Mechanical Behaviour of Materials
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Lecture - 23
Introduction to Plastic Deformation – II

Hello I am Professor S. Sankaran in the Department of Metallurgical and Materials Engineering.

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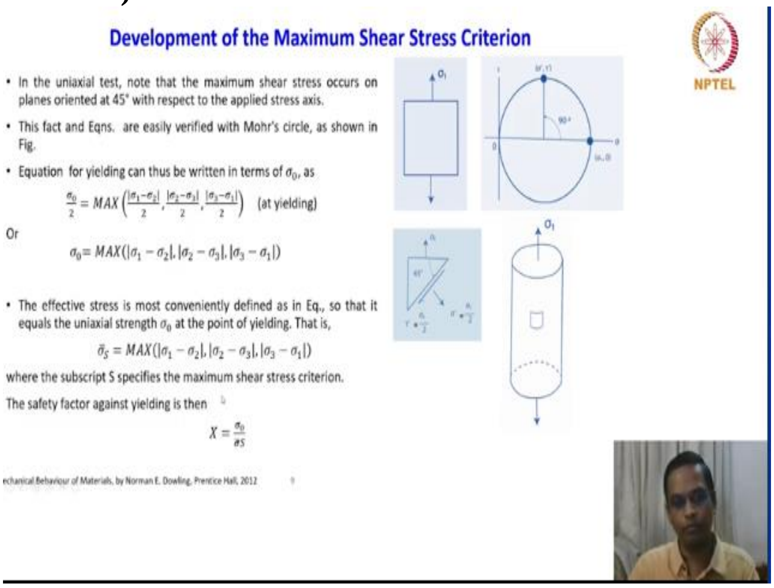

So, how to develop this criterion in the uniaxial tests note that the maximum shear stress occurs on the planes oriented at 45 degree with respect to applied stress. This fact and equation are easily verified with Mohr's circle as shown in figure, so this is just to reinforce our understanding not that anything we are finding it new here. Just plot this ideas into a Mohr's circle and we will get to know see the visual experience of the maximum shear stress criterion.

So, what is that we are seeing we are now considering the uniaxial deformation tensile deformation whether it could be a cylindrical member or it is a square member? And we are saying that the maximum shear stress occurs on the planes oriented 45 degrees. So, this is the squared member, so according to this geometry this is the plane which you are talking about which is 45 degrees. So, on this plane the shear stresses τ' is σ_1 /2 which is nothing but τ_0 = σ_1 /2. So, same thing is plotted here.

So, since it is a uniaxial tension, so you have only σ_1 others are 0 and you remember that this is maximum shear and in the Mohr circle we always plot 2α if you recall those concepts. So, it is exactly 45 degree 2α is 45 degree so, you have the 2α is 90. So, the maximum shear stress τ is here and then this is the MAX I mean the principal σ_1 so, this is very nicely we can represent a lot more so that is what I am showing.

So, the equation for yielding can thus be written in terms of σ_0 as $\sigma_0/2$ is equal to MAX times τ_1, τ_2, and τ_3, or you can rewrite in this form. The effective stress is most conveniently defined in these equations. So, that it equals the uniaxial strength σ_0 at a point of yielding that is so we are interested in relative in this into simple yield strength in a uniaxial tension. So, it can be written like this $\bar{\sigma_S}$= MAX times σ τ_1, τ_2, τ_3.

But the subscript S specifies the maximum shear stress criterion. Similarly, similar to what we have seen in the normal fracture criterion here also the safety factor against the yielding is then X = $\sigma_0/\bar{\sigma_S}$ what it is a subscript there is a type error here $\bar{\sigma_S}$, so this is a safety.

