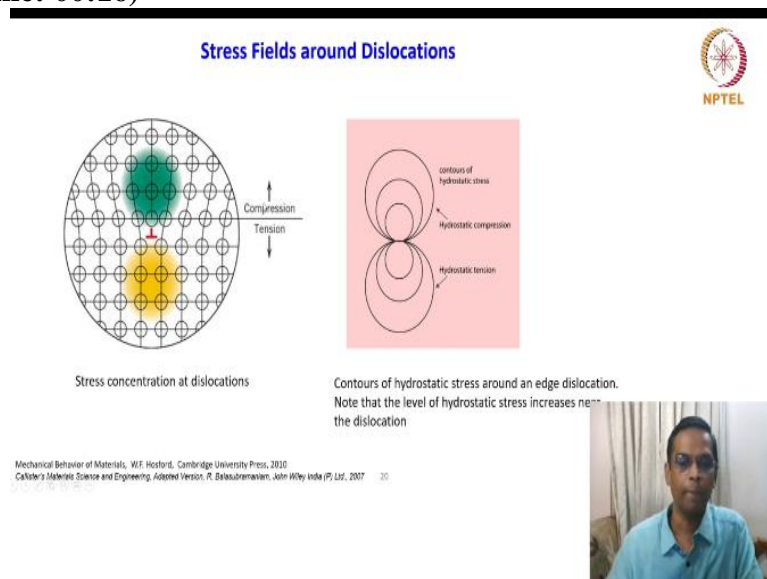


**Mechanical Behaviour of Materials**  
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**Indian Institute of Technology - Madras**

**Lecture - 15**  
**Introduction to Dislocations - III**

Hello I am Professor S. Sankaran in the Department of Metallurgical and Materials Engineering.

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Let us continue our discussion on dislocations, in the last class we looked at detailed geometry of dislocations and types of dislocations and we also looked at the energy of dislocations. And in this lecture we will see stress fields around the dislocation. So, if you look at this slide what is shown here on the left hand side is nice image which clearly shows that you know the extra half plane is there and then there is a dislocation the positive edge dislocation.

And then you see that second half of the lattice and there is some you known like in the previous slide I showed some shade this green shaded nicely gave some kind of a stress field around the edge dislocation. What you have to appreciate here is this the extra half plane this particular the above which the stress field is quite different compared to the bottom of the edge dislocation that is they above planes they above half I would say will have a experience a compressive stress and the bottom of experiences tensile stress.

And it is not that simple I mean we have just nicely shown in the picture like this, but if we get into the details, it is much more interesting to note that contours of the stress fields are shown here. And it is not simple compression simple tension it is hydrostatic stress. So, you know the meaning of hydrostatic stress now, because we have in a background for that. So, the above one is hydrostatic compression and the bottom one is hydrostatic tension. And we will now see that how complicated it becomes when it comes to the edge dislocation stress rates as compared to screw dislocation.

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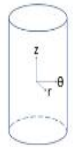
### Stress Fields around Dislocations

- Around a dislocation, the atoms are displaced from their normal positions.
- These displacements are equivalent to the displacements caused by elastic strains arising from external stresses.
- The equations given here assume of isotropic elasticity, T
- For a screw dislocation with a Burgers vector,  $b$ , parallel to the  $z$ -axis,

$$\tau_{\theta\theta} = -\frac{Gb}{2\pi r} \text{ and } \tau_{r\theta} = \tau_{rz} = \sigma_r = \sigma_\theta = \sigma_z = 0$$

where  $G$  is the shear modulus and  $r$  is the radial distance from the dislocation.

- The minus sign indicates that the repulsive force is inversely proportional to the distance between the dislocations.



Coordinate system for describing a location near a screw dislocation.  
The dislocation is parallel to  $z$  and at a distance,  $r = 0$ .

He Structure and properties of Materials, John Woll, 1965

Mechanical Behavior of Materials, W.F. Hosford, Cambridge University Press, 2010

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So, around a dislocation, the atoms are displaced from their normal positions, these displacements are equivalent to the displacements caused by elastic strains arising from the external stresses. So, to describe these stress fields and then this is a coordinate we are choosing just a reference and you can see that polar coordinates. So, these coordinate systems for describing your location near a screw dislocation.

