Powder Metallurgy
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Lecture-23
Particle size distribution

Hello and welcome back. In the previous class we talked about particle size distribution and how you can measure it.

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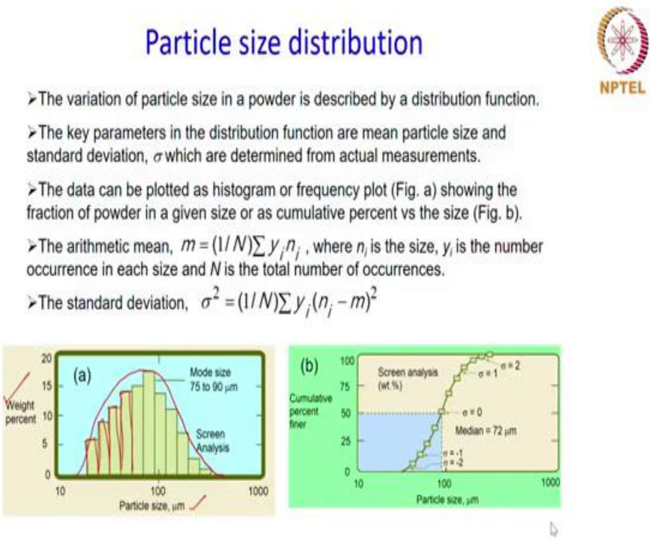

And if you remember we were talking about the two kinds of representation of the same data, when you describe the particle size distribution. One is through a histogram or a frequency plot like this and another is a cumulative plot like this (slide above).

The cumulative plot can be obtained if you add up the previous sizes and then plot it as a function of the cumulative percent finer. The same data can also be plotted the other way for example, as a function of cumulative percent coarser.

So, either the size above a given size or the size below a given size can be considered to construct the cumulative plot. We can take an example and then see how it is done.

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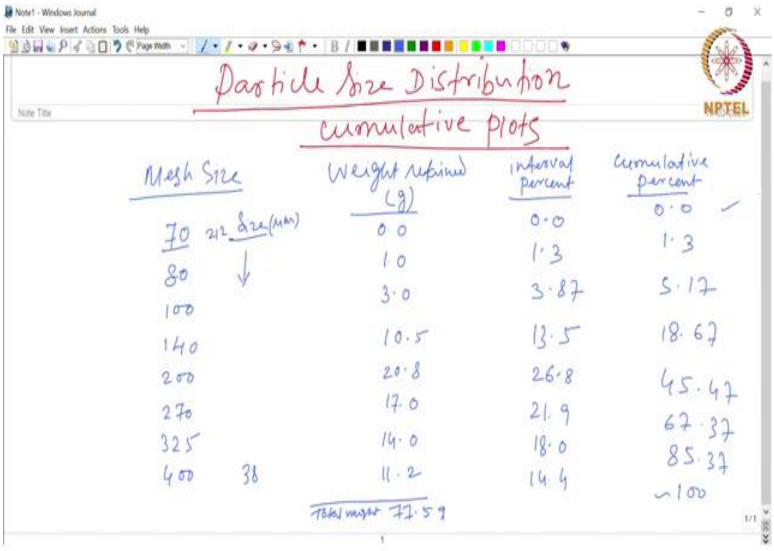

So, let us take this example (slide above). Let us say we have done the sieve analysis and collected this data. So, we have different mesh sizes. Let us say these are 70, 80, 100, 140, 200 and so on. The quantity of powder retained on each of the sieves is obtained from the sieve analysis. So, here you can see the total weight is 77.5 gram. So, if you calculate the interval percentages; that means, the fraction of a given size this is 0. So, this will also be 0. This one (2nd row 3rd column) is around 1.3 percent. And now, if you take the cumulative percent, it is obtained by the sum of terms in the third column up to the corresponding row. For example, 3rd row in cumulative percent column is filled by summing up the terms in interval percent column up to the 3rd row (0 + 1.3 + 3.87). 4th row is filled as (0.0 + 1.3 + 3.87 + 13.5) and so on.

So, here you can see, as the mesh size is increasing, the particle size is actually decreasing as you go down. For example, the corresponding size in micron for 70 is 212 microns and for 400 mesh it is 38 microns.

Thus cumulative percentages are calculated and plotted as shown below:

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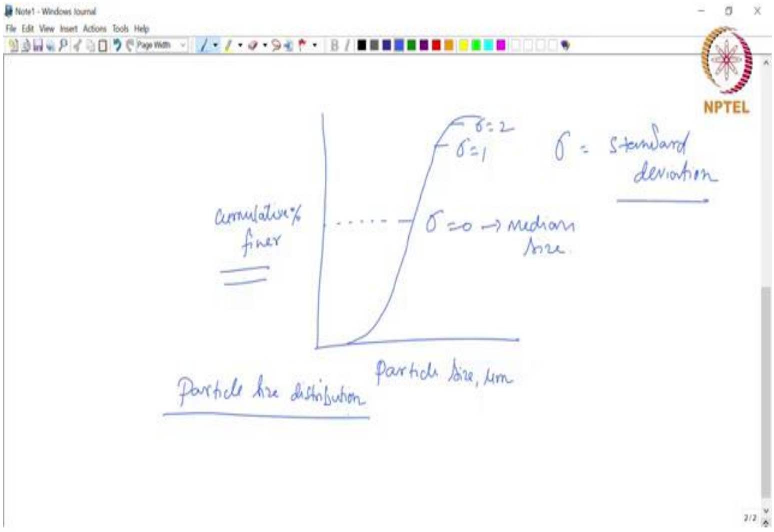

And it is typically an ‘S’ shaped curve like this and from this you can also derive the statistical information like the median size corresponding to $\sigma = 0$, $\sigma = 1$, $\sigma = 2$ and so on where sigma is the standard deviation.

