Elementary Stereology for Quantitative Microscopy

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Lecture – 15
Counting of Grains and Particles – Part 2

Hello, everyone. Welcome to this yet another session of the tutorial class of this Elementary Sterology course. In the last tutorial what we have seen in the demonstration of how to calculate the sum of the basic stereological parameters and one or two derived parameters using the microstructure of the graphite nodulars the cast iron microstructures.

So, today I am going to show you some of the similar calculations on a completely filled microstructures. I would say a polycrystalline a single phase microstructure completely phase filling grains something like this.

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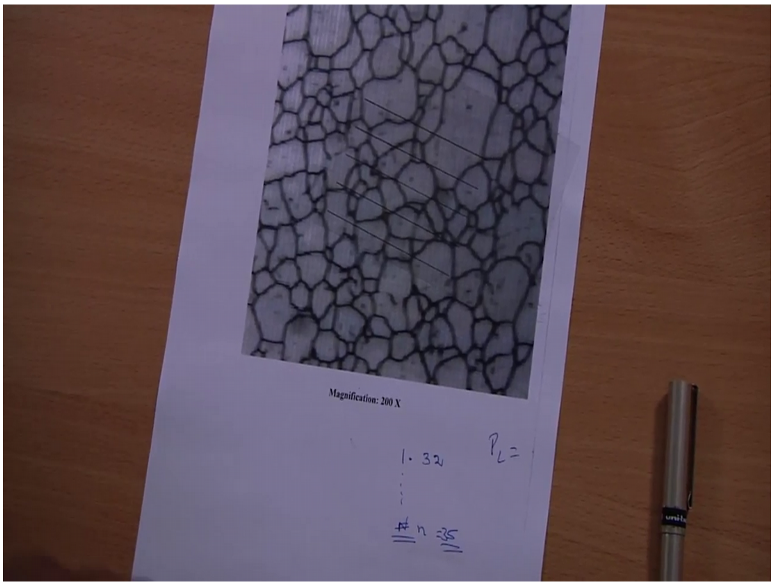

So, what I am going to do is we are going to perform a some of the stereological parameters using this polycrystalline microstructure.

So, the one of the first parameter we are interested is to calculate the mean intercept length.

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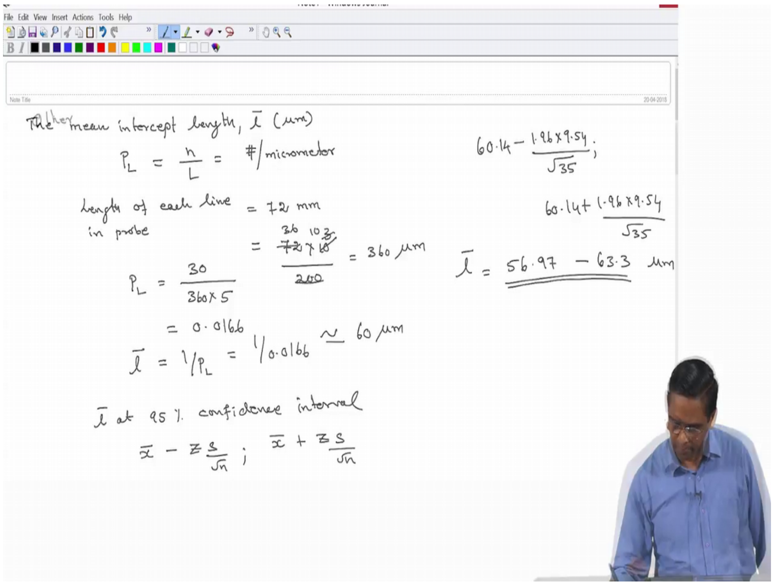

So, let us write mean intercept length l bar in micrometer. So, if you recall we have done this for the graphite nodules yesterday. Today, we will try to measure this on this completely space filling grains. So, what I will do is; so, what we need to do we need to calculate P L which is nothing, but number of intersections per micrometer. So, how do we perform this? So, how to calculate this? This is what I am going to just demonstrate. So, let us now go to this microstructure, ok.

