Mechanical Behavior of Materials

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Lecture - 22
Introduction to Plastic Deformation - I

Hello I am Professor S. Sankaran in the Department of Metallurgical and Materials Engineering. Hello everyone welcome to this NPTEL course of mechanical behaviour of materials. Today we will move on to a new topic called plastic deformation. If you recall what all we have seen so far you will agree with me that we dealt with subjects, which are mostly described by theory of elasticity if you look at from the fundamental perspectives as well as the core theory.

And description of elastic properties and elastic stress strain relations and then the description of dislocations, energy stress fields and everything so on. So, we extensively used elasticity so now, we will move on to plastic deformation. So that means we need to get into the theory of plasticity we will have some introductory aspects of theory of plasticity we will not get into the core idea because our intention here is to just to get the concept in describing the yielding phenomenon in a polycrystalline materials.

And of course we will see how we can extend this idea of the yield phenomenon or a yield criteria or theory of yielding to different kinds of materials. So that is our intention to get into this theory of plasticity.

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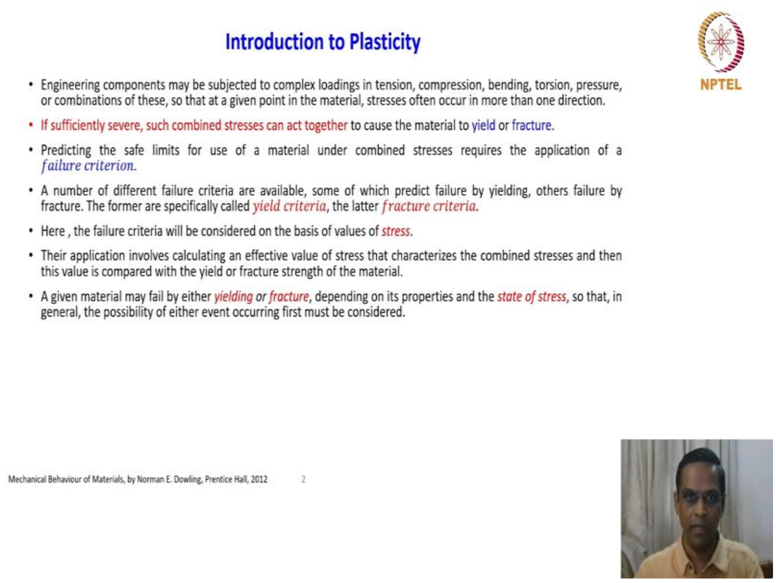

So, if you look at this, the kind of situations we have in engineering environment the engineering components may be subjected to complex loading intention, compression, bending, torsion, pressure or combinations of these. So, you all will agree that this is the case in reality most of the components especially in engineering application, structures they are all subjected to different complex loading.

It could be combinations of tension, tension or tension compression or bending, torsion simply pressures or any one of the combinations of these two. So that the given point in the material stresses often occur in more than one direction. You see we have now looked at some of the very basic definition of stress at a point stress in a direction like that and we also looked at the three dimensional state of stress and you know biaxial stress, triaxial stress and so on.

But in reality we exactly, seeing the similar complex combinations of stresses in a day to day life. So, if sufficiently severe such combined stresses can act together to cause the material to yield or fracture. So, such a complex type of stresses or loading can lead to either the material will yield or fracture. So, we are introducing two words here yield and fracture. Predicting the safe limits for use of material under combined stresses requires the application of failure criterion.

So, everyone offers want to avoid failure so, in an engineering situation this is very, very important in terms of you know safety and economy and so on. So, you need to know the failure criterion on what basis we will you know apply load on the structural members. So, there should be some guidelines otherwise there could be some failures I mean unexpected failures, sudden failures and so on.

So, predicting the safe limits is a primary concern here that is what we are going to look at it a number of different failure criteria are available some of which predict failure by yielding others failure by fracture. So, we are talking about two different things either your material can yield and or fracture. So, yield is better than fracture because you know fracture is complete total failure yield is a partial failure that is kind of a warning yielding is kind of a warning signal.