The dislocation is parallel to  $z$  axis and at a distance  $r = 0$ , so it is parallel to  $z$ . So, the  $r$  component is 0 there, the equations given here asking the isotropic elasticity T. For a screw dislocation with a Burgers vector  $b$  parallel to  $z$  axis, these are the shear stress  $\tau$  in polar coordinates shear stress you know  $Gb / 2\pi r$  we have already seen this one, but since it is  $\tau_{\theta\theta}$  and  $\tau_{rz}$  which is equal to  $\sigma_r$  which is equal to  $\sigma_\theta = \sigma_z = 0$ .

Because  $r$  is 0 where  $G$  is a shear modulus and  $r$  is the radial distance from the dislocation. The minus sign indicates that the repulsive force is inversely proportional to the distance


between the dislocations. So, we are talking about the repulsive force of the dislocation we will in a minute we will see what are the detail forces?

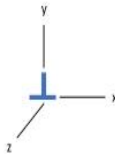
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### Stress Fields around Dislocations

- Equation indicates that a screw dislocation creates no hydrostatic tension or compression because  

$$\sigma_H = (\sigma_r + \sigma_\theta + \sigma_z)/3$$
- Therefore, there should be no dilatation (volume strain) associated with a screw dislocation. (However, real crystals are elastically anisotropic, so there may be small dilatations associated with screw dislocations.)
- For an edge dislocation that lies parallel to z and that has its Burgers vector parallel to x





$$\tau_{xy} = \frac{Dx(x^2 - y^2)}{(x^2 + y^2)^2}$$


$$\sigma_x = \frac{Dy(3x^2 - y^2)}{(x^2 + y^2)^2}$$

$$\sigma_y = \frac{Dy(x^2 - y^2)}{(x^2 + y^2)^2}$$

$$\sigma_z = \nu(\sigma_x + \sigma_y) = -2D\nu y / (x^2 + y^2), \tau_{yz} = \tau_{zx} = 0.$$

Mechanical Behavior of Materials, W.F. Hosford, Cambridge University Press, 2010

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So, the above equation indicates that the screw dislocation creates no hydrostatic tension or compression because the hydrostatic stress is defined like this  $(\sigma_r + \sigma_\theta + \sigma_z)/3$ . So, here we are talking about  $r = 0$ . Therefore, there should be no dilatation or volume strain associated with the screw dislocation. However, real crystals are elastically anisotropic we have to remember the equations whatever we showed here is through isotropic elasticity and in real time crystal system, we will have elastic anisotropy this elastically is valid and then we may require some special expressions.

So, that it will describe the elastic modulus with respect to each direction of the disorientation or atomic density and so on. So, there may be small dilations associated with the screw dislocation in real time that is what it says. For an edge dislocation that lies parallel to z and that has its Burgers vector parallel to the x axis then the situation is slightly different. So, the shear stress is defined like this in a Cartesian coordinate.

And what you see here is there are two normal stresses,  $\sigma_x$  and  $\sigma_y$  and the shear stress all of them are there and notice that there is another  $\sigma$  that is a third normal stress also existing this case of edge dislocations. So, we are not getting into the details of how to derive these equations, my intention of bringing here is to give an idea that the stress field around the edge dislocations are quite complicated. It is quite complicated that is the idea and then these all

the equations support that we will also go to visual impression on how the stress distribution around this edge dislocations are perceived.

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### Stress Fields around Dislocations

where  $x$  and  $y$  are the coordinates of the dislocation as shown in Figure and  $D = Gb / (2\pi (1 - \nu))$ .


- One of the important features of these equations is that there is a hydrostatic stress,  $\sigma_H = (\sigma_x + \sigma_y + \sigma_z)/3$ , around an edge dislocation.
- Combining equations,

$$\sigma_H = -\frac{\left(\frac{2}{3}\right) Dy(1 + \nu)}{x^2 + y^2} \quad \text{or}$$


$$\sigma_H = -A(y)/(x^2 + y^2)$$

where  $A = \frac{Gb(1 + \nu)}{3\pi(1 - \nu)}$

- There is **hydrostatic compression** (negative  $\sigma_H$ ) above the edge dislocation (positive  $y$ ) and **hydrostatic tension** below it.