Now, let us start looking at this how we can do this intersection counting. See what I am holding in my hand is like a probe which is measuring about 72 mm the length of the probe is 72 mm. So, you have about 5 lines. So, I am going to use this probe and place it on this microstructure like this and I will just keep it in a random orientation and then start counting the intersections. So, we can I will just show you some of the counting then I will go to that excel sheet.

For example, if you start counting like this. So, we can start from here 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32. So, I will write the event number 1, the number of count is 32. So, like that I can keep on going up to n count n number of events of different different orientations like this. In fact, what I have done is I have done similar counting up to the n value up to not capital N; it is small n up to 35 which I have tabulated in the excel sheet now I will go back to this calculation, ok.

So, now what is that we have measured the length of length of each line in the probe the probe is about 72 mm, ok. So, we have to remember that micrograph was taken at 200 X. Probably if you if you have noted that carefully that micrograph does not have the micron marker, but there was a mention that it has taken at 200 X magnification. So, we can convert that into this into 1 X what how can we do this is so, you can do that 72 we are interested in micron. So, you can do this calculation. So, this will cancel out and this is about 36. So, you get around 360 micrometer.

So, if you can substitute this into this formulae for example, the first count I have made as in the exercise is 30 suppose if I take that value and I have about the each line length is 360; So, 360 into 5 lines. So, so, I will get the P L value equal to 0.0166. So, I like that I will generate P L, but our interest is l bar. So, what is l bar? l bar is nothing, but 1 upon P L this is what we have learnt it from the lecture. So, you see one upon 0.0166 which is equal to approximately 60 micrometer. So, this is how you have to calculate for each case.

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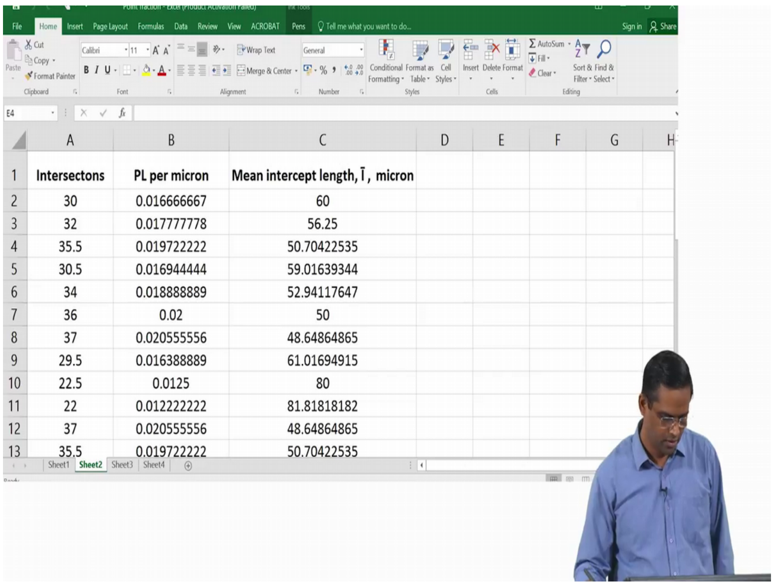

	A	B	C
1	Intersectons	PL per micron	Mean intercept length, $\bar{l}$, micron
2	30	0.016666667	60
3	32	0.017777778	56.25
4	35.5	0.019722222	50.70422535
5	30.5	0.016944444	59.01639344
6	34	0.018888889	52.94117647
7	36	0.02	50
8	37	0.020555556	48.64864865
9	29.5	0.016388889	61.01694915
10	22.5	0.0125	80
11	22	0.012222222	81.81818182
12	37	0.020555556	48.64864865
13	35.5	0.019722222	50.70422535

Now, what I do is I will just show you what I have done for about 35 measurements, yeah.