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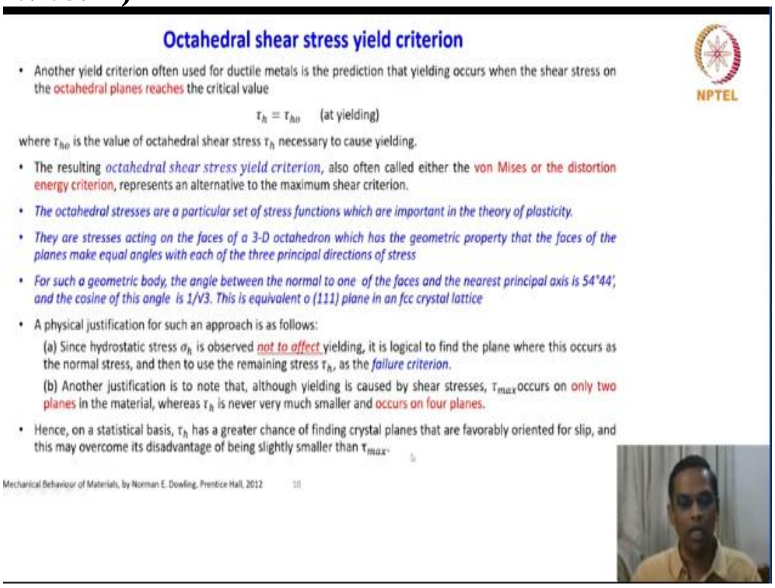

So, we will now move on to another criterion called octahedral shear stress yield criterion this is again for ductile materials only similar to a Tresca yield criterion like I mentioned in the beginning there are different yield criterion are proposed several depending upon the materials and the state of stress and so on. We are looking at some of the important one and the same time simple ones which can be related to simple this criterion can be related to the parameter which you can normally obtain by the simple uniaxial tension test that is the idea.

So, the other criterion shear stress yield criterion for ductile material is octahedral shear stress yield criterion. So, what does it say? Another yield criterion often used for ductile metals is the prediction that yielding occurs when the shear stress on the octahedral planes reaches that critical value $\tau_h = \tau_{h0}$ at yielding. So, now we have to understand what is this octahedral plane you know our crystal system we will see it in a minute where $\tau_{h0}$is the value of octahedral shear stress τ_h necessary to cause yielding.

The resulting octahedral shear stress it criterion also often called either the von Mises or the distortion energy criterion, represents an alternative to the maximum shear stress criteria. So, this is another way of predicting the yield criterion similar to maximum shear stress criterion or Tresca yield criterion. So, what are this octahedral stresses? The octahedral stresses are a particular set of stress functions which are important in the theory of plasticity.

They are stresses acting on the faces of a 3D octahedron which has a geometry property that the faces of the planes make an equal angle with each of the three principal direction of the stress. So, if you assume that in a cube when we define state of stress and cube and then we looked at all three principal directions. So, if you recall that cube describing the stress then you can imagine this.

For such a geometric body and angle between the normal to one of the faces and that nearest principal axis is 54 ° 44 ‘ and the cosine of this angle is $1/\sqrt{3}$ this is equivalent to (111) plane in an fcc crystal lattice. It is simple cube the (111) plane which is nothing but an octahedral plane. So, that is what is talked about here. A physical justification for such an approach is follows, what is the physical justification to look at the octahedral stresses in an octahedral plane what is the physical justification.

Since the hydrostatic stress σ_h observed not to affect yielding, it is logical to find the plane where this occurs as a normal stress and then to use the remaining stress τ_h as the failure criterion. So, what does it say? We are now saying that the hydrostatic stress is not going to contribute to the yielding. So, in a total stress; so, if you recall that we have looked at one particular lectures exclusively, how to separate this hydrostatic component of a stress under stress deviate.

So, this here exactly we are using that stress deviate we are talking about nothing but the stress deviator here, which is very useful now. So, another justification is to note that, although yielding is caused by the shear stresses τ_{MAX} occurs on only two planes in the material. Whereas, τ_h is never very much smaller and occurs on four planes very important you have to remember although yielding is caused by shear stresses τ_{MAX} occurs only two planes in the material.

We have two planes if you recall that image what we have just discussed in the previous slides, where τ_h is never very much smaller and occurs on four other planes. So, τ_{MAX} are two planes but τ_h is also a occurring other four planes. So, totally we have seen six planes. So that is what we are talking about, so they are not very much smaller so we have to look at those. Hence, on a statistical basis τ_h has a greater chance of finding a crystal planes that are favourably oriented for a slip.

And this may overcome its disadvantage of being slightly smaller than τ_{MAX}. So, the shear stress which is going to cause the slip also depending upon several other factors like orientation of the crystal with respect to access of loading and so on. We will see those things when we look at I mean plastic deformation much more detail and why we take shear stress for a criterion to predict the yielding this is one of the reason.

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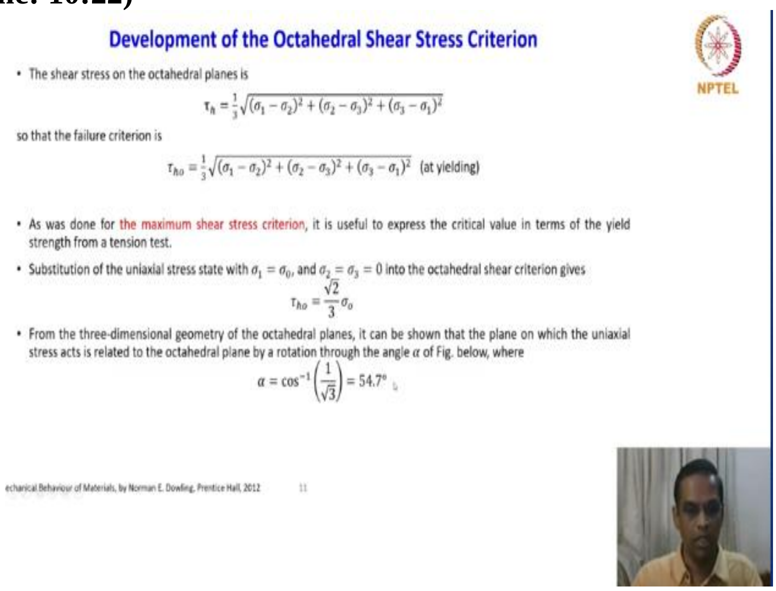

So, the shear stress on octahedral planes is

$$\tau_h = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

So that the failure criterion is

$$\tau_{h0} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$ at yielding.

As was done for the maximum shear stress criterion it is useful to express the critical value in terms of yield strength from a tensile test.

Substitution of the uniaxial stress state with $\sigma_1 = \sigma_0$ and $\sigma_2 = \sigma_3 = 0$ into the octahedral shears criterion gives

$$\tau_{h0} = \frac{\sqrt{2}}{3}\sigma_0$$

so we got now an expression for the octahedral shear stress. From this three dimensional geometry of the octahedral planes it can be shown that the plane on which the uniaxial stress acts is related to a octahedral plane by a rotation through an angle α of the figure what we have seen here below

$$\alpha = cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

which is nothing but 54.7 degree. So, this is an angle which it is oriented with respect to tensile axis.

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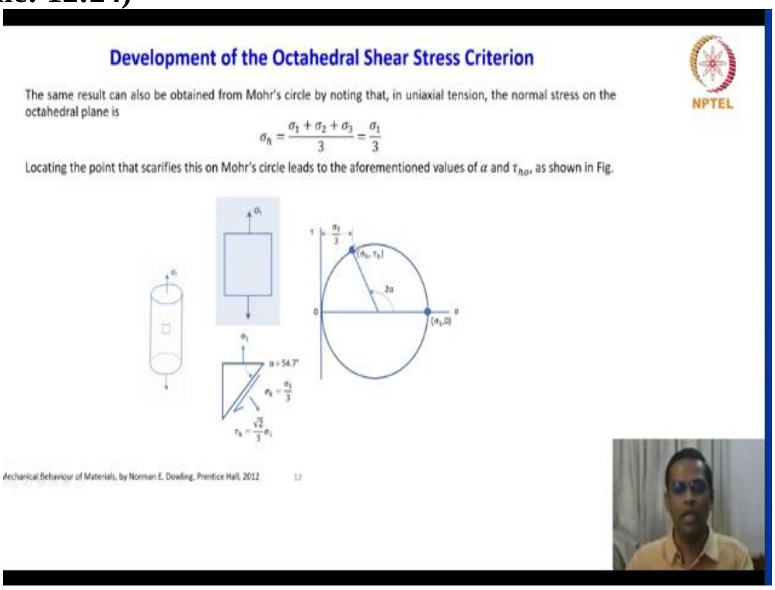

So, again we will plug in all this values into the Mohr's circle the same result can also be obtained from Mohr's circle by noting that in uniaxial tension the normal stress on the octahedral planes is nothing but $\sigma_1/3$. Locating the point that sacrifices are Mohr's circle leads to the aforementioned the values of α and τ_{h0} how do we do that. So, this is again uniaxial tension this is a cylindrical member we are looking at $\alpha = 54.7$ that is the orientation of this plane with respect to σ_1.

