Elementary Stereology for Quantitative Microscopy
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Lecture – 19
Stereology of Anisotropic Microstructures

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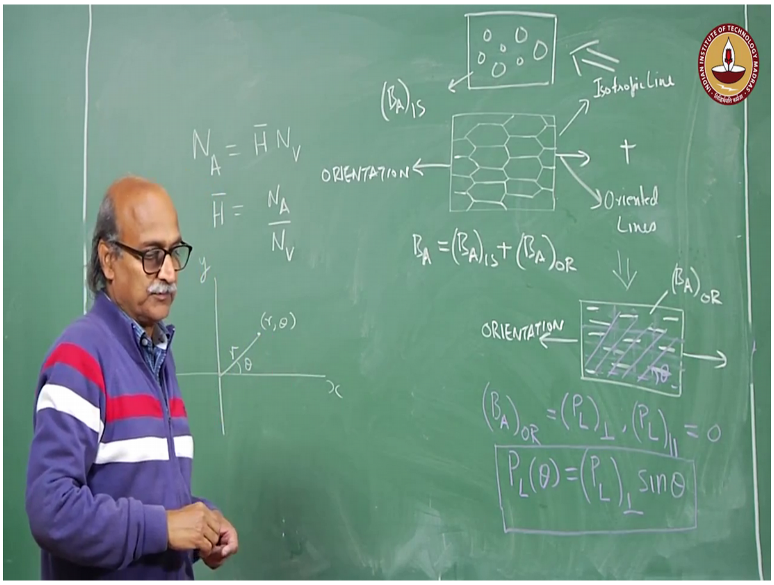

So, we want to see stereology of anisotropic microstructures and in this case I am just going to take only one kind of microstructure on a 2 dimensional section, just to get an get a feel for it that how do you deal with such structures and then similar ideas can be extended to the 3 dimensions.

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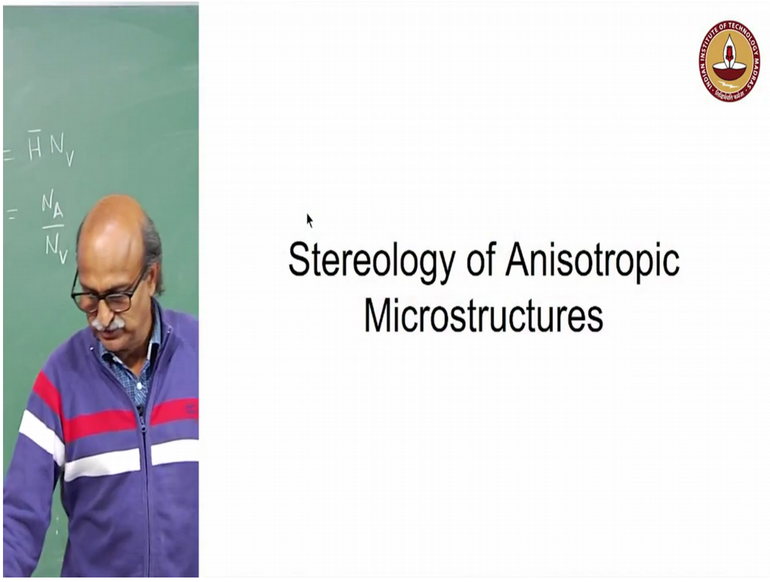

One good example of an anisotropic structure is the pearlitic colonies in the steel.

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So, if you take a particular colony they are all oriented in one direction the ferrite and the cementite lamellate plates are oriented in a direction. So, one has to suppose I want to measure interlamellar spacing one will have to be careful in order to measure the average interlamellar spacing for such structures, now here is the polycrystal structure which has been rolled.

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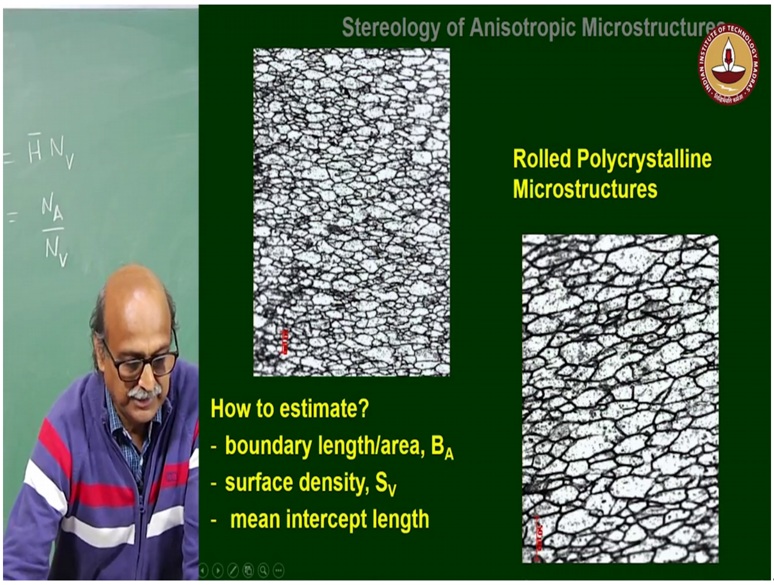

So, here again there is a clear direction along which the grain boundaries are oriented. Suppose I want to estimate what is the boundary length per unit area, if I take let us say this particular microstructure, I want to estimate boundary length per unit area I may want to and estimate S V I may want to estimate mean intercept length and so on.

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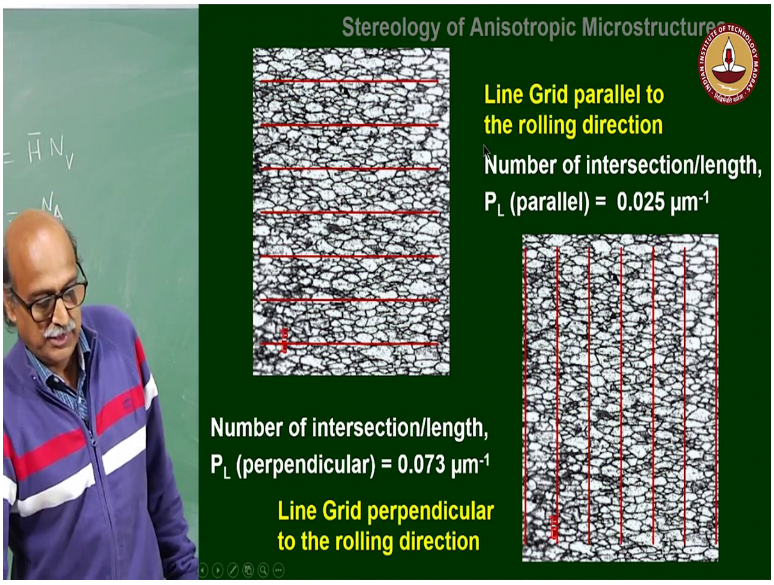

My usual technique of estimating P L number of intersections per unit length and from there I can then get my boundary length per unit area. I can put my intercepts like this or on the same structure I can put my intercepts like this, both will give the same result or a different result both will give a different result, which way you are going to get more intersections per unit length is the this particular direction the vertical direction as oppose the horizontal direction.

