Mechanical Behaviour of Materials
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Lecture - 11
Elastic Properties Part II

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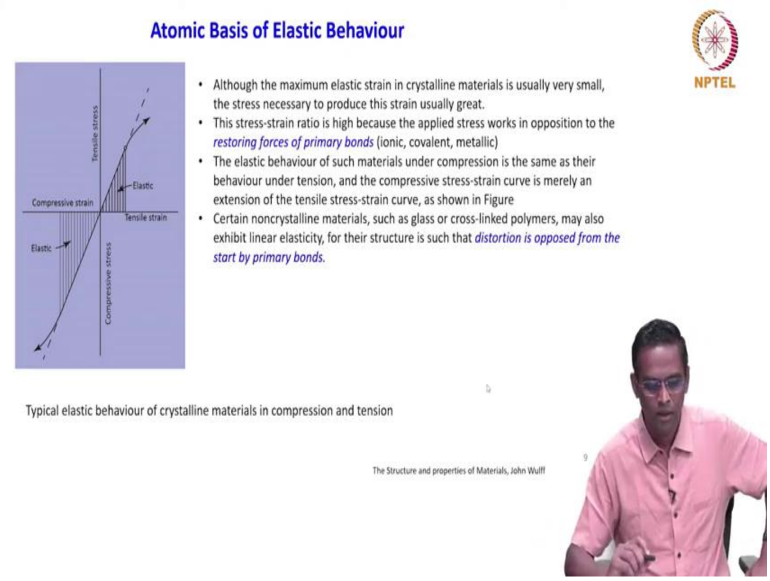

Atomic Basis of Elastic Behaviour

- Although the maximum elastic strain in crystalline materials is usually very small, the stress necessary to produce this strain usually great.
- This stress-strain ratio is high because the applied stress works in opposition to the *restoring forces of primary bonds* (ionic, covalent, metallic)
- The elastic behaviour of such materials under compression is the same as their behaviour under tension, and the compressive stress-strain curve is merely an extension of the tensile stress-strain curve, as shown in Figure
- Certain noncrystalline materials, such as glass or cross-linked polymers, may also exhibit linear elasticity, for their structure is such that *distortion is opposed from the start by primary bonds.*

Typical elastic behaviour of crystalline materials in compression and tension

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Hello, I am Professor S Sankaran in the Department of Metallurgical and Materials Engineering, let us continue our discussion on the atomic basis of elastic behaviour. What we have seen yesterday is we have gone through some of the elastic behaviour of crystalline materials in tension as well as compression mode what we have seen as something this is the kind of stress-strain behaviour we have seen.

And then we have come to conclusion that this stress-strain ratio is high because the applied stress works in opposition to the restoring force of primary bonds for example ionic, covalent, metallic bonds. Similarly, we also started our discussion on the, a non-crystalline materials such as glass, cross-linked polymers may also exhibit linear elasticity, for their structure is such that distortion is opposed from the start by primary bonds.

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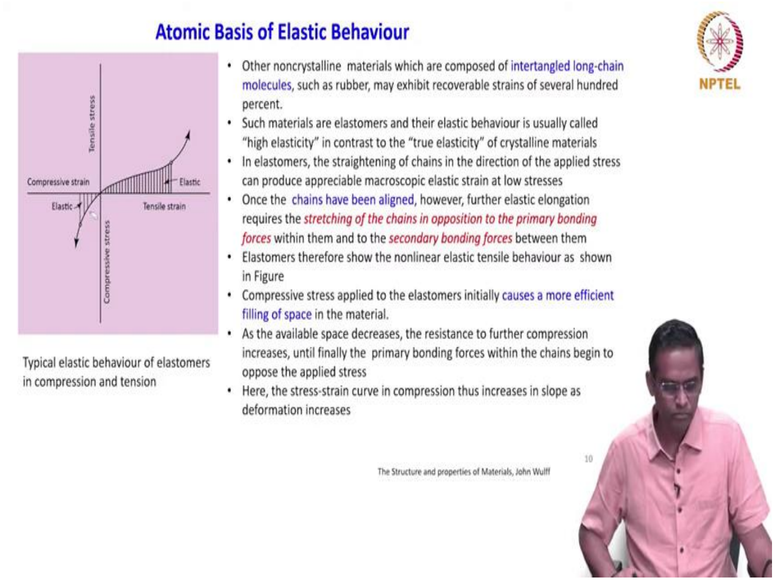

Atomic Basis of Elastic Behaviour

- Other noncrystalline materials which are composed of intertangled long-chain molecules, such as rubber, may exhibit recoverable strains of several hundred percent.
- Such materials are elastomers and their elastic behaviour is usually called "high elasticity" in contrast to the "true elasticity" of crystalline materials
- In elastomers, the straightening of chains in the direction of the applied stress can produce appreciable macroscopic elastic strain at low stresses
- Once the chains have been aligned, however, further elastic elongation requires the *stretching of the chains in opposition to the primary bonding forces* within them and to the *secondary bonding forces* between them
- Elastomers therefore show the nonlinear elastic tensile behaviour as shown in Figure
- Compressive stress applied to the elastomers initially causes a more efficient filling of space in the material.
- As the available space decreases, the resistance to further compression increases, until finally the primary bonding forces within the chains begin to oppose the applied stress
- Here, the stress-strain curve in compression thus increases in slope as deformation increases

Typical elastic behaviour of elastomers in compression and tension

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So, again we looked at the typical elastic behaviour of elastomers and we have seen that the first physical activity in this tension is stretching of chains in, sorry first activity is alignment in the direction of tension force and then further elastic elongation request stretching of the chains in opposition to primary bonding forces and we have also seen that there is a difference significant difference between elastic response in tension versus compression in this class of material and that we have attributed that to the compressive stress applied to the elastomers initially causes a more efficient filling of space and then the moment the space available for filling then the resistant for the compression increases significantly that is reason, why you see the significant increase in the slope initially as compared to tension forces here. So, this is what we have seen we stopped here and then we ended our lecture yesterday. So, we will continue this discussion in this direction.

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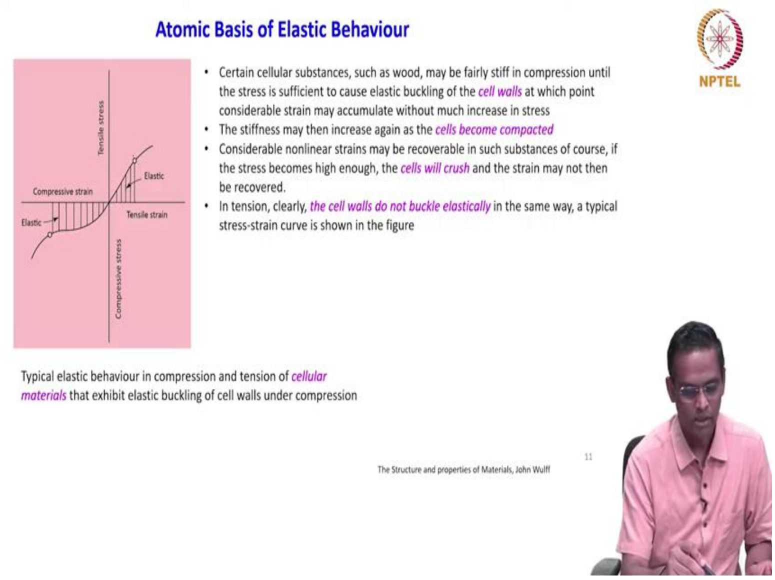

Atomic Basis of Elastic Behaviour

- Certain cellular substances, such as wood, may be fairly stiff in compression until the stress is sufficient to cause elastic buckling of the *cell walls* at which point considerable strain may accumulate without much increase in stress
- The stiffness may then increase again as the *cells become compacted*
- Considerable nonlinear strains may be recoverable in such substances of course, if the stress becomes high enough, the *cells will crush* and the strain may not then be recovered.
- In tension, clearly, *the cell walls do not buckle elastically* in the same way, a typical stress-strain curve is shown in the figure

Typical elastic behaviour in compression and tension of *cellular materials* that exhibit elastic buckling of cell walls under compression

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Now, I will show the other type of elastic deformation behaviour and what you can see, it appears exactly opposite to what we have just seen yesterday. So, you see this elastic response in a tension domain like you know, tensile force is much steeper as compared to in a compression mode. So, very interesting behaviour. So, what kind of material will exhibit this kind of behaviour.

So, we are now going to talk about elastic behaviour in a compression and tension of cellular materials that exhibit elastic buckling of cell walls under compression. So, the other class of material is cellular materials, what are cellular materials, the name itself has some answer the structure is made up of cells, the microstructure of these kind of materials made up of cells, there are lot of examples even natural material wood is also considered as a cellular material several you have a lot of cells.

The cells could be open cells or it could be closed cells could be vacuum inside the cell or it could be some gas filled and depending upon what we do in the processing procedure. So, we will just talk about, those kind of material. So, the one example classical example is the natural wood may be fairly stiff in compression until the stress is sufficient to cause elastic buckling of cell walls at which point considerable strain may accumulate without much increase in stress.

So, what is shown here is we are talking about a fairly stiff in compression until the stress is sufficient to cause elastic buckling of cell walls. So, we are talking about not primary bonds like the other materials, we are talking about the buckling of cell walls, I said the microstructure consists of cells. So, the cell wall elastic deformation of cell wall that is called buckling is considered the energy absorbing event here.

So, once the start becoming or cells start to buckle then it also tried to get compacted. The stiffness may then increase again as the cells become compacted and considerable nonlinear strains may be recoverable in such substances of course, if the stress becomes high enough the cell walls will crush and the strain may not then be recovered. So, that is also possible, but most of the, you would have seen that damping materials these cellular materials are used for damping application.

So, that means, it can take lot of strain which may be recoverable large amount of strain considerable non-linear strains can be recoverable from such substances beyond I mean up to a certain point till the cells will start crushing. In tension, clearly, the cell walls do not buckle elastically in the same way, as a typical stress-strain curve is shown in the figure. So, this is the reason why it looks I mean the energy absorbed in the elastic region is compensatively small because of the buckling mechanism is not operational in the tension mode. So, the cell walls do not buckle elastically. So, it will be difficult.

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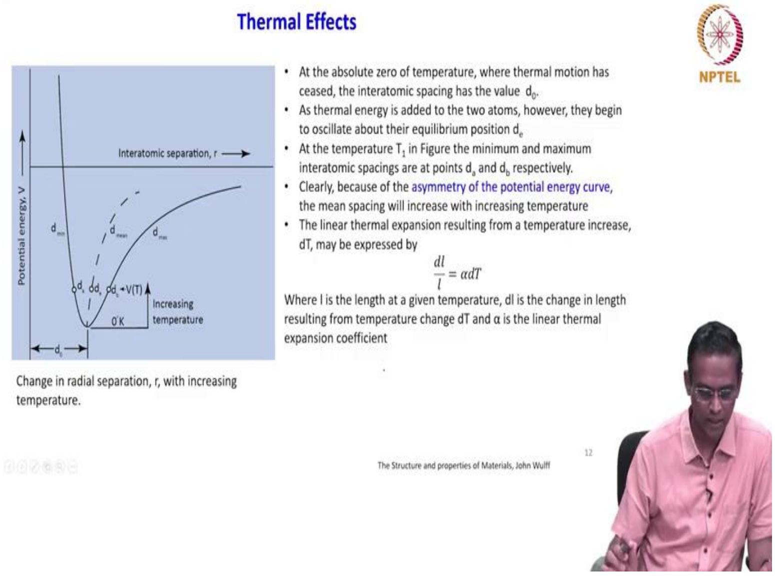

Thermal Effects

Change in radial separation, r, with increasing temperature.

- At the absolute zero of temperature, where thermal motion has ceased, the interatomic spacing has the value d_0.
- As thermal energy is added to the two atoms, however, they begin to oscillate about their equilibrium position d_e
- At the temperature T_1 in Figure the minimum and maximum interatomic spacings are at points d_a and d_b respectively.
- Clearly, because of the asymmetry of the potential energy curve, the mean spacing will increase with increasing temperature
- The linear thermal expansion resulting from a temperature increase, dT, may be expressed by

$$\frac{dl}{l} = \alpha dT$$

Where l is the length at a given temperature, dl is the change in length resulting from temperature change dT and α is the linear thermal expansion coefficient

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Coming back to our Condon-Morse curve once again just to give a completion of thermal effects, so, we just talked about elastic response by stress as well as temperature we said, in order to change the shape or volume of a material initially we said that there are two things which we can do either we can load the material or apply the stress or change the temperature these two things can bring some change in shape and volume of the material.

So, we have seen this diagram already, just to complete our elastic property also we need to bring this aspect because temperature also plays a significant role. So, what is what is shown here, potential energy versus distance diagram and this you are not very much familiar that is d_0 is in equilibrium position and if you start increasing the temperature and of course, this equilibrium position d_0 is valid at 0 Kelvin and as move from 0 K to higher temperatures. Then the material will tend to oscillate between these two positions, the a and b atoms will try to oscillate and as the temperature increases, then the oscillation distance also increases, this is what we are seeing. This is what we have seen already. So, clearly because of the asymmetry of the potential energy curve. So, this is asymmetric nature the mean spacing will increase with increasing temperature this aspect also we have already seen just for completion here I am doing it here.

The linear thermal expansion resulting from the temperature increase dT, may be expressed as dl / l = αdT, where α is thermal expansion coefficient and l is the length at which the given temperature, dl is the change in length resulting from the temperature change dT, this also we have seen.