So, the former are specifically called yield criteria the latter is fracture criteria. So, we are looking at two different criterions here yield criterion and fracture criterion. Here the failure criteria will be considered on the basis of values of stress. So, we know what is stress; now and we have sufficient background to understand the word stress. So, this failure criterion will be given in terms of stress.

We are not talking about a load here we are talking about a stress so please pay attention to this rewrites their application involves calculating an effective value of stress that characterizes the combined stresses and then this value is compared with the yield or fracture strength of the material. So, what does it mean is you try to come to a safe limit of stresses by make some prediction or calculation.

And then compare that value with the what is that experimental value reported or determined in our end then these two values be compared and then you can take a decision, a given material may fail either by yielding a fracture depending upon its properties and the state of stress. So that in general the possibility of either even occurring first must be considered. So, it is not that we want to reach the fracture of the material.

It can be the prediction can just lead to yield point or fracture point. So, yield and fracture need not be this final goal because it depends primarily on the material property not necessarily all the material will exhibit yield. Only ductile materials will exhibit yield not the brittle materials will exhibit sufficient yielding.

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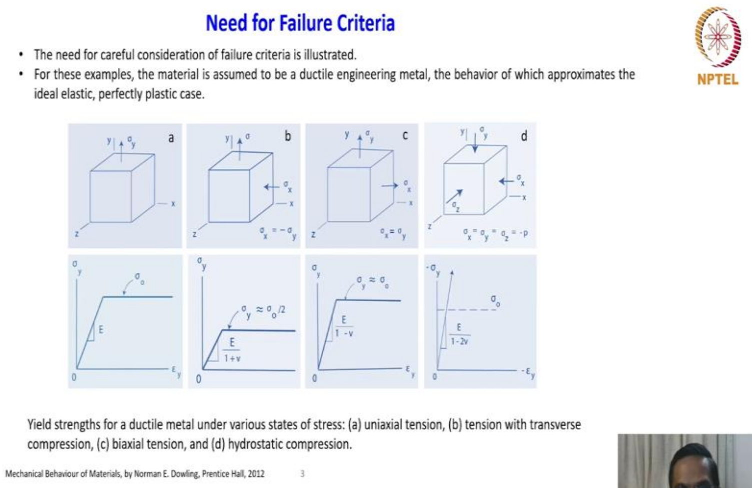

So, we will now look at the yield criteria. The need for a failure criteria why do we need this criteria? So, you look at this schematic shown here what is shown here is a cube which is subjected to uniaxial tension and this is the stress strain curve for this. So, the assumption here is the material is assumed to be ductile engineering material the behaviour of which approximates the ideal elastic perfectly plastic case.

So, just for our own consideration a ductile material which exhibits ideal elastic that means it is just elastic behavior and then perfect plastic please understand this, this is for our own understand just for assumption in reality there is nothing like this you all know that. But what is that we are seeing now we are seeing that elastic and then plastic that means we are getting some value here σ_0 that is a yield stress.

Now in the same geometry we will subject them to different type of loading here. That is what is shown here suppose if the load is not only applied in unidirectional but one direction of the stress is reverse or a transverse stress what happens to be σ_0 that is the idea. So, what is that we are seeing here? It is quite interesting the moment you change the state of stress you see that accordingly the Young’s model is also getting modified this we know this we have background to understand this.

But what happens to the σ_0, σ_0 becomes that is σ_y is approximately becomes half this is very important to note. So, when the σ_x becomes -y then the yield stress is becoming nearly half of the value as compared to the uniaxial tensile force. The third one what you are seeing here is it is a biaxial tension what you are seeing is biaxial tension $\sigma_x = \sigma_y$.

And what is that we are seeing here surprisingly there is no change in the yield stress σ_y is approximately equal to σ_0. So, looking at the previous experience one would have expected some change whether it could increase the strength or decrease the strength but nothing has happened fortunately. So, this is quite surprising and then finally what will happen if you subject this component to a hydrostatic stress.

That means you know the meaning of hydrostatic stress now that means σ_x, σ_y and σ_z will be of equal magnitude and in compression that is why it is -p. So, what happens quite interestingly it is very linear behaviour the material does not yield one important point we have to remember here is in all these a, b, c can be experimentally verified by some means we have a means of doing this.