And in fact so, number of intersections per unit length P L parallel means parallel to the orientation direction and this is my orientation direction then I get P L to be 0.0 to 5 per micron and in the vertical direction I get almost 3 times this number 0.073 per micron; obviously, both I use if I use the parallel orientation I will get some value of boundary length per unit area if I get use perpendicular I will get some length boundary, length per unit area and both will be incorrect.

Now, one way to tackle this is use a different kind of probe do not use straight lines or if you have to use straight lines then you have to put straight lines in different orientations this is one kind of probe you can put these are circles. So, the line probes are these red circles that have been superimposed on the structure.

Now, this probe is ensuring that you are measuring in intersections per unit length in all possible directions and then I get number of intersections per unit length as 0.046 per micron and if you compare it from the previous thing it is in between these two values 0.025 and 0.073. Along the parallel direction you is the minimum possible value you can get and perpendicular direction is the maximum possible value you can get so, you get an in between value.

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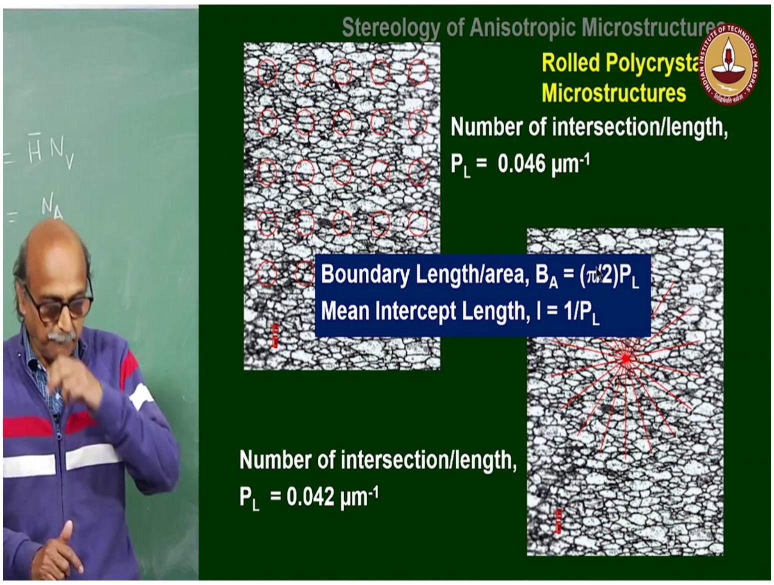

And this is an alternative probe you can use which are you can say it is a varying orientation probe where you are using again straight lines, but you have lines which are in different orientation. In fact, these lines are oriented with the angular spacing of 20 degrees between each line each pair of lines.

If I use this particular probe I get the number of intersections per unit length as 0.042 per micron. So, I get similar number of intersections with both of these probes and then you can calculate boundary length per unit area as before by the relationship of multiplying P L by a factor of pi by 2.

So, that gives the I am able to get boundary length per unit area this way, I can also able to get some kind of a mean intercept length as well. So, kind of a grain size, but I may get these as correct values, but then again these values do they reflect what kind of a microstructure it is not really it just giving me a number, but this not telling me that you know there is a significant amount of orientation in the structures.

So, in order to reveal the orientation and the extent of orientation there is in a structure we can use now a different methodology and just to understand this and may be let me use the board that I have a microstructure with elongated grains like this I can understand the structure as found of isotropic lines and oriented lines ok. This is kind of a addition of 2 structures there are lines which are oriented and there are lines which are isotropic and what I want to do is, I want to isolate these two components of the structure one can conceptually view this oriented lines as lines along the particular orientation. So, this is my orientation axis and here this is my orientation axis.

So, the or oriented component can be viewed like this the isotropic component could be viewed like this and this structure is a addition of this and this, what I am try want to find out in the end is, I want to find out how much of it how much boundary length is per unit area is of oriented lines and how much boundary length per unit area of these grain boundaries can be regarded as isotropic lines and the total boundary length which I will just call it B A would be the some total of B A isotropic plus B A oriented.

So, if I am able to get a separate out these two components from here then I will be in a better position to say something on the extent of orientation that is there, if there is a large fraction of the total boundary length is of oriented then we will say that this is re highly oriented and if there is only a very small fraction which is oriented then it is not it is more towards isotropic and I will be able to define then a single parameter to describe such a structure.

So, first let us understand if I had a structure like this which is 100 percent aligned 100 percent oriented, how can I measure on this the boundary length per unit area. So, first of all this is something that now you would be able to prove I am going to give you a relationship that, if I put my line probe perpendicular to the direction of orientation and obtain P L number of intersections per unit length and I will call it P L perpendicular, perpendicular to the orientation. Then this estimates directly the boundary length per unit area of all of these elements there is no isotropic component here if there was an isotropic component of course, then this P L will also include that.

If I put the grid not in this direction if I put the grid in the parallel direction along the orientation which let me call as an P L parallel, what would be this number, this would be 0 because probability of intersection of the line with these line elements aligned in the same direction is 0 because line widths are 0. Now, finally, if I align my grid at some angle theta, then P L theta is equal to P L perpendicular times sin theta and as I vary theta this number of intersection counts per unit length will vary from 0 at theta is equal to 0 to P L perpendicular at theta equals pi by 2.

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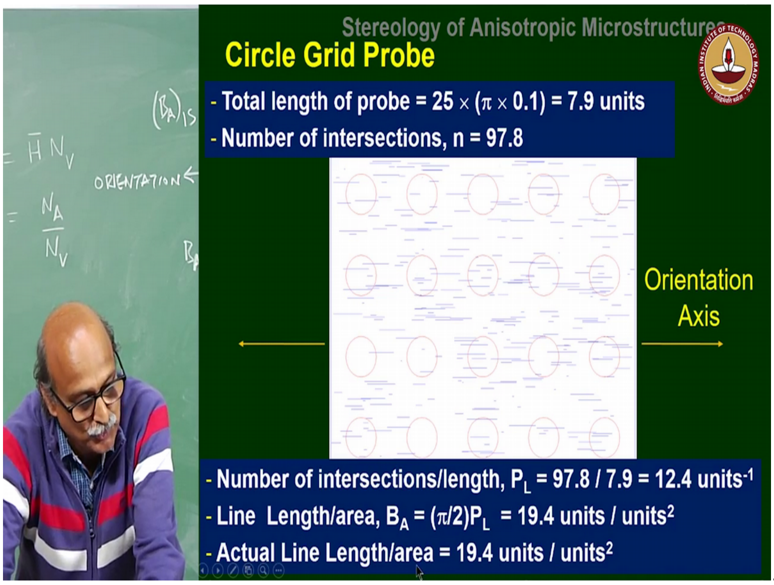

So, this is what we do, we introduce this.