And $\sigma_h = \sigma_1/3$ and $\tau_h = \sqrt{2}/3$ times σ_1 this is what the field criterion shows. And if you look at the Mohr's circle what is shown here so, this is uniaxial tension. So, it is σ_1 here and this is 2α that means, it is about 108 something it is 2 times the 54.7, so that is shown here. So, this is where the maximum shear stress will lay here and that is if you look at the corresponding σ_1 it is $\sigma_1/3$ and this is the τ_{MAX} here or the value of τ_h is this value and this is the $\sigma_1/3$.

So, this is now very easy, when you I mean this Mohr's circle concept is comes very handy to visualize the maximum stress normal stress versus a shear stress values. So, this is for the octahedral shear stress criteria.

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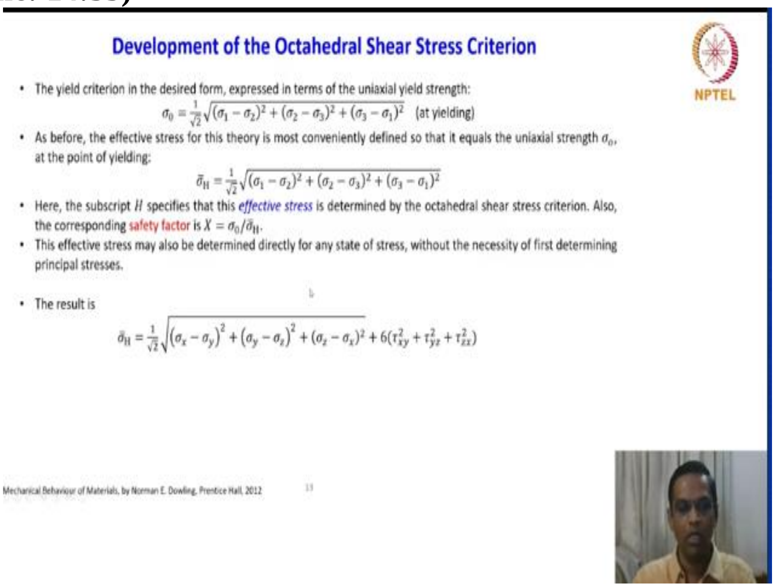

So, the yield criterion in the desired form expressed in terms of the uniaxial yield strength this

$$\sigma_0 = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

at yielding. We are interested in to express this criterion in terms of yield, that is why you are writing in here again we will now arrive at a safety factor.

So, we are interested in the effective stress for this theory is most conveniently defined so that it equals the uniaxial strength σ_0 at the point of yielding, So, $\overline{\sigma_H}$ times this same expression here and the subscript H specifies that this effective stress is determined by the octahedral shear stress criterion also the corresponding safety factor X = $\sigma_0/\overline{\sigma_H}$ so, similar to the previous cases.

This effective stress may also be determined directly for any state of stress without the necessity of first determining the principal stresses, so the result is

$$\overline{\sigma_H} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

So, what is this expression?

If you recall when we opened up the strain matrix we looked at all the invariants no stress invariants and strain invariants if you recall the our world variation if we go back to the our old derivation j_1 j_2 j_3 these are the three invariant things we derive if you look at the j_2. So, this expression is nothing but the j_2 invariant. So, instead of doing this we can also can determine this $\overline{\sigma_H}$ by directly substituting into this equation that is why this statement is given.

This effective stress may also be determined directly for any state of stress using this expression. So, just given an idea know why we looked at the earlier expressions and how it is useful here now to predict the yield criteria that is why this shown here.

