Mechanical Behaviour of Materials
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Lecture-12
Anelasticity

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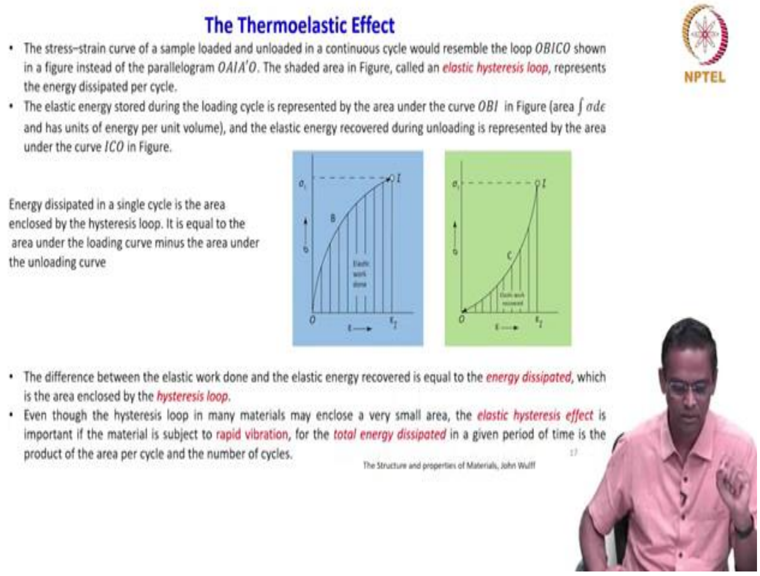

Hello, I am Prof. S. Sankaran in the Department of Metallurgical and Materials Engineering; we will now look at some other details. The stress-strain curve of a sample loaded and unloaded in a continuous cycle would resemble the loop OBICO shown in the figure instead of a parallelogram OAIA'O. Of course, they shaded this I have already told, the elastic energy stored during the loading cycle is represented by the area under OBI.

This is a one way of looking at it. So, OBI this is the area elastic work done and this is nothing but $\int \sigma dE$ and has got the unit of energy per unit volume, the energy recovered during unloading is represented by the area under ICO. So, this area, so, work recovered, work done. So, this, minus this will give what so, the energy dissipated in a single cycle in the area enclosed by the hysteresis loop.

It is equal to the area under the loading curve minus the area under the unloading curve. So, this minus this will give the small hysteresis loop which is just shown in the elastic hysteresis loop. So, that is what it is, so, this is what is written here. So, the difference between elastic work done and the elastic energy recovered is equal to the energy dissipated, which is the area enclosed by the hysteresis loop.

So, any event we can correlate so, many events we can talk about like migration of atoms, thermal energy we said stress and strain and so on. This is a general description of energy dissipation or energy elastic work done in a hysteresis loop, so, that is general description. Even though the hysteresis loop in many materials may enclose a very small area, the elastic hysteresis effect is important if the material is subject to rapid vibration.

So, we have to remember that even though the area enclosed in this kind of curve is very small like this, it is important when the material is subject to rapid vibration for the total energy dissipated in a given period of time is the product of area per cycle and the number of cycles. So, this also we will see as we proceed, we will look at some of the case studies.

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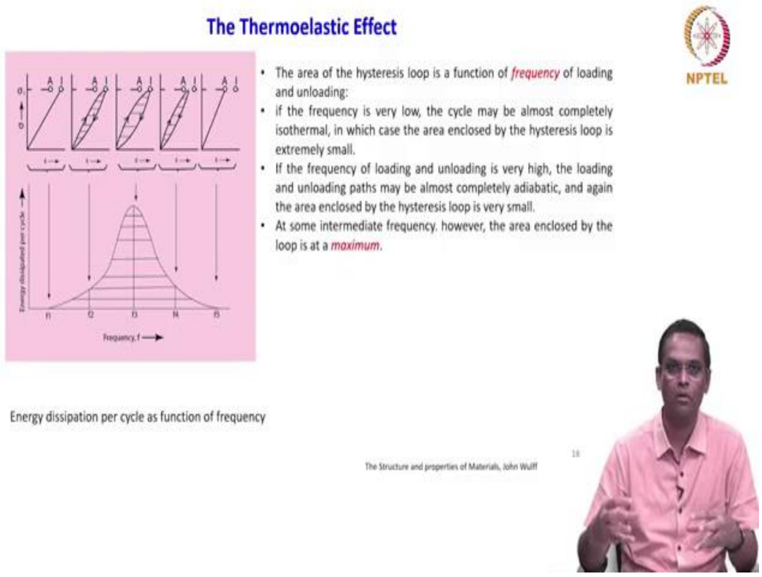

