Creep Deformation of Materials
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Part 2
Creep and different Factors that Influence Creep Deformation-Part 2.

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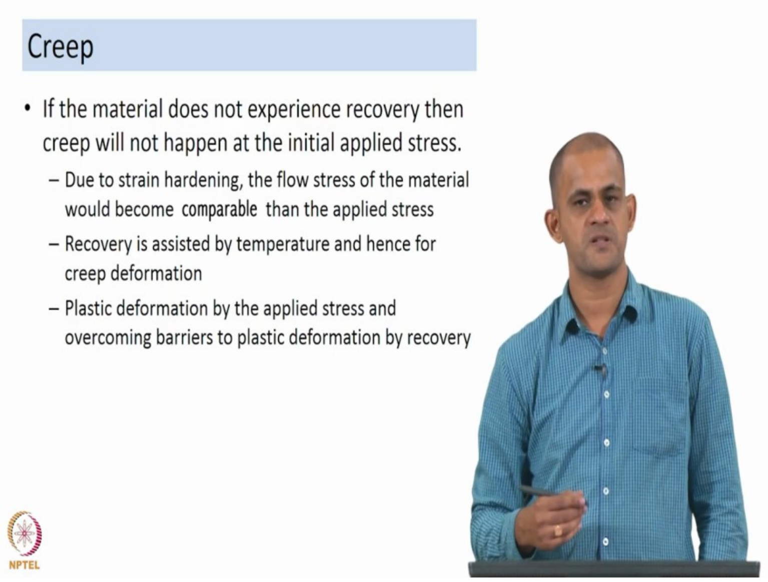

Creep

- If the material does not experience recovery then creep will not happen at the initial applied stress.
  - Due to strain hardening, the flow stress of the material would become comparable than the applied stress
  - Recovery is assisted by temperature and hence for creep deformation
  - Plastic deformation by the applied stress and overcoming barriers to plastic deformation by recovery

So, up to the last point, so we were talking about the sub grain formation in titanium alloys during the process of creep and the sub grain formation happens because the dislocations that are getting hindered during the motion try to overcome those barriers by recovery. So the recovery process requires high temperature. And that is how you have this subgrains where the boundaries of the sub grains are consistent, consisting of arrays of dislocations and the sub grain region is a more or less strain free.

Now the question that arises is what will happen if the material would not experience recovery. Well, the point is if the material does not experience recovery, then creep may or may not happen at the initial applied stress. Most probably it will stop or the creep deformation rates will be so low that for all practical purposes it will be imperceptible. Now, the reason behind this is that, creep as we understand is a competition between the mechanisms of strain hardening and the mechanisms of recovery.

So as creep is a plastic deformation process, so when creep is happening, you have dislocation motion, dislocations getting generated as well as dislocation motion. And this leads to strain hardening and when strain hardening happens, the strength of the material increases, that is the flow stress of the material increases and at some point of time the flow stress would become equal to the initial applied stress and so the material would stop creeping. Unless, unless there is recovery which is assisted by temperature and recovery allows these dislocations to overcome the barriers to motion.

So recovery is important for creep deformation and plastic deformation by the applied stress and overcoming barriers to plastic deformation by recovery, this is more or less what creep is. Now, one of the earliest studies on creep, the time dependence of strain was carried out by Andrade. And Andrade, based on his experiments, he proposed the following equation.