Mechanical Behavior of Materials, W.F. Hosford, Cambridge University Press, 2010



So, here where  $x$  and  $y$  are the coordinates of the dislocation as shown in the figure, where the  $D$  is  $Gb / 2\pi(1 - \nu)$  this I have already told in the previous class, whenever you are talking about into dislocation, you have to look at this elastic constants this  $1 - \nu$  comes because this is treated as a plane problems and especially when strain problems, can you just go back and verify what is that we have a drive for planes stress in plane strain conditions the different elastic constants we have derived and you will be able to appreciate this.

One of the important features of this equation is that there is a hydrostatic stress  $\sigma_H$  this term of you will know in the earlier class we have seen  $(\sigma_x + \sigma_y + \sigma_z) / 3$  this is in a Cartesian coordinate around an edge dislocation. So, the hydrostatic stress can be written like this by the previous expressions what we have for it  $\sigma_x$  and  $\sigma_y$  and  $\sigma_z$  and there

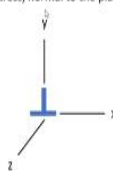
$$A = \frac{Gb(1 + \nu)}{3\pi(1 - \nu)}$$

So, there is a hydrostatic compression above the edge dislocation that is positive  $y$  and hydrostatic tension below it this is what we have already seen it.

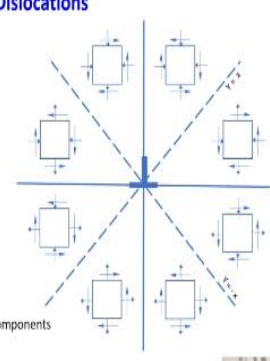
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**Stress Fields around Dislocations**


- The dilatational and shear stresses and the signs of these stresses are illustrated in Figure.
- There are no shear stresses acting in the direction parallel to the dislocation line
- There is however, a third dilatational stress, normal to the plane of the figure




The sense of the shear ( $\tau_{xy}$ ) and dilatational ( $\sigma_x, \sigma_y$ ) stress components of an edge dislocation



Mechanical Behaviour of Materials, Thomas H. Courtney, Wiley-Interscience, 2005 24





And what I am trying to show is another nice illustration of how these stress fields are distributed around this edge dislocation. So, all of you know this notation now, the way of representing shear stress in the strength of material notations, we have already discussed this how the shear stress and how positive shear stresses symbolized like this or negative this is positive this is negative. So, what we are trying to do by showing this here is as such we said that there is a hydrostatic compression and hydrostatic tension here.

When the stress fields are too complex like this, how this dislocation will respond to the external stimulates for example, a stress within the crystal itself how it will what type of stress whether it is a positive sign or negative sign? Whether it will attract or repel or just annihilated or it will just come to equilibrium or it will further increase energy. So, all these questions can be answered or at least make an attempt to co-relate make an attempt to interpret this will help so, that is a reason.

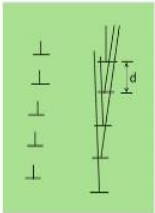
So, if you look at this is a coordinate system we will try to know explain this, the relational and shear stresses and the signs of the stresses are illustrated here. So, you know the shear stress this is positive and this is negative and again this is all compressive you can look at the normal stresses they are compressive more and here again biaxial, this is tension, this is compression, this direction, it is tension, this is compression, this is positive shear, this is negative shear.

So, what is the net that is what is shown here the sense of shear stress  $\tau_{xy}$  and the dilatational  $\sigma_x$  and  $\sigma_y$  is stress components of an edge dislocations. So, we will come back to this diagram

in a minute where we exactly see what this particular stresses will in each quadrant, how they are going to behave in this quadrant is it going to behave completely tension or what kind of effect to create to an external force or external object is that it is going to attract something or they are going to repel something.


What kind of force it is going to exhibit in each quadrant that will also give you much more idea, we will come back to the diagram. There are no shear stresses acting in the direction parallel to the dislocation line that means this location line you know this is a line which is going through this perpendicular to this plane of this image. And there is however a third relational stress normal to the plane of the figure, there is no shear stress but there is a normal stress.

