Elementary Stereology For Quantitative Microscopy

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Lecture – 14
Description of Polycrystalline Microstructures derived measures

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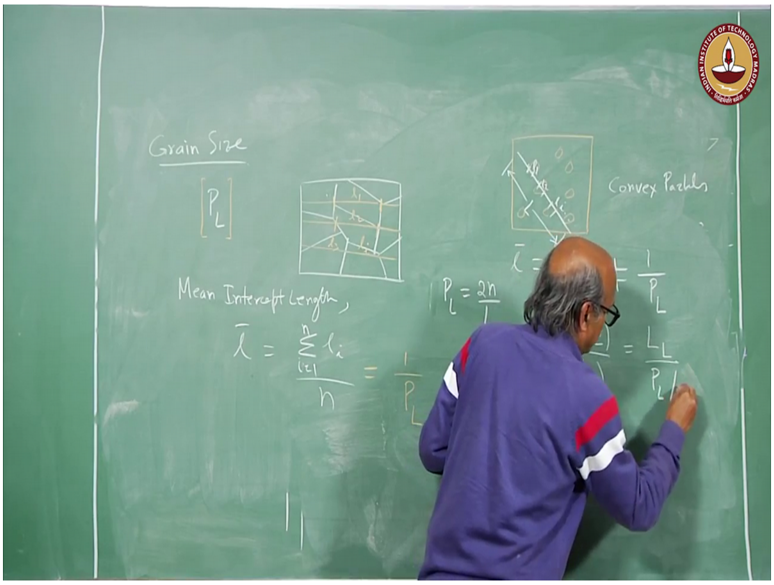

So, here if I look at this.

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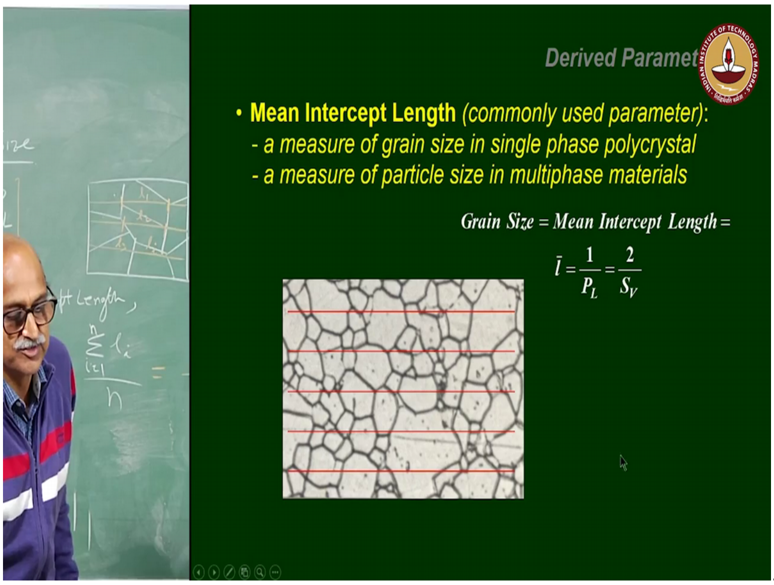

I superimpose a set of parallel lines and get the measure P L, number of intersections of the lines with the grain boundaries per unit length, using this line grid. One measure of grain size can be and that is actually a important measure which could be that when I put this grid of parallel lines, I measure these lengths intersected within the grain ok. So, I measure these intersects l 1, l 2, this is l 3, this is l i and so on.

And I can define a grain size measure as what is called as the Mean Intercept Length, l bar which is just nothing but summation l i; if I have n number of intercepts taken average value. This is the mean intercept length, this is very commonly used where particularly, where if you are doing structure property correlation, this is the very important measure for grain size as the mean intercept length.

Now, this thing is how do I measure this mean intercept length? Do I individually measure these lengths or can I have a counting, a simple counting procedure. We try in sterology to always go wherever possible go for a counting procedure where I simply have to do counting because then there is less error prone, it is very fast to get the grain size. Now this should be very clear that, if I measure P L, then mean intercept length would be nothing but simply 1 upon P L.