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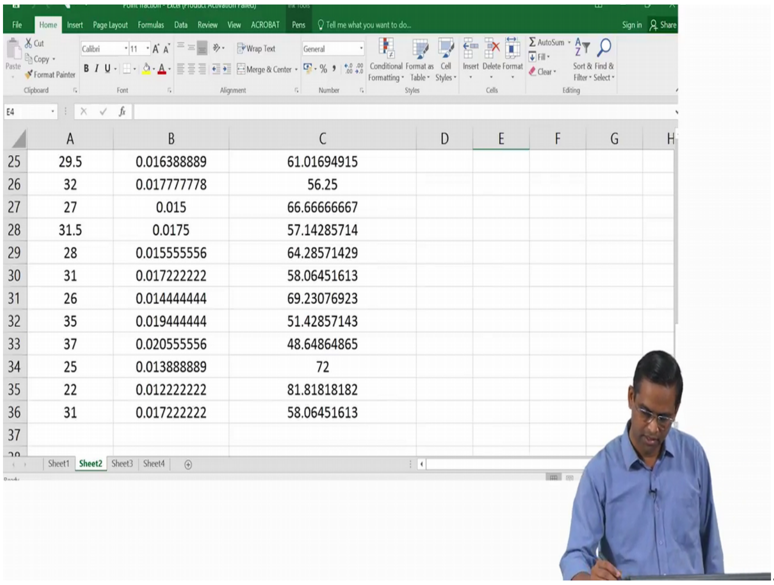

	A	B	C
25	29.5	0.016388889	61.01694915
26	32	0.017777778	56.25
27	27	0.015	66.66666667
28	31.5	0.0175	57.14285714
29	28	0.015555556	64.28571429
30	31	0.017222222	58.06451613
31	26	0.014444444	69.23076923
32	35	0.019444444	51.42857143
33	37	0.020555556	48.64864865
34	25	0.013888889	72
35	22	0.012222222	81.81818182
36	31	0.017222222	58.06451613

So, for 35 measurements I have done this. So, this is a P L this is per micron and then of course, mean intercept length is 1 by P L.

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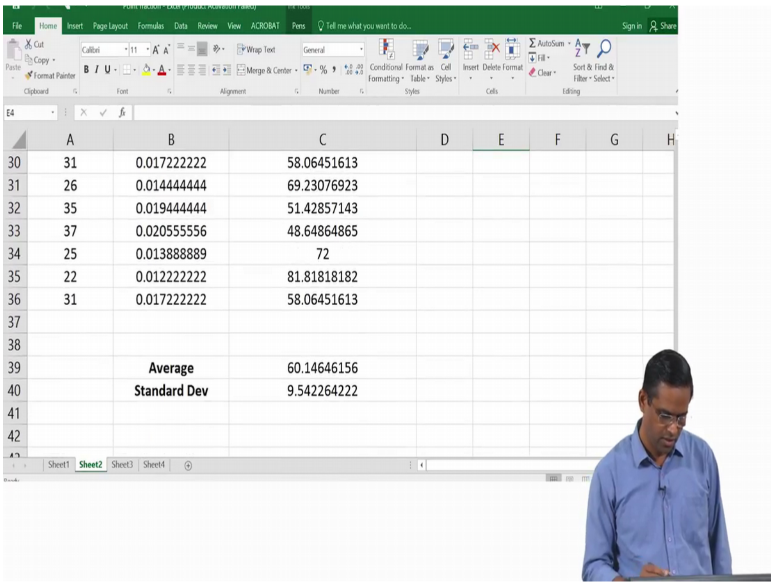

	A	B	C
30	31	0.017222222	58.06451613
31	26	0.014444444	69.23076923
32	35	0.019444444	51.42857143
33	37	0.020555556	48.64864865
34	25	0.013888889	72
35	22	0.012222222	81.81818182
36	31	0.017222222	58.06451613
39		Average	60.14646156
40		Standard Dev	9.542264222

So, if you look at this I have done the calculation of average mean intercept length is about 60.14 and the standard deviation is about 9.54.

So, similar to our reporting format what we can do we will now say that we will use normal distribution and then we will report l bar at 95 percent confidence interval like we have done yesterday. So, what we do, we substitute it is this formula, this is a lower interval and this is for higher interval. So, when you substitute this what you get you can write the average is 1.64 minus 1.96 that is a z value into the standard deviation is 9.54 divided by total number of events is 35. So, this is about lower limit and then you can do it for the higher limit.