So, what is shown here, the energy dissipation per cycle as a function of frequency. So, we were talking about loading, unloading this kind of hysteresis we arrived at a physical meaning of what is hysteresis loop or elastic energy hysteresis loop but that has to happen in particular frequency. So, what is the frequency at which things will happen or in other way at what frequency the maximum energy dissipation will happen that is also important depending upon what you are looking at.

So, this is what we are going to discuss now. So, this is energy dissipated per cycle versus a number of I mean different frequencies and then you can see that this particular a small graph which shows that the different positions of the faster diagram you can just go back and relate A and I position and then how the frequency is different from different different positions. For example, if it is, what we can do is we can just look at where this 0 to I position and 0 to I position can be of enclosing a small hysteresis loop at a different frequency.

This is the lowest frequency, slightly increased frequency in some medium range frequency and again it is coming to the very high frequency. So, what do you see from this, in general the lowest frequency and highest frequency is not dissipating maximum energy. So, only at particular medium range of frequency your energy dissipation is maximum. So, we will see some of the physical events how they are relating to this concept and how it is helpful.

If the frequency is very low, the cycle may be almost completely isothermal so, in our case what we just said a crystalline material is very slowly stretched from 0 to the position I in which case the area enclosed by the hysteresis loop is extremely small. So, in fact it is a straight line almost here. If the frequency of loading or unloading is very high the loading and unloading paths may be almost completely adiabatic and again the area enclosed by the hysteresis loop is very small.

So, here so, this is isothermal I this is adiabatic A, in both the cases the hysteresis loop is extremely small. At some intermediate frequency the area enclosed by the loop is at a maximum. So, this is some intermediate frequency. So, now, how do we understand this graph it is a very general description we are doing. So, now we have to look at some physical events for example, you know atom gems, defect migration, thermal energy migration, whether it fits into any what kind of frequency and how much of energy can be.

So, we can easily relate all the physical events to the energy dissipation it could which involve in any part of the heat treatment or processing sequence and so on, this is very important idea.

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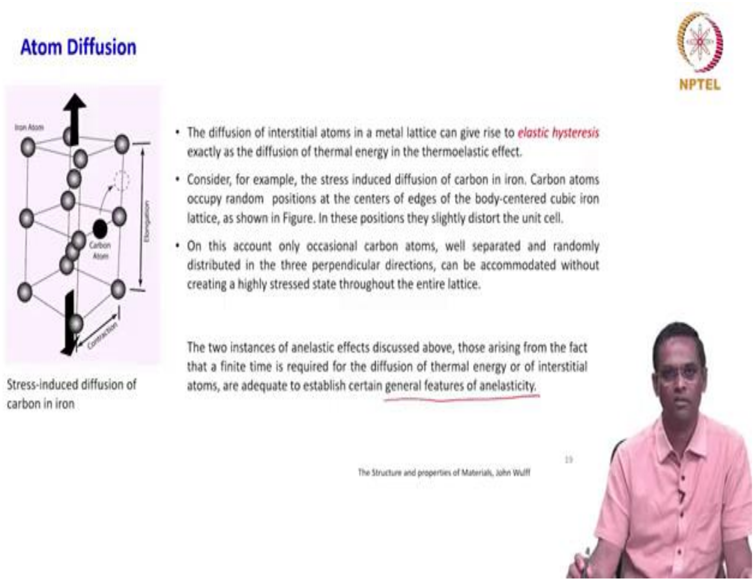

So, one classical example we will take the stress-induced diffusion of carbon in iron very well-known phenomenon here, this is a BCC lattice in fact, it is a unit cell, two unit cell which is tacked one upon the other it is a BCC unit cell and where you have this carbon atomicity here in the centre and then this unit cell is getting stretched in a tension mode then what can happen that is the general description.