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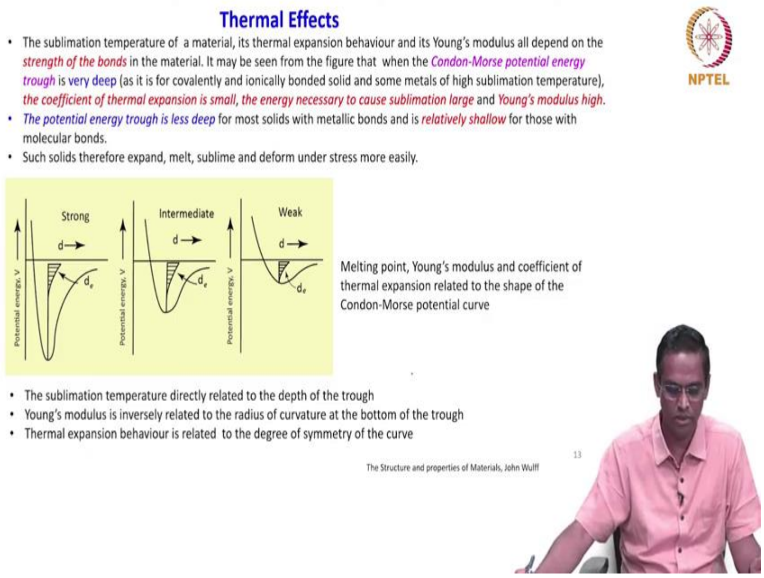

Thermal Effects

- The sublimation temperature of a material, its thermal expansion behaviour and its Young's modulus all depend on the *strength of the bonds* in the material. It may be seen from the figure that when the *Condon-Morse potential energy trough* is very deep (as it is for covalently and ionically bonded solid and some metals of high sublimation temperature), *the coefficient of thermal expansion is small*, *the energy necessary to cause sublimation large* and *Young's modulus high*.
- *The potential energy trough is less deep* for most solids with metallic bonds and is *relatively shallow* for those with molecular bonds.
- Such solids therefore expand, melt, sublime and deform under stress more easily.

Melting point, Young's modulus and coefficient of thermal expansion related to the shape of the Condon-Morse potential curve

- The sublimation temperature directly related to the depth of the trough
- Young's modulus is inversely related to the radius of curvature at the bottom of the trough
- Thermal expansion behaviour is related to the degree of symmetry of the curve

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So, now; getting into the again physical properties because of this temperature response. The sublimation temperature of material and its thermal expansion behaviour and its Young's modulus all depend on the strength of bonds in the material. So, we were just relating the bond strength with elastic constants earlier stiffness and all we have just seen. Now, we are also seeing that sublimation temperature of a material is also connected to the strength of the material.

So, it may be seen from the figure that when a Condon-Morse potential energy trough is very deep. We are now talking about this diagram is interesting diagram you can see that, so, the Condon-Morse curve is of different nature here. So, this is for what is written here it is written strong that means, very deep. So, the trough is very deep then what it means, it is acid for covalently ironically bonded solid and some metals of high sublimation temperature.

So, a material with stronger bonds, ionic covalent or high melting point materials, they are Condon-Morse curve will have a deep, very deep trough like this. So, similarly, the coefficient of thermal expansion is small and the energy necessary to cost the sublimation large and Young's modulus is high. So, these are the characteristic of when you have this potential well diagram is by just looking at the potential well diagram.

You will be able to relate this to thermal expansion coefficient energy required to cause sublimation and Young's modulus. So, they will be very high for a material of this nature. So, if the potential energy trough is less deep for most solid with the metallic bonds and it is relatively shallow for those with the molecular bonds, so, see for the metallic bond, we can relate this for molecular bond like polymers, this can be like this.

So, such solids therefore expand, melt, sublime and deform under stress more easily. So, now, we have we are now relating just by looking at the potential energy curve we are talking about deformation. So, this is idea for which you know we have been discussing all these days. So, when you have a shallow potential energy trough, then it can the material can easily melt, it can easily sublimate or it undergoes sublimation or it can undergo deformation with more easily.

So, this is very important point you have to keep in mind. So, the sublimation temperature is directly related to the depth of the trough, Young's modulus is inversely related to the radius of the curvature at the bottom of the trough and thermal expansion behaviour is related to the degree of symmetry of the curve. So, these are the additional points, we have some general remarks whatever we have seen whether it is a deep or shallow or you know weak and so on.

And further you can relate this symmetry of this curve to sublimation temperature Young's modulus and thermal expansion behaviour, through these three points. So, what we are seeing here is the important message what we are trying to give from this discussion is the bonding nature has a strong connection to its mechanical behaviour, whether it is elastic or plastic and so on. Right now, we are discussing elastic deformation; we will also see the plastic deformation in due course.