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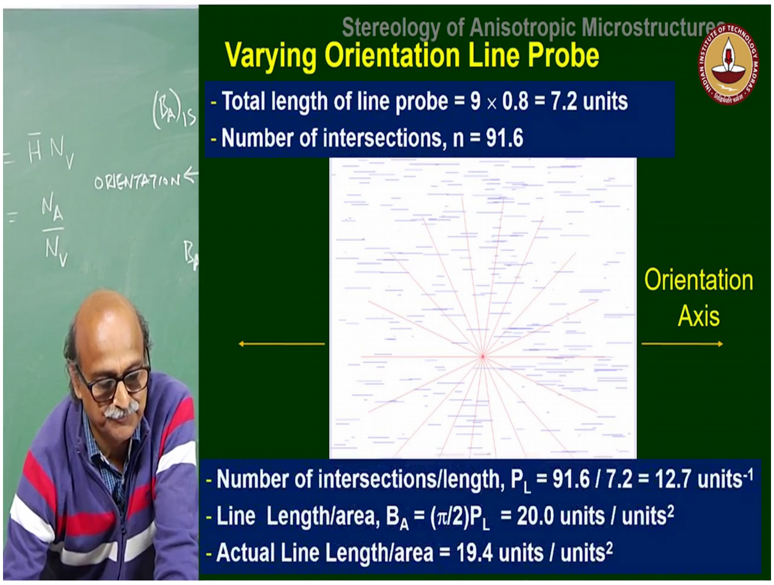

Varying orientation line probe P L and then measure number of intersections per unit length individually along each of these lines to get a variation of number of intersections per unit length as a function of theta. Now, this was done on these are all simulated structures this was done on several fields and let me show you this on excel that the data that you see here along 0 degrees.

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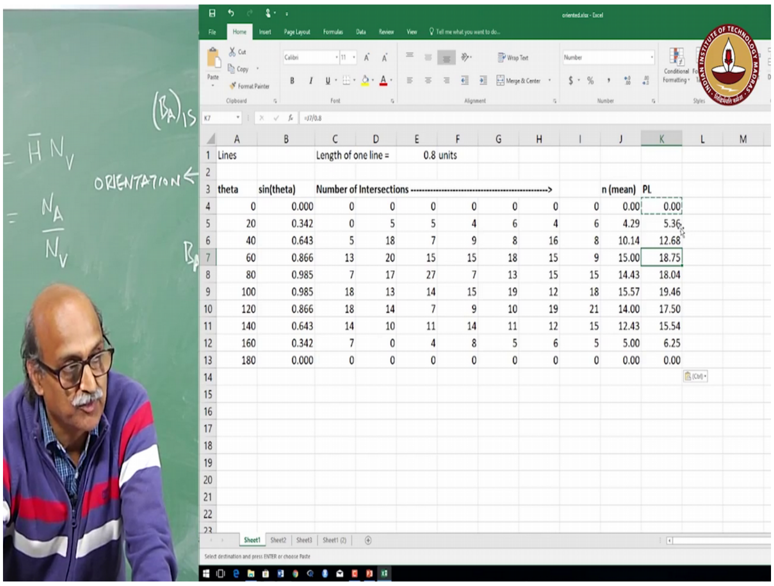

Lines		Length of one line =		0.8 units						
theta	sin(theta)	Number of Intersections ------------------------------------->							n (mean)	PL
0	0.000	0	0	0	0	0	0	0	0.00	0.00
20	0.342	0	5	5	4	6	4	6	4.29	5.36
40	0.643	5	18	7	9	8	16	8	10.14	12.68
60	0.866	13	20	15	15	18	15	9	15.00	18.75
80	0.985	7	17	27	7	13	15	15	14.43	18.04
100	0.985	18	13	14	15	19	12	18	15.57	19.46
120	0.866	18	14	7	9	10	19	21	14.00	17.50
140	0.643	14	10	11	14	11	12	15	12.43	15.54
160	0.342	7	0	4	8	5	6	5	5.00	6.25
180	0.000	0	0	0	0	0	0	0	0.00	0.00

As I said the number of count will be 0 then at 20 degree intervals we get counts. So, here each column here represents one field that was taken for total of 1 2 3 4 5 6 7 fields have been taken and this and this should also be 0. So, I have this data what I do first, let me calculate the average number of intersections per unit length along different orientation.

So, I will just show you on excel itself that for example, I will first introduce the formula for the 0 degree theta is equal to 0 degree just use the function average in excel that is already there and select the row which contains all the measured quantity for that particular angle and then the brackets closed this of course, will give you 0, but now I can copy this formula to the rest of the cells and paste it there and I will get basically average intersection count per unit length as we vary theta. So, for example, at 0 degrees it is 0, at 20 degree orientation I get 4.29, at 40 degrees I get 40.14, at 60 I get 15, then 14 again at 80, 100 15.57 and so, on I will I keep I get this count.

Ah in the previous structure the, these are simulated structures with arbitrary units. So, the each length of the line is simply 0.8 unit which is specified here. So, then number of intersections per unit length would be the average number of intersections per unit length would be this mean count divided by length of those individual lines all the lines are of equal length so, it is 0.8 and then copy this formula to the rest of the cells this gives me now number of intersections per unit length now as a function of theta ok.

Now, this should follow this relationship if is not it. So, why do not we try and plot this in excel itself. So, let me add data to this graph so, for my x axis this theta values and for my y axis is P L.

And I get a graph which looks something like this.

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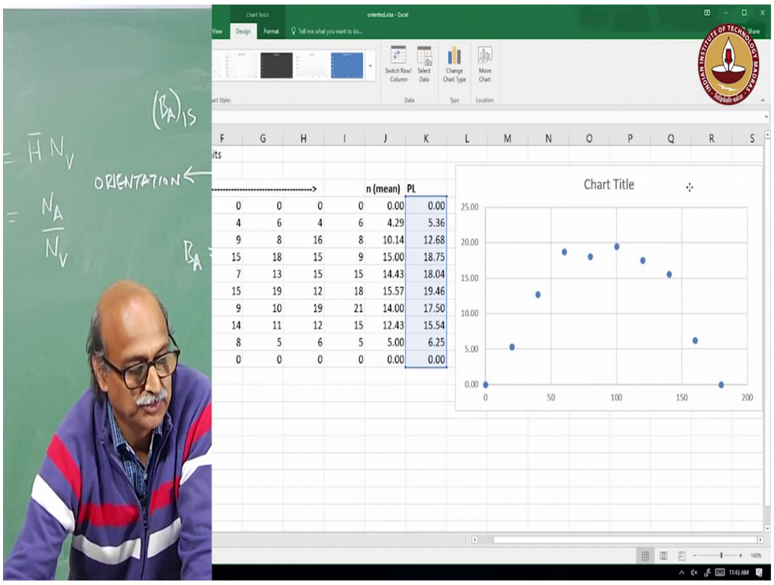

Now, what I can do is, I have to I want to find out this particular relationship which means I want to find out P L perpendicular from all this data ok, if the if I try to find this out what I could do is, I can instead of plotting P L versus theta I plot P L versus sin theta then what kind of a graph I should get, if I plot P L versus sin theta one graph do you think I should get not sure what graph I will get if I plot P L versus sin theta, looking at this relationship.

Well let us see we will plot it choose x axis this time I am going to choose sin theta values I choose y and y is P L I expect to get a straight line graph is not it with the slope of P L perpendicular.

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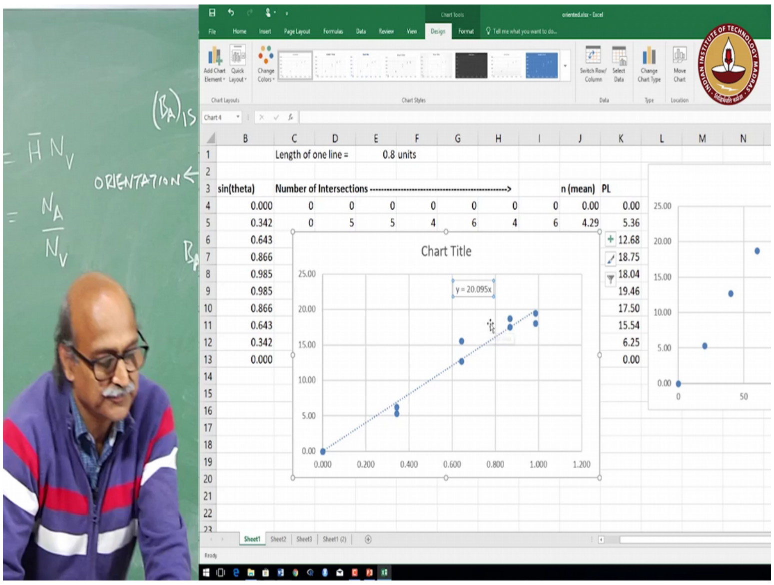

So, excel will let me determine this let me try to fit the best curve to this, how do you fit the best curve there is a technique called least squares. So, least square technique which we are not going to how the least squares work, we can just go and fit this add trend line in excel what excel does is actually gives us the least square method to fit the best line and I will set the intercept to 0 because this line should pass through the intercept and let me display the equation and that is it I guess. So, this is the equation of the graph this is giving me when the slope is 20.1 ok.

So, P L perpendicular should be 20.1 I could have directly measured P L perpendicular also and it should come close to 20.1 for such a structure now let me just go back. So, what this means is num the total line length per unit area of these oriented lines should be equal to P L perpendicular which means it should be equal to 20.1 units per unit square those arbitrary units.

If I go back to go back to this and just go back a little to this side which I had skipped earlier where I was actually trying to measure the boundary length per unit area using the circular probes. Then I had got these total length of this probe was 25 into perimeter of each, each circle because each I have 25, I have total of 25 circles here each circle has a diameter of 0.1 units. So, 25 into pi d will give me the total length of these lines. So, number of intersections I get is 97.8. So, number of intersections per unit length I will get 12.4 and then I apply pi by 2 P L because I have measured in all directions I get 19.4, actual line length because this is a simulated structure is 19.4.

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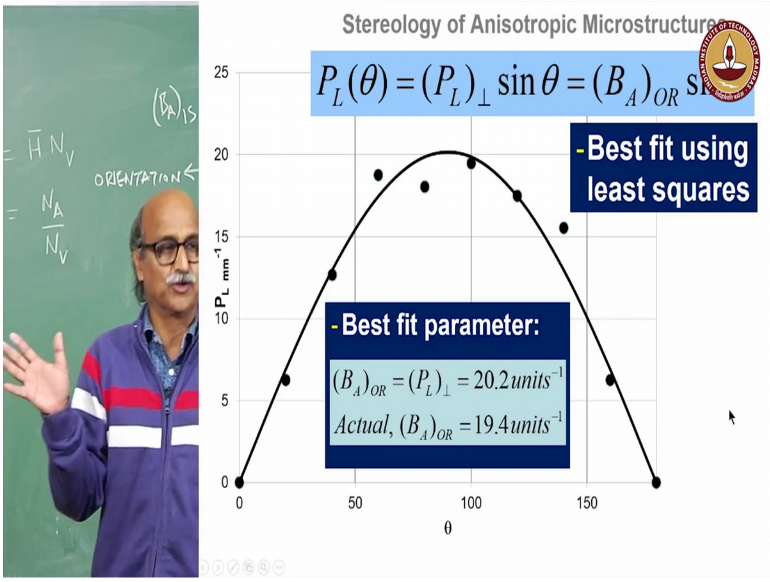

And through this process of fitting using least squares I got order the order of 20.1 or 20.2 per unit and hence and the actual value I should get is of the order of 19.4. So, I am very close to the correct value and this also shows that there is validity to this particular relationship that is there.

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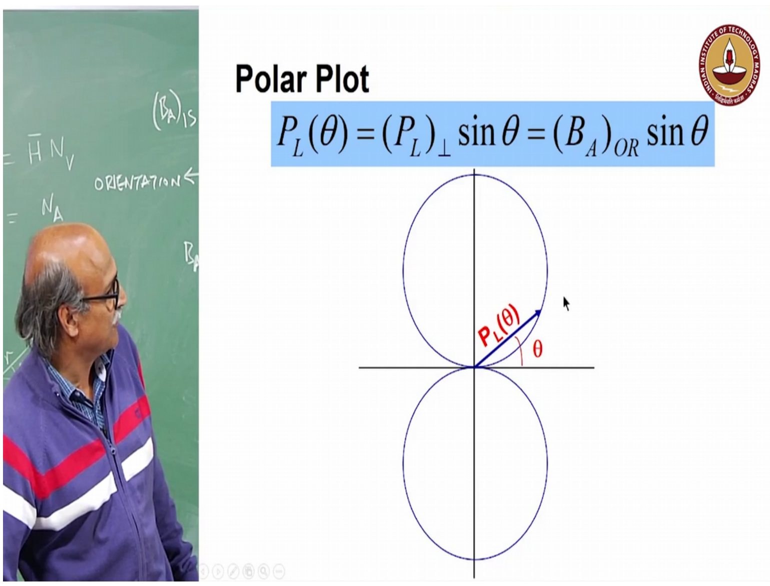

Now, what we do is we make a different kind of plot, instead of plotting P L versus theta, theta is put on the x axis and P L is put on the y axis I make a polar plot of this ok. So, what is a polar plot? How do I represent a point on a polar plot? A point on a polar plot has the coordinate's r theta , where r is the distance from the origin and theta is the angle that this vector makes to that point from the origin with respect to the x axis so, that is the kind of plot which represents polar plots.

So, what I will do is basically, I will be looking at individual P L values versus theta and plotting the points in that manner.

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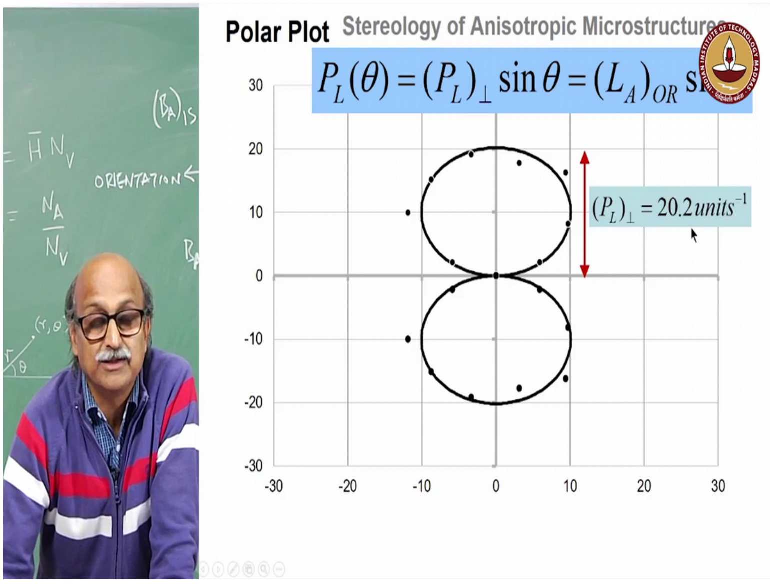

So, the end of this vector so length of this vector is P L theta and the angle this vector makes with the x axis is theta. So, this point is represent so, all the points I have plot like this, then what kind of a curve I am going to get. So, this is what I am plotting, P L is the length of the vector theta is the angle the vector makes with the x axis, when the equation is this when the equation is this what kind of a curve do you expect you will get from theta varying from 0 to 180 degrees you will get a circle like this.