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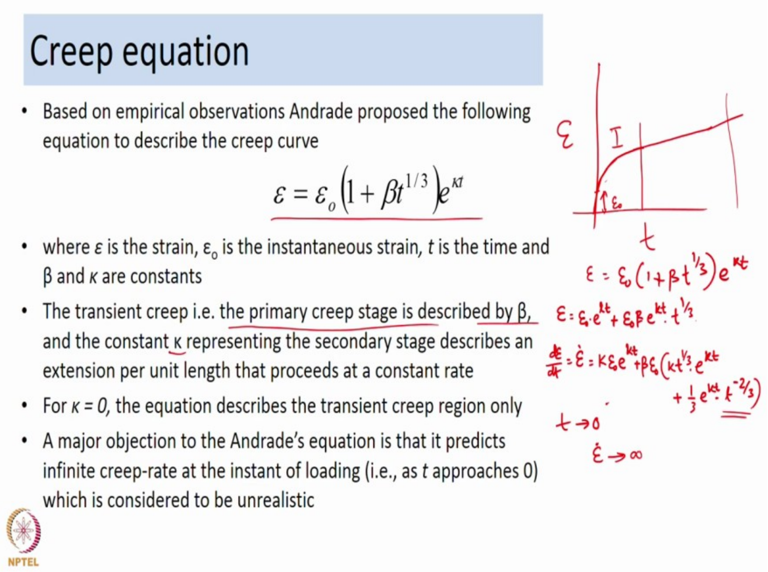

Creep equation

- Based on empirical observations Andrade proposed the following equation to describe the creep curve

$$\underline{\varepsilon = \varepsilon_0\left(1 + \beta t^{1/3}\right)e^{\kappa t}}$$

- where ε is the strain, ε_0 is the instantaneous strain, t is the time and β and κ are constants
- The transient creep i.e. the primary creep stage is described by β, and the constant κ representing the secondary stage describes an extension per unit length that proceeds at a constant rate
- For $\kappa = 0$, the equation describes the transient creep region only
- A major objection to the Andrade's equation is that it predicts infinite creep-rate at the instant of loading (i.e., as t approaches 0) which is considered to be unrealistic

$$\varepsilon = \varepsilon_0\left(1+\beta t^{1/3}\right)e^{kt}$$

$$\varepsilon = \varepsilon_0 \cdot e^{kt} + \varepsilon_0 \beta e^{kt} \cdot t^{1/3}$$

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon} = k\varepsilon_0 e^{kt} + \beta\varepsilon_0\left(kt^{1/3} \cdot e^{kt} + \frac{1}{3}e^{kt} \cdot \underline{\underline{t^{-2/3}}}\right)$$

$$t \to 0$$

$$\dot{\varepsilon} \to \infty$$

So Andrade was trying to understand the time dependence of plastic strain and what he found out was that whenever a material is subjected to a certain load, there is, initially there is some instantaneous strain and then there is some, you have some instantaneous strain and then there is some initial flow which gradually disappears and then finally the material undergoes deformation under constant flow conditions and experiences elongation under constant flow conditions. And based on these observations, he proposed the following, he proposed the following equation.

Now here in this equation, Epsilon is the creep strain, Epsilon0 is the instantaneous strain that the material is the strain that the material exhibits or experiences, right at the application of the stress. And t is the time or the duration of creep and beta and Kappa are constants. Now, if you recall, we are saying a creep curve, technically looks like that, you have an instantaneous strain, then you have a primary creep region and then it goes to a secondary creep region and then following it goes to failure in the tertiary creep region.

But in the Andrade's experiments, he only found out or looked at the instantaneous strain and the primary creep region and then looked at the steady-state region. Now, in his equations, the primary creep stage is discussed by the constant beta and the constant K or the Kappa represent the secondary creep stage. Where the secondary creep stage is basically talking about extension per unit length, that proceeds at a constant rate.

And if Kappa is equal to 0, then basically the curve will be limited only up to the stage one or only up to the primary or transient creep region only. While this was one of the earliest equations to describe creep, there were some objections or objections raised to the Andrade's equation. And one of the major objection to the Andrade's equation is that it predicts infinite creep rate at the instant of loading. that is, when the t is very small when t approaches 0, then the creep rate approaches a value of infinity, which is considered to be unrealistic.