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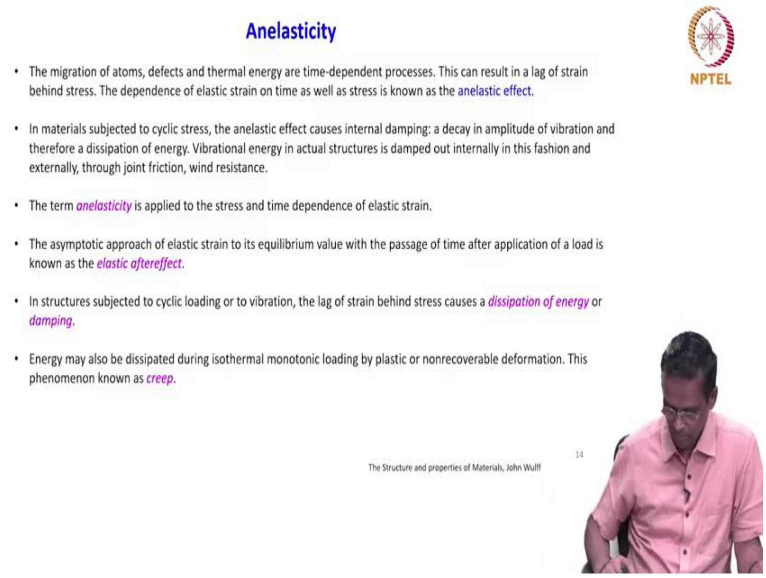

Anelasticity

- The migration of atoms, defects and thermal energy are time-dependent processes. This can result in a lag of strain behind stress. The dependence of elastic strain on time as well as stress is known as the anelastic effect.
- In materials subjected to cyclic stress, the anelastic effect causes internal damping: a decay in amplitude of vibration and therefore a dissipation of energy. Vibrational energy in actual structures is damped out internally in this fashion and externally, through joint friction, wind resistance.
- The term *anelasticity* is applied to the stress and time dependence of elastic strain.
- The asymptotic approach of elastic strain to its equilibrium value with the passage of time after application of a load is known as the *elastic aftereffect*.
- In structures subjected to cyclic loading or to vibration, the lag of strain behind stress causes a *dissipation of energy* or *damping*.
- Energy may also be dissipated during isothermal monotonic loading by plastic or nonrecoverable deformation. This phenomenon known as *creep*.

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So, next fundamental aspect I want to discuss is a term called anelasticity very important terminology phenomenon we all know this terminology, but we may be just looking at the behaviour directly in a day-to-day life or and we may not be realizing that you know what is happening to this material and so on. And this section of this lecture will be little descriptive, but conceptually It is very important.

So, I want to mention that you know, we will just discuss but slowly so that you will not miss any important points. The migration of atoms, defects and thermal energy, we are talking about migration of atoms, defects, thermal energy or time-dependent processes. This can result in a lag of strain behind stress. The dependence of elastic strain on time as well as stress is known as anelastic effect.

So, we are throwing a lot of terms here each term is very tricky and important also we are talking about time-dependent processes, migration of atoms, defects thermal energy and so on this can result in a lag of strain behind stress. So, what does it mean you may apply stress rapidly you will not see the strain effect immediately, it may happen the strain effect will come as a time pass by that is what that is the meaning of the lag of strain behind the stress.

You may do certain activity loading, unloading, heating, cooling we can do all these activities in a much higher rate. So, the all this activity is supposed to create some strain in the material, but the strain is not happening immediately then we say that or if the strain happens or the strain experienced with the material you can observe as a function of time, then we say that the lag of strain behind the stress so, that is the meaning of this.

So, we are now going to discuss a lot of physical events which may we know it already but we would not have connected to this concept quite often and we are going to look at all those physical events and then connect this with anelastic property. So, the definition is the dependence of elastic strain on time as well as to stress is known as anelastic effect. So, we are going to look at some physical event of anelastic effect.

The material subjected to cyclic stress the anelastic effect causes internal damping, what is this internal damping? A decay in amplitude of vibration and therefore, a dissipation of energy, vibrational energy in actual structures is damped out internally in this fashion and externally, through joint friction, wind resistance etcetera. So, we are now trying to give some direct physical events for this thermo elastic effect.

So, one of them is like the one of the consequences I would say is the internal damping behaviour. So, you all know what is damping just now we have just discussed about the cellular material which is also being used for the damping you know it that means what it absorbs lot of energy. So, what it means, so, it takes load in a very high strain it can take a quite recoverable elastic strain but then it can also you know relax as a function of time.