This turns out to be the equation of a circle, with center on the y axis and the radius of the circle; obviously, is P L perpendicular divided by 2 or P L perpendicular is a diameter of the circle and if I go from theta is equal to pi to 2 pi I am simply measuring again the orientations that I have already measured. So, I should get a mirror image of the circle at the bottom going from theta is equal to pi to 2 pi.

So, this is the graphical way of representing this orientation. So, if you get completely oriented structure then this would be the graphical representation of such a structure ok. Now what we do, this is the actual data now plotted as a polar plot so, I get these points represented now as a as a form of a polar plot. So, this is all these points 0 to pi and then these points are simply reflected from the top because it is just a mirror image of the top.

You once I have got P L perpendicular then I can now draw a smooth circle through this points and that is what one would get as the polar plot passing through the points; obviously, not all points are falling exactly on the circle because this is an experimentally measured data. So, there is statistical fluctuation about the circles and the diameter of the circle is P L perpendicular and this has been estimated as 20.2 per unit.

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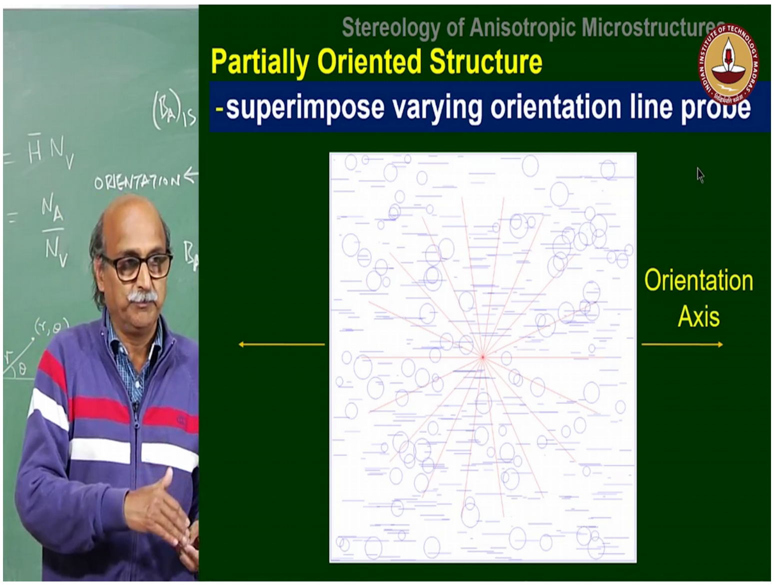

Now, consider a structure now which is actually a combination of these circles and these straight lines aligned in a given direction this represents is a representation of such kind of a structure, where you have isotropic elements and you have elements which are aligned along the orientation axis. On this I place now the varying orientation line probe. So, superimpose the varying orientation line probe and what we do let us collect this data and see it in excel first.

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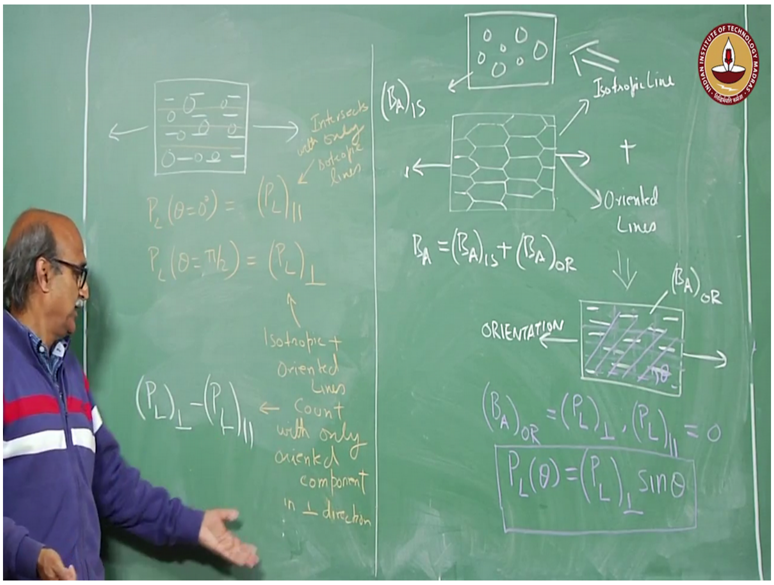

But before even I see it in excel let us try to understand that I have a structure where there are isotropic lines and there are oriented lines. If I superimpose on it a line probe like this and get P L at theta equals 0 degrees with which elements this line probe will going to intersect it is only going to intersect with the isotropic lines it will not intercept with the. So, this is the way of isolation from the oriented component and the isotropic component so, let me call this as P L parallel.

What about P L where theta is equal to pi by 2 and I will call that as P L perpendicular. So, this one intersects with only isotropic lines, what about this one now, this count will have this will count will have both, this count will have isotropic plus oriented lines. Now then what about what about the difference between these 2 counts, P L perpendicular minus P L parallel, this will represent a count in the perpendicular direction with only the oriented lines.

So, this represents a count with only oriented component with only the oriented component in perpendicular direction. Because the isotropic component does not change as the orientation of the line probe is changed that remains the same. So, all I have done is I have removed the isotropic component and extracted the oriented component out of the structure.

So, if I had a structure like this then what kind of a polar plot I will get well I will get some kind of a polar draft plot again.

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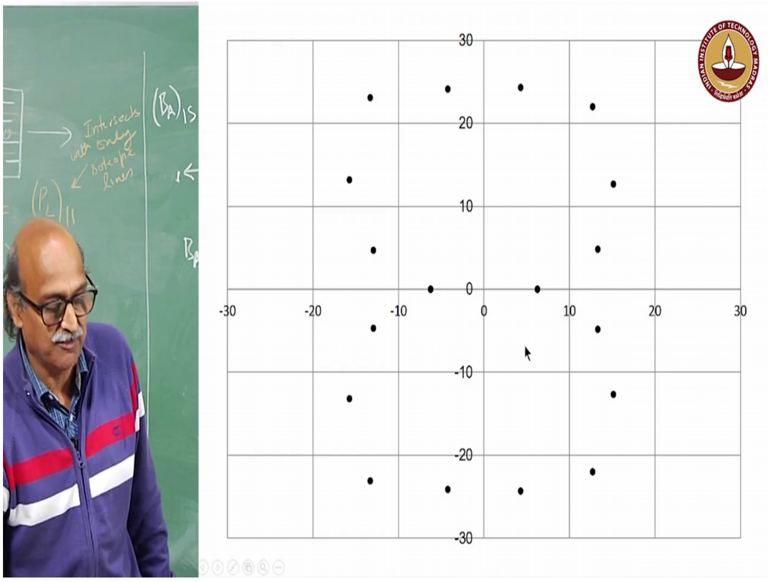

But it will not be quite the same because it will have both an isotropic and oriented component in different directions. So, we can visualize this polar plot in the following way for the completely oriented component I should get this.

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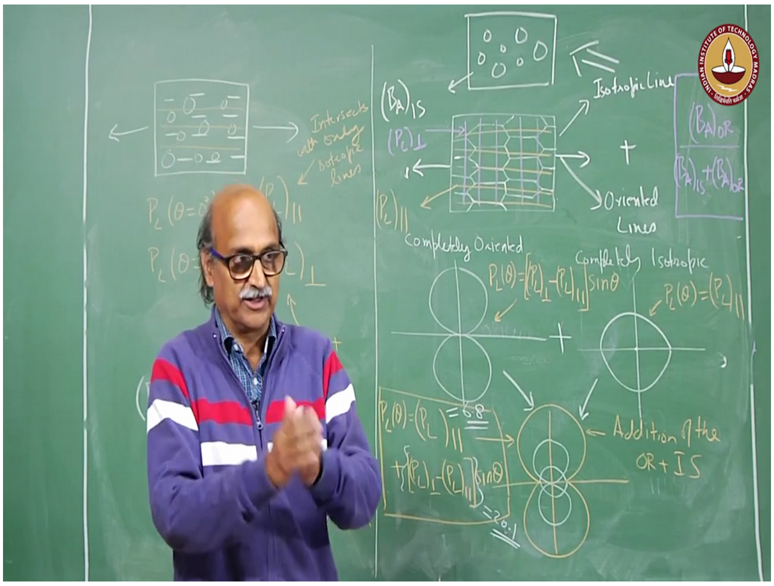

What about for the completely isotropic component, for the completely isotropic components of this is completely oriented, this is completely isotropic, which means as you vary theta from 0 to pi I should get basically the same value of the number of intersections per unit length.

So, if I plot make a polar plot my vector length remains the same like from 0 to pi I will get like this, then pi to 2 pi will simply be the mirror image I will get like this, I add these 2 components. So, this one is represented as P L theta equals P L perpendicular minus P L parallel giving me the only the oriented component times sin theta out here this one is P L theta equals P L parallel that is it right if P L parallel intersects only with the isotropic lines and it should remain the same in all orientations with the isotropic lines.

Adding these 2 would give me the sum total of the polar plot that I will get. So, adding this what I will get for the polar plot is like this I have the completely oriented part superimpose on this the isotropic part and then add up these 2 polar plots and I should get a polar plot like this. So, this is the addition of the 2 of the oriented plus isotropic and what would be the equation for such a plot , well it is just a sum total of these 2. So, this would be P L theta would be equal to P L parallel plus P L perpendicular minus P L parallel times sin theta ok.

So, all I have to do then is first make the polar plot and then fit the curve to this data so, how to fit the curve we go back to excel.

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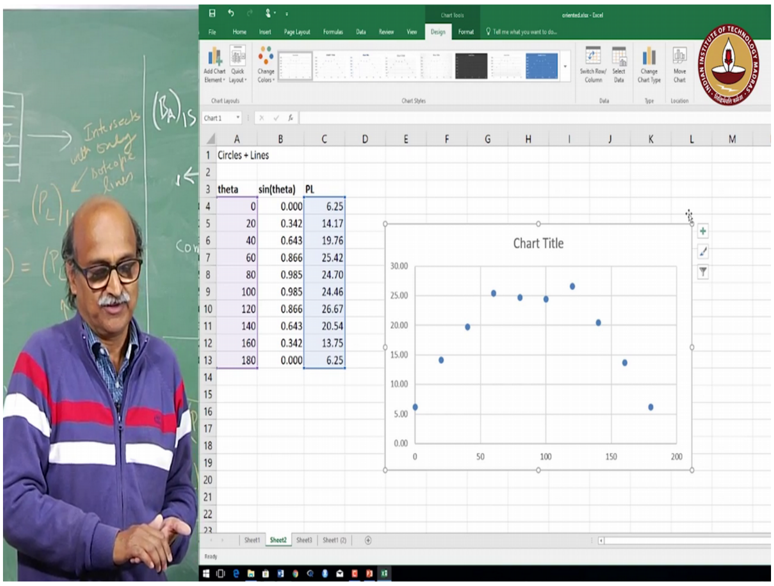

Now, this is a structure consisting of circles and lines I have again obtained data from number of fields and what you are looking at our average values of number of intersections per unit length how they vary with theta and this was basically the structure in which some circles have been added to the previous structure. So, some isotropic component has been added.

Now, let us try to plot this as before if I let me insert let me add data to this. So, x axis is theta values and y axis is P L values oops sorry theta values y axis P L values and I got a curve which looks something similar to what I had. So, this curve does not tell me enough so, instead of plotting this let me plot P L versus sin theta.

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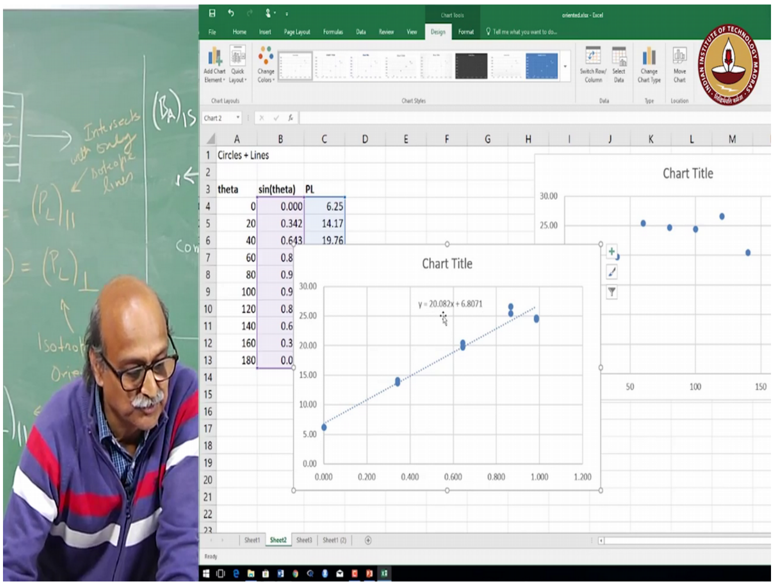

I think I have plotted this reverse I will just change it now. So, sorry about this let me remove this add x values should be sin theta and y values yeah. So, I get this data points again I should get a straight line looking at this equation I should get a straight line with an intercept of P L parallel and slope of P L perpendicular minus P L parallel.

So, add trend line linear selected I would display the equation and that is it. So, I have this equation here. So, y is equal to 20.082 plus x times. So, intercept is 6.8 and the slope is 20.1. So, what basically we are saying is that intercept is this 6.8 and the slope is this one 20.1, then if I take this data then and then plotted there is a polar plot. So, this is the equation I have just fitted.