So the way to look at it is, if you write Epsilon is equal to Epsilon0 (1+ Beta t ^1/3)*Epsilon Kappa t. So Epsilon can be written as Epsilon 0 into e to the power Kappa t + Epsilon 0 beta e to the power Kappa t into t to the power 1 by 3. So d Epsilon by dt which is equal to Epsilon dot then becomes Kappa Epsilon 0 e to the power Kappa t + beta Epsilon 0 Kappa t to the power 1 by 3 into e to the power Kappa t +1 by 3 e to the power Kappa t into key to the power -2 by 3.

So if you look at this equation, if t tends to 0, then since Epsilon dot has a term that includes t to the power -2 by 3, Epsilon dot will automatically tend to infinity. And that is the primary objection raised to the and its equation because it is not, it is unable to explain how Epsilon dot will tend to infinity which is not true.

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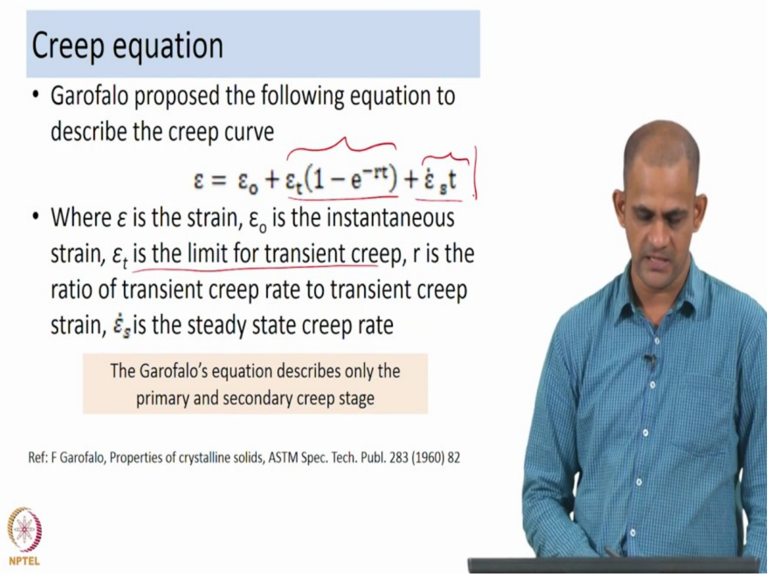

Creep equation

- Garofalo proposed the following equation to describe the creep curve

$$\varepsilon = \varepsilon_0 + \varepsilon_t(1 - e^{-rt}) + \dot{\varepsilon}_s t$$

- Where ε is the strain, ε_0 is the instantaneous strain, ε_t is the limit for transient creep, r is the ratio of transient creep rate to transient creep strain, $\dot{\varepsilon}_s$ is the steady state creep rate

The Garofalo's equation describes only the primary and secondary creep stage

Ref: F Garofalo, Properties of crystalline solids, ASTM Spec. Tech. Publ. 283 (1960) 82

Subsequent years, Garofalo proposed the following equation to describe the creep curve and this equation was supposed to overcome the limitations of the Andrade's equation. So in this equation Garofalo propose that Epsilon is equal to Epsilon0 and then there is a primary creep term and there is a steady-state creep term. So here Epsilon is the creep strain, Epsilon0 is an instantaneous strain, Epsilon-t is the limit for transient creep or the primary creep region and "r" is the ratio of transient creep rate to the transient creep strain and Epsilon-dot S, Epsilon-dot S is the steady-state creep rate.