So, what can happen if you just stretch it, you know the effect now, please remember we are talking about elastic deformation we have not crossed that elastic we are still in elastic deformation thermo-elastic effect. So, if you stretch it elastically what happens with, this axis will get elongated and the Poisson's ratio effect will be there. So, this will get contracted. So, what will happen to the atom sitting here it will also try come subjected to some constraint.

So, it will try to escape from the constraint or relax to a very comfortable position wherever this know this is pushing from there. So, this is getting elongated. So, it there is one possibility that this carbon atom can jump from this position to this position when we stretch it in a axis perpendicular to this is perpendicular to this section. So, here what is that we are talking about cycling.

The cycling is the atom has to jump from one position to the other position and relax come back. So, that is the event we are taking it up. So, the diffusion of interstitial atoms in a metal lattice can give rise to elastic hysteresis exactly as the diffusion of thermal energy in that thermo elastic effect and here the stress induced diffusion of carbon and iron, carbon atoms occupy a random position at the centres of edges of the body-centred cubic iron lattice in the positions they slightly distort the unit cell.

So, of course, when you have this any atom you know foreign atom which sits there in the interstitial positions then it will try to distort a little bit. On this account only occasional carbon atoms, well separated and randomly distributed in three perpendicular directions can be accommodated without creating highly stressed state throughout the entire lattice. So, what does it mean?

For this, not all the carbon atoms will undergo a jump what we are discussing above only a carbon atom which is subjected to constraint will jump and also will try to go to the new position where there is no other constraints. So, that is what it is shown here. So, this is also one of the classical example of this thermo elastic effect when these two when this kind of lattice is stretched the diffusion of carbon atom from the constraint position to the new position is also as a function of elastic strain.

And then it relax back to its original position is also a cycle in a cycle depending upon the temperature, or it can happen with the wind temperature also a phase transformation temperature, elastic strain, thermal energy or vacancy motion, that is what we said you know migration of defence thermal energy migration or the atoms and so on, so, this is one classical example. The two instances of anelastic effects discussed above what are this, this is one the other one is that crystalline material getting stretched up.

Those arising from the fact that the finite time is required for the diffusion of thermal energy or the interstitial atoms are adequate to establish certain general features of anelasticity. So, looking at these kinds of events, physical events that are salient features, one can list out what are the general features of anelasticity.

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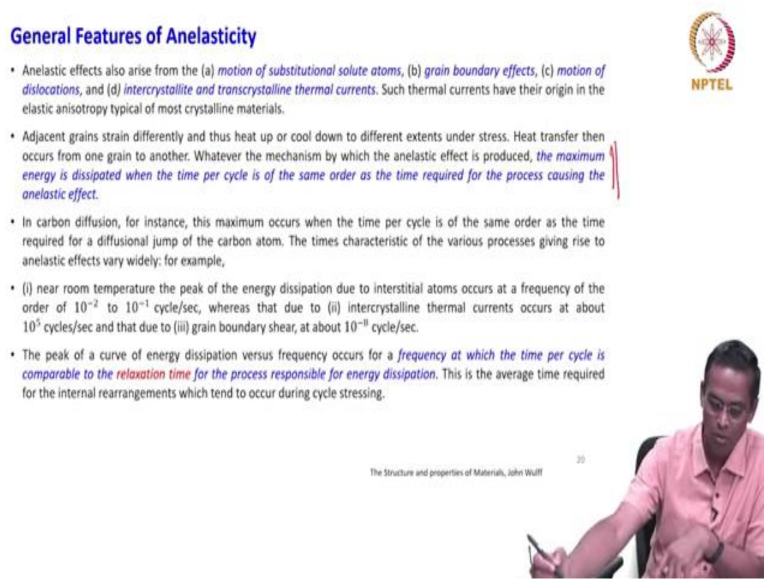