So, that is how the cumulative data can be represented for particle size distribution.

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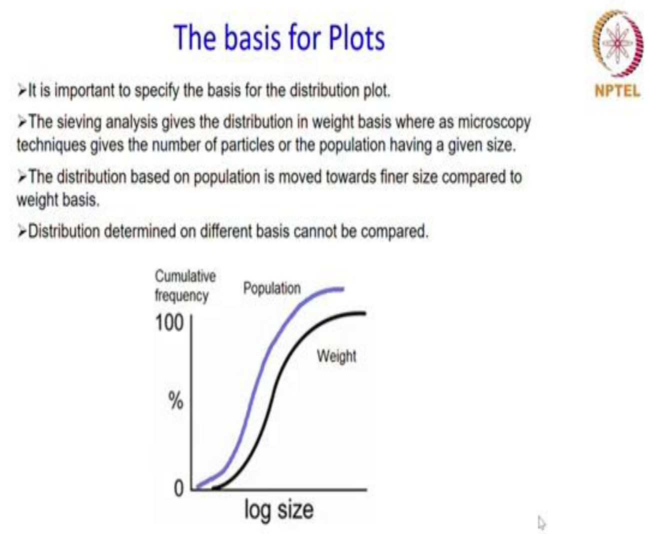

And one thing I must mention over here is that when you are representing the cumulative plots, it is important to specify the basis for the distribution. The data collection can be a population based or it can be weight based.

So, that is why when you are representing the cumulative distribution plots, it is important to specify what is the basis, because a given basis will give you a particular type of data which may not be identical when the same data is being recorded using a different basis.

Like here for example (slide above), you can see the two plots which represent the same data of particle size distribution of a given powder; since the basis is different (one is population, another is weight) they are not exactly the same. Although, the trend can be same, because it is collected from the same powder sample.

But since, the basis is different they are not exactly identical. So, this is why when you plot it and represent the data you should also mention the basis. And therefore, it is also true that the distribution determined on different basis cannot be compared.

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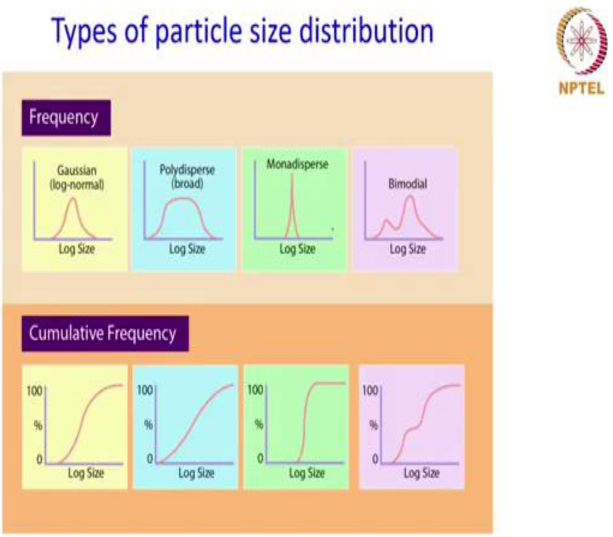
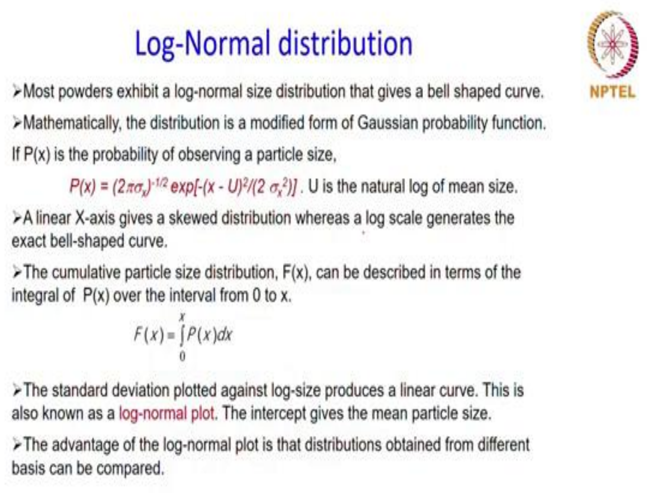


The log normal is a kind the distribution which is encountered very often. And that is why we are going to look at it in little more detail; because as I said most powders exhibit a log normal size distribution that will give you the bell shaped curve.