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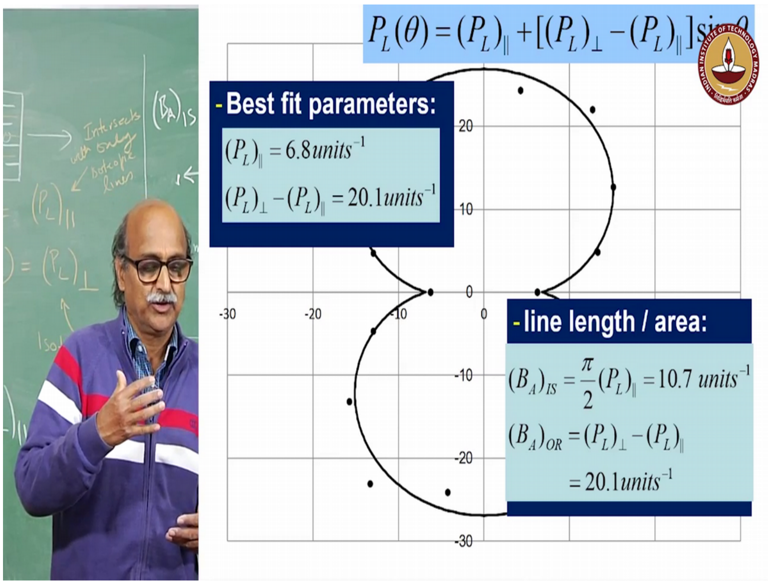

And if I plot that equation now I am going to get curves like this or these are not complete circles as before. We got the intercept 6.8 per unit and the slope P L perpendicular minus P L parallel is 20.1 unit and remember that P L parallel represents the isotropic component P L perpendicular minus, P L parallel represents the only the oriented component. So, if I calculate now boundary length per unit area of the ice isotropic component boundary length per unit area of the oriented component this is what I am going to get.

For the isotropic component this relationship is the one that is to be used pi by 2 times P L parallel and I will get 10.7 per unit, for oriented component the same relationship still holds that number of intersections per unit length with the oriented component with the same as the boundary length per unit area and that is 20.1 per unit exactly the same result as we had got before. So, we have been successfully able to extract the oriented and the isotropic components separately.

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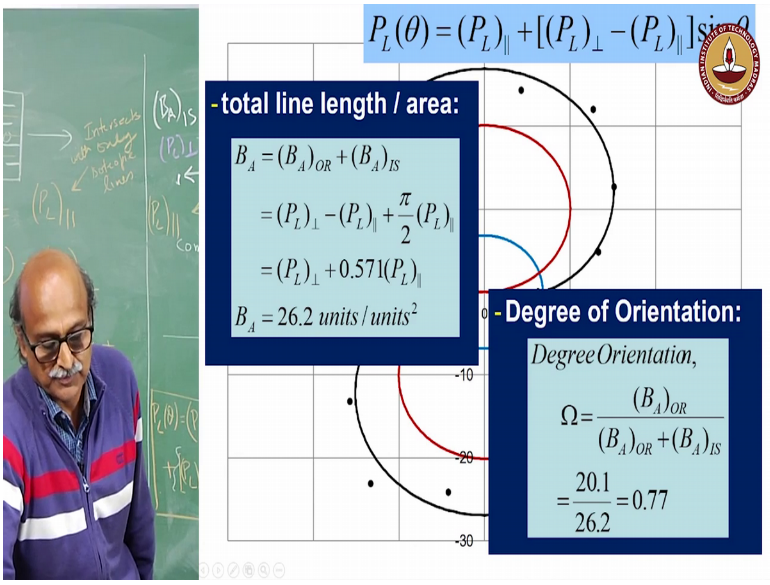

Plus if I now want to know what is the total boundary length per unit area well all I have to do is add up the two components B A oriented plus B A isotropic, in the end if you notice this I can write the total boundary length per unit area in terms of I substitute for oriented P L perpendicular minus P L parallel for B A isotropic I introduce pi by 2 P L parallel then just P L parallel components I can add up. So, I will get P L perpendicular plus 0.57 P L parallel getting me total of 26.2 units per unit square total isotropic as well as oriented.

The other thing that I want to tell you from this is I do not really need to measure the complete polar dropped to understand the structure all I have to do is once put my grid of line in the perpendicular direction to the orientation axis get P L perpendicular and another measure I do in the same structure is put them in the parallel direction and obtain P L parallel without measuring other orientation , then I would be able to get the oriented as well as the isotropic component just by making 2 measurements on such a structure.

Now, one last thing on this oh that can I express the extent of orientation that is there, one can express the extent of orientation now as a ratio, ratio of the fraction of the oriented component, how much fraction of the oriented component is there which can be expressed as B A oriented divided by B A isotropic plus B A oriented right and this can be expressed like this. Degree of orientation is what we will call this call that as omega let us say so; it is B A oriented upon the total boundary length per unit area which is B A oriented plus B A isotropic.

So, B A oriented is nothing, but P L perpendicular minus P L parallel which from the previous slide is 20.1 and total of the 2 is 26.2 so, that gives me 0.77. So, let us see what thus this be that 77 percent of the component is oriented in this microstructure. So, very high degree of orientation and this degree of orientation will vary from if you had only isotropic no oriented then omega will be 0 because B A oriented will be 0 and if you had only oriented and no isotropic then b a isotropic will be 0 and omega will be 1 so, the degree of orientation will vary from 0 to 1 ok.

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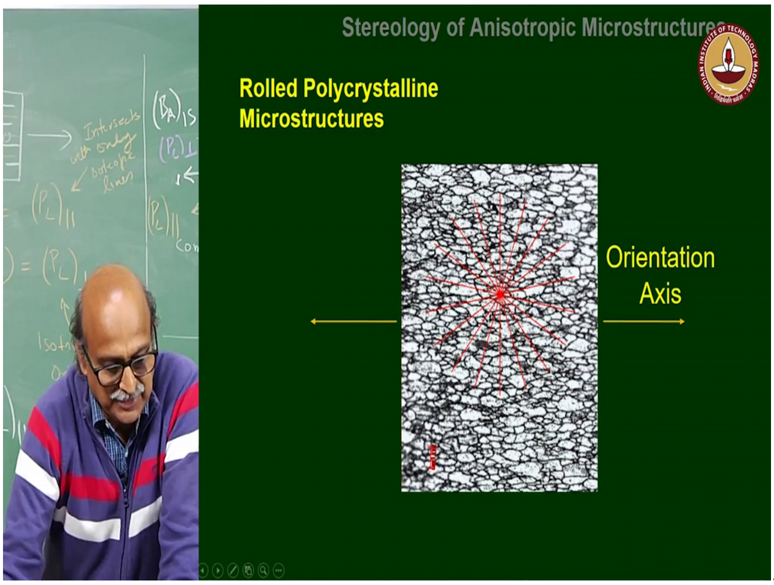

Now, this idea now can be straight away applied to a microstructure like this.

