microyielding. So, the first yielding is going to take place in this grain and the other grain are not going to yield similar extent where it is favourably oriented that is why it is

called microyielding in a grain favourably oriented for slip may proceed macroscopic yielding.

So, this will all the this is just one grain we have shown in a polycrystalline so like that several grains may or may have similar situation, but some other rest on the grains may not have the similar situation to start with. So, the macroscopic flow requires dislocation activity in all grains and this may be induced by the internal stress caused by the dislocation pileup at the boundary in grain 1.


So, now we are to have the continuous you know plastic flow, the cooperative you know displacement of all the boundary is mandatory, that is what we have seen if that is not going to happen, it is going to create a why not overlap and so, on this is we have already seen we have a sufficient background to discuss a kind of a concept. So, this is what is shown here. So, the stress concentration which is generated because of this pileup against the boundary will induce you know the plastic deformation in the other grain like this.

This stress may cause a dislocation emission from the boundary and may activate the dislocation source at a point  $r$  in grain 2. So, this boundary is going to activate this source because of the stress concentration against it. The magnitude of stress concentration depends on the number of dislocations the pileup and increases with the grain diameter  $d$ . So, this is quite obvious. So, the more the grain diameter the more the dislocation will more number of dislocation will join the pileup and the stresses will be more.


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### Boundary Strengthening

- The process can be described semi quantitatively. Suppose  $\tau^*$  is the stress required to activate dislocation motion in the unfavorably oriented grain, and these dislocations are located a distance  $r$  from the boundary.
- The stress concentration due to the dislocation pile-up in the active grain increases with the number of dislocations that come up against the impenetrable boundary
- In turn, this number increases as the mean slip distance (the grain size,  $d$ ) in the active grain increases. Analysis shows that dislocation activation in the second grain occurs when
 
$$(\tau_{app} - \tau_0) \left( \frac{d}{4r} \right)^{1/2} = \tau^*$$
- In Eq. ,  $\tau_{app}$  is the applied shear stress at which this activation occurs and  $\tau_0$  is the intrinsic stress resisting dislocation motion in the deforming grain. The parameter  $\left( \frac{d}{4r} \right)^{1/2}$  represents the stress concentration arising from the pileup; this increases with the number of dislocations in it (i.e., with  $d$ ).
- Rearrangement of above Eq. allows the applied shear stress to be expressed in terms of the grain diameter
 
$$\tau_{app} = \tau_0 + 2\tau^* r^{1/2} d^{-1/2} = \tau_0 + k_y' d^{-1/2}$$
- In terms of tensile yield strength, above Eq.'s analog is
 
$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$
- This equation is known as the *Hall-Petch equation* and its prediction that the yield strength of a polycrystal increases linearly with  $d^{-1/2}$  has been substantiated in several materials.



Physical Behaviour of Materials, Thomas H. Courtney, Wiley Press Inc., 2005



So, this process can be described semi quantitatively so like I said all this mathematical treatment in this mechanisms will be semi quantitative in nature. Suppose,  $\tau^*$  is the stress required to activate dislocation motion in the unfavourably oriented grain and these dislocations are located at a distance  $r$  from the boundary. The stress concentration due to a dislocation pileup in an active grain increases with the number of dislocations that come up against the impenetrable boundary.

In turn, this number increases at the mean slip distance that is the grain size  $d$  in an active grain increases. Analysis shows that the dislocation activation in the second grain occurs when  $\tau_{\text{applied}} - \tau_0$  times  $(d / 4r)^{1/2}$  which is equal to  $\tau^*$ . So, what is  $\tau^*$ ?  $\tau^*$  is stress required to activate dislocation motion in the unfavourably oriented grain that should be equivalent to this. What are these parameters?

In the equation  $\tau_{\text{app}}$  is the applied shear stress at which the activation occurs and  $\tau_0$  is the intrinsic stress assisting dislocation motion in deforming grain the parameter  $(d / 4r)^{1/2}$  represents the stress concentration arising from the pileup very important. So, we have been looking at the shear stress which is required to move the dislocation and obstacles and so on. So, what is the pileup effect? The pileup effect is this let say the parameters  $(d / 4r)^{1/2}$  represents the stress concentration arising from the pileup and this increases with number of dislocation in it with the  $d$ . So, this factor parameter is going to contribute as the grain size is going to increase that is how we should look at it. So, if you rearrange this expression which allows the applied shear stress to be expressed in terms of grain diameter.

$$\text{So, } \tau_{\text{app}} = \tau_0 + 2 \tau^* r^{1/2} d^{-1/2} = \tau_0 + k_y' d^{-1/2}$$

If you write in terms of tensile yield strength above equation analog is  $\tau_{\text{app}}$  becomes  $\sigma_y$  and  $\tau_0$  become  $\sigma_0$ ,  $k_y'$  becomes  $k_y$  and  $d^{-1/2}$  remains. So, what is this equation this equation is quite popular.

This equation is known as Hall-Petch equation and its prediction that the yield strength of a polycrystal increases linearly with the  $d^{-1/2}$  has been substantiated in several materials. So, in the boundary strengthening mechanisms Hall-Petch equation clearly demonstrated that the yield strength can have profound influence on the grain sizes. So, as the grain size decreases yield strength will be increased that is what this equation shows. We will come back to this equation in a much later stage.

When we talk about deformation of polycrystals are when you do mechanical testing what happens we will have few more parameters you can modify this and then we can discuss about what is the significance of  $k$ ? And what is the significance of  $d$  and so on. And you can just imagine in this equation suppose if  $d$  is 0 what happens it becomes  $\sigma_y = \sigma_0$  that means they become single crystal deformation but the moment it becomes you are bringing in a boundary then you have the influencing parameter called  $k$  here.

That will give the much more narrative about how the grain size influences which is characteristic of individual systems.