Mechanical Behavior of Materials
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Lecture - 10
Elastic Properties Part - I

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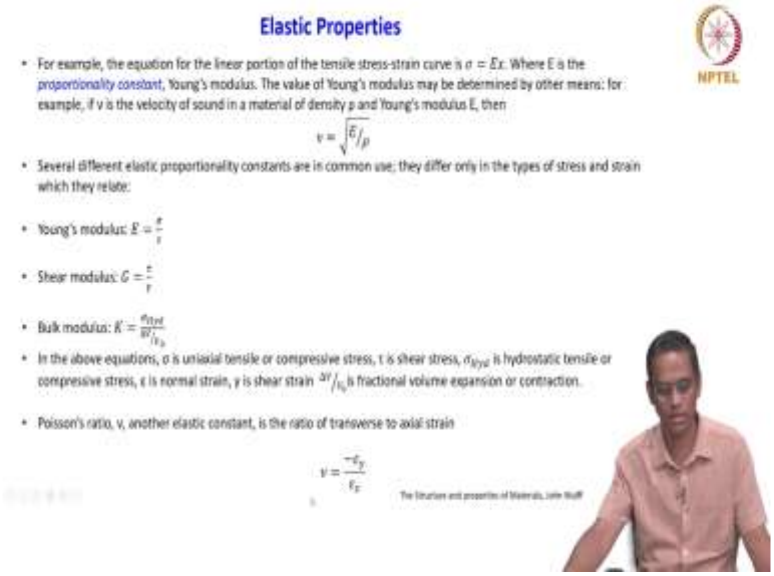

Hello I am Professor S. Sankaran in the department of metallurgical and materials engineering. For example the equation for the linear portion of this tensile stress strain curve is

$\sigma = Ex$

where E is the proportionality constant that is Young's modulus. The value of Young's modulus may be determined by other means for example if ν is the velocity of sound in a material of density ρ and Young's modulus E, then there is a relation

$\nu = (E/\rho)^{1/2}$. This is one way of finding an Young's modulus.

The other way is the usual tensile deformation within the elastic region the slope is equal to Young's modulus. And similarly we have we are looking at E here as a proportionality constants as per the linear elastic theory that is what the previous slide last statement said. And we are now looking at what are the proportionality constants in terms of elastic behaviour. So, several different elastic proportionality constants are in common use.

They differ only in the type of stress and strain which they relate. This also we have seen already just to give a perspective of what is proportionality constants I have brought it again. So, this is Young's modulus which is E = σ /ε and shear modulus G = τ / γ, bulk modulus K = σ (hydrostatic) /volume change in volume. The above equation σ is uniaxial tensile or compressive stress; τ is shear stress, σ hydrostatic tensile or compressive stress.

ε is normal strain, γ is a shear strain, $\Delta v / v_0$ is a fractional volume expansion or contraction. Poisson's ratio ν , another elastic constant, it is the ratio of transverse to axial strain

$\nu = - \varepsilon_y / \varepsilon_z$.

This equation we have used extensively in the principle of superposition and then from there we looked at generalized Hooke's law that is very familiar to you know.

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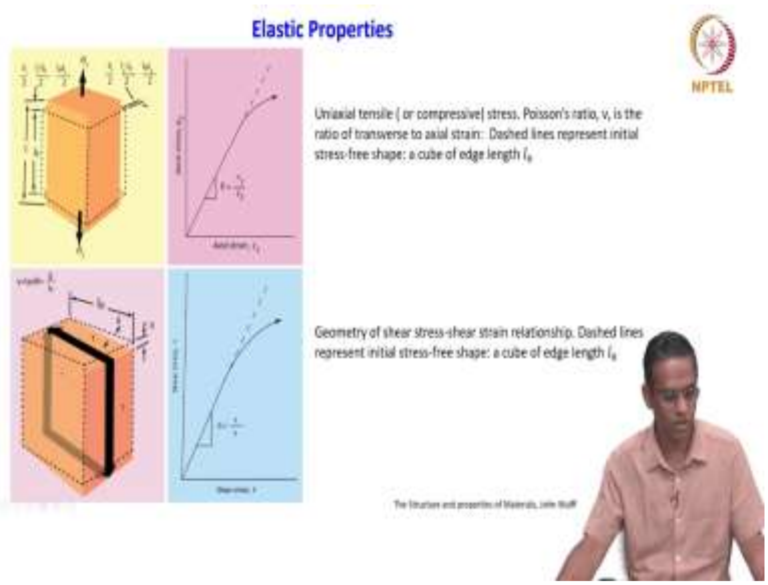