So, this is same thing 6.14 plus 1.96 into 9.54 square root of 35. So, you have a range which is coming out to be 56.97 to 63.3 micrometer. So, they mean intercept length of the grains what you have seen is will be lie in this range at least 95 percent of the grains will fall in this range that is the physical meaning of this mean intercept length calculation.

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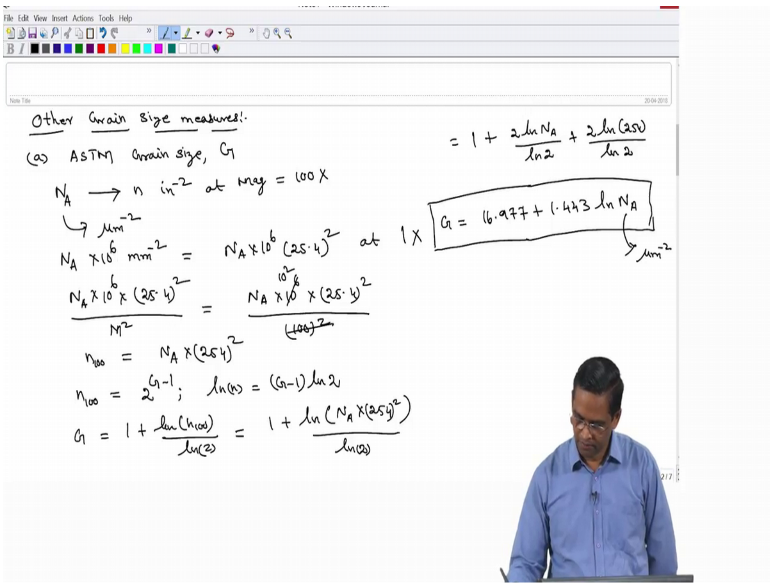

So, now we will move on to the next topic other grain size measures. Like it is mentioned in the lecture there are popular grain size measures available. So, we will first take up the ASTM grain size G very popular in the industry and very widely used even in academics. So, what does it say the ASTM grain size number? It says that number of grains per unit area at a given x. So, basically it is number per unit area which is given as number of grains per inch square at magnification of 100 times. So, this is what ASTM grain size is defined, but we can look that look at that for a micrometer also. So, then it becomes N A into 10 to the power 6 per mm square also we can represent.

So, you can write if you convert this into a comfortable units like this is inch square. So, you at 1 x; So, you can rewrite this N A times 10 to the power 6 into 25.4 whole square divided by this M square which is nothing, but N A into 10 to the power 6 into 25.4 whole square divided by 100 square. So, you can just now cancel this out then it becomes 10 square and the you can write n at 100 x is equal to N A into 254 whole square, ok. So, again you can rewrite this at n is equal to 2 G 2 power G minus 1 this is what. So, you can rewrite like this lon n is equal to G minus 1 lon 2.

So, you can conveniently make some rearrangement using simple mathematics which can be written as 1 plus lon N A into 254 whole square divided by lon 2 which can be written like this. 1 plus 2 lon N A by lon 2 plus 2 lon 254 divided by lon 2 which can be further simplified, where G is equal to 16.977 plus 1.443 lon N A. So, this is one way of simplifying this G and you can where N A is represented per micrometer that is area the number of grains per unit area.

This is when you use micrometer this is what you will get and what we can do is this is one way of looking at the ASTM grain size number and if you use mm square this expression is slightly modified for the sake of a clarity I will do that again.

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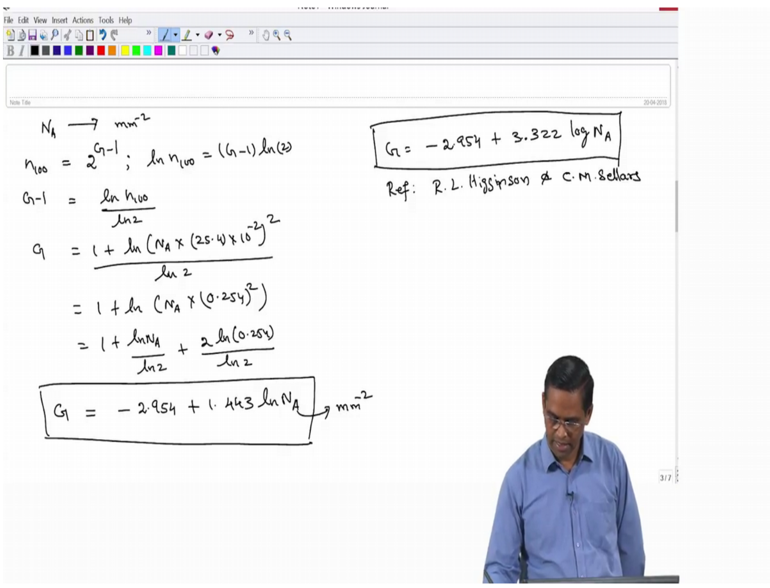