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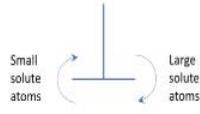


### Stress Fields around Dislocations

- The dilatation causes interactions between edge dislocations.
- Given sufficient mobility, edge dislocations of **like signs** tend to form walls with one dislocation directly over another as shown.
- The hydrostatic tension caused by one dislocation is particularly **annihilated by the hydrostatic compression of its neighbour**.
- The relatively low energy and therefore **stable configuration** forms a low-angle grain boundary




- In solid solutions, the solute atoms that are larger than the solvent atoms are attracted to the region just **below** the edge dislocation where their larger size helps **relieve the hydrostatic tension**.
- Solute atoms that smaller than the **solvent atoms** are attracted to region **above** the edge.
- In either case, **edge dislocation will attract solute atoms**.
- In interstitial solid solutions, all solute atoms are attracted to the region just below the edge dislocation where they help **relieve the tension, Fe-C system**



Small solute atoms      Large solute atoms

Mechanical Behavior of Materials, W.F. Hosford, Cambridge University Press, 2010      25



This is where we have seen the stress field around the dislocation tells us something what is this we shows this image shows the lining of the edged dislocation one over the other. Please remember we just described that the extra half plane wherever it is there it is hydrostatic compression and below that hydrostatic tension and when the dislocations are lined up like this, what can happen.

So, you should just so the forces software in a compression and tension that tries to align together. So that is why this kind of alignment is possible which is also called a low angle boundary. So that dilatation causes interaction between edge edge dislocations. Given sufficient mobility edge dislocations of light signs tend to form walls with one dislocation directly over

the other as shown. So that is an interaction of this edge dislocation with the light signs that is same sign here as shown here they try to form a wall.

The hydrostatic tension caused by one dislocation is particularly annihilated by the hydrostatic compression of its neighbour. So, this is compression and this is tension. So, it goes just aligns very interesting. So, the relatively low energy and therefore stable configuration forms and low angle grain boundary very important idea. So, you see looking at the stress fields or on the dislocation already gives you a perspective of how the boundary forms if at all forms a boundary whether it is going to remain same or not.


We are getting some idea already and because of the stress field around dislocation and what are the other aspects of this. So, you have the dislocation like this and it has got hydrostatic compression and tension and it can attract large solute atoms below this and small solute atoms above this. In solid solutions, the solute atoms that are larger than the solvent atoms are attracted to the region just below edge dislocation, where the larger size helps relieve the hydrostatic tension.

So, the larger atoms in a solid solution larger than the solvent will come here and that relieve the hydrostatic tension very important idea. And solute atoms that are smaller than the solvent atoms are attracted to the region above the edge, so here hydrostatic compression region. In either case, edge dislocation will attract solute atoms very important point metal scale point of view. So, edge dislocation attracted solute atoms because of the complex just being around it, not the screw dislocation.

In interstitial solid solutions, all solute atoms are attracted to the region just below the edge dislocation where they help relieve the tension. For the classical example is iron carbon system where the carbon is an interstitial solid solution, forms an interstitial solid solution. So, carbon is an interstitial there and then it grows and gets settled here.

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**Forces on Dislocations**



- A stress in crystals causes a force on dislocations.
- Consider a dislocation of length,  $L$ , and Burgers vector,  $b$ , on a plane, as shown in Figure. A shear stress,  $\tau$  acting on that plane will cause a force on the dislocation, per unit length,  $F_L$ , of

$$F_L = -\tau \cdot b$$

Note that a dot product is possible here because once the plane of the stress is fixed, the stress can be treated as a vector (force).

- The stress,  $\tau$ , may result from the stress field of another dislocation.
- Thus, two screw dislocations exert an attractive force on each other of

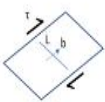
$$F_L = -Gb_1 \cdot b_2 / (2\pi r)$$

where  $b$  is the Burgers vector of the dislocation of concern.

- The minus sign means that they repel one another if the dot product is positive.
- An equivalent statement is two dislocations repel each other if Frank's rule predicts that their combination would result in an energy increase.
- If the angle between  $b_1$  and  $b_2$  is greater than 90 degrees,  $|b_1 + b_2| > |b_1| + |b_2|$

Mechanical Behavior of Materials, W.F. Hosford, Cambridge University Press, 2010

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So, now we look at the forces, what are the forces on dislocations? When it is forces that could be several possibilities, either it is applied load or the stresses stored in the crystal itself. And what this stresses will have influence on this dislocations. A stress in crystals cause a force on dislocation, what kind of force suppose if you take a plane like this and there is a dislocation line and this is a Burgers vector the dislocation line of the length  $L$  and the shear stress  $\tau$  is applied being applied.