So, the diagrams which are going to show now is going to explain geometrically whatever we have just seen in terms of you know the elastic this is a member which undergoes tensile deformation. So, you see that l_0 becomes l after the deformation and then you see that change in length and then you try to calculate the displacement and then from there to calculate the stress and strain.

Similarly this is for a shear stress shear strain relationship geometrically to show what you arrive at here is

$E = \sigma_z / \varepsilon_z$

this is the z direction. Similarly here

$G = \tau / \gamma$. So, all, these shear I mean how the dimensionally it can vary this is already familiar to you, we have already discussed so I just brought it because we are talking about elastic properties and this is how it is related.

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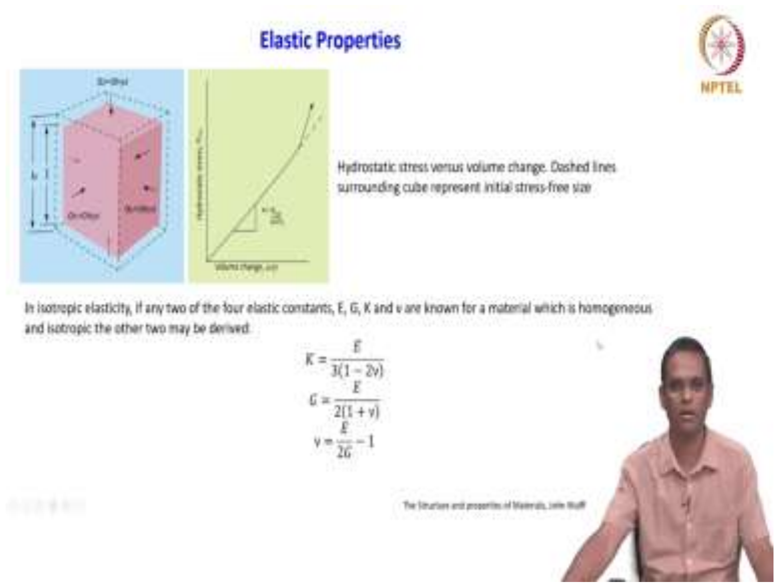

And finally this is hydrostatic stress versus volume change, the slope is

K = σ hydrostatic divided by a fraction of volume change and you see that geometry the stresses equally applied from all over the place here it is a compression. So, the initial dimension l_0 becomes l the final dimension. So, it is not in one direction all other directions are getting compressed so it is hydrostatic compression.

So, coming back to this isotropic elasticity if any two of the four elastic constants that is E, G, K and ν are known for a material which is homogeneous and isotropic the other two maybe derive this also we have already seen some of the relations we have demonstrated but then these are the other relations we will be using this relationship in the fracture problems in solving fracture problems or any failure analysis problems will be it is quite useful.

