Elementary Stereology for Quantitative Microscopy
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Lecture – 13
Counting of grains and particles Part - 1

So, we looked at the basic stereological parameters and how to derive them. And, in this lecture we will look at some more parameters, which we can derive from what we have already done, ok. So, we will call these as derived parameters. One of the last things that we were doing in the last lecture was a numerical density, when there is a problem of that we have 2 unknowns we have the mean tangent diameter, we do not know because we do not know the shape of the particle very often and we also have to estimate of course, the numerical density and the only measure that we can do on the 2 dimensional microstructure is the number of particles per unit area. But if we assume some shapes then we can arrive at a numerical density.

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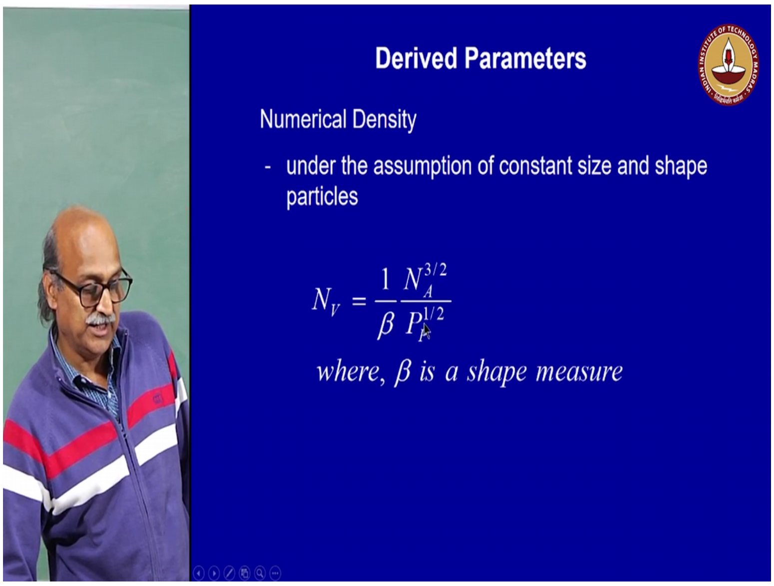

So, let us first look at the first derived parameter then, for numerical density and it will be under the assumption of constant size and shape of the particles. What that shape is, if we are able to assume that, then we can estimate number of particles per unit volume from a measure of number of particles per unit area. Under this assumption this is the first derived relation I can write that N V the number per unit volume is equal to 1 upon beta and is the multiplication factor to two measures N A and P's of P. N A is number of particles we observed on the 2 dimensional structure per unit area this is raise to 3 by 2 and the point fraction raise to power half.

The interesting thing about this measure is that we only have to do counting. Here number of particles in the microstructure and here just a counting the number of points that fall inside those particles with using a point grid. Now, and beta is a shape measure this is where; so, this is the general expression. If you have particles of constant size and shape and beta is a shape measure, which is going to be function of if I assume spherical particles then beta will have some value, if I use cylindrical particles beta will have some values, ellipsoidal it will have another value and so on.

So, if you are able to make an assumption on shape then this is a the kind of expression we can use, but here what I am going to do now is I am going to show you how to arrive at this expression from whatever we have done. We are not going to do anything new now just from those basic stereological relationship.

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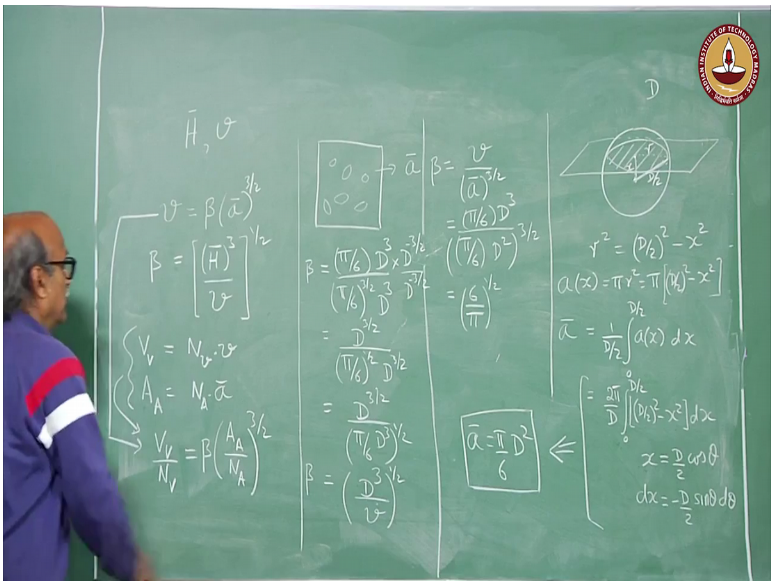

So, I will do that on the board if we are assuming that these particles are of constant size and shape, which means all particles will have the c same mean tangent diameter, they would have the same volume and one expression that I can write for this is a relationship between volume and the average particle area on the 2 dimensional section. So, I have for instance a microstructure or a micro structural section and I will get depending on what kind of shape by having the 3 dimensional structure. I will get particles of varying shape and they could also be for varying size as well because even though particles are constant shape constant size I might get something very different.

For example, if I have cylindrical particles of constant size and shape, if I section it you can get a circle, you can get an ellipse; you can get a truncated ellipse; if the section passes through one of the ends of the cylinder so you will have different shapes. We can get from here an average area of these particles.

So, you measure the areas take an average so, you get a bar. Now, for particles of constant size and shape I can write the following expression for volume which is a shape measure beta for the particular shape, the average profile cut to the power 3 by 2, ok. This if I if you look at this expression, this balance is only the units because a bar would be length square so, to power 3 by 2 will make it length cube and hence the volume has to be length cube and beta is then a dimension less shape measure.

But, how to I get this expression? I will just show you for a simple case of sphere and then this is applicable for any shape. So, if I show this for a simple case of a sphere you take a take a sphere of diameter D, suppose my sectioning plane my 2 D probe cuts this sphere at some location, at some distance from the center of the sphere. So, this is my center and this probe is at a distance x from the center and obviously, when this section cuts a sphere it cuts it into a circle, which will have some radius and this radius will vary as a function of distance at which the plane cuts the sphere.

Now, what is the area? That is cut out let us look at this. Area that is cut out is if this is r and this is clearly the radius of the sphere or half the diameter. So, the radius that is cut out can be written as D by 2 square minus x square. Hence the area that cuts at a distance of x from the center, becomes pi r square or pi times D by 2 square minus x square, ok.

Now, if you were to repeatedly cut this sphere by random sections, then what is going to be the average area and incident that is what you would get on a 2 dimensional microstructure. There is a repetitive cutting of large number of spheres, which is similar to repeatedly cutting one single sphere. So, average section area then we can obtