Elementary Stereology for Quantitative Microscopy
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Lecture – 08
Volume Fraction and Particle Size Part 4

Hello everyone, welcome back to this Elementary of Stereology for Quantitative Metallographic course. And so, far you have seen the lectures given by Professor Sangal, I was also showing some demonstrations about calculation of all these stereological parameters on some of the nodular cast iron microstructure.

So, in this tutorial what I am going to do is, I am going to continue the calculation what I have demonstrated in the lecture itself. So, some of the aspects which I felt that what I showed in the form of excel sheet was little fast and then I thought let me come back and then work it out in a much slow manner so, that you can appreciate them in a better way.

So, what I am going to do today is, just going to continue that exercise. What did we do the other day? We took the cast iron nodular cast iron micro structures. In fact, that was performed inside the class, almost all the students performed that events of point counting to calculate the point fraction and then we just entered all the data into the excel sheet and then we continue to use the data as and when the lecture progresses and for demonstration of new stereological parameters as we progresses.

So, this is what we have been doing. So, today what I will do is, if you recall what we have done the point fraction P P and P L are represented by 95 percent confidence interval.

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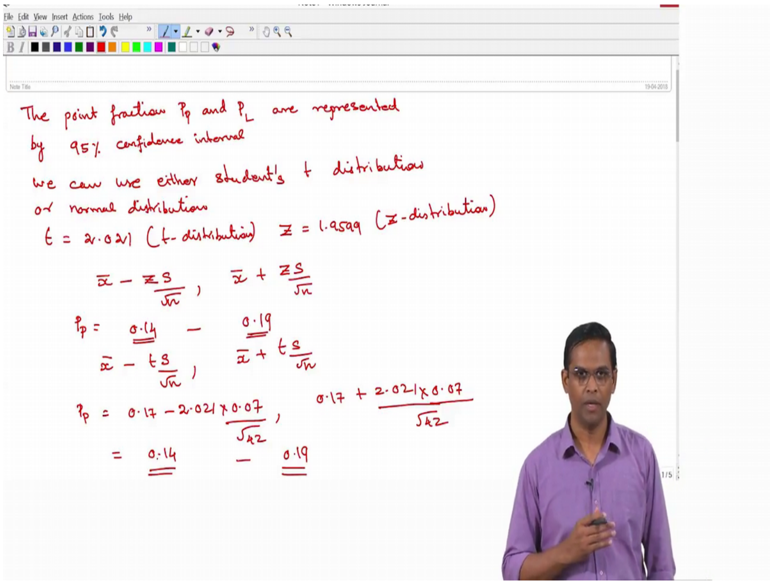

So, what I am going to do is, we are going to format the data whatever we have got. So, if you look back that excel sheet we have about 42 data points or 42 events a small n and then we collected number of point intersecting these nodular cast iron and microstructure and also we calculated the P L. So, if you look at this 95 percent confident interval there are 2 possibilities we can use either student's t distribution or a normal distribution.

So, if you recall the classroom, I mean the exercise what we have performed there we are showed 2 kind of a table. So, one is a t table and another is a z table, where you choose the number of degrees of freedom and then and take the value of z or t. So, if you recall for the 95 percent confidence level the t is correspond to 2.021 and for z it is 1.9599 this is z distribution or normal distribution.

If you recall this formula what is that this is x bar minus z s upon root small n and then x bar plus z s upon square root of n. So, for a P P if you recall what we have collected from the, what I will do is, I will just go back to this excel sheet and come back.