Suppose if you do N A is expressed in per mm square then you rewrite this equation n at 100 mag is equal to 2 G to the power 1 sorry d 2 power G 2 to the power G minus 1 then again same what we have done previously. So, it is a simple manipulation lon n by lon 2 and you can G.

Again rewrite this in terms of N A we can substitute N A. So, G is equal to 1 plus lon N A into 25.4 into 10 to the power minus 2 whole square divided by lon 2 because we have to take care of that 100. So, that is 10 to the power minus 2 and if it comes to N A into 0.254 whole square and this can be rewritten as 1 plus lon N A by lon 2 plus 2 lon 0.254 by lon 2.

So, you will get G is equal to minus 2.954 plus 1.443 lon N A. So, this is another way of writing G if N A is represented by per mm square ok. So, there are one more relation reported in the literature something like you have G is equal to minus 2.954 plus 3.322 log base 10 times N A. So, this is reported in one of the references what we have shown in the introductory slide; I will just give the authors name probably you can refer that book the authors name is R. L. Higginson; Higginson and C. M. Sellars worked out examples of quantitative metallography. So, they have reported this kind of expression. So, you can find out them.

So, now the question is how are we going to find the N A because we are now relating ASTM grain size by calculation of N A; So, how do we do that? So, again what I am going to do is I will bring back the microstructure I am going to use the probe of this kind and I am just going to count the number of grains. You know that in the theory lecture it has been demonstrated how it should be counted, how to count the particles using these kind of a probe.

There is a methodology which has been demonstrated we will adopt that and once I show how it is counted then I will come back to the excel sheet and look at the spread the data spread then we will come back to the calculations.

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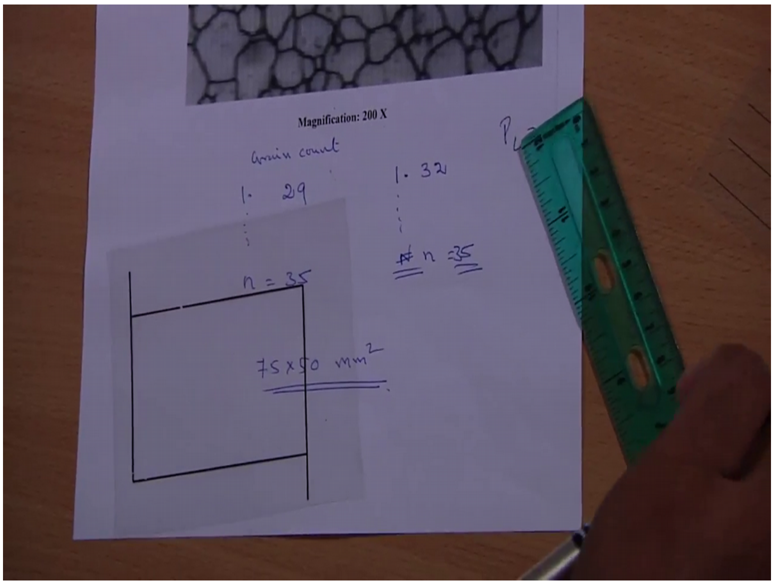

So, let us now go and do the count. So, we can place this probe at an arbitrary orientation. So, the what is the rule of using this kind of probe this is an unbiased probe and then that unbiased way of counting is you have to eliminate the grain which is intersected by this two lines and then we have to include the grains which is intersected by this line and this line.