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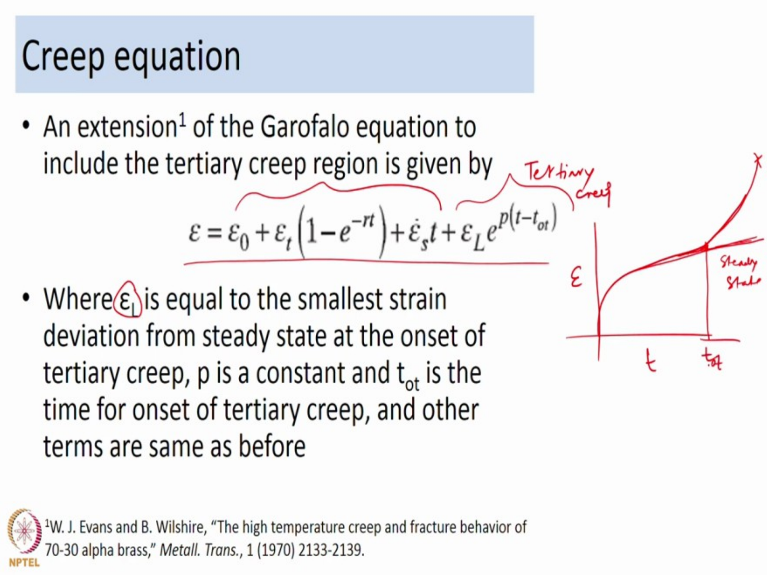

Creep equation

- An extension[1] of the Garofalo equation to include the tertiary creep region is given by

$$\varepsilon = \varepsilon_0 + \varepsilon_t\left(1 - e^{-rt}\right) + \dot{\varepsilon}_s t + \varepsilon_L e^{p(t - t_{ot})}$$

- Where ε_L is equal to the smallest strain deviation from steady state at the onset of tertiary creep, p is a constant and t_{ot} is the time for onset of tertiary creep, and other terms are same as before

[1] W. J. Evans and B. Wilshire, "The high temperature creep and fracture behavior of 70-30 alpha brass," *Metall. Trans.*, 1 (1970) 2133-2139.

However what is very clear from this equation is that the Garofalo equation is only describing only the primary and the secondary creep stage and it does not talk anything about the tertiary creep stage. However an extension of the Garofalo equation can be made to include the tertiary creep region and this was done later by Evans and Wilshire who showed that the creep curve could be described with the following equation. So you could have Epsilon is equal to Epsilon 0 and this is basically upto this point the Garofalo equation and then they added an additional term for the tertiary creep region.

Here Epsilon L, which is a new term that they introduce, Epsilon L is equal to the smallest strain deviation from the steady-state at the onset of tertiary creep. So if you look at the curve Epsilon versus t. So the point or the smallest strain that takes the material away from steady-state. So if this is an extrapolation of steady-state, so the smallest strain deviation which takes the material from, away from steady-state into tertiary creep, that is what Epsilon L is. And the onset of tertiary creep and P is a constant and t OT is the time for the onset of tertiary creep. So, probably somewhere around here, this will be t OT, and the other times are the same as before.

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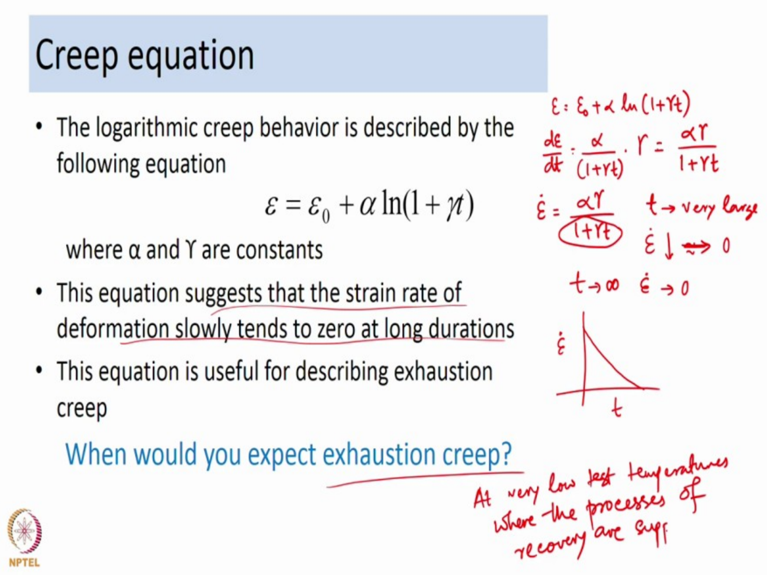

Creep equation

- The logarithmic creep behavior is described by the following equation

$$\varepsilon = \varepsilon_0 + \alpha \ln(1+\gamma t)$$

where α and ϒ are constants

- This equation suggests that the strain rate of deformation slowly tends to zero at long durations
- This equation is useful for describing exhaustion creep

When would you expect exhaustion creep?