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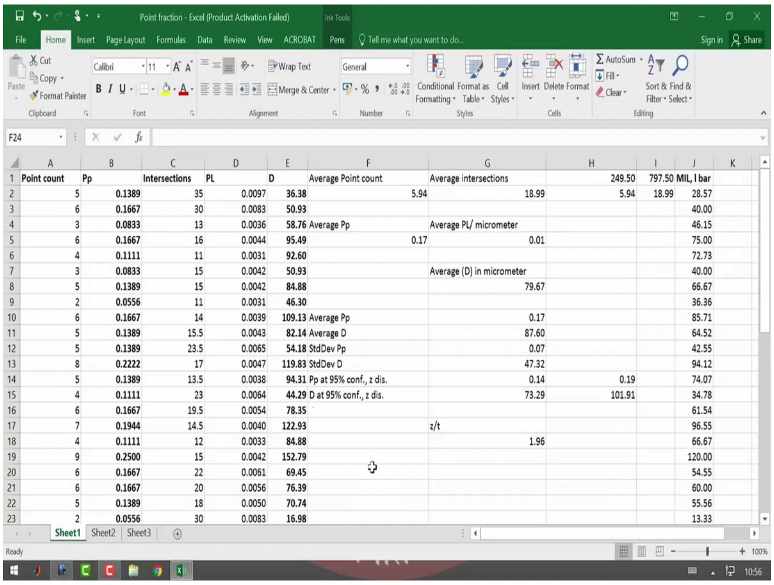

	A	B	C	D	E	F	G	H	I	J
1	Point count	Pp	Intersections	PL	D	Average Point count	Average intersections	249.50	797.50	MIL, l bar
2	5	0.1389	35	0.0097	36.38	5.94	18.99	5.94	18.99	28.57
3	6	0.1667	30	0.0083	50.93					40.00
4	3	0.0833	13	0.0036	58.76	Average Pp	Average PL/ micrometer			46.15
5	6	0.1667	16	0.0044	95.49	0.17	0.01			75.00
6	4	0.1111	11	0.0031	92.60					72.73
7	3	0.0833	15	0.0042	50.93		Average (D) in micrometer			40.00
8	5	0.1389	15	0.0042	84.88		79.67			66.67
9	2	0.0556	11	0.0031	46.30					36.36
10	6	0.1667	14	0.0039	109.13	Average Pp	0.17			85.71
11	5	0.1389	15.5	0.0043	82.14	Average D	87.60			64.52
12	5	0.1389	23.5	0.0065	54.18	StdDev Pp	0.07			42.55
13	8	0.2222	17	0.0047	119.83	StdDev D	47.32			94.12
14	5	0.1389	13.5	0.0038	94.31	Pp at 95% conf., z dis.	0.14	0.19		74.07
15	4	0.1111	23	0.0064	44.29	D at 95% conf., z dis.	73.29	101.91		34.78
16	6	0.1667	19.5	0.0054	78.35					61.54
17	7	0.1944	14.5	0.0040	122.93		z/t			96.55
18	4	0.1111	12	0.0033	84.88		1.96			66.67
19	9	0.2500	15	0.0042	152.79					120.00
20	6	0.1667	22	0.0061	69.45					54.55
21	6	0.1667	20	0.0056	76.39					60.00
22	5	0.1389	18	0.0050	70.74					55.56
23	2	0.0556	30	0.0083	16.98					13.33

If you look at this P P what we have got as an average P P 0.17 and then average P L is 0.01 right. So, we are going to use this data again and then we have also calculated the standard deviation for P P and we will use this data for this calculation. So, if you go back and if you substitute all these values so, for P P what is that you want to get you will be getting we have already calculated in the excel sheet this is 0.19. So, this is the interval and for a t distribution we calculate. So, again we will write so in fact, when the average P P is 0.17 minus 2.021 into 0.07 that is the standard deviation square root of 42 and in this case 0.17 plus 2.021 into 0.07 square root of 42.

So, here also you can write like that I have directly put the answers because it has worked out in the excel sheet. So, we got that value so, I have just simply taken these 2 interval values the lower limit and then higher limit and this z distribution this is what we will get. So, the value would be 0.14 and then and here it is 0.19.

So, if you look at this values for a t distribution as well as the z distribution there is no difference here that is what you have to appreciate, but in general the t distribution student t distribution is preferred for where the n is small in general that is what it is you have to appreciate. So, you see that the both the values are same whether you use z distribution or t distribution the 95 percent confidence interval for arriving at P P is within 0.14 to 0.19.

So, now what we can do is we can go back to this excel sheet where I have shown we have also computed this D this average diameter average diameter D and for each case we have calculated this D for all 42 events you can see that. So, now, we can perform a similar exercise we have already computed this average D and standard deviation and so on. So, this data is here so, let us complete this confidence interval exercise for D as well.

