

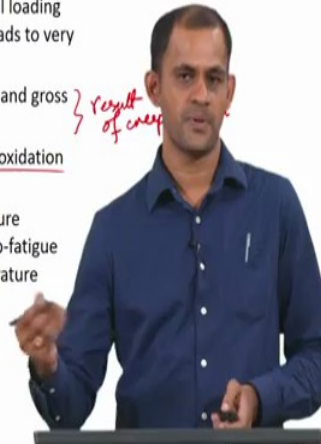
Creep Deformation of Materials

Modeling the useful Creep Life of Materials/Components Part 3

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Modeling creep life using continuum damage mechanics

- The irreversible material damage caused by mechanical loading and environmental features during creep eventually leads to very high strain rates of deformation and failure.
- Damage could be due to cavity formation, microcracks and gross deformation such as strain or aging induced.
- Other mechanisms of damage such as fatigue, surface oxidation and internal corrosion are also important.
- Although some of these phenomena are not temperature dependent, their interactions with creep, such as creep-fatigue interaction, can have significant effects on high temperature damage accumulation.



So we are talking about theta projection concept method that was developed by Wilshire and coworkers as a way of modeling the creep behavior of materials, so Wilshire and coworkers they said that instead of relying on transitions in creep mechanisms which may not be completely correct they said the single dislocation based mechanism with some parameters which accurately capture the creep curve together they can be used to predict the creep life of materials and in fact the theta projection concept is even able to account for the transitions which are coming from conventional creep analysis methods of explaining transitions as change in deformation mechanisms.

So moving away from the theta projection method now we will go we are going to talk about an approach which involves modeling of creep life using continuum damage mechanics. So in the continuum damage mechanics essentially as you know in a creep curve we you move from primary to secondary from secondary to tertiary and in the tertiary region we are saying there is this generation of defects with in the material why it is and cracks which start growing as a function of time and when they start growing the creep rate of deformation is accelerated.

Now this aspect is used in continuum damage mechanics, so what they are saying is this material damage that happens within the material the damage that happens within the

material is irreversible and is caused by mechanical loading and other environmental features experienced by the material during creep and this damage eventually leads to very high strain rate of deformation and failure which is what we associate as happening in the material during tertiary creep.

Now the damage that they are talking of is the damage that we already know in tertiary creep which is cavity formation, voids, microcracks et cetera, or it could be just gross plastic deformation like slip happening over certain preferred slip planes or it could be strain or aging induced coarsening of precipitate, so dissolution of precipitate etcetera which can render the material weak.

So these are result of creep deformation but in addition to that you could have a other mechanisms of damage happening within the material such as you could have the material undergoing fatigue while also experiencing creep or it could there could be some amount of surface oxidation, surface oxidation which leads to loss of load bearing material or there could also be internal corrosion if the environment is corrosive in nature, so all these other mechanisms of damage are also introducing defects and damage into the material.

Now some of these mechanisms such as fatigue may not be temperature dependent but interactions with creep such as creep fatigue interaction can also have significant effects on high temperature damage accumulation. So the continuum damage mechanics approach takes into account all the damage that is happening because of the different mechanism operating while the material or component is in service.

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Modeling creep life using continuum damage mechanics

- The different damage processes are ductile creep rupture, intergranular cavitation during creep, continuum creep rupture, continuum fatigue damage, environmental damage, and age and strain induced hardening and softening.
- Continuum damage mechanics (CDM) takes a holistic view of the damage process and models creep accordingly
- CDM provides greater accuracy in creep life estimation comparison to models based on single mechanism of creep, viz grain boundary sliding or dislocation creep.



So the different damage processes are such as ductile creep rupture or you can have intergranular cavitation during creep or you could have continuum creep rupture, continuum fatigue damage, environmental damage and like I mentioned earlier also you could have aging induced hardening and softening within the material. Now in continuum damage approach the continuum damage mechanics approach you basically take a holistic view of the damage process and you take into account all these different damages that are happening within the material and then model creep accordingly.

Now in that sense it is more powerful than modeling creep life based on strain rates of deformation associated with a certain creep mechanism, because that is only looking at deformation rates within the steady state where defect generation has not happened yet, and also it is only looking at the role of creep and the importance of creep on the or the influence of creep on the service life of the material.

In contrast to that in continuum damage mechanics you are basically looking at damage as a process or damage occurring due to all this different processes such as fatigue, creep, oxidation act etcetera acting together. So in that sense CDM or continuum damage mechanics is going to provide you greater accuracy in creep life estimation. So that is the advantage of CDM.