So, I do not have to go and measure these individual lengths. I will simply measure count the number of intersections I get with the grain boundaries divided by the total length of the grid line, I will get P L reciprocal of that would simply give me the Mean Intercept

Length ok. So, you see the grain size measure here, Mean Intercept Length is 1 upon P L which is also now related; as you can see, this measure is related to surface density of grain boundaries because surface area per unit volume, S V is equal to 2 times sphere.

Hence, you will get Mean Intercept Length to be equal to 2 upon S V. So, Mean Intercept Length is inversely proportional to surface area per unit volume.

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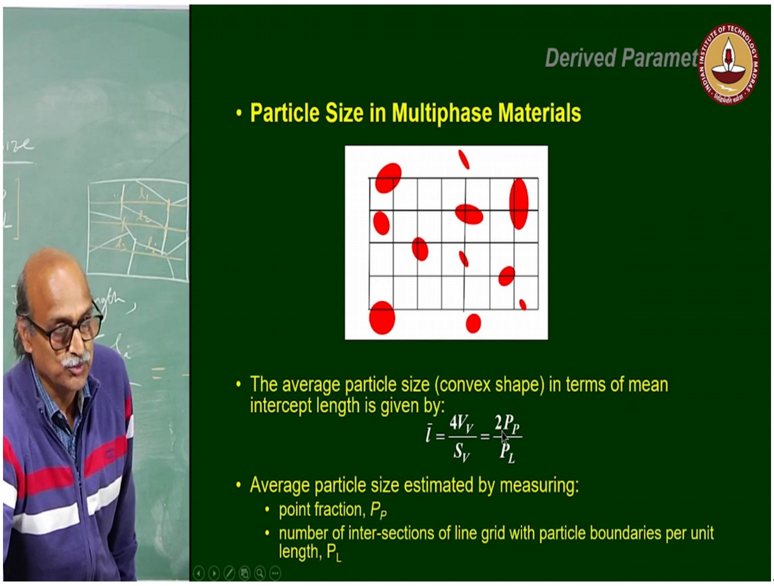

Now I would like to use the same mean intercept length for this particles ok. If I have, now particles and I want to get some kind of a average size for these particles and these particles are of irregular shape. So, how do I, so I can again put let us say, we will explain this in this manner. I can put a grid of lines or I can simply put one line. I measure these lengths ok. Again l 1, l 2, l i and so on, then I can get a measure of the size of these particles as Mean Intercept Length as summation l i upon n.

Now, here the Mean Intercept Length will not be equal to 1 upon P L. It this will work only when you have space filling grains. Here you have this other phase this particles are embedded in some phase. Now, but I would like to still evolve some kind of a counting procedure for this. Now this is done in the following way. Let this total length of this line be capital L and what I do here, I divide the numerator and the denominator by this length of my line ok.

This is what I do. Now let us look at these two quantities separately. What are these quantities, what is this quantity in the numerator? The lengths intercepted within the particle, some total of those divided by the total length of the line that I have superimposed. That is what, that is called the length fraction. So, let us write this. That is the linear fraction.

What is n upon L? We are going to assume here that these are all convex particles. How many; if there are i intersect n particles; how many intersections I have with the, with the interphase boundary, 2 times n right. This is one intersection with the particle, but it produces 2 intersection with the phase boundary, is not it? So, if I want to measure P L, what is P L equal to? P L would be equal to 2 n upon L right. So, n upon L, this term would become what P L upon 2.

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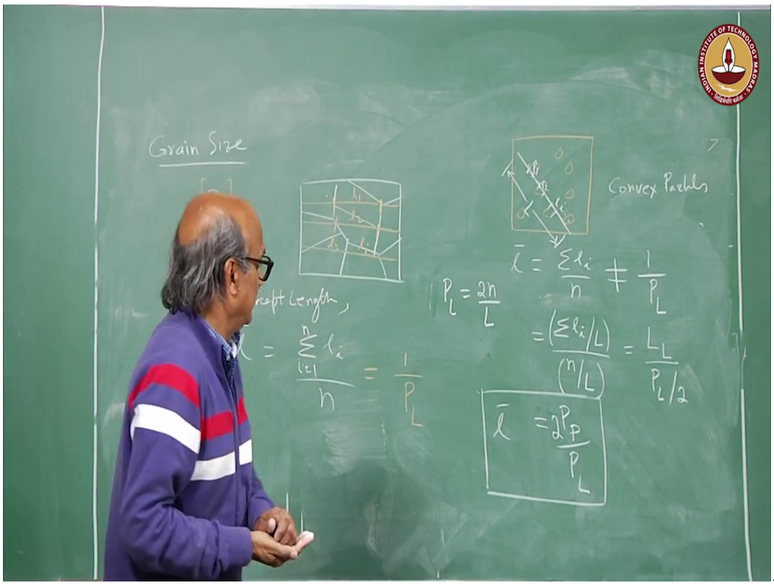