So, that is what we are just going to look at, it is more descriptive but very important. So, that is why I am trying to stop and then repeat certain statement again and again. Anelastic effects also arise from motion of substitutional solute atoms, we are just discussed about interstitial atoms that is carbon in iron lattice, it can also arise from motion of substitutional solid atoms, grain boundary effects, motion of dislocations, inter crystallite and the trans crystalline thermal currents.

So, we made general statements in the previous slides. Now, we are getting to a little more specific we just say thermal energy now, we are talking about inter crystallite and trans crystalline thermal energy currents we are talking about we just generally said defects now, we are talking about dislocation motions of this location we just mentioned about atoms now, we are talking about substitutional solute atoms in addition to interstitial atoms.

So, we are getting to specific ideas such thermal currents have their origin in elastic anisotropy typical of most crystalline materials. So, this is more relevant to crystalline materials. Adjacent grains strain differently and thus heat up or cool down to different extents under stress. So, this also we will see; when we go to the plastic deformation or polycrystalline deformation, when you have two different you know.

Several grains will be oriented at different directions and then you are going to stretch it in one particular direction or apply the Load in particular direction all the grains will try to rotate, accommodate, you know, strain each other and that is what it is, adjacent grains strain differently. So, depending upon the slip characteristics, we will see you know, depending upon the particular results, shear stress and so on.

Everything gets realigned or stretched towards stress axis or loading axis in that activity they get heated up or cooled down to different extent under stress heat transfer then occurs from one grain to the other. So, that is one event, what are the mechanism by which anelastic effect is produced, the maximum energy dissipated when the time per cycle is of the same order as the time required for the process causing the anelastic effect very important.

It looks a little confusing, but then, when we just look at a concept like relaxation time and all this statement will become much more straightforward. The time per cycle is of the same order it should be the same order of the time required for the process causing the anelastic effect. So, very important ideas we have to just relate this to physical events then it is interesting, otherwise it looks more kind of a text.

In carbon diffusion, for instance, the maximum occurs that is anelastic effect when the time per cycle is of the same order as the time required for the diffusional jump of the carbon atom. The time characteristic of the previous processes giving rise to anelastic effects vary widely. For example, near room temperature the peak of energy dissipation due to interstitial atoms occurs at a frequency are of the order of 10^{-2} to 10^{-1} cycle/second.

So, what is that we are talking about the maximum energy that peak of energy happens when the frequency of this order. What is the physical event? Interstitial atom jumps, whereas, inter crystalline thermal currents occurs at about 10^{5} cycles/second look at the different frequencies. See, now, you can just relate these frequencies with that diagram what we have shown some we have just seen that you know the energy dissipation at two different frequencies.

So, this is one classical example. So, this frequency is very different compared to this frequency and the grain boundary shear, at about 10^{-8} cycle/sec. So, every event will have a different range of frequencies, but even if you take grain boundary shear alone this one particular physical event alone also will have the relevance to that diagram not all the frequencies will produce a similar kind of dissipation.

Only you know particular range of frequency will dissipate the maximum energy so, that is that something you have to remember. The peak of a curve of energy dissipation versus frequency, this is what we have seen already occurs for a frequency at which the time per cycle is comparable to the relaxation time for the process responsible for the energy dissipation.

So, this point is almost now what we have just made a statement just before. So, now, to have some more clarity we are bringing another parameter called relaxation time. So, relaxation time of what relaxation time further which is causing that as a whole thermo-elastic effect that is a cycle. So, that is relaxation time to the average time required for the internal rearrangement, which tend to occur during the cyclic stressing.