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Modeling creep life using continuum damage mechanics

- A well known CDM model is the one proposed by Kachanov and Rabotnov
 - Kachanov described continuum damage as an effective loss in material cross-section due to the formation and growth of internal voids.
 - The internal stress corresponding to a nominal externally applied load increases with increasing damage.
 - Kachanov assumed that damage could be described by a quantity which he called the 'continuity'. The continuity is essentially the ratio of the remaining effective area A to the original area A_0 .
- $$\sigma_i = \sigma_o \frac{A_0}{A}$$
- Where σ_i is the internal stress and σ_o is the initial applied stress



Now a well-known CDM model, so there are several models that have been proposed for continuum damage mechanics but one of the popular CDM model is the one proposed by the Kachanov and subsequently refined by Rabotnov, so you can call it as the Kachanov Rabotnov model. So Kachanov Rabotnov basically describes continuum damage as an effective loss in material cross-section due to the formation and growth of internal voids.

So when you have formation and growth of internal voids there is a loss of material connection and as a result certain sections of the material are going to sustain higher loads or they are going to share more of the load compare to the other regions. So naturally the sections which are taking up the load are going to be under higher stress.

So he introduced the concept of an internal stress as the internal stress corresponds to and nominally external applied load increases with increasing damage, so the point is as damage increases the material cross-section is coming down and because the material cross-section is coming down for the same initial applied load now the material is under higher stress, so there is a increase in the stress with the increasing damage and so Kachanov said that the damage the material is experiencing could be described by quantity which he called the continuity and the continuity is basically the ratio of the effect remaining effective area to the original area.

Like I said because of the generation of defects or the change in cross-section of the material the effective area of the material is coming down the effective area is the area which is bearing the load, so that is coming down and it is coming down with respect to the original

starting cross-sectional area of the material and so he said the continuity is essentially the ratio of the effective remaining effective area to the original area and this is how the internal stress will depend on this two terms one is the initial applied stress and the second term is the continuity.

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Modeling creep life using continuum damage mechanics

- The continuity term was later modified by Rabotnov and was called the damage parameter ω , where

$$\omega = \left(1 - \frac{A}{A_0} \right)$$

- By assuming a power law dependence of stress, the creep strain rate is described by

$$\dot{\epsilon}_c = \frac{k\sigma_0^m}{(1-\omega)^p}$$

- Where m and p are material parameters
- The creep rate is inversely dependent on ω



Now the continuity term was originally proposed by Kachanov but later modified by Rabotnov and he called it that the damage parameter where damage parameter omega where omega is equal to 1 minus A over A₀ and by assuming a power law dependence of stress the creep strain rate was described by the following equation. So first Rabotnov modified this and then he introduced this parameter called omega and then he said a power law dependence of the stress, the creep strain rate can be described by the following relations says he said there is a relationship between the creep rate and the stress through a power law and these are the terms that were introduced where m and p are the material parameters and as we can see here the creep rate is inversely dependent on omega which means if omega is increasing then 1 minus omega is coming down, so the creep rate becomes higher as omega becomes higher and as we said omega is a damage parameters so as omega becomes higher you have more and more creep rate.

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Modeling creep life using continuum damage mechanics

- Kachanov assumed that damage is a function of the initial stress σ_0 . This was later generalized by Rabotnov who assumed that the damage is instead a function of the instantaneous stress and described the rate of change of damage through the following

$$\frac{d\omega}{dt} = \frac{B\sigma_0^k}{(1-\omega)^\lambda}$$

- The creep strain is then obtained as

$$\left\{ \frac{\epsilon_c}{\epsilon_R} = \left[1 - \left(1 - \frac{t}{t_R} \right)^{\frac{1}{\lambda}} \right] \right\}$$

where ϵ_c is the instantaneous creep strain, ϵ_R is the rupture strain, t is the time and t_R is the time to rupture and λ is damage tolerance factor.



Modeling creep life using continuum damage mechanics

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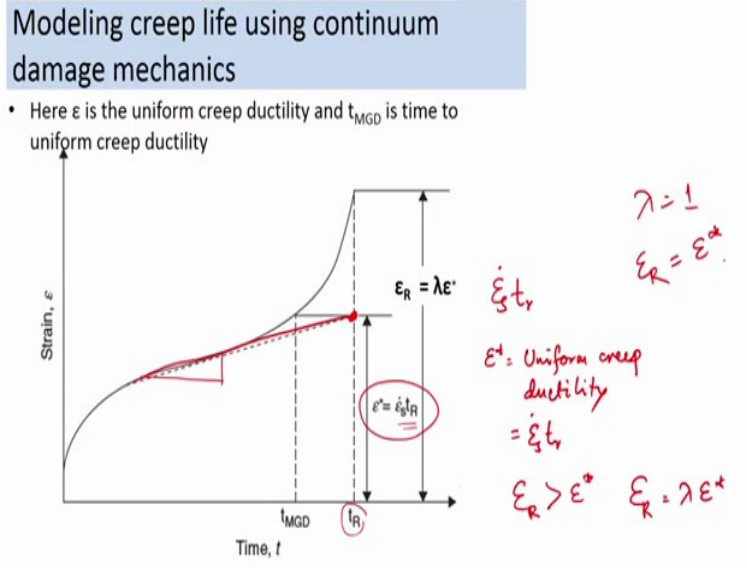