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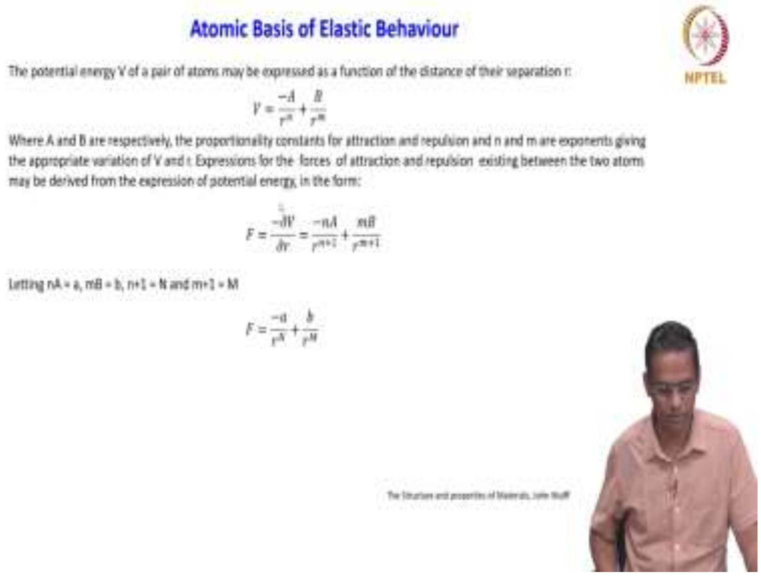

Now we will go back to what we have just discussed atomic basis of elastic behaviour I just said in the beginning we will come back to this topic little later and then I also mentioned that we will discussed in much more detail about this atomistic basis for elastic behaviour. So, the potential energy V of pair of atoms may be expressed as a function of distance of their separation r where $V = - A / r^{n} + B/r^{m}$.

So, we have talked about this exponents n and m initially, so have to save time I am just going little fast where A, B are constants and proportionate constants for attraction repulsion and n and m exponents giving appropriate variation of V and r. The expressions for the forces of attraction repulsion existing between two atoms may be derived from the expression of potential energy in the form. So, this also we have already seen. So, from this is a potential, energy this is a force, so this is just a simplification.

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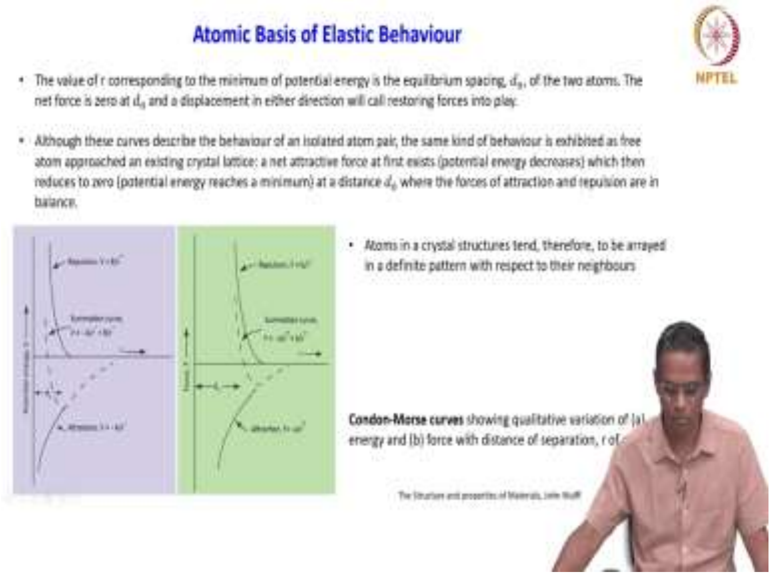

We will now plot these two, potential energy versus distance and this is a force versus distance and we know that this is repulsion and this is attraction and this is a net dashed lines are net. And what is to be noted here is importantly the value of r corresponding to the minimum of potential energy is the equilibrium spacing d_0. So, here it is the minimum the potential well here and this distances d_0 that is the equilibrium spacing.

And remember the net force is 0 here the net forces 0 at d 0, d_0 for this is d_0 and this is net force is a 0. So, although these curves describe the behaviour of an isolated atom pair the same kind of behaviour is exhibited as a free atom approached an existing crystal lattice and net attractive force at first exist potential energy decreases which then reduces to 0 potential energy reaches a minimum at a distance d_0 where the forces of attraction repulsion are in balance.

So, this is some kind of a description about this region. Atoms in a crystal structures tend, therefore to be arrayed in a definite pattern with respect to their neighbors. So, because of this force balance the atoms are trying to be in their respective I mean in equilibrium with the respective neighbours. So, I have already introduced this name Condon Mores curve these type of curves.