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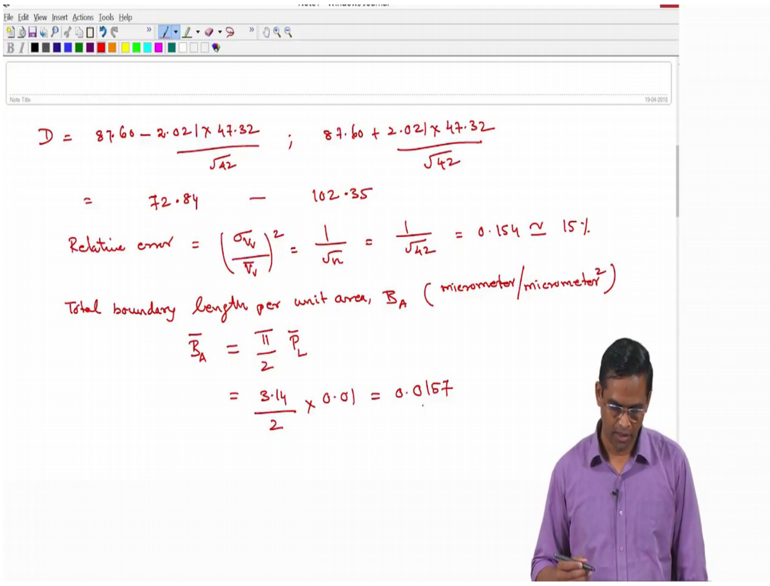

So, you write for t distribution so, you can write 87.60 into 2.021 into 47.32 which is the standard deviation divided by square root of n which is nothing, but here a 42 and there is an error here. So, this supposed to be a minus sign and here 87.60 plus 2.021 into 47.32 square root of 42. So, you will get the interval of 72.84 and this is 102.35.

So, you can calculate the same thing for the z distribution also. So, which I think we have already reported in the, if not we can leave it there. So, similarly we can report this 95 percent confident interval for calculating the mean average mean diameter of the natural graphite in the microstructures.

So, the other important point I want to mention is, we also talk about the relative error, we wrote if you look at the class lecture videos lecture it is like we have process and the best prove that the relative error for a point count could be 1 upon square root of n. So, if you see that here we have which is 0.154 symmetry 15 percent error.

So, the whatever the calculation we have done for the point fraction which is equivalent to the volume fraction if you say that volume fraction of the graphite nodules in this microstructures about 17 percent, then it will you should expect that it is with approximately about 15 percent of an error.

So, you recall that if you want to eliminate much more error in this estimation then as demonstrated in that the lecture you have to count more for example, if you want to have the error within 5 percent then you have to count more than 3000 particles in the microstructures. So, I have just shown how the data format is finally, appears.

So, the data appears with the 95 percent confident interval range plus with some error estimations, but normally this error estimation is given to check whether the methodology is correct or not for example, the grid size you are chosen the particle size you have chosen whether is there any change do you need to change this or not.

So, in this case what you can now be confident about is within 50, it comes within 15 percent. So, you can be just assured that the methodology you have adopted is good it holds good. So, that is the way I want to show and the next parameter what we can derive using this P P and P L is the total boundary length per unit area that is B A, which is you can write in the units micrometer by per micrometer square, which is very important unless you write this units you will not appreciate the meaning of this numbers significantly.

So, you have to this is boundary length is represented in a micrograph by micrometer and which is this is per area so, ratio of 2 quantities. So, from the relation we know that the average B A can be given by pi by 2 times the average P L. So, we have estimated P L already. So, this P L you can just substitute like this. So, 3.14 divided by 2 into 0.01 that is what we have calculated as an average P L which comes out to be 0.0157.

So, of course the, you can also represent this data because it is taken again overall the forty 2 events of measurements and then you can represent this is similar to what we are representing with the 95 conference level with 15 percent of relative errors. So, since they are all coming from the similar event so, that holds good here also.

See the other parameter what we are planning to use or estimate using this P P and P L is surface area per unit volume.