So, this would become P L upon 2; number of intersections with the phase boundaries. What can I replace this linear fraction with? I can replace the linear fraction with volume fraction which I can replace with point fraction. So, my numerator will become point fraction; this 2 will go up and at the bottom P L.

So now, my measure of particle size in terms of mean intercept length is given by an expression 2 times for point fraction upon number of intersections per unit length of the test line. This again is a, I have replaced this method with a counting procedure. So, if I look at the slide here. So, it is 2 P sub P upon P sub L, which since point fraction is volume fraction, I can also write it in terms of volume fraction and P L remember that, instead of P L, I can write surface area per unit volume divided by 2. So, this becomes 4 times volume fraction upon surface area per unit volume ok.

So, here to get this average particle size, I just have to measure point fraction, I just have to measure P L. I do not have to do anything else; very simple method of getting some kind of an estimate of particle size. So, if I am doing some work in which I am may be trying to see how my particle size is evolving; as a function of some processing treatment I am giving that this will be quick measure for measuring the particle size, only counting is required. So, what you do you put a grid here. The same grid you use to get P P and the same grid you use to get P sub L that is it.

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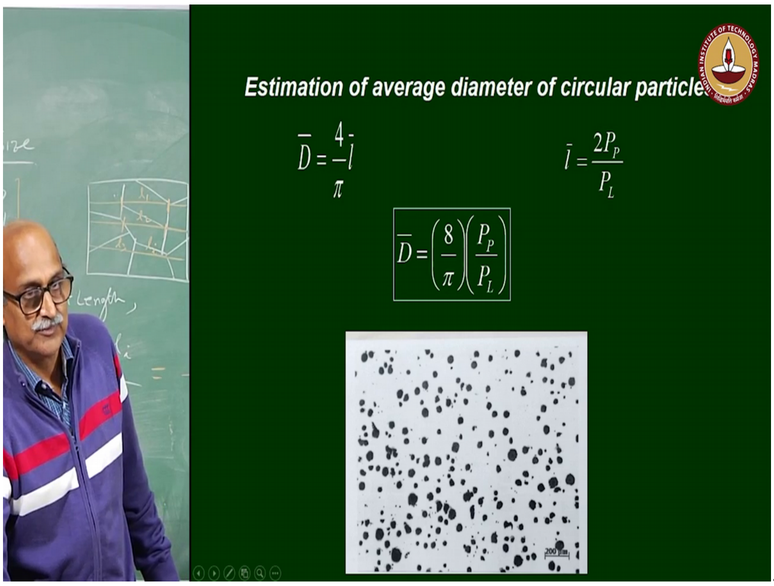

And let us look at now something where the relationship was given to you and let us see how we got this particular relationship. The average diameter of the graphite nodules is given by 8 upon pi times the point fraction over P L ok.

Here, if you look at the slide, this is the expression that we just now got for the mean intercept length of particles. Now I am trying to get this; in order to get this, I have to somehow relate the particle size to the mean intercept length. Now if I have circular particles, I am assuming circular particles here, then what is the relationship between the diameter of the particles and the mean intercept length. So, just the way we did for sphere, remember we got average particle area. Here we are talking about an average intercept length.