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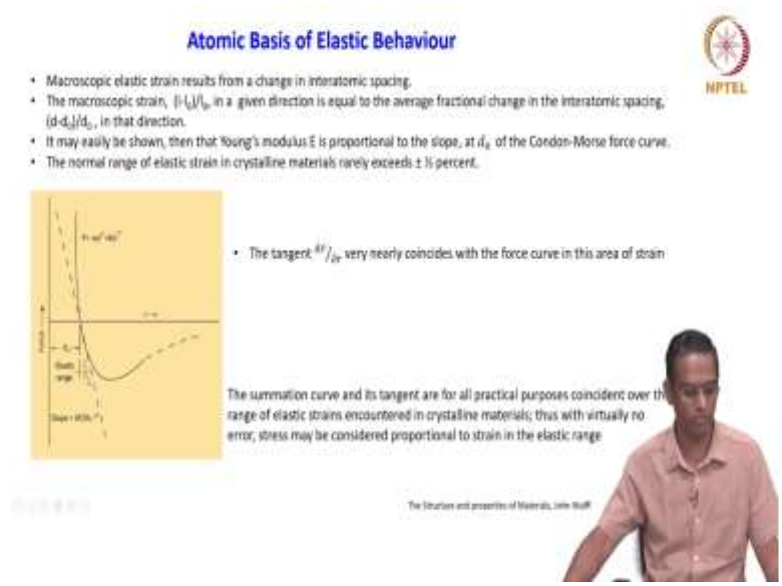

And now we will try to relate this with the Young's modulus, so what is that now we are not taking a force versus r plot, we are only showing the net force, we are not showing that attraction repulsion here it is only a net force and we know that it is a d_0 which is an equilibrium. So, the slope at this point at this point d_0 is showing the elastic range that is nothing but $\partial F / \partial r$ that force what we have just seen in the previous slide is from this idea.

So, this elastic range is marked here, so you can appreciate this is the atomic basis for the elastic behavior. So, we will just see the salient features, so macroscopic elastic strain results from a change in inter atomic spacing. So, we are now talking about macroscopic strainplease note it is not just two atoms now, we are talking about a bulk property now the that is macroscopic strain, elastic strain, please elastic strain that is resulting from the change in interatomic.

So, macroscopic strain is what this is what we know $l - l_0 / l_0$ in a given direction is equal to average fractional change in the interatomic spacing that is $d - d_0 / d_0$ in that direction. So, this is valid for a given direction in fact in the beginning of this course we have also looked at some normal strain and average strain and so on. But here we are also talking about the macroscopic strain similarly but then if you are specifically interested in the direction then you have to relate that way that is the idea.

This is already we have seen so the normal range of elastic strain in crystalline materials rarely exceeds ±1/2%. So, this is a thumb rule just to have an idea so, the tangent $\partial F / \partial r$ here very nearly coincides with the force curve in this area of the strain. So, this is one confirmation to show that this assumption is not bad this is good. So, the elastic range really coincides with this.

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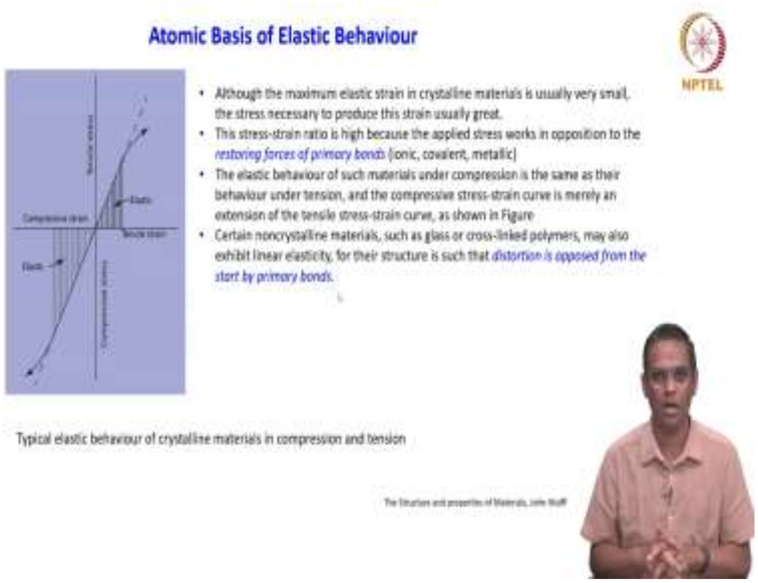