So in the case of Kachanov's approach, Kachanov assumed that damage is basically a function of the initial stress but Rabotnov believes that they should rather be generalized by saying that the damage is not just dependent on the initial stress but it is dependent on the instantaneous stress and so he described the rate of change of damage, so Rabotnov describes the rate of change of damage as a function, basically gave the following equation, so the rate of change of damage is a function of ω , σ_0 and things like that, and if you integrate this equation after taking into account $\dot{\epsilon}_c$ is stress dependence on $k\sigma_0^m$, what we eventually get is the expression for the creep strain.

So ϵ_c is the instantaneous creep strain and it depends upon the following terms where ϵ_c is the instantaneous creep strain, ϵ_R is the rupture strain, t is the time and t_R is

the time to rupture and lambda is known as the damage tolerance factor. So this is the equation for creep strain that can eventually be arrived at after looking at all the equations, so if you look at this equation and then take it in along with this then one can eventually arrived at the following equation.

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Modeling creep life using continuum damage mechanics

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$$\frac{d\omega}{dt} = \frac{B\sigma_0^k}{(1-\omega)^\lambda}$$

- The creep strain is then obtained as

$$\left\{ \begin{array}{l} \epsilon_c \\ \epsilon_R \end{array} \right\} = \left[1 - \left(1 - \frac{t}{t_R} \right)^{\frac{1}{\lambda}} \right]$$

where ϵ_c is the instantaneous creep strain, ϵ_R is the rupture strain, t is the time and t_R is the time to rupture and λ is damage tolerance factor.

Modeling creep life using continuum damage mechanics

- When $\lambda = 1$, the material fails in steady state creep regime
- Ashby and Dyson have observed that every damage micromechanism has its own characteristic λ value and a characteristic shape of the creep curve
- Phaniraj *et al.* have established a correlation between the ratio of time to Monkman-Grant ductility, t_{MGD} and time to rupture, t_R and the damage tolerance factor as given by

$$\frac{t_{MGD}}{t_R} = 1 - \left[\frac{\lambda - 1}{\lambda} \right]^\lambda$$

Ref: M. F. Ashby and B. F. Dyson, *Advances in Fracture Research*, Valluri et al. (Eds.), Oxford: Pergamon Press, 1984.
 C. Phaniraj, B.K. Choudhary, K.B.S. Rao and B. Raj, "Relationship between time to reach Monkman-Grant ductility and rupture life," *Scripta Mater*, 48 (2003) 1313-1318.

Using this equation using this particular equation you can develop a creep curve, so basically strain versus time curve using the previous equation. So in this creep curve there are a few terms that need to be introduced, first you have t_R , so t_R like we said is the time to rupture and ϵ_R is the strain to rupture corresponding to t_R , if we use the steady state creep rate like it is generally used, so then the steady state creep rate would actually predict a strain of $\dot{\epsilon} S$ into t_R , so up to t_R the creep strain coming from the steady state process.

So t_R is the point where the material has failed and at the point of failure we know in the tertiary creep region you have lot of these defects generated and all that. So that is lot of strain getting concentrated at localized regions, so you cannot call it as uniform strain. So the contribution of uniform strain which is coming from steady state creep rate can be determined by the following equation, so the amount of uniform strain up to t_R will be given by $\dot{\epsilon} t_R$, so this term the uniform creep ductility is equal to $\dot{\epsilon} t_R$ and as we can see from here this is ϵ^* and ϵ_R which is a strain to rupture ϵ_R is greater than ϵ^* , so and one way of looking describing it is ϵ_R is equal to λ times ϵ^* .

So now if we go by this view point then when λ is equal to 1, so if you have λ is equal to 1 then basically ϵ_R is equal to ϵ^* which means the material has failed within steady state itself, the material has not really gone into the tertiary creep region, so the material fails within steady state if λ is equal to 1 and depending on the value of λ you can have different amounts of tertiary creep strain contributing to the process.