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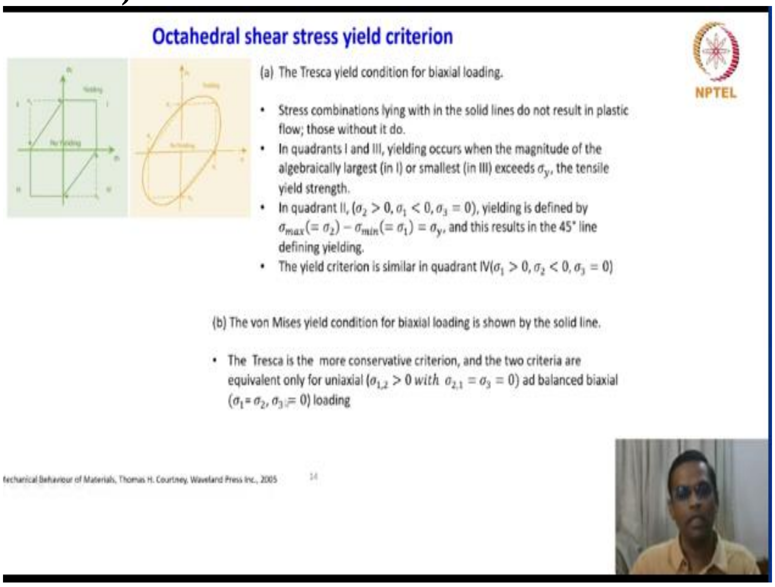

Now come to be the most important idea we are now created the surfaces the yield surfaces or the fracture surfaces here it is yield surfaces for biaxial loading, this is a shear stress yield criterion and which shows that four quadrants I II III IV and then you have σ_1 and σ_2 and the boundary demarks are no yielding and then yielding. Stress combinations lying within the solid lines do not result in plastic flow those without it do. So, that means, within the solid line the combination of stresses σ_1 and σ_2 will not cause adding as per the Tresca yield criterion.

In quadrants I and III yielding occurs when the magnitude of the algebraically largest in I and smallest in III exceeds σ_y the tensile yield strength, so this is quite obvious from looking at this. In quadrant 2 on the other hand, $\sigma_2 > 0$ and $\sigma_1 < 0$ and $\sigma_3 = 0$ yielding is defined by σ_{max} that is nothing but $\sigma_2 - \sigma_{min}$ that is $\sigma_1 = \sigma_y$. So, $\sigma_{max} - \sigma_{min}$ is equal to yield strength. And this results in the 45 degree line defining the yielding.

So, this is what it is shown here 45 degree line here and in the yield criterion is similar important for except that it is just reversed here. So, $\sigma_1 > 0$ and $\sigma_2 < 0$ and $\sigma_3 = 0$ so that is shown here. So, what is this image now shows it is showing the same in criterion here, but also it plotted the von Mises yield criterion, which is a solid line this is again the von Mises yield condition for biaxial loading is shown by the solid line.

What you can see is the Tresca is the more conservative criterion and the two criteria are equal and only for uniaxial that is σ_1 or $\sigma_2 > 0$ with σ_2 and $\sigma_1 = \sigma_3 = 0$ and balanced biaxial $\sigma_1 = \sigma_2$ and $\sigma_3 = 0$ loading. So, only it is uniaxial and balanced biaxial loading these two criterion are the equivalent otherwise the one means is a criterion shows slight deviation when you move away from the principal axis.

So, this particular image shows the experimental findings you can see that it is quite interesting these are the experimental data obtained for steel, copper and nickel and then you see that most of the data points lies just close to von Mises yield criterion, but not very different from Tresca but then it is close to von Mises yield criterion for biaxial loading.

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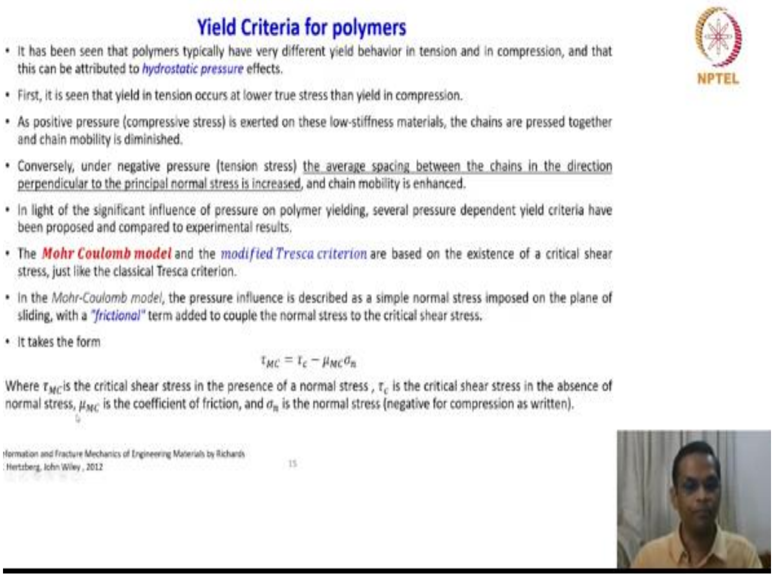

Now, we have to turn our attention to the yield criteria for other than crystal line materials or semi crystalline materials, quiet polymers. So, for all this yield criterion represented the failures of yielding of crystalline material or ductile materials or even a brittle materials where we looked at the maximum normal principle stress or fracture stress criterion. So, those things those criteria will not work for polymers or semi crystalline or amorphous material.