So that was kind of atomic basis discussion on elastic modulus of materials. So, now we go to the bulk material behaviour but we are still talking about elastic properties but we will talk about bulk behaviour. What is this diagram which is shown here this is very interesting diagram you can see that a tensile stress and this stress is Y axis strain is X axis and it is further divided by four compartments tensile stress, compressive stress, tensile strain, compressive strain.

And then the stress strain plot is going through the center that means the material is showing elastic behavior in tensile region as well as in compression region that means it is something like you take a material elastic material pull it in one direction and then compress it. So, how the stress strain behaves or what is that linear relationship you are seeing, something like that so very interesting. So, a typical elastic behavior of crystalline materials in compression and tension, we are now talking about crystalline materials please remember that. Crystalline material elastic both tension and compression although the maximum elastic strain in crystalline material is usually very small the stress necessary to produce this strain usually great very important point, you have to pay attention to this details.

It looks all very you know familiar to us but then if you just pay a little more attention the new information you will get what is that? Though the elastic strain maximum, elastic strain is very small even though they are very small elastic strain the material experiences. The stress necessary to produce the strain, even the small strain will be much more very important. This stress-strain ratio is high because the applied stress works in opposition to the restoring forces of primary bonds.

See now you see immediately we are not talking about mechanical behaviour but we are now talking about bonding. So, now you will realize why we started with chemical bonding. So, this strain ratio is high because the applied stress works in opposition to the restoring force of primary bonds (ionic, covalent and metallic). So, what it means is you are applying some load that load has to work against the primary force restoring what is the restoring force?

That is a chemical bonding for any material. So that is the restoring force it has to work against the restoring force, that means unless you have some idea about the restoring force which is a chemical bond and we are talking about load that means bond energy and bond strength unless you have some idea this cannot be related directly. So, now you appreciate that point the elastic behaviour of such materials under compression is the same as their behaviour under the tension.

And the compressive stress strain curve is merely an extension of the tensile stress strain curve as shown in the figure, very important aspect. We are talking about elastic behavior of a crystalline material both in tension and compression and this particular graph shows that the linear behaviour exactly the same in compression region as well as a tensile region. Certain non-crystalline materials such as glass or cross linked polymers may also exhibit linear elasticity for the structure is such that distortion is opposed from the start by primary bonds.

Some of the non-crystalline solids, glass or cross linked polymers they may also exhibit similar as points but not always the case. So, again the; how to work against the restoring force, like primary bonds and secondary bonds and so on. Especially in a Polymeric chain and all you have not just a primary bond like metallic bond or ionic bond alone it has also got secondary bonds like hydrogen bond, Vander Waals bond and so on.

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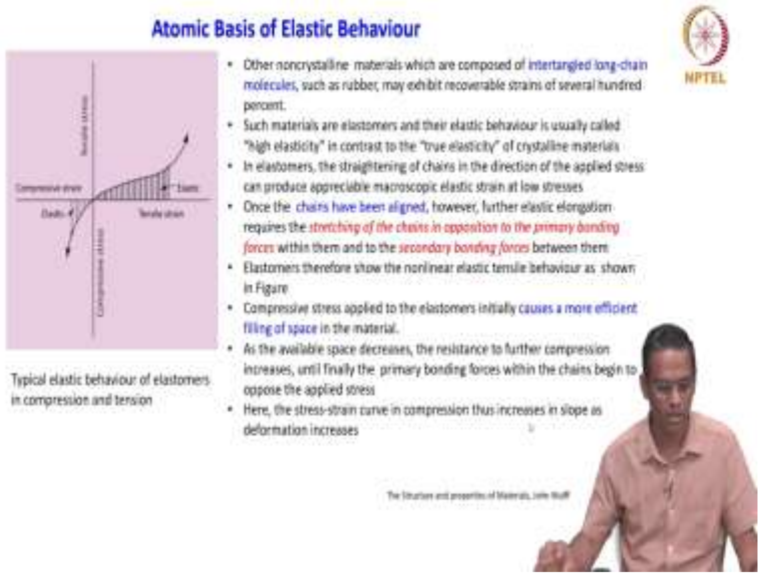