And, if you want to describe this distribution mathematically, Gaussian probability function can be use to describe it and which is given over here.

$$P(x) = (2\pi\sigma_x)^{-\frac{1}{2}}\exp\left(\frac{-(x-U)^2}{2\sigma_x^2}\right)$$

Wherein P(x) is the probability of observing a particular size and U is the natural log of the mean size. And if you use a linear X-axis that will give you a kind of a skewed distribution whereas, a log scale will generate the exact bell-shaped curve. And the cumulative distribution can be represented by this.



Cumulative plot is a summed up representation of the data. Here too, the data is summed up from a particular size to a given size, x, or over a particular interval from 0 to x.

$$F(x) = \int_0^x P(x)dx$$

And if you plot the standard deviation against the log of size that will give you a linear curve and that particular curve is known as a log-normal plot (slide below).

And from this curve we can derive some information. For example, the intercept will give you the mean particle size. And the advantage of the log-normal plot is that the distribution obtained from different basis can be compared, which is not possible with the cumulative plots.

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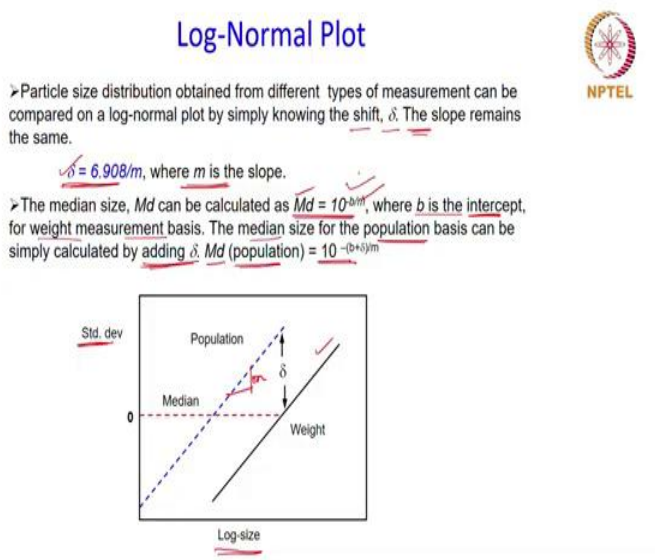

It will give you a straight line. And now, in order to compare the data which is collected from different using different basis, this is what you can do.

You can simply note the value of δ, or the difference between these two plots. And once you note the shift, the data can be easily compared. And the delta is given by this expression that you see over here. δ is given by,

$$\delta = \frac{6.908}{m}$$

where m is the slope.

So, now that you know the slope, you can easily calculate δ and you can simply compare different data collected using different basis. The median size Md is calculated with the help of the relationship below,

$$Md = 10^{-\frac{b}{m}}$$

where b is the intercept for weight measurement basis and m is the slope of the straight line.

Now, if you want to calculate the median size for another basis for example, the population basis; that can be simply obtained by adding this δ value. And therefore, the Md or the median size for the population based data would be

$$Md = 10^{-\frac{b+\delta}{m}}$$

So, that is the advantage of this plot. As I said you can compare data from different basis by simply knowing the shift between them or δ. So, you have seen in last couple of classes now, that how the particle size and the particle size distribution can be obtained for a given powder sample using certain techniques. You can see that the population based data move towards finer size compare to weight basis.

So, far we have only discussed about two techniques just to demonstrate on what basis the data is collected. So, we have seen two basis of determining the particle size distribution, one is weight based (screen size analysis) and another is the population based analysis (microscopy).

And you have seen how you derive the particle size and the particle size distribution from the data. So, before we close today's class, let us quickly summarize it. First of all, the particle size techniques use a particular basis to do the analysis of particle size measurement. That is the first important point to note about particle size measurement. And we have seen two examples of that. One was microscopy, which is population based and the other one was the screen or the sieving technique, which is weight-based.

In one case, you plot the data as the weight percent for a given size. And in the other case for the microscopy-based technique, you plot the count. So, that is exactly why it is known as the population-based method, because you actually report the population or the count that is the number of occurrences of a given size. So, that count is reported or plotted as a function of the particle size.

And then, as far as the particle size distribution is concerned, we have seen that there are two ways of representing it; one is through the histogram or the frequency plot and another is the cumulative plot where you sum up the previous particle sizes or particle size above or below a given particle size.

A typical ‘S’ shaped plot is obtained when the cumulative data is plotted. And here also, the important point to note is that the basis has to be specified, because the particle size distribution data determined from different basis cannot be compared, as they are not identical. And therefore, as I said it is important to specify the basis when you are representing particle size distribution data.

And the other thing that we discuss today was this log-normal distribution, because this is the most commonly occurring distribution that you can encounter in a given powder sample. And mathematical expressions were also discussed.

When you plot the standard deviation as a function of the particle size, you get a straight line. And in that case the data from different basis can be easily compared, because you can get δ or the shift between them.

So, once you know δ, the size obtained from one particular basis can be easily converted to other basis by simply adding up δ. And that is how you can represent different basis and obtain the same information of particle size using different techniques using different basis. So, with this we come to the end of this class.

Thank you for your attention.