$\varepsilon = \varepsilon_0 + \alpha \ln(1+\gamma t)$

$\frac{d\varepsilon}{dt} = \frac{\alpha}{(1+\gamma t)} \cdot \gamma = \frac{\alpha\gamma}{1+\gamma t}$

$\dot{\varepsilon} = \frac{\alpha\gamma}{\boxed{1+\gamma t}}$ t → very large

$\dot{\varepsilon} \downarrow \Rightarrow 0$

$t \to \infty \quad \dot{\varepsilon} \to 0$

At very low test temperatures where the processes of recovery are supp

However there are other kinds of creep behaviour also, creep equations that people have observed. One such creep equation is a logarithmic creep equations. this equation is mentioned here, so Epsilon is equal to Epsilon 0+ Alpha lon is, lon of 1+ gamma t where Alpha and gamma constants. So, what this equation suggests is that the strain rate of deformation slowly tends to 0 at long duration. So and this kind of an equation is useful for describing exhaustion creep.

So if you look at the, this point, the equation suggests that the strain rate of deformation slowly tends to 0 at long durations, that is very clear if we take this equation Epsilon is equal to Epsilon 0+ Alpha lon of 1+ gamma t, so d Epsilon over dt is equal to Alpha over 1+ gamma t into gamma, so that becomes Alpha Gamma by 1+ gamma t. So the strain rate in during logarithmic creep, the strain rate of deformation is basically alpha gamma by 1+ gamma t. So when t tends to very high values, so, say t is very large, then the denominator the, this becomes very large, so Epsilon dot decreases and probably look and say it tends towards, it tends towards 0.

So the Epsilon dot eventually tends to 0, so t is very large, say for example, t tends to infinity, then Epsilon dot tends to 0. So this kind of behaviour is also known as the exhaustion creep behaviour. And the point is when would you expect exhaustion creep. So if you plot Epsilon dot versus t, what you eventually see is something like that, Epsilon dot tending to 0 and one could expect this kind of a behaviour where under creep conditions where the temperature is low.

So if your temperature is low, then what would happen if the material will not be able to recover. So since creep involves a competition between mechanisms of strain hardening and mechanisms of recovery. So when if, if the material is unable to recover, then what would happen is, because of strain hardening, the flow stress of the material exceeds the applied stress and at that point of time, the material will stop creeping or the creep rate would tend towards 0.

So that is when you would expect exhaustion creep. So exhaustion creep can be expected at very low test temperatures, where the processes of recovery are suppressed. So we looked at 3 equations, Andrade’s equation, we looked at the Garofalo's equation and we also have looked at exhaustion creep, which is the logarithmic equation. So the question that arises is what is the basis of creep equation. Well, when we talk of Andrade's equation, Andrade used an empirical approach to come up with his equations, so he basically relied on his experimental observations, the observations that he made during his creep tests and based on those observations he came up with his equation.