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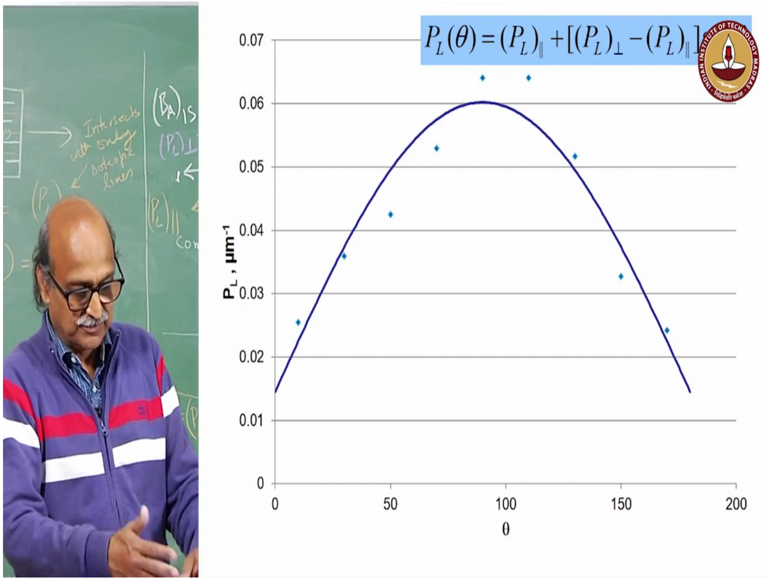

And we get this P L versus theta plot.

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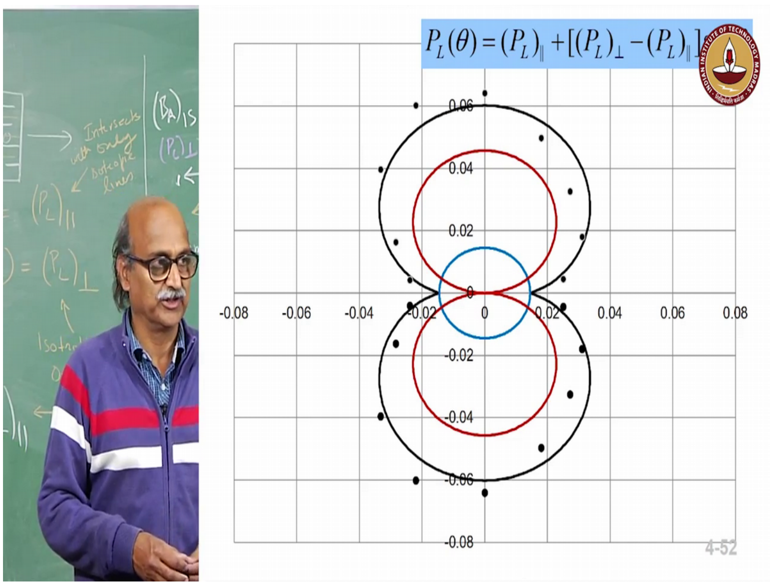

We get this polar plot, we figure out what are P L parallel, P L perpendicular minus P L parallel etcetera from this and this can be left. So, this can be left as an assignment problem, for this particular microstructure to figure out you know what is the degree of orientation in the microstructure.

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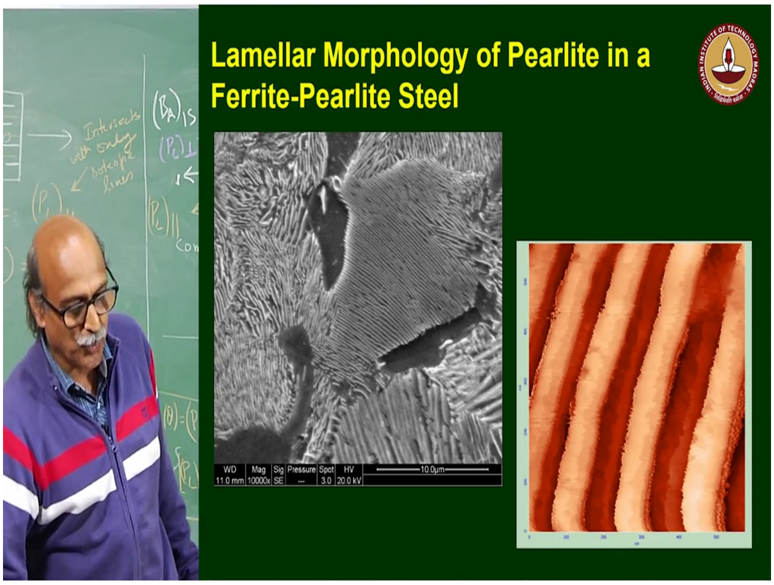

Now, the same probe that we have looked at can also be used to find the average interlamellar spacing of a structure like structure of pearlite or any other lamellar morphology.

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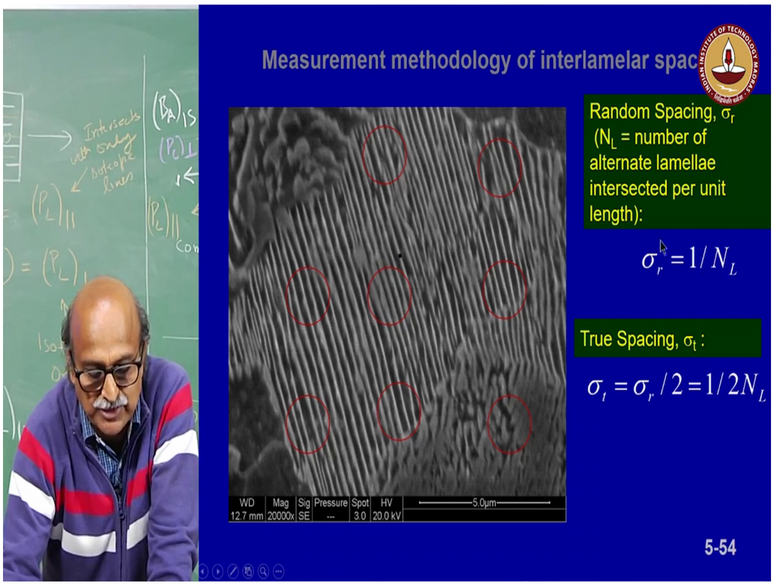

So, what you would do, you could put this varying orientation line probe and get an average number of intersections per unit length with alternate lamellae ok. So, I am using either I can use this average orient varying orientation line probe or I can use the circular line probes to get an L, the number of alternate lamellae intersected per unit length reciprocal of that would give me what is called as the random spacing between the alternate lamellae ok. Interlamelar spacing would be defined line that cementite to cementite plate or ferrite to ferrite plate.

So, this sigma r becomes the random spacing then it can be shown again through stereology and I am not going to derive it here that the true interlamellar spacing on an average is half the random spacing. So, sigma t which is the true spacing is given by sigma r the random spacing divided by 2 or it is simply 1 upon 2 N L. So, that is fairly straight forward and the derivation can be seen in some of the references that are available in the literature on how to are, how this particular relationship was arrived at again by stereological procedures that we have seen earlier.

We are going to use some of the older technique only to derive this relation as well with this I with this we terminate this lecture on how to deal with oriented structures and this lecture has been limited only to the 2 dimensional case.

But we can also extend these ideas into the 3 dimensions where you can have a more complicated orientation that may be present in the microstructure, depending on what kind of processing that has been given. Whether, if the structure that extruded it may give you a different kind of orientation versus if a structure is rolled or the structure is undergone more complicated thermo mechanical processing then you can have quite complex orientation relationships ok.

Thank you.