So, then what happens the shear stress  $\tau$  acting on that plane will cause a force on the dislocation per unit length that is  $F_L$ . So, we are when we measured the energy also be measured the energy per unit length. So, here again force per unit length  $F_L$  because this location we are measuring in length. So,  $F_L = -\tau \cdot b$  that is a force. Note that dot product is possible here because, once the plane of the stress is fixed, the stress can be treated as a vector force there is a force.

The stress  $\tau$  may result from a stress field of another dislocation so, it could merely by the stress field of other dislocation as well not necessarily applying the stress. So, if you consider two screw dislocations exert an attractive force on each other than the force will be  $F_L = -Gb_1 \cdot b_2 / 2\pi r$ . So, what is that we are now doing additionally we know that  $Gb / 2\pi r$  we know that term already, the new term is  $b_1 \cdot b_2$  because we are considering two dislocations the stress field of one dislocation how it influences the other.

The minus sign means that they repel one another if the dot product is positive that could be a possibility. So, each dislocation we are now saying that the way in which it is represented by

is in a positive dislocation or negative dislocation positive positive come together what happens the positive negative comes from together what happens positive positive comes together what happens there are possibilities.

So, if they repel one another either a dot product becomes positive then it is denoted by the negative sign. An equivalent statement is two dislocations repel each other if Frank's rule predicts that their combination would result in an energy increase. So, what is Frank's rule? Frank's rule is you know that the energy of the dislocation is proportional to  $b^2$  that is what we have seen the last class.


Suppose, a dislocation can dissociate or to partial dislocation can combine to form an new dislocation or a unit dislocation can also can split into two partial dissolution, but this can happen only when the energy is favourable that is what the Frank's rules says. So, what does it mean suppose, if the unit dislocation want to dissociate into two partial dislocations they energy requirement this suppose the unit dislocations Burgers vector is  $b_1$  and  $b_2$  partial dislocations being  $b_2$  and  $b_3$ .

Then  $b_1$  there is a condition for you know the dissociation is both sides should be Burgers one side should be favourable to the energy. We will see some of the examples when I show some example then it is easier so, that is what the Frank's rules says. If the angle between  $b_1$  and  $b_2$  is greater than 90 degrees, then this is the condition for the dislocation reactions  $|b_1 + b_2| > |b_1 + b_2|$ .

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### Forces on Dislocations

- The interaction of two parallel edge dislocations is somewhat more complex.
- The shear stress field for one edge dislocation that lies parallel to  $z$  with a Burgers vector parallel to  $x$  is given by equation,



$$\tau_{xy} = Dx(x^2 - y^2)/(x^2 + y^2)^2,$$


where  $D = Gb/(2\pi(1 - \nu))$ .

- The mutual force on that plane is

$$F_L = - \frac{G \left( \frac{b_1 b_2}{2\pi(1 - \nu)} \right) x(x^2 - y^2)}{(x^2 + y^2)^2}$$

- For dislocations with like sign ( $b_1, b_2 > 0$ ), there is mutual repulsion in the region  $x > y$  and attraction in the region  $x < y$ .

This is equivalent to saying that there is mutual repulsion if Frank's rule predicts that a reaction would cause an increase of energy and mutual attraction if it would cause a decrease of energy.