So, this is very important, we will also see the parameter little later something called relaxation times. So, this is very important aspect to focus on. So, that is internal damping, the decay in amplitude of vibration and therefore, the dissipation of energy. So, this also we will see now and these are all vibrational energy in actual structures is damped out internally in this fashion and then externally through joint friction, wind resistance and so on.

So, the term and elasticity is applied to stress and time dependence of elastic strain. So, time dependence of elastic strain is anelasticity. We will be using this terminology later in plastic deformation also, like you know, we will be using this again the same concept in when we discuss high temperature deformation behaviour called creep and so on. So, this is a fundamental starting point the asymptotic approach of elastic strain to its equilibrium value with the passage of time after application of load is known as elastic aftereffect.

So, there are different terminologies people have given but the physical event is the same. What is the physical event? Asymptotic approach of elastic strain that means, what it the strain will start slowly coming down and it will never touch the 0 it will just go parallel to the lowest possible it will try to go and you know these two curves will never meet it will meet the infinity it will go towards infinity very slowly it will come down. So, that is asymptotic approach of elastic strain to its equilibrium value with the passage of time.

So, that is important for example, in structure subjected to cyclic loading or to vibration the lag of strain behind stress cause dissipation of energy or damping. So, this is what I just said this very simple example is damping vibration damping or it could be any other damping as well. But it is more easy to realize when the structure is subjected to cyclic loading. Vibration is also as kind of a cyclic it is not just constant amplitude vibration; it will go with some particular frequency.

So, the energy may also be dissipated during isothermal monotonic loading by plastic or non-recoverable deformation. This phenomenon is known as creep. I just mentioned, we will look at this plastic deformation, high temperature plastic deformation this also involves a non-recoverable deformation that also can happen as a function of time.

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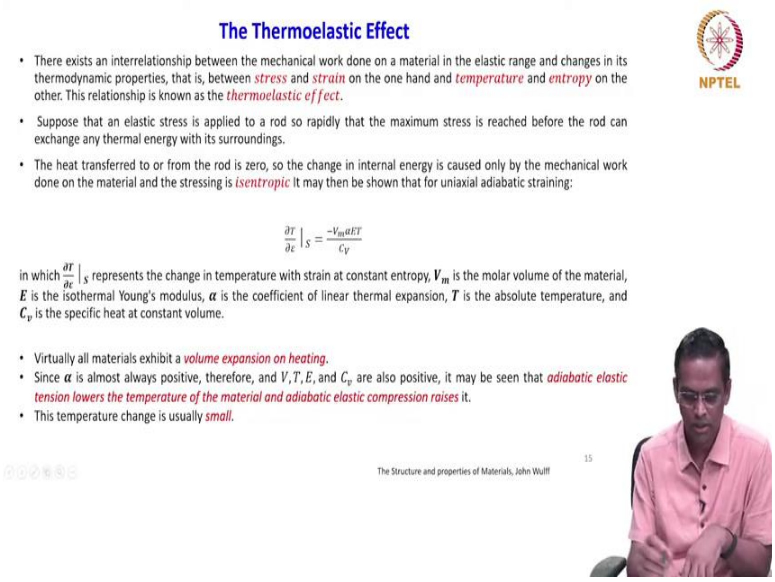

The Thermoelastic Effect

- There exists an interrelationship between the mechanical work done on a material in the elastic range and changes in its thermodynamic properties, that is, between *stress* and *strain* on the one hand and *temperature* and *entropy* on the other. This relationship is known as the *thermoelastic effect*.
- Suppose that an elastic stress is applied to a rod so rapidly that the maximum stress is reached before the rod can exchange any thermal energy with its surroundings.
- The heat transferred to or from the rod is zero, so the change in internal energy is caused only by the mechanical work done on the material and the stressing is *isentropic* It may then be shown that for uniaxial adiabatic straining:

$$\frac{\partial T}{\partial \varepsilon}\Big|_S = \frac{-V_m \alpha E T}{C_V}$$

in which $\frac{\partial T}{\partial \varepsilon}\Big|_S$ represents the change in temperature with strain at constant entropy, V_m is the molar volume of the material, E is the isothermal Young's modulus, α is the coefficient of linear thermal expansion, T is the absolute temperature, and C_v is the specific heat at constant volume.

- Virtually all materials exhibit a *volume expansion on heating*.
- Since α is almost always positive, therefore, and $V, T, E,$ and C_v are also positive, it may be seen that *adiabatic elastic tension lowers the temperature of the material and adiabatic elastic compression raises* it.
- This temperature change is usually *small*.