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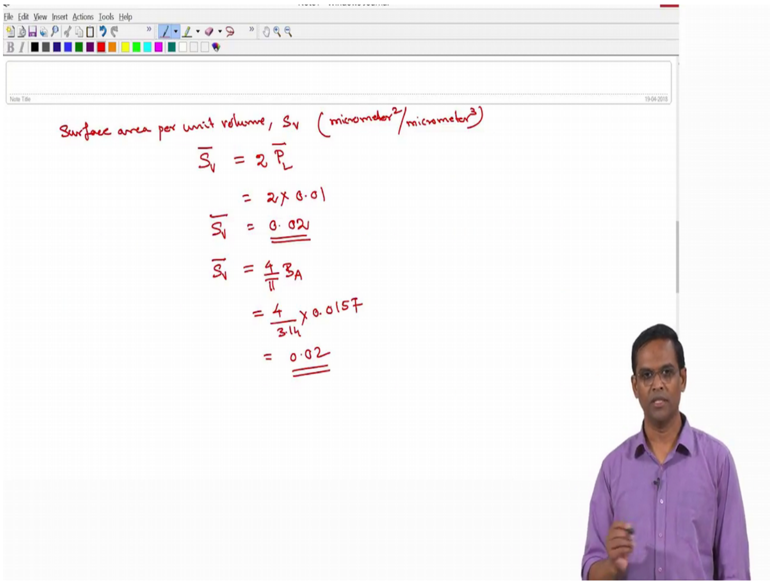

This is S V so, you say that here it is micrometer square by micro meter cube. So, surface area per unit volume so, you can recall this, the relation 2 P L. In fact, there are 2 relations available so, one is this. So, we will try to calculate this 0.01 which is equal to 0.02. So, this is the surface area per unit volume for this microstructure.

And of course, the same if you use the other formula like S V is equal to 4 by pi B A. So, that also you can see that previously you have done that. What you have done? You can see that this is the B A we have given. So, you use this and then see it will be of same value. So, you will get again if you 4 by 3.14 into what is that value have got, it is 0.157.

So, 0.0157 which is again so, this is what you will get for the, you can verify this. So, again if you want to represent this S V by a 9 5 confidence interval you can do that and of course, the error estimate also will holds good here. So, now, the other application of this measurement like using P P and P L the other important measurement is what we have done, the mean intercept length that is l bar micrometer.

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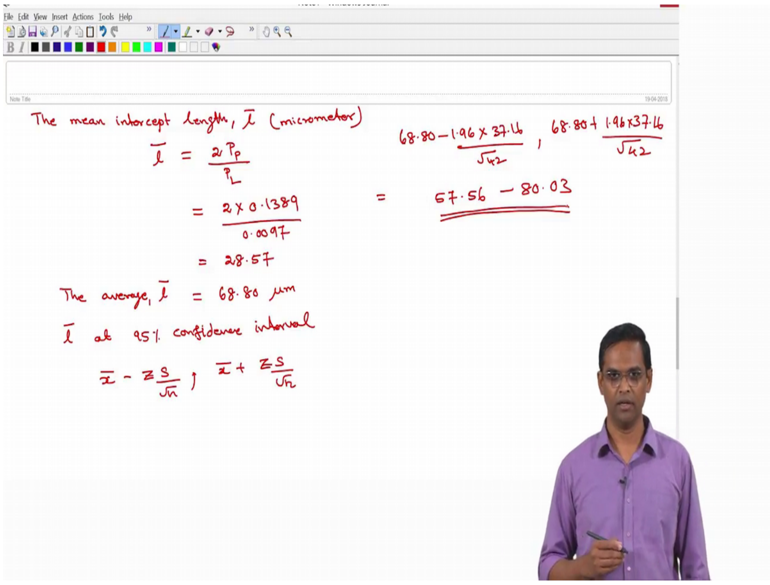

So, if you recall this formula this is 2 P P by P L. So, what we can do is, suppose if I take the first measurement what we have done in the class last time the P P first value is 0.1389 that is a P P of the first event and this is the P L 0.0097, which is equal to 28.57, is the average.

So, if I go back to this excel sheet what I am going to show. So, if you see that this is what I have taken this and these 2 I have taken and this is a value, I have got I have shown in the calculation suppose if I in an excel sheet you can simply paste this formulae for all this 42 events of measurements.