So, that is the unbiased counting. So, you have to exclude lines and then this is include lines. So, let us do that counting now I would start counting like this 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29.

So, I have the count you can write it like this grain count 1 number of event 1 I have about 29. So, like that I will now go keep on moving up to 35, I have done that. By keeping this make sure that you are not going like this make sure that the boxes inside this and then you have to take a different different orientation on this way several things you can do and finally, what you have to do is you have to measure this because we are interested in area.

So, this is about 75 mm length and 50 m 50 mm width. So, note down this. So, 75 into 50 mm square, ok.

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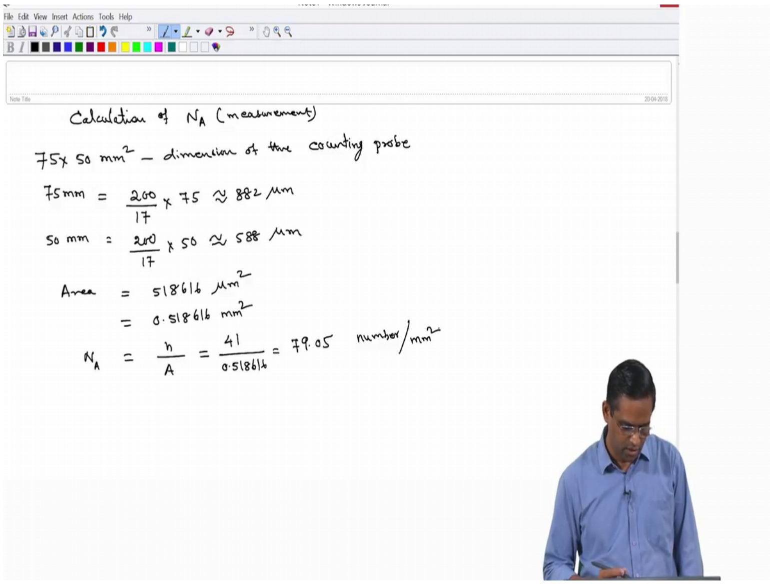

So, now let us go back to the calculation. So, the area what we have measured in the let us write calculation of N A are basically a measurement not exactly a calculation. So, we will see we have taken the probe of a dimension 75 into 50 mm square, let us write as a dimension of the counting probe. So, again we have to convert them into micron because we are interested in reporting the results at micron meter.

So, this is similar exercise what we are doing. So, this is again 200 x magnification. So, similar exercise you have to do into 75 which is approximately 882 micron meter and 50 mm this is we have done it earlier. So, we can do it this way 200 by 17 into 50, 588 micrometer. So, the area is 518616 micrometer square and if you want to report this into mm square it is 0.518616 mm square.

So, N A is nothing, but you have small n by area. So, in our first event the which I have calculated earlier the first data point is 41 and the area is. So, I will use mm 518616 which is 79.05 which is a number per mm square. So, number per unit area. So, this is when I use mm square. So, I will you can use that micrometer scale also for the doing grain size calculation for example, the ASTM grain size and so on. So, what we can do is I will just show how I have used that calculation.

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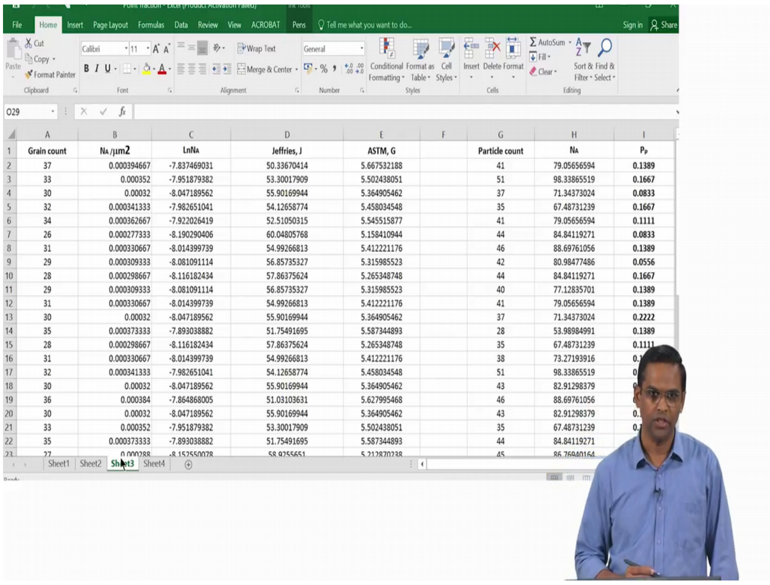