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So, first we will see some basic physics behind this thermo elastic effect. There exists an interrelationship between the mechanical work done on the material in the elastic range. Please understand, we are now talking about only elastic property do not think that we are talking about plastic property we are we are still talking about elastic properties and all this load stress we are talking about still in elastic region.

So, the material is in elastic range and changes in its thermodynamic properties, what are the unique thermodynamic properties here? We know lot of thermodynamic properties in enthalpy, free energy, temperature, entropy and so on. So, the relationship here, what are the interrelationship here we are talking about between stress and strain that is mechanical work parameter on the other hand temperature and entropy of the material.

This relationship is known as thermo elastic effect. So, the interrelationship between mechanical work and thermal dynamic properties, here it is a stress and strain and temperature and entropy. So, how does it how do they relate suppose, that anelastic stress is applied to a rod so, rapidly that the maximum stress is reached before the rod can exchange any thermal energy with its surroundings.

This is exactly we are discussing about we rapid loading then what happens to the material. So, in this case, the elastic stress is applied to a rod so rapidly that the maximum stress is reached before the rod can exchange any thermal energy with its surroundings. Then what happens? The heat transferred to and from the rod is 0. So, the change in internal energy is caused only by the mechanical work done on the material and the stressing is isentropic.

So, it may then be shown that for uniaxial adiabatic straining, because it is there is no thermal exchange it is considered adiabatic straining then it can be written as

$$\frac{\partial \mathrm{T}}{\partial \mathcal{E}} | s = \frac{-\mathrm{Vm}\alpha\mathrm{ET}}{\mathrm{Cv}}$$

of the parameters you know $\frac{\partial \mathrm{T}}{\partial \mathcal{E}} | s$ represent the change in temperature with respect to strain at constant entropy Vm is the molar volume of the material, E is the isothermal Young’s modulus, α is coefficient of linear thermal expansion, T is the absolute temperature and C_v is a specific heat at constant volume. So, this is the relation where thermo elastic effect virtually all material exhibits a volume expansion on heating this we all know all materials in general exhibit volume expansion on heating, but certain material gives opposite you should just guess or you would have seen this already.

Since α is almost always positive therefore, V, T, E and C_v are also positive it may be seen that adiabatic elastic tension lowers the temperature of the material and adiabatic elastic compression raises it. So, these are the some of the salient features we have to note down, then we will just look at some examples, the temperature what we discuss here is the temperature changes usually small you should not think that it is a huge, the temperature effect which we talked about in all this is small.

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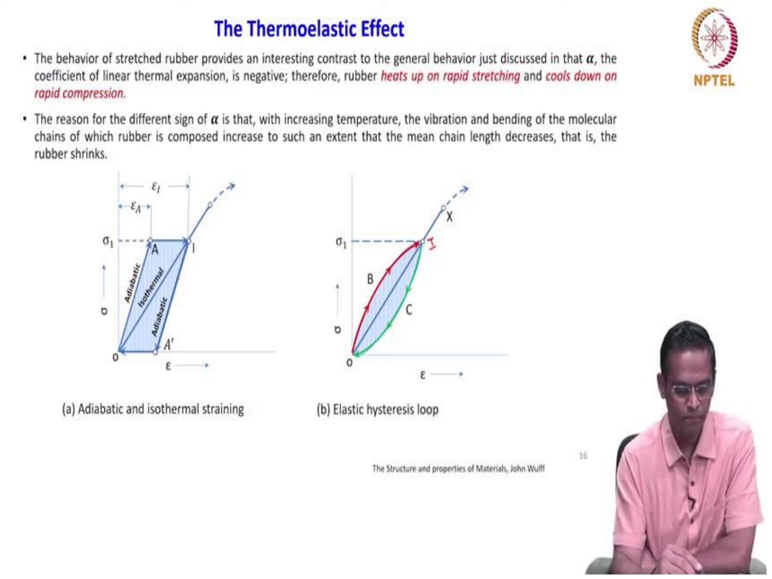

The Thermoelastic Effect

- The behavior of stretched rubber provides an interesting contrast to the general behavior just discussed in that α, the coefficient of linear thermal expansion, is negative; therefore, rubber *heats up on rapid stretching* and *cools down on rapid compression.*
- The reason for the different sign of α is that, with increasing temperature, the vibration and bending of the molecular chains of which rubber is composed increase to such an extent that the mean chain length decreases, that is, the rubber shrinks.

(a) Adiabatic and isothermal straining

(b) Elastic hysteresis loop

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So, the behaviour of stretched rubber provides an interesting contrast to the general behaviour just discussed in that α the coefficient of linear expansion is negative therefore, rubber heats up on rapid stretching and cools down on rapid compression. So, this is something you know contradict to what just we just said we said in general most of the material tend to expand on heating, but there are exceptions.