So, another plot is very interesting plot which looks very different from what we have just seen before let us try to describe this, what is that this plot shows. So, first of all this is not-linear both in compression and tension it is a non-linear elastic behaviour that itself quite interesting. But in tension it is quite I mean the kind of amount of elastic energy, the material takes is quite significant as compared to the compression region that elastic energy.

So, what could be the reason? So that again can be correlated with the type of bonding, so what is this? Typical elastic behaviour of elastomers in compression and tension. So, now we are getting to specific material type, previously we just talked about general crystalline materials now we are talking about elastomer that means you have to think about what is the type of bonding, what is the type of crystalline, nature and so on and so forth.

So, the other non crystalline materials which are composed of intertangled long chain molecules such as rubber may exhibit recoverable strains of several 100%. So, now we are not just talking about long molecular chain alone, there is another complication to that that is intertangled it is not just long molecular chain but also a tangled in nature, which will exhibit a recoverable strain of several 100%.

So that means lot of elastic energy it can absorb it is not just that the same thing can be recovered, which is very good. So, rubber is one example I also shown in the introduction video one eraser thick eraser I could bend it to any degree and then it could come back. Such materials are elastomers and their elastic behavior is usually called high elasticity in contrast to the true elasticity of crystalline materials.

So, this is just a simple nomenclature, In elastomers the straightening of chains in the direction of applied stress can produce appreciable macroscopic elastic strain at low stresses very important points we are talking about elastomers and we are talking about straightening, straightening of what? Straightening of the intertangled long chain molecules. So, that itself takes a lot of initial elastic energy or load whatever it is in a direction of applied stress to produce appreciable macroscopic elastic strain at low stresses.

Once the chains have been aligned however further elastic elongation requires stretching of the chains in opposition to the primary bonding forces within them and to the secondary bonding forces between them. So, the previous case the crystalline material we just talked about just a restoring force the applied load or a force have to act against the restoring force here it is slightly different.

Again before you go to restoring force there is some other obstacles are there, that is intertangled long chain molecules. So, first of all those intertangled long chain molecules have to become straight, so that itself takes some amount of energy and then further stretching it in elastic region. So, which will again act against the restoring force like primary bond and secondary bond so on.

So, therefore elastomers that show a nonlinear elastic tensile behavior as shown, so this is about elastic tensile behavior but why compression is a little different that we are to just see compressive stress applied to the elastomers initially causes a more efficient filling of space in the material. So, in entangled molecule if you try to pull it, you try to align first but instead of a tensile force you try to compress, then you can imagine a lot of you know already it is intertangled so all the gaps get filled. So that takes a lot of energy I mean but then but that energy is quite different from what we just talked about in a tension mode. So, in compressive stress takes or I would say cause a more efficient space filling of the material. So that is why it is quite steeper, it completely steeper as compared to the tension mode.

So, you can see that this curve is quite steeper compared to this tensile curve.

As the available space decreases the resistance to further compression increases, until finally the primary bonding forces within the chain begin to oppose the applied stress. So, all this space you know, get filled or the space available for further compression decreases increases the stress, compression stress that is why the slope is quite steeper as compared to the tension mode.

Here the stress strain curve in compression thus increases in slope as a deformation increases. So, similarly we will go to some other material and then look at how this stress strain behaviour looks like, maybe I will stop here and we will continue in the next class. Thank you.