So, we look at it what are the changes required to predict the yield criteria for polymers. So, it has been seen that polymers typically have very different yield behaviour intention and in compression and this can be attributed to hydrostatic pressure effects. So, if you recall when we looked at the elastic properties in the earlier lectures, we have clearly shown how this you know semi crystalline materials or you know long chain molecular materials how they behave in tension as well as compression.

Very nice illustration we had also discussion about how this compression behaviour is entirely different from the tensile behaviour when it comes to deformation of poly I mean long chain molecular materials are mainly polymers. So, that is exactly is brought here so the hydrostatic pressure effects has to be taken into out. First it is seen that yield in tension occurs at lower true than that yield in compression, this also been our seemed.

The slope of the stress strain curve even in elastic region in tension and compression for this long molecule chain molecular materials, they are not same they are very different. Since we have already seen this, I am just going little fast that is the purpose we have different some time on the fundamentals. So, you can easily recollect and even if you have doubts, you can go back to those sections and then read them then it will be easily.

As positive pressure that is compressive stresses exerted on these low stiffness material the chains are pressed together and the chain mobility is diminished. Conversely, under the negative pressure, that is a tension stress, the average spacing between the change in the direction perpendicular to the principal normal stresses increased on the chain mobility enhanced, this point also we have already seen.

So, in the light of the significant influence of pressure on polymer yielding several pressure dependent yield criteria have been proposed and compared to the experimental results. So, one thing is very clear, we are going to look at the yield criteria almost similar time, but the one parameter which is very new here is the pressure. So, the Mohr Coulomb model which is going to describe this and the modified Tresca criterion so, are the based on the existence of a critical shear stress just like the classical Tresca criterion.

We are going to use the same criteria like what we have seen for no tactile materials, we will simply use the pressure effect on them and which is called Mohr Coulomb model. In Mohr Coulomb model the pressure influences described as a simple normal stress imposed on the plane of sliding with the frictional term added to couple the normal stress to that critical shear stress very important. So, this model brings the pressure influence where the normal stress is imposed on a plane of sliding.

So, we are talking about now molecular chains which are sliding it is sliding of this molecule chain with a fictional term added to couple the normal stress just to the critical shear stress. So, it takes the form τ_{MC}, MC is Mohr Coulomb model is equal to τ_C critical shear stress minus μ_{MC} σ_m

$$\tau_{MC} = \tau_{C-} \mu_{MC}\sigma_n$$

where μ_{MC} is the critical shear stress in the presence of normal stress and τ_C is the critical shear stress in the absence of normal stress, μ_{MC} is the coefficient of friction and σ_n is the normal stress negative for compression as written.

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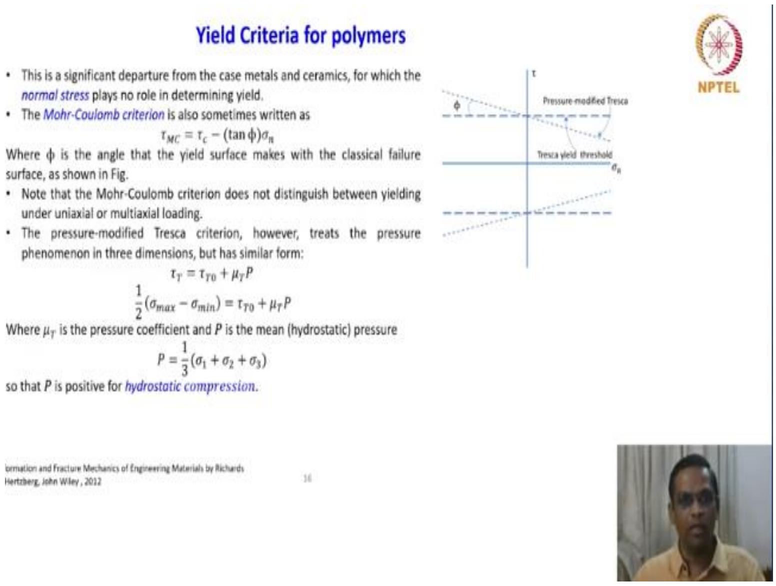