But we cannot create a hydrostatic pressure and then I mean in simple experiments we cannot realize this. So, nevertheless this is what it is shown here there is nothing happens to this material. You recall that you know in previous elasticity theory also we have seen that hydrostatic stress does not contribute to plastic deformation that also you should remember. So, what we have seen here is the yield strength for a ductile metal under various state of stresses namely uniaxial tension, tension with a transverse compression biaxial tension.

And hydrostatic compression you see that different, different type of state of stresses a type of stresses the material exhibits a different behaviour in terms of yielding and now you see the question what is the need for failure criteria? So, the answer comes from these experiments. So, unless we have this number that failure criterion not namely yield or fracture criterion we do not know how this material is going to respond when the stress is raised to the higher level. So that is the core idea of looking at the failure criteria or the need for the failure criteria.

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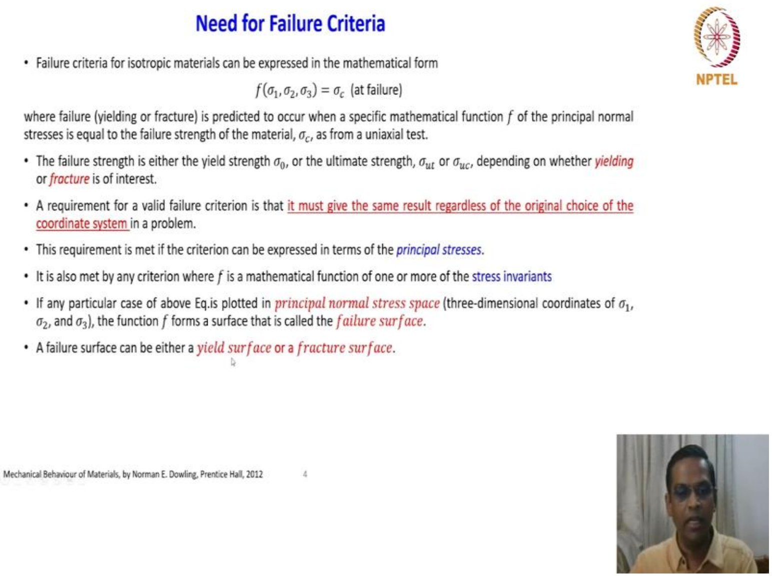

So, now we look at how do we go about this, failure criteria for isotropic material can be expressed in the mathematical form that is a $f(\sigma_1, \sigma_2, \sigma_3) = \sigma_c$ at failure. So, what is σ_1, σ_2, σ_3, you all know principal stresses. So, the principal stresses a function which is you know depends on this principal stresses should predict some critical stress σ_c.

In terms of we are interested in calculating the critical stress in terms of principal stresses which we already know. So that is the idea so where failure yielding our fracture is predicted to occur when a specific mathematical function f of the principal normal stress is equal to failure strength of the material σ_c as from the uniaxial test this is very important. So, you know what is critical stress?

That is not important whether you can relate that quantity with a common test what is the common test? We do a simple uniaxial test whether we can find out or the critical stress whatever the yield theory is predicting whether we can relate that quantity with the what we already or we are able to do it with the uniaxial tension test that is the question the failure strength is either the yield strength σ_0 or the ultimate strength σ_{ut} or σ_{uc} depending upon whether the yielding or fracture is of interest.

So, here depending upon the interest we can relate not just to yield strength that is the another term now ultimate strength this also we will see when we look at the mechanical testing syllabus but this is another term which normally one can estimate in the simple tensile test and this is σ_{uc} is the fracture at the time of fracture what is the stress? A requirement for a valid failure criterion is that it must give same result regardless of the original choice of the coordinate system no problem this is important.

This requirement is met if the criterion can be expressed in terms of principal stresses. So, only when you express the failure criterion or yield criterion in terms of principal stresses then you do not have to worry about whether it is represented in the different, different coordinate forms. It is also met by any criterion where f is a mathematical function of one or more of the stress invariants.