So, here we see that the stretch rubber, the moment you stretch the rubber the linear expansion, thermal expansion is negative what could be the reason, the reason for a different sign of α is that with increasing temperature the vibration and bending of the molecular chains of which the rubber is composed of increased to such an extent that the mean chain length decreases and that is the rubber shrinks.

So, we have looked at some of these basic molecular chains in the previous classes. So, the vibration and bending of the molecular chains causes a mean chain length decrease in the mean chain length. So, which attributes to the shrinkage of this rubber while heating, so, that is one observation why it is giving negative behaviour. Now, what I am going to do is we are going to talk about this thermoelastic effect in a much more realistic manner.

And for that, I am just going to describe the two types of stress-strain behaviour and what is shown here is σ versus ε and you just assume that any crystalline material is now being, you know, warmed up or stretched. So, when it stretched, it is supposed to warm up that is the general behaviour. So, you just think such kind of a situation when the material is stretched. There are two kind of stretching is possible like as I said very slow stretching that means also equilibrium stretching you can say that or we can do it very fast.

Like I, we describe we described certain events before we can do it very rapidly. So, what you can see here is there is two situations here adiabatic straining or isothermal straining, suppose if I am straining a material slowly. So, this there is no problem with any thermodynamic aspects, this is called isothermal straining. On the other hand, if I stretch it very fast, rapidly and I am not going to get the final strain which I am supposed to reach ε_I here instead I land up in ε_A.

Instead of ε_I, this is my original plastic strain if I stretch it slowly and if I stretch it abruptly then I reach a point A where I get only ε_A of the strain. So, that means this is that is no temperature exchange with these surroundings. So, this is adiabatic process and then if I allow that material to relax and then it warms up and then it relaxes to the strain ε_I. So, that is one way of looking at it.

Similarly, instead of unloading slowly, which is also again will pass through this line, I do it abruptly here then again it is exactly the opposite you know things will happen thermodynamic properties, I mean temperature exchange and so on. It will just it will not go to 0 or that means it will not recover all the strain instead it will go to A’ as the time passed by it will just slowly reaches its original or equilibrium strain.

So, this kind of a cyclic loading or cyclic event like stretching and stretching or stretching relaxation involves some energy dissipation. So, that is energy shown here. So, which is called you know elastic hysteresis loop the hashed region here is the amount of energy involved in this process whatever I just shown this is a hypothetical situation, but what is shown here is an actual energy per cycle. 0 to 0 OBX or 0BX or 0, this again it can be I also you can put I so, 0BI and an ICO cycle. So, it could be either isothermal cycling or adiabatic cycling. So, that is the idea.

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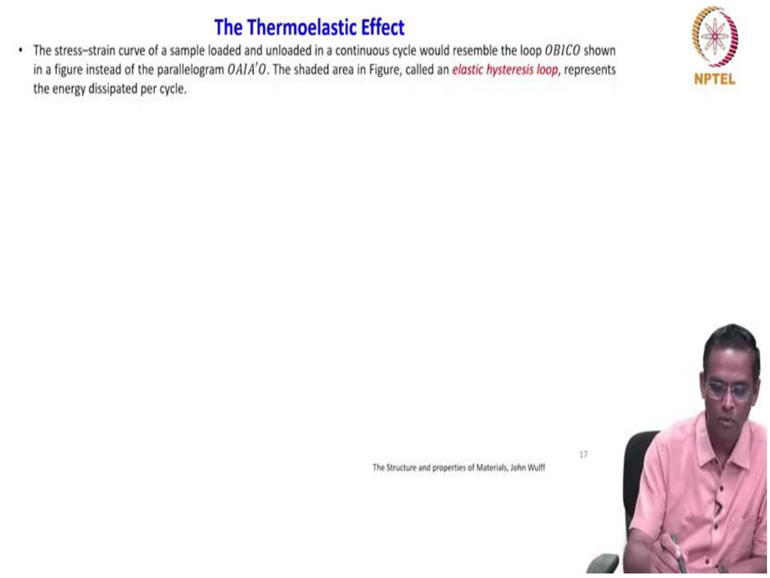

The Thermoelastic Effect

- The stress–strain curve of a sample loaded and unloaded in a continuous cycle would resemble the loop $OBICO$ shown in a figure instead of the parallelogram $OAIA'O$. The shaded area in Figure, called an *elastic hysteresis loop*, represents the energy dissipated per cycle.

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So, with this background we will now look at some other details the stress-strain curve of a sample loaded and unloaded in a continuous cycle would resemble the loop OBICO shown in the figure instead of parallelogram OAIA’O.