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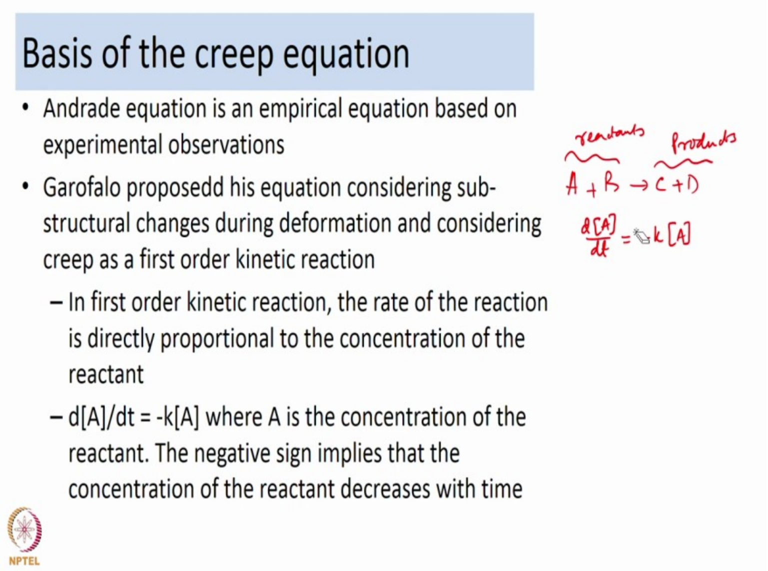

Basis of the creep equation

- Andrade equation is an empirical equation based on experimental observations
- Garofalo proposedd his equation considering sub-structural changes during deformation and considering creep as a first order kinetic reaction
  - In first order kinetic reaction, the rate of the reaction is directly proportional to the concentration of the reactant
  - d[A]/dt = -k[A] where A is the concentration of the reactant. The negative sign implies that the concentration of the reactant decreases with time

$$\underbrace{A + B}_{\text{reactants}} \rightarrow \underbrace{C + D}_{\text{products}}$$

$$\frac{d[A]}{dt} = k[A]$$

Whereas in the case Garofalo, Garofalo proposed this equation considering sub structural changes occurring during the creep deformation and by considering creep as a first-order kinetic reaction. So Garofalo looked at creep as a first-order kinetics process and based on that he suggested his equation. Now what is a first-order kinetic rejection? Well, in first-order kinetic reaction, the rate of the reaction is directly proportional to the concentration of the reactant.

So if you have reaction like this going on A + B Giving us C + D, where A and B are the reactants and an C + D are the products, C and D are the products, that the rate of the reaction, if it turns out to be of this form, so dA over dt, and the changing concentration of A, if it turns out to be directly proportional to the concentration of A itself, then that is known as the first-order kinetic reaction. Of course the negative term actually implies that the concentration of A is decreasing with time.

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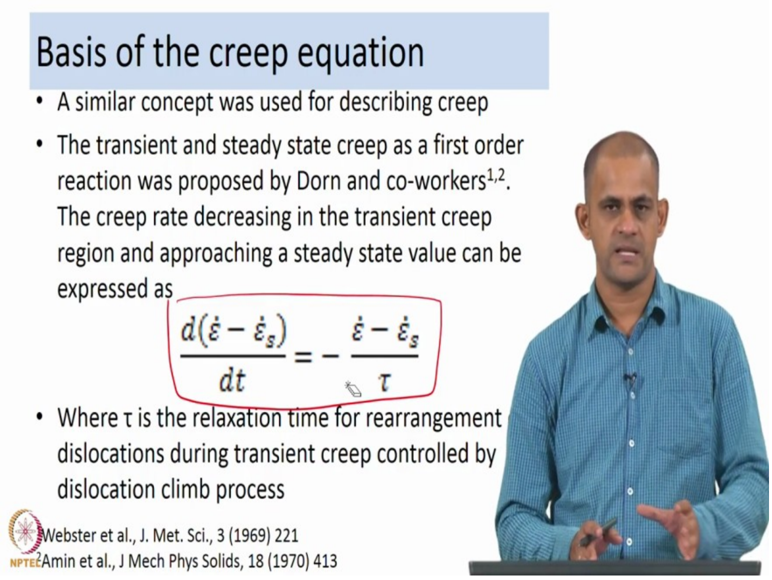