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Modeling creep life using continuum damage mechanics

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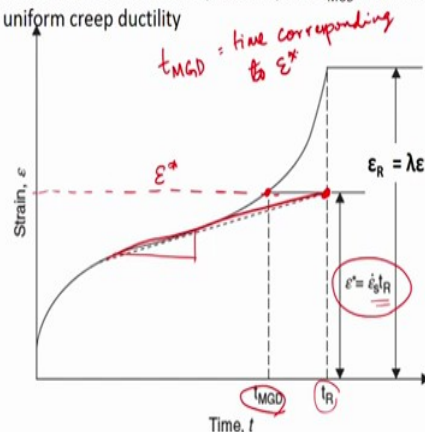
$$\frac{t_{MGD}}{t_R} = 1 - \left[\frac{\lambda - 1}{\lambda} \right]^\lambda$$

$$\frac{t_{MGD}}{t_R}$$

Ref: M. F. Ashby and B. F. Dyson, *Advances in Fracture Research*, Valluri *et al.* (Eds.), Oxford: Pergamon Press, 1984.
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Modeling creep life using continuum damage mechanics

- Here ϵ is the uniform creep ductility and t_{MGD} is time to uniform creep ductility



$$\lambda = 1$$

$$\epsilon_R = \epsilon^*$$

$$\dot{\epsilon} t_r$$

$$\epsilon^* = \text{Uniform creep ductility} = \dot{\epsilon} t_r$$

$$\epsilon_R > \epsilon^* \quad \epsilon_R = \lambda \epsilon^*$$

Now Ashby and Dyson they have observed that every damage micromechanism has its own characteristic lambda value and naturally depending on the characteristic lambda value you can have a characteristic shape of the creep curve and in a separate study Phaniraj *et al.* have established a core relationship between the ratio of time to Monkman-Grant ductility which is t_{MGD} and time to rupture t_R and the damage tolerance factor lambda, so they came up with a relation which was basically t_{MGD} which is a time to Monkman-Grant ductility, so if you look at t_{MGD} , t_{MGD} is the point corresponding to epsilon, so the uniform ductility the uniform creep ductility is epsilon star and the time corresponding to epsilon star is t_{MGD} , so t_{MGD} is equal to the time corresponding to epsilon star.


So it is basically the amount of time that the material requires to achieve a strain of (tr) uniform creep ductility of epsilon star. So they took this t_{MGD} and they took t_R which is

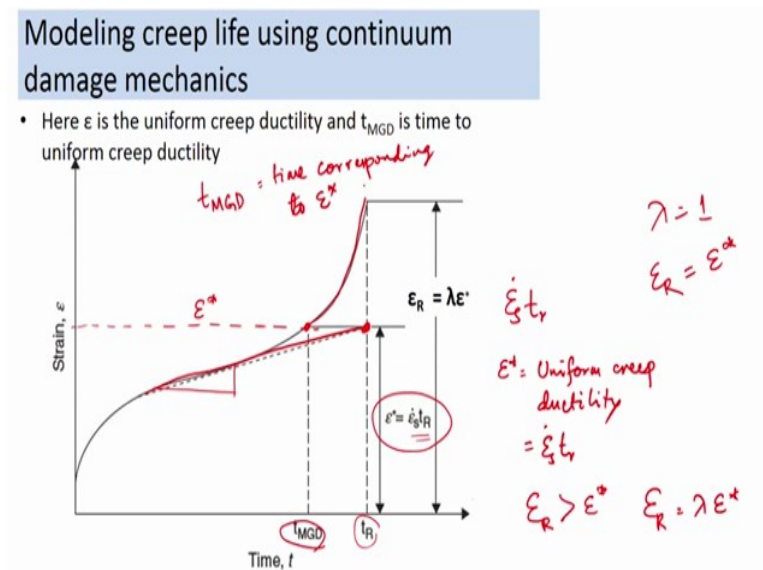
time to rupture and then they proposed the following relation and t_{MGD} by t_R is dependent on λ which is a damage tolerance factor.

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Modeling creep life using continuum damage mechanics

- The t_{MGD} was suggested as time for onset of true tertiary creep damage and was considered to be an important parameter in identifying the useful creep life of a material.
- It also describes the time till which minimum creep ductility is ensured. Hence Phaniraj *et al.* contend that the stress to cause t_{MGD} in 10^5 h can be used as a useful design criterion for creep of elevated temperature components.