If any particular case of the above equation that is the f = σ_c is plotted in a principal normal stress space that means the it is in a three dimensional coordinates of σ_1, σ_2, σ_3 the function forms a surface that is called failure surface. So, if you have this values σ_1, σ_2, σ_3 that means a mathematical function predicts the critical in all these three stresses. Then we can construct the surface whether it is safe or whether our data points or the stress values fall inside the surface or outside the surface that is the idea your failure surface can be either a yield surface or a fracture surface.

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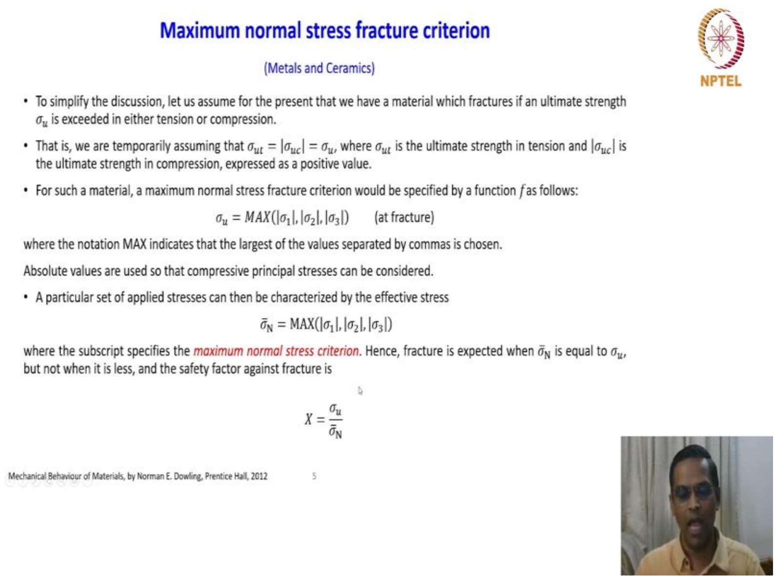

So, we now look at something called maximum normal stress fracture criterion which is mostly applicable to metals and ceramics. So, to simplify the discussion let us assume for the present that we have a material which fractures if an ultimate strength sigma u is exceeded in either tension or compression that is we are temporarily assuming that $\sigma_{ut} = |\sigma_{uc}| = \sigma_u$ where σ_{ut} is the ultimate strength intention.

And $|\sigma_{uc}|$ is the ultimate strength in compression expressed as a positive value. So, here the criterion that is the maximum normal stress fracture criterion assumes and predicts that this critical stress is ultimate strength in tension or compression. So, you should not reach uc or ut otherwise the material will fail. So that is the criteria for such a material the maximum normal stress factor criterion would be specified by a function f as follows.

$\sigma_u = MAX(|\sigma_1|, |\sigma_2|, |\sigma_3|)$ at fracture so either one of them or it is a combination of them where the notation MAX indicates that the largest of the values separated by commas is chosen any one of them also could be chosen absolute values are used so that compressive principles stresses can be considered. Please understand that we are now talking about principal normal stress.

You know now the meaning of principal normal stress we are not talking about shear stress here. A particular set of applied stresses can be characterized by the effective stress. So, what is effective stress? We are now talking about $\bar{\sigma}_N = MAX(|\sigma_1|, |\sigma_2|, |\sigma_3|)$ anyone have this where the subscript to specify the maximum normal stress criteria.

So, capital N denotes a normal stress criterion. Hence the fracture is expected when $\bar{\sigma}_N = \sigma_u$ but not when it is less and the safety factor against the fracture

$$X = \frac{\sigma_u}{\bar{\sigma}_N}$$

So, suppose if the safety factor is known like this then we have a safe limit by which you can allow the material to take the load or a stress so that is the idea before it fractures.