So, you get this and the average mean intercept length is about 68.80 and the standard deviation is around 37.16. So, what we can now do is, we will again do the exercise of let us write the average mean intercept length and as per the excel sheet it is 68.80 micrometer and you would like to represent l bar at 95 percent confidence interval.

So, we will follow this, a normal distribution here. So, we know that equation which is just to recall this is a lower limit and this will be the higher limit. If you substitute this, you will get you can write 68.80 minus 1.96 into the standard deviation is 37.16 upon 42 and the other limit is 68.80 plus 1.96 into 37.16 divided by square root of 42.

So, you get 2 values a similar to the earlier cases so, you have 57.56 to 80.03 micrometer. So, this is a range of l bar what we can calculate is 57.56 to 80.03. So, what is the meaning of this so, 95 of the particles the graphite natural particles in the microstructure will fall within this range that is the, a meaning of this 95 percent confidence level for the, if you use the normal distribution.

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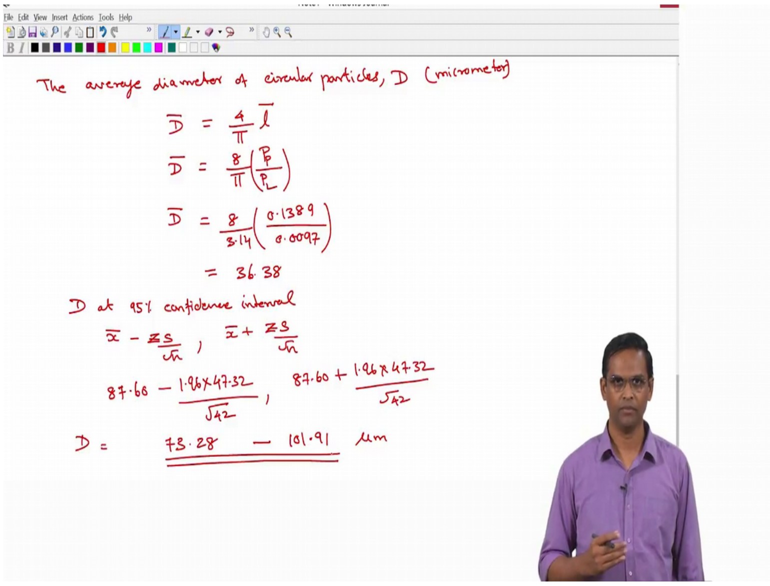

So, then next thing I want to just do is we can do the similar exercise for the D. The average diameter of the circular particles, particle D which is also in micrometer in this case it is micrometer. So, what are the formulas we, the D bar which is equal to 4 by pi l bar or I think D is also can be written as 8 by pi into P P by P L. So, we can do one calculations for example, to try take one first reading from the excel sheet the first reading is 0.1389 divided by 0.0097.

So, you get around 36.38 as the average diameter and I will go back to this excel sheet this is for just one event we have done and if you so, this column is populated with the D for each of this event. So, you have P P and this is a point count P P and you have this P L and this is intersections you know that and we are populating this column with the D and we have also estimated this average D and the standard deviation a for this D. So, we can now again put them in the form of confidence interval; that means, D at 95 percent confidence interval using the normal distribution let us again use this is for lower limit and this is for higher limit.

So, the average D is a 86.60 minus 1.96 is as per the normal distribution the standard deviation is 47.32 divided by square root of 42 the other limit is 87.60 plus 1.96 into

47.32 divided by the square root of sorry square root of 42. So, you get again a 2 values the range of this will worked out to be 73.28 to this one will be 101.291 micrometer.

So, this is the average diameter of the nodular cast iron microstructure they are graphite nodules. So, we have estimated l bar as well as D as well as volume fraction. So, all these things we were able to calculate simply by estimating the point counter. So, so in this exercises we have not only calculated all these 4 parameters. So, simply we have also shown that it is more efficient as per the theory what been taught in the course and this point count is very effective and you can derive so, many other related parameters also in this case.

So, what I will do in the next class I will take the, a poly crystalline microstructure which will have a completely spaced field grains. So, we will again go through this exercise of calculating the grain size are the mean intercept length then how the other quantities like derived quantities we can estimate in the next class with this I close this a tutorial.

Thank you.