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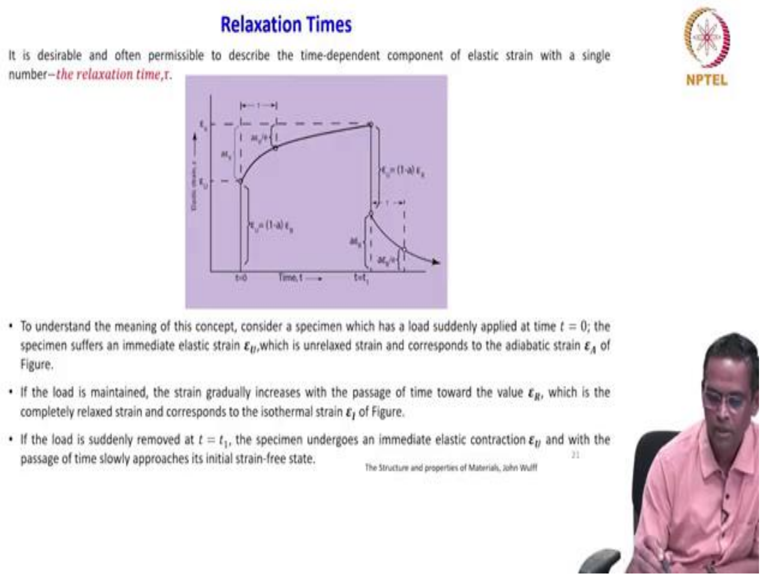

So, what is relaxation time? We will just see now; it is desirable and often permissible to describe the time-dependent component of elastic strain. So, this is time dependent component of elastic strain with a single number, we would like to give some number for this quantity relaxation time τ which is designated as a τ. So, how do we understand this concept? So, look at this diagram.

So, the diagram will look little complicated in the very fast look, because too many writings on them. So, do not get confused. So, we should also relax. So, then you will be able to catch this, what is shown here elastic strain ε versus time, do not pay any attention to the writings. So, now, we have two types of I mean two different part of the curve, one is t = 0 and it is almost like you know, constant t is equal to here and then and slowly it relaxes to one point and again unloading and then relaxes.

So, instant loading or adiabatic loading, you may can whatever the way we can look at it, then it the elastic strain ε_u is reached all of a sudden that is $t = 0$ and then it relaxes, it continuously the load is on. So, but it relaxes the material relaxes to the Maximum that is ε_r is relaxing strain and then suddenly unloaded $t = t_1$ and then slowly relaxes asymptotic, this is asymptotic we just discussed about this behaviour asymptotic.

So, that is this is a physical event you have to just first understand, the next point is this is what I just said, to understand the meaning of this concept, consider the specimen which has a load suddenly applied at $t = 0$; the specimen suffers an immediate elastic strain, which is unrelaxed strain and correspond to adiabatic strain ε_A of the figure and then the load is maintained the strain gradually increases with the passage of time toward the value ε_R.

Actually, what I think it is adiabatic strain ε_A is also is equal to ε_U that you can either you use it as one ε_U or ε_A is fine. There is some inconsistencies here immediate elastic strain ε_U and this is also equal to ε_A of the original figure we have the introduction we have just used to explain the concept of isothermal loading and adiabatic loading we have there we have used ε_A that is why it is brought here.

It is equal to ε_U so you do not have to get confused with that. So, if the Load is maintained the strain gradually increases with the passage of time toward the value ε_R, which is the completely relaxed strain. So, that is why I said it is the relaxed strain and correspond to the isothermal strain ε_I of the figure, the load is suddenly removed at the security one the specimen undergoes an intermediate elastic contraction. This is intermediate contraction again ε_U and with the passage of time slowly approaches its initial strain-free state. So, this is asymptotically coming down.

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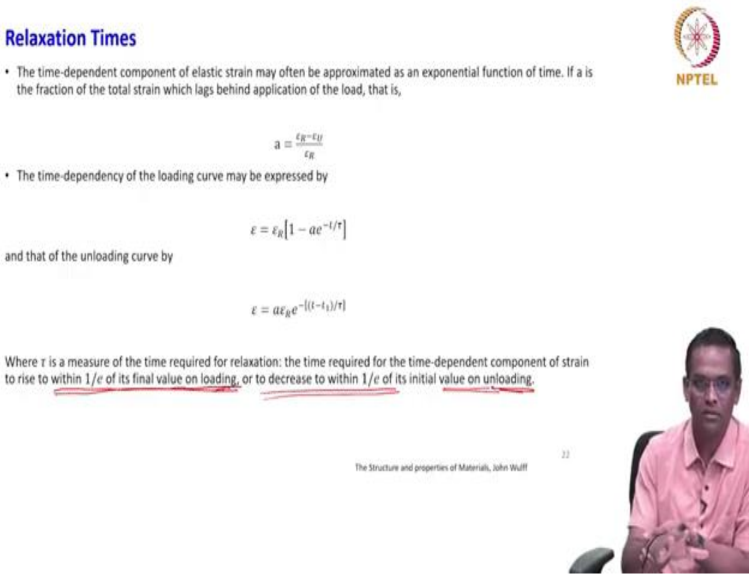

So, now, we will just look at little more description of this how, do we understand that relaxation time. The time dependent component of elastic strain may often be approximated as an exponential function of time. So, this is one assumption we have to understand here. If a is a fraction of the total strain which lags behind the application of load that is;

$$a = \frac{\varepsilon R - \varepsilon U}{\varepsilon R}$$

The time- dependency of the loading curve may be expressed as by this expression

$$\varepsilon = \varepsilon_R[1 - ae^{-t/\tau}]$$

this is for the loading. For unloading

$$\varepsilon = a\varepsilon_R e^{-[(t-t1)/\tau]}$$

this is for unloading. Now, suppose, we are talking about a cyclic loading the relaxation time that means, any physical event which is going to complete loading and unloading that means, it has to attain the equilibrium strain and also isothermal strain.

And it has to come back to the strain free region so, that that cycle it has to complete. So, the relaxation time is the time dependent component of elastic strain is related to relaxation time. So, if you just take this particular equation suppose if I am putting $t = 0$, $t = t_1$, these are the two times we have just looked at in that figure. Suppose, if I replace t/τ that means what that production physical event as completed.

So, if you replace $t = \tau$, what happens then it becomes 1/e times the rest of the whatever it is, so, we just look at this point alone, then you will get the complete idea, then will go back to the figure then understand. Suppose if you replace if the t is if the complete event is over then τ is t = τ or before that it could be anything it could be t = 0, t = t_1, t = anything, but the event is over that physical event is over then t = τ we can put then it is 1/e.

That is what it is shown here, where τ is the measure of the time required for the relaxation the time required for the time-dependent component of this strain to raise to within 1/e of its final value of the loading or to decrease to within 1/e of its initial value on unloading. So, what is the meaning that is what is shown in that figure? Now, we will go back to the figure you see, we have now just put suppose, I am interested here this is the τ this is a measure of τ here.

So, this is a relaxation time it is not completely relaxed sustained it has not reached here, but it is somewhere here. So, here if I want to look at it that will have so, much of deep fraction of 1/e that is what it is, the strain is of a fraction but it is within the 1/e it is not completed it is a 1/e is after completion. So, here it at this point a τ is a fraction of 1/e. Similarly, you can see here also it is relaxed here.

So, here also it is a fraction of 1/e and in between these sudden changes even it will be equal into this expression. So, that is how you should look at it then the meaning of relaxation time is easily visualized from this. So, this can be of any range. So, τ can be here τ can be here or tau can be here, but still, it will be within that, I will go back to the show, it will be still within the 1/e of it is final value on either unloading or unloading.

So, this is what we are just seen in the figure in the loading part as well as unloading part. So, that is quite important concepts relaxation times and this is we will be using these parameters in the deformation behaviour of non-crystalline materials, semi crystalline materials and so on later and elasticity wherever we discuss this property, these parameters will be much useful.

So, if you know from the beginning, then you do not have to just think I will be there you can just concentrate on the material and microstructure and so on. So, these are the some inputs you require the beginning. So, we will stop here and we will continue in the next class. Thank you.