So, if you look at this excel sheet in a in the first column I have just put the grain count using all these measures up to 35 I have populated this column up to 35 count I have done and I have measured this N A, but in this case for calculating the grain size ASTM and other measures I have used a micrometer scale. So, you should not get confused with these two depending upon I will tell you why I use the micron meter and millimeter just in another case study I will tell you will also agree with me when we go to that scale difference.

So, let us now assume that I am using only a micrometer scale here. So, that N A is populated like this then I have estimated the ASTM grain size like this. So, we will just look at the calculations how I get this and I have also used the other grain size measures like Jeffries grain size which is nothing, but a square root of 1 by N A. So, it all related. So, I have just populated here and what we can do is now we will go back to this sheet and then we will try to work it out how I arrived at this.

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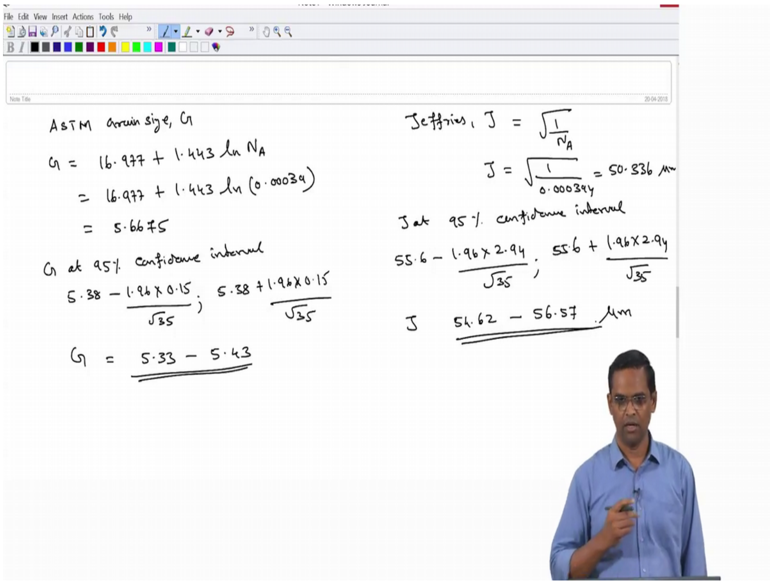

So, let us do this as I said that ASTM grain size number or rates in grain size G the expression we got as G is equal to 16.977 plus 1.443 into lon N A. So, this is the expression when you take the micrometer scale. So, if you substitute this 16.977 plus 1.443 lon the first measurement I have taken in the excel sheet is 39 which will comes out to be 5.6675. So, this is the ASTM grain size number. So, you report only 5.6 or you can even say it is approximately 6 and so on.

So, you can just again we can do this 95 percent interval because you have enough data to speculate and what I have done is you can just look at this G at 95 percent confident level confidence interval. So, what I have done is this is I directly substitute 1.96 into 0.15 divided by square root of 35 and 5.38 plus 1.96 into 0.15 square root of 35. So, I get the range ASTM grain size number about 5.33 to 5.43 at this interval. So, this is for ASTM grain size number.

If I use Jeffries so, the Jeffries grain size J which is nothing, but a square root of 1 by N A we can check whether how do I get that. So, you simply substitute this one of the N A what we have measured I will put this 0.00394 which is nothing, but 50.336 micrometer. So, you will get this. So, in an excel sheet I have populated this for all the 29 counts. So, you just you can report this again J at 95 percent confidence interval. This is just for a repetitive exercise, but then it will give you some exercise to practice. This is 2.94 this is

35 and this is 45.6, this is an average plus 1.96 into 2.94 divided by square root of 35; so, which comes to in the range of 54.62 and 56.57 micrometer. So, this is how the.