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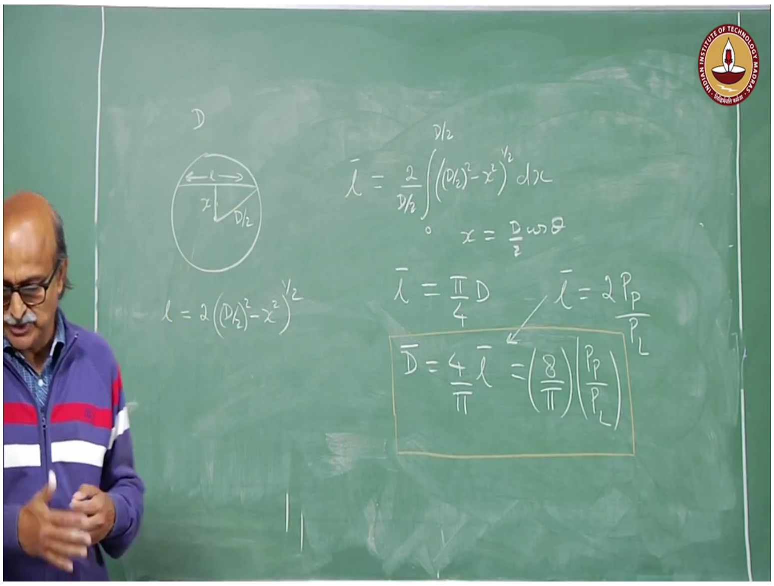

So, again the intercept length, what it would be would depend on when the line is passing through the particle, how far away it is from the center of the circular particle ok. So, let this be x, this is D by 2; D being the diameter and this length is l, the intercept length. So, clearly I can write the intercept length in terms of D and x. So right, 2 times D by 2 square minus x square to power half and twice of that because this will give me just the half the length. So, make it twice.

Then we go through the exercise of finding out the mean intercept length l bar. I need to integrate over all possible values of x going from 0 to D by 2. This factor of 2 can come out and out here is D upon 2. I just have to integrate this again. This is a simple really a simple integral. You just have to substitute. Again do this substitution, it becomes straight forward; x is equal to D by 2 cos theta, just plug it in this and you will get mean intercept length to be equal to pi by 4 D ok. So, mean intercept length is equal to pi by 4 times the diameter and we already know that mean intercept length can be written in terms of 2 times the point fraction upon P L.

So, diameter is actually 4 by pi l bar, substitute for the mean intercept length in this and this will become 8 upon pi times point fraction upon P L. So, this is a another of the derived relation where you can get diameter average. And In fact, this is particles are of varying diameter, this will give me average diameter of all those graphite nodules, if we assume them to be circular ok. So, that is one more of the derived parameters.

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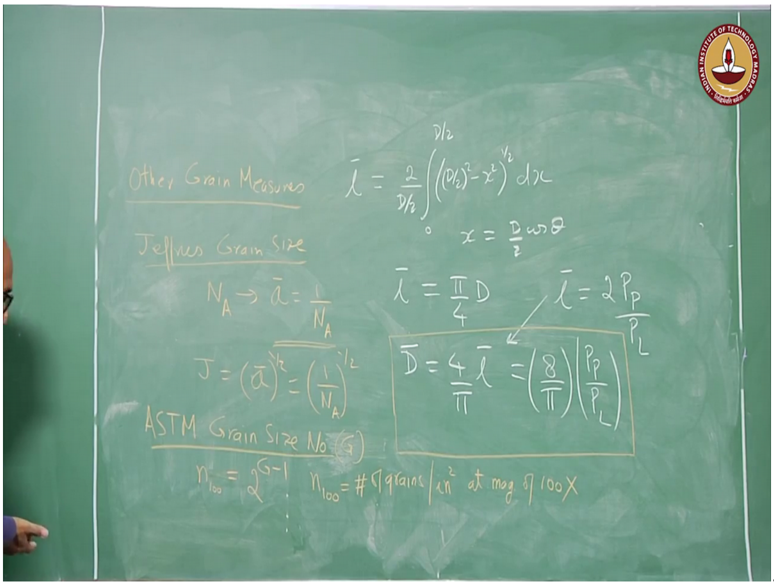

While we are at this particle size and we will also discuss grain size, I should give you 2 more gain measures which have been used some other grain measures; one is what is called as the other than the mean intercept length; one is what is called as the Jeffries Grain Size. In Jeffries Grain Size, you measure N A, number of grains per unit area. This would give me an average grain area to be simply the reciprocal. This is I am talking about a single phase polycrystalline structure with their space filling grains.