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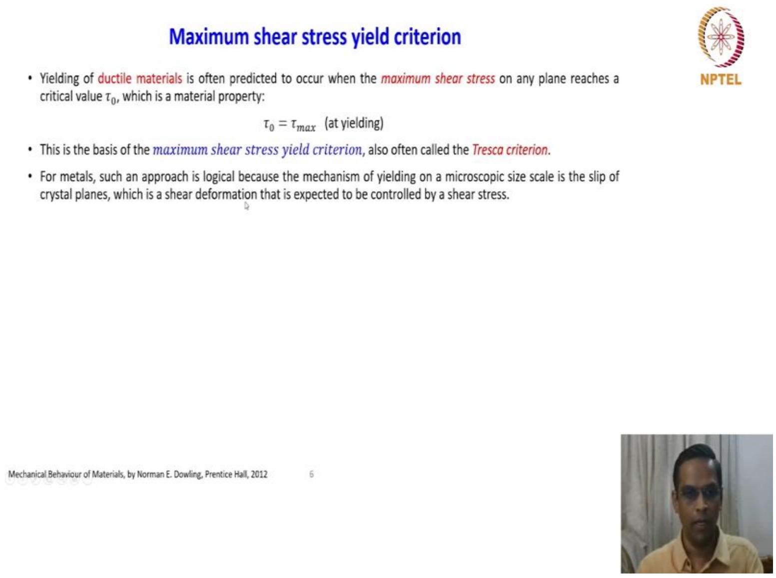

So, then the next point is maximum shear stress yield criterion we have now seen maximum normal stress fracture criterion. So that name itself tells us that we are now predicting the fracture that means the material is not supposed to yield or the other way is the material it is a criterion for brittle materials for example ceramics that is why I said very hard metals and brittle materials like ceramics and composites the maximum normal principal stress fracture criterion will be valid.

But now we are talking about maximum shear yield criterion so the name itself suggests that yielding of ductile materials often predicted to occur when the maximum shear stress on any plane reaches that critical value τ_0 which is a material property so $\tau_0 = \tau_{max}$ at yielding. So, for a ductile material we are now talking about a shear stress yield criterion. And what we are talking here the maximum shear stress on any plane reaches a critical value τ_0.

So, now you should recall we looked at the shear stress I mean we resolve the total stress into normal stress in shear stress in the theory of elasticity lectures and then we also looked at these shear stresses are maximum only at certain planes that planes also we have shown how it bisects the normal principal axis stress I mean the planes or the axis and so on. I will bring back that slide to recall those concepts what we have seen.

But before that let us see what this criterion says this is the basis of the maximum shear stress yield criterion also often called the Tresca criterion. For metals such an approach is logical because the mechanism of a yielding on a microscopic size scale is the slip of crystal planes which is a shear deformation that is expected to be controlled by your shear stress. So, we are repeating the same concept that plastic deformation is caused by a shear stress that we have already shown. And this is exactly every you know crystal planes will experience the shear deformations when they are subjected to tensile loading or compression loading etcetera.

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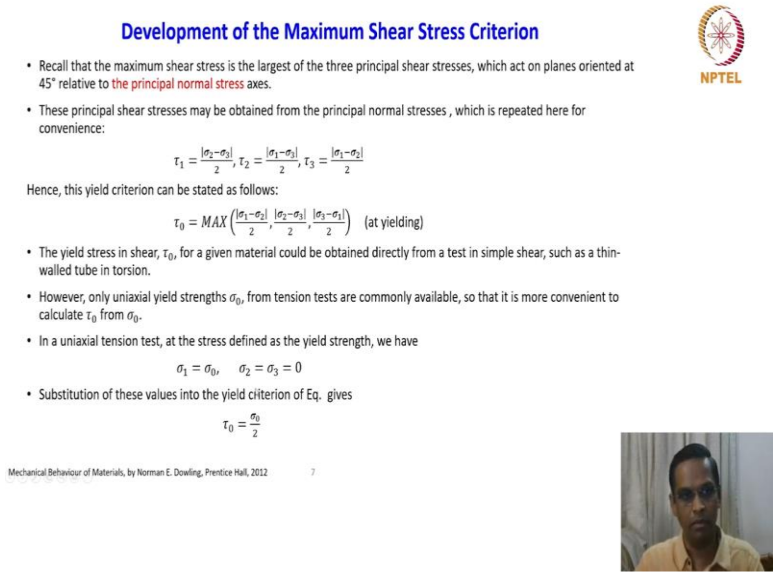

So, recall that the maximum shear stress is the largest of the three principal shear stresses which act on planes oriented at 45 degree relate to principal normal stress axes this is what just I mentioned. So, we know the three principal shear stresses τ_1, τ_2, τ_3 we looked at it so that is what it is these principles shear stresses may be considered which are obtained from principal normal stresses which is repeated here.