Basis of the creep equation

- A similar concept was used for describing creep
- The transient and steady state creep as a first order reaction was proposed by Dorn and co-workers[1,2]. The creep rate decreasing in the transient creep region and approaching a steady state value can be expressed as

$$\boxed{\frac{d(\dot{\varepsilon}-\dot{\varepsilon}_s)}{dt} = -\frac{\dot{\varepsilon}-\dot{\varepsilon}_s}{\tau}}$$

- Where τ is the relaxation time for rearrangement dislocations during transient creep controlled by dislocation climb process

[1] Webster et al., J. Met. Sci., 3 (1969) 221
[2] Amin et al., J Mech Phys Solids, 18 (1970) 413

So when you have something like this, that that is known as the first-order kinetic reaction. this similar concept was use for describing creep. So if we use first-order kinetic approach to understanding creep, then we will be able to derive the equation that we that we know about creep. What Don and co-workers did was to look at transient and steady-state creep as a first-order reaction and by using that they came up with the creep equation. So the creep rate was, is known to be decreasing in the transient creep region and to approach a steady-state value.

So that kind of a behaviour can be modelled by using an equation like this. So the decreasing rate of strain rate in the primary creep eventually leading to a steady-state creep, that kind of a behaviour can be captured by the following equation. So here you have a term tao. So, tao is a relaxation time for rearrangement of dislocations during transient creep controlled by dislocation climb process. So the underlying assumption here is the plastic deformation is being carried out by dislocation, and dislocation climb is a necessary mechanism for recovery to happen. And when dislocation climb is happening, there is some amount of rearrangement of dislocations that are happening and that rearrangement of dislocations has a certain relaxation time tao.

So this is basically the equation and Epsilon dot is instantaneous creep rate and Epsilon F.is the steady-state creep rate. So if we use this equation, then we will be able to derive the entire creep equation. So let us go through the derivation and we can, we can find out how it will lead us to the Garofalo equation.

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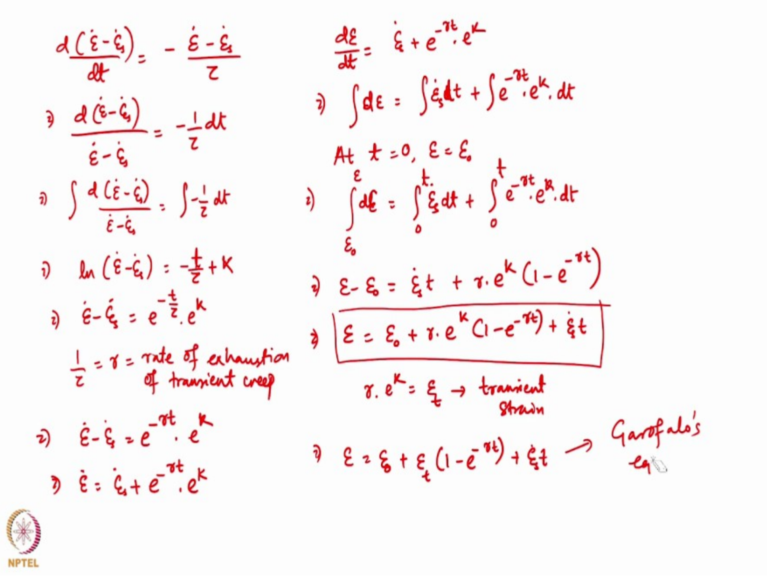

$$\frac{d(\dot{\varepsilon}-\dot{\varepsilon}_s)}{dt} = -\frac{\dot{\varepsilon}-\dot{\varepsilon}_s}{\tau}$$

$$\Rightarrow \frac{d(\dot{\varepsilon}-\dot{\varepsilon}_s)}{\dot{\varepsilon}-\dot{\varepsilon}_s} = -\frac{1}{\tau}dt$$

$$\Rightarrow \int \frac{d(\dot{\varepsilon}-\dot{\varepsilon}_s)}{\dot{\varepsilon}-\dot{\varepsilon}_s} = \int -\frac{1}{\tau}dt$$

$$\Rightarrow \ln(\dot{\varepsilon}-\dot{\varepsilon}_s) = -\frac{t}{\tau}+K$$

$$\Rightarrow \dot{\varepsilon}-\dot{\varepsilon}_s = e^{-\frac{t}{\tau}}.e^{K}$$

$\frac{1}{\tau} = r =$ rate of exhaustion of transient creep

$$\Rightarrow \dot{\varepsilon}-\dot{\varepsilon}_s = e^{-rt}.e^{K}$$

$$\Rightarrow \dot{\varepsilon} = \dot{\varepsilon}_s + e^{-rt}.e^{K}$$

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon}_s + e^{-rt}.e^{K}$$

$$\Rightarrow \int d\varepsilon = \int \dot{\varepsilon}_s dt + \int e^{-rt}.e^{K}.dt$$

At $t=0$, $\varepsilon = \varepsilon_0$

$$\Rightarrow \int_{\varepsilon_0}^{\varepsilon} d\varepsilon = \int_0^t \dot{\varepsilon}_s dt + \int_0^t e^{-rt}.e^{K}.dt$$

$$\Rightarrow \varepsilon - \varepsilon_0 = \dot{\varepsilon}_s t + r.e^{K}(1-e^{-rt})$$

$$\Rightarrow \boxed{\varepsilon = \varepsilon_0 + r.e^{K}(1-e^{-rt}) + \dot{\varepsilon}_s t}$$

$r.e^{K} = \varepsilon_t \rightarrow$ transient strain

$$\Rightarrow \varepsilon = \varepsilon_0 + \varepsilon_t(1-e^{-rt}) + \dot{\varepsilon}_s t \rightarrow \text{Garofalo's eq}$$

So if we take d Epsilon dot - d Epsilon S dot by dt is equal to - Epsilon dot - Epsilon dot S over tao. So this is the first-order kinetic equation, so this implies d Epsilon, that is equal to -1 by tao dt. And now if we carry out integration of this, so that will lead us to lon of Epsilon dot - Epsilon dot S is equal to - t by tao + some constant K. So, this implies Epsilon dot - Epsilon S dot is equal to e to the power t over tao into e to the power K.

And so tao as I mentioned is a relaxation time for rearrangement of dislocations and 1 over tao can also be written as r which is the rate of exhaustion of transient creep. So, this implies Epsilon dot - Epsilon dot F is equal to e to the power - rt into e to the power K. So this implies Epsilon dot is equal to Epsilon S dot e to the power - rt into e to the power K. So, d Epsilon by dt is equal to, this implies integration of d Epsilon is equal to integration of Epsilon S dot.

So at time t is equal to 0, they get the instantaneous strain Epsilon 0. So at t is equal to 0, we get the instantaneous strain, Epsilon is equal to Epsilon 0, so therefore the bounds of the integration will be Epsilon 0 to Epsilon is equal to 0 to t Epsilon S dot dt +0 to t e to the power -rt e to the power K into dt. So this gives Epsilon - Epsilon 0 is equal to Epsilon dot s into t. And the other integration gives us r into e to the power K into 1 - e to the power - rt.

So this implies Epsilon is equal to Epsilon 0+ r into e to the power K1 - e to the power - rt + Epsilon S dot into t. So, this is very similar, this equation that we have arrived at following this derivation is very similar to the Garofalo's equation. And if you notice r into e to the power K can be written as Epsilon t which is the transient strain. So then this becomes the Garofalo's equation, Epsilon equal to Epsilon 0 into Epsilon t into 1 - e to the power - rt + Epsilon S dot into t.

So this is how using the 1st order kinetic approach one can derive the Garofalo's equation. Of the creep equation to be understood by physics-based mechanism. So this is basically the foundation, the physics-based mechanisms for the foundations of this equation.