Then the average grain area would simply be the reciprocal of this. A Jeffries Grain Size simply tells me that you take the square root of this average area as a grain size measure. There is another important measure; this one is probably more important than Jeffries is what is called as the ASTM Grain Size Number. This is one very old measure. It is not very useful if you doing some structure property correlation stuff but why I am mentioning this is that this is still used in industry. Many industry still use it as a quality control measure and this is defined in the following way. ASTM Grain Size Number as designate that as G, then this is given by the following expression.

n is equal to, I should write n 100 is equal to 2 to power G minus 1, G is the ASTM Grain Size Number, n 100 is the number of grains per inch square at a at a magnification of 100 X ok. This is the very old measure and since it is still used in industry, so you should know this. This measure tells me that; when I get larger number of grains per unit area, per inch square, then my G will be larger and if I n is small, then G will be small.

Now, if n is large which means grains are smaller or bigger, grains are smaller. So, larger is this grain size number, smaller is the grain size ok. You will learn more about this in the demonstration exercise that will be followed.

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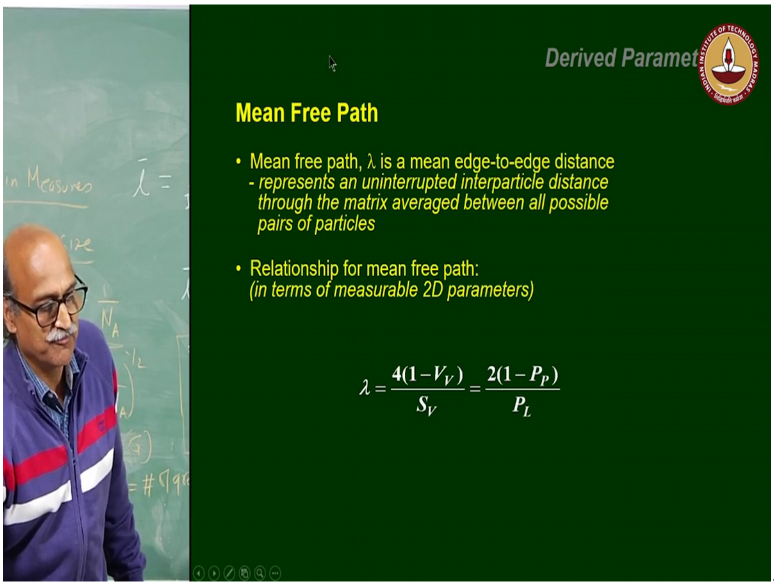

One final derived parameter that I will just mention, the mean free path, so if you have particles which are dispersed, precipitates let us say which are dispersed in a matrix, what is the kind of a mean free path, which kind of represents may be dislocation movement from one particle to another particle where it is hindered. So, mean free path becomes important from that point of view.

It represents an uninterrupted inter particle distance through the matrix; average over all possible pairs of particles ok. And the relationship for this mean free path is again can be obtained by simply a counting exercise. So, the mean free path lambda is given by 2 times 1 minus the point fraction divided by P L, the number of intersections per unit length ok. So, this is an important, can be an important parameter if you, let us say doing some mechanical behavior studies.

Now, before I complete this lecture, I just want to introduce something that in many of our measures, we have to do this N A, counting of number of grains, counting of number of particles. So, we must know the proper method of counting, an unbiased way of counting particles because the problem comes when in any region that we are trying to count where only half the grain is inside. So, part of the grain is inside.

So, methodology for counting of grains and particles

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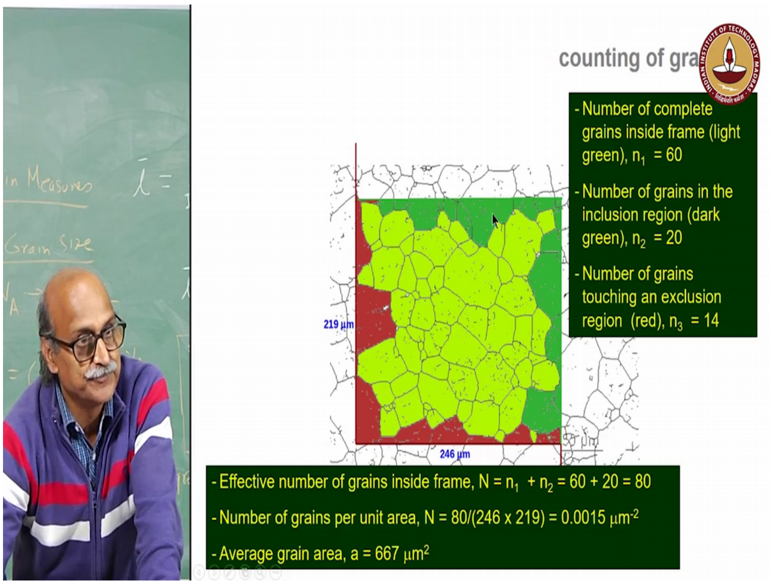