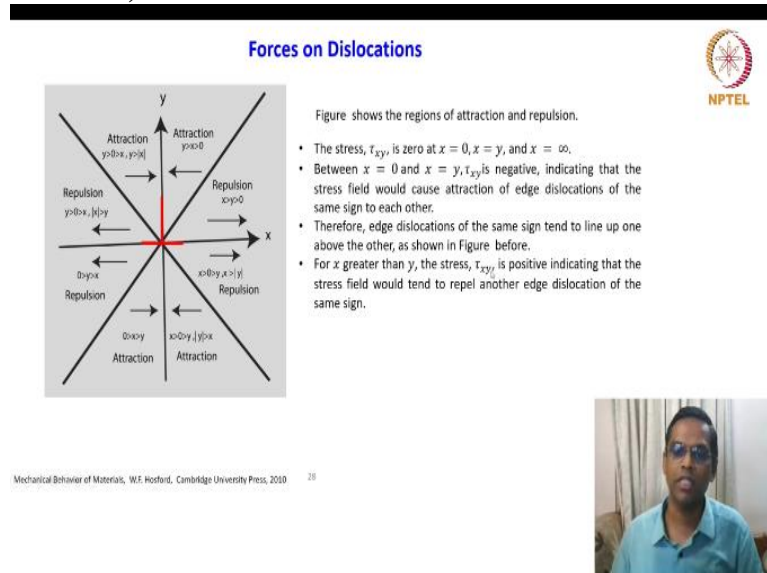
Mechanical Behavior of Materials, W.A. Hosford, Cambridge University Press, 2010
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The interaction of two parallel edge dislocation is somewhat more complex this seen already we can now see why it is the shear stress field around the dislocation itself is too complex then the interaction between them definitely much more complex. The shear stress field for one edge dislocation that lies parallel to  $z$  with a Burgers vector parallel to  $x$  is given by the equation. This is the same equation what we just seen the couple of slides before shear stress.

The mutual force on that plane will be of this nature, see earlier we just looked at the stress field alone. But now we are talking about two dislocations coming together a parallel dislocation that is why come to two Burgers vectors and dot product. For the dislocation with the like sign that means the dot product is greater than 0 there is a mutual repulsion in the region  $x > y$  and it will be an attraction in the region if  $x < y$ . How do we understand this?

Like I said when I showed an image which shows all the complex stressful states. I said we will come back to this we are now going back to the diagram and it different forms. So, this is equivalent to saying that there is a mutual repulsion if the Frank's rule predicts that a reaction would cause an increase in of energy and mutual attraction, if it cause a decrease of energy so, we will qualify these statements what are these statements?

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So, now you look at this image much more carefully, we will spend some time on this some time is exactly similar diagram what we have shown a few slides back. So, what we have just divided them into four quadrants and each quadrant also has been bifurcated with some straight line here. So, what does it mean? So, in the initial description, we said that the top

half is the hydrostatic compression and the bottom is hydrostatic tension. So, within that we have now some classification.

If  $x = 0$  or  $x = \infty$  or  $x = y$  in all these cases the stress is 0 you can see that  $x = \infty$ ,  $x = 0$ ,  $x = y$  means you are traveling this way the stresses the force is 0. Any value other than these three will have some influence, suppose to the  $x > y$  and  $y > 0$  then it is repulsion. So, in this region but if the  $x$  and  $y > x > 0$  then it is an attraction. So, within the quadrant you have two scenarios on this repulsion attraction.

So, this is between the two dislocations that attraction I mean, the stress field is like that, but in order with another dislocation comes then we have to see and that is what the Frank's rule helps, whether we can consider the energy whether that will dictate, whether that kind of a combination of dislocation will be stable or not. So, this is shear stress field around the dislocation. But you have to just if the other dislocation is coming then that is where the Frank's rule helps in terms of energy that is you have to look at it.

We will see some examples, the stress  $\tau_{xy}$  is 0 at  $x = 0$ ,  $x = y$  or  $x = \infty$ , between  $x = 0$  and  $x = y$ ,  $\tau_{xy}$  is negative indicating that the stress field would cause attraction of edge dislocations of the same sign to each other. So, here we are talking about specifically the another dislocations. So, it could be here dislocation interaction, but the same stress field is also will act on a second phase particular also not necessarily a dislocation, but here we are giving dislocation description.

But you can also later you will see that this stress field also will be useful in explaining how it interacts with the second phase quarter. Therefore, edge dislocations of the same sign tend to line up one above the other as shown in the figure before what is that before that, I have shown the low angle grain boundary how it forms and that is also because of this you know attraction.

For  $x > y$  and the; stress  $\tau_{xy}$  is positive indicating that the stress field would tend to repel another edge dislocation on the same sign. So, it is like opposite to the previous case. So, this is correspond to this this quadrant this region they above statement one is first log attraction this is a repulsion. So, for just for clarity, I want to just so, what I want to say tell you is so,

you just tried to compare that diagram with this then it will make much more sense to connect.

What is this symbols indicate and then how those attraction and repulsion and attractive force. You can put one side by side and then you will be able to appreciate this. So, what each symbol means that will be very clear that is why I want to try.