For the convenience

$$\tau_1 = \frac{|\sigma_2 - \sigma_3|}{2}, \tau_2 = \frac{|\sigma_3 - \sigma_1|}{2}, \tau_3 = \frac{|\sigma_1 - \sigma_2|}{2}$$

So, now you should recall out of these three which one is the maximum that also you know we have seen and how it can be resolved from the principle normal stresses that also we will recollect all of them as we proceed. Hence this yield criterion can be stated as follows. So, now talking about yield criterion so τ_0 we have to define

$$\tau_0 = MAX\left(\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2}\right)$$

Like what we have seen in the normal principal facture criterion and here we are putting the τ_1, τ_2, τ_3 at yielding whenever this τ_0 reaches or reaches a maximum of any one of these shear stresses the material will yield that is the criterion the yield stresses shear τ_0 for a given material could be obtained directly from a test in simple shear such as a thin wall tube in torsion either we can obtain this quantity by a simple torsion test or even a simple tensile test.

We can take we can do it because we have the relation we will see what is that however only a uniaxial yield strength σ_0 from tension tests are commonly available. So that it is more convenient to calculate τ_0 from σ_0 in a uniaxial test at the stress defined at the yield strength we have $\sigma_1 = \sigma_0$ σ_2 and σ_3 is 0 because it is uniaxial substitution of these values into the yield criterion but the equation gives $\tau_0 = \sigma_0/2$. So, if you substitute this into this equation this relation so finally you get $\tau_0 = \sigma_0/2$ this is the yield criterion or the ductile materials are Tresca yield criterion.

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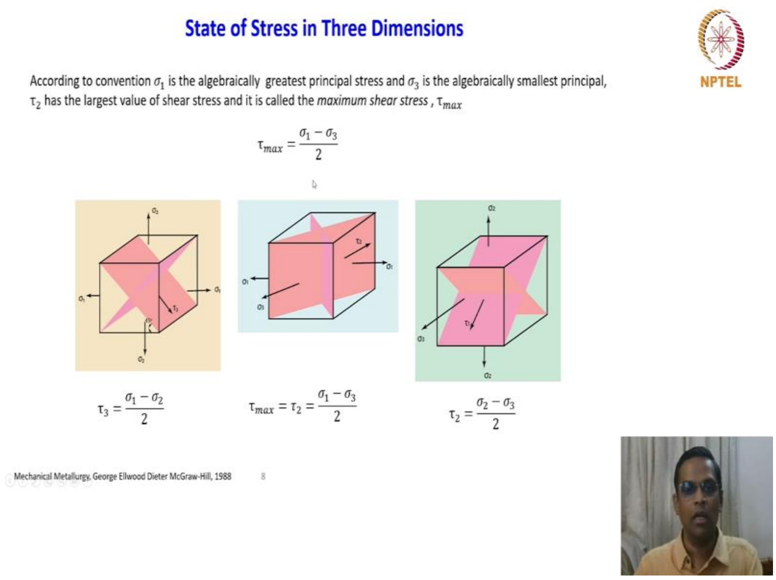

According to the convention σ_1 is the algebraically greatest principles stress and σ_3 is the algebraically smallest principle stress τ_2 has the largest value of shear stress and it is called a maximum shear stress τ_{MAX} this also we have already seen just for convenience we are rewriting here $\tau_{MAX} = (\sigma_1 - \sigma_3)/2$. So, this is what I just said I will bring this three I mean the maximum shear stress planes which we have already seen.

If you look at this τ_3 , τ_2, and τ_1 these are the exact planes on which these shear stress will act and this is the corresponding equations which relates the normal stresses and then you clearly see that the

$$\tau_{MAX} = \tau_2 = \frac{\sigma_1 - \sigma_2}{2}$$

this is exactly the yield criterion also predicts so when this max reaches the material which is a τ_{MAX} of this value then it will start yielding.