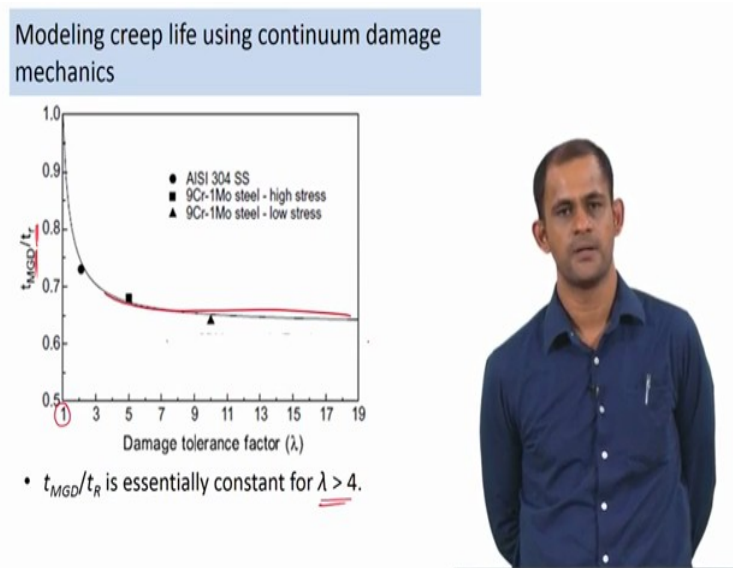
And t_{MGD} was suggested as a time Phaniraj *et al* basically suggested t_{MGD} as a time for onset of true tertiary creep damage and they hence said this can be considered as an important parameter in identifying the useful creep life of a material. Now so for all practical purpose we would like our material to be deforming within the steady state creep region only because the moment we enter into tertiary creep region, creep strain rates are accelerated so the material is going to fail faster.

So what Phaniraj et al are saying is if we stay within t_{MGD} , t_{MGD} basically it is the (stra) time corresponding to uniform creep ductility or the amount of uniform creep ductility that can happen because of the steady state creep rate, so the t_{MGD} corresponds to that so should be t_{MGD} can be taken as an onset of tertiary creep region.

So if you look at the figure here this is the point where the creep curve is taking of and that is because of the contribution of tertiary creep damage or that is where the tertiary processes have started or the creep defect generation is now accelerated, so they are saying that this t_{MGD} can actually be taken as the time for the onset of true tertiary creep damage and hence it is an important parameter in identifying the useful creep life of a material, and it also describes the time till which minimum creep ductility is ensure and hence Phaniraj et al are proposing a design parameter they are saying t_{MGD} the stress to cause t_{MGD} and 10 to the power 5 hours can be used as a useful design criteria for creep of elevated temperature components.

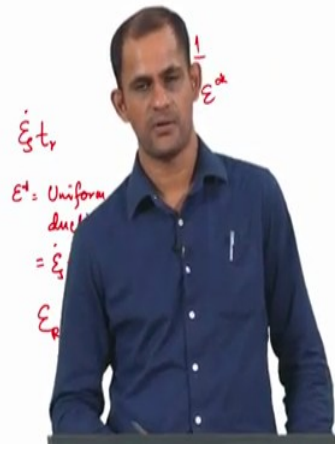
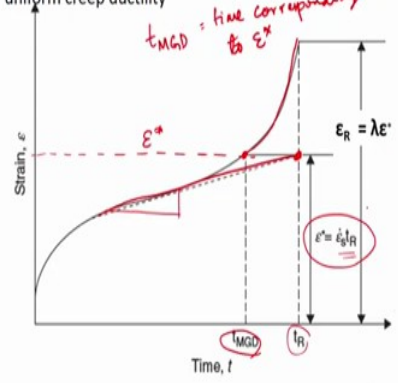
So this is a different way of basically looking at things compare to what we discussed in terms of Larson-Miller parameter and all that, so this is coming from continuum damage mechanic.

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Modeling creep life using continuum damage mechanics

- Here ϵ is the uniform creep ductility and t_{MGD} is time to uniform creep ductility



Phaniraj et al made a plot of t_{MGD} by t_R versus the damage tolerance factor and when you have a very small damage tolerance factor such as 1 your t_{MGD} is equal to t_R which means the failure will happen within steady state creep region only whereas if you have a large lambda then your t_{MGD} is basically coming down, so if your t_{MGD} is coming down that means your onset of tertiary creep will be faster but beyond a point for a lambda greater than 4 t_{MGD} by t_R is essentially constant, so if there is no change in t_{MGD} with respect to t_R after lambda values greater than 4.

So this is basically introducing you to the continuum damage mechanics approach of modeling creep life and the different types of parameter is that you can extract using the continuum damage mechanics approach. So t_{MGD} is something that we have got and from the creep curve you can also determine the lambda value and then use it for modeling the creep behavior of materials that brings me to the end to the portion on modeling creep life, the different models are approaches for predicting the useful creep life of a material.