So, this is a significant departure from the case of metals and ceramics for which the normal stress plays no role in determining the yield so very important idea, where we are bringing a normal stress into any especially a stress yield criteria. So, what is that we have seen so far interest gap. So, this is what this part shows clearly this is a normal Tresca yield threshold is shown in a dotted line here a thick dotted line and this is τ and this is σ_n.

And we have now put another slanting line which is having an angle with respect to this horizontal line and that we call it as pressure modified Tresca. So, the Mohr Coulomb criterion is also sometimes written as

$$\tau_{MC} = \tau_{C-} (\tan \emptyset)\sigma_n$$

where Φ is the angle that the yield surface makes with the classical failure surface as shown in the figure. Note that the Mohr Coulomb criterion does not distinguish between yielding under the uniaxial or multiaxial loading this is one limitation.

The pressure modified Tresca criterion however, treats pressure phenomenon in three dimensions, but it has the similar form. So,

$$\tau_T = \tau_{T0} + \mu_T P$$

which is also written as

$$\frac{1}{2}(\sigma_{max} - \sigma_{min}) = \tau_{T0} + \mu_T P$$

where μ_T is the pressure coefficient and P is the mean hydrostatic pressure that is

$$P = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

So, that P is positive for hydrostatic compression.

So, what do you know c is here we using the similar Tresca criterion, where the pressure effect is very nicely brought in to accommodate the polymeric or long chain molecular behaviour under compression.

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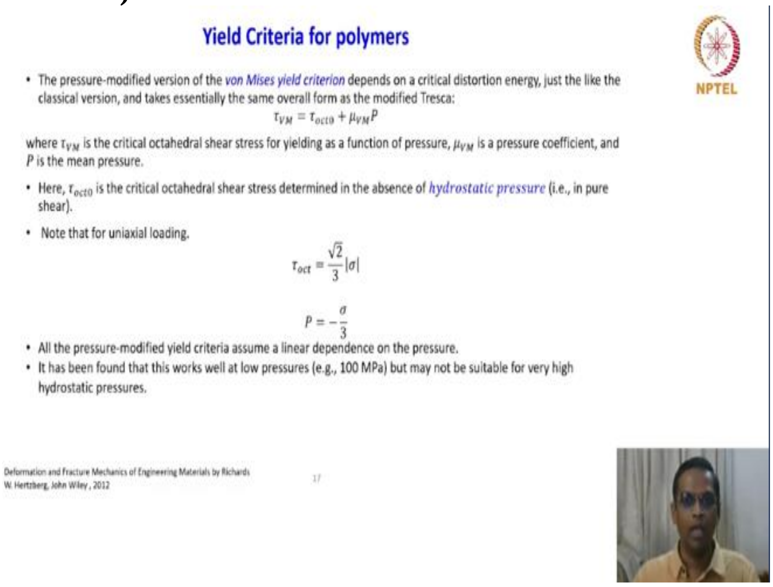

The pressure modified version of von Mises yield criterion depends upon the critical distortion energy, just like the classical version and takes essentially the same overall form as the modified Tresca, which is $\tau_{VM} = \tau_{oct0} + \mu_{VM}P$ so, this is von Mises yield criterion. So, τ_{VM} is the critical octahedral shear stress for anything as a function of pressure, μ_{VM} is the pressure coefficient and P is the mean pressure. Here τ_{oct0} is the critical octahedral shear stress determined in the absence of hydrostatic pressure.

Note that for uniaxial loading

$$\tau_{oct} = \frac{\sqrt{2}}{3}|\sigma|$$

where $P = -\sigma / 3$. All the pressure modified yield criteria assume a linear dependence on the pressure this is one thing we have to note. It has been found that this works well at low pressures that is 100 MPa, but may not be suitable for very high hydrostatic pressures. So, for that some other empirical models have to be looked at. So, with that I would like to stop here, we will continue our lectures in the next class.