So, I have let us say a polycrystalline microstructure. I need to put some kind of a frame, a counting frame on this to get number of grains per unit area let us say and this is the kind of counting frame that you put ok. This is the little peculiar look looking frame. This is the red line and there is a green line specified here, these are the dimensions of the frame. So you know what is the area of the frame from these dimensions. Now I have to count the, which grains, I count which grains I do not count what happens to the grains at the edges. So, here the red line represents what is called the exclusion zone, that any grain which touches the red line will not be counted.

All grains which are completely inside count as whole. All grains which touch only the green line will be counted as whole. So, going with this, I will not count this grain. All of these grains which are marked red will not be counted. These grains will be completely counted as whole grains completely inside will be counted, going with this you would get for this particular area number of complete grains inside the frame which are light green are 60, number grains in the intrusion region dark green is 20, number of grains touching and exclusion region, red ones and 3 is 14. So, 14 I will not count, but I will add 60 and 20 together.

To get a total of effective number of grains as 60, this if I divide now by the area of the frame which his 246 into 219 micrometer square, then I will get this result of 0.0015 per micrometer square.

If I take the reciprocal of this remember, this is actually essentially N A is what I have done one upon N A is the average grain area. So, 1 upon 0.005 gives me about 667 micrometer square as the average area of the grain.

So, from here you can go to ASTM Grain Size and any other parameter that you wish to.

A similar methodology you use for counting of particles.

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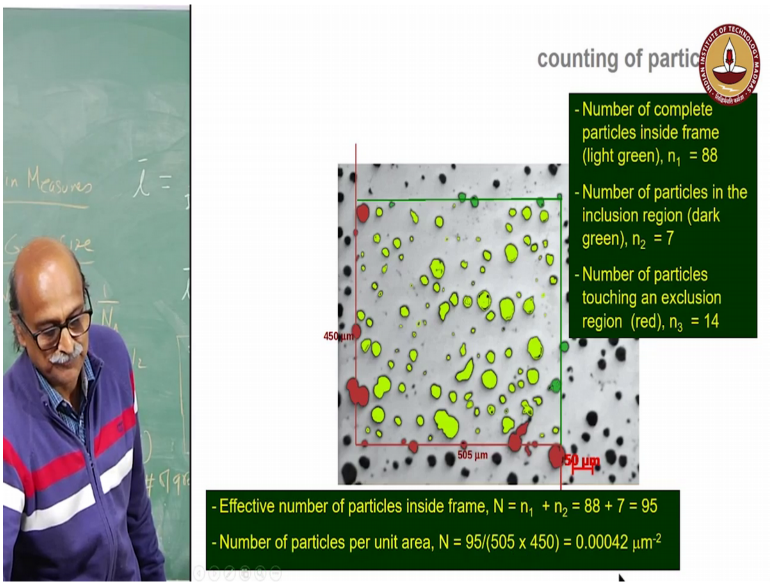

If I have now these particles, dark particles put again counting frame of the similar nature, again these red ones which are marked as grey particles which are touching the exclusion zone and they will not be counted. The dark green ones will be counted because they are only touching the intrusion zone and of course all particles which are completely inside the frame will be counted. And hence you get 88 light green particles, 7 dark green these are to be counted, the red ones not to be counted there are 14. You get effective number of particles inside the frame as 95.

You want particles per unit area, then it is 95 divided by the area of the frame, 505 into 450 giving me 0.0042 per micrometer square. So, this counting methodology gives you there are other methodologies you might find for counting but this methodology turns out to be the best methodology, but gives you the unbiased, there is no bias involved in this counting, that is a very objective methodology to count and it is an easy methodology to count.

So with this, we I complete this lecture and. In the next lecture you will be seeing a, you will be doing an exercise on the various things that we have done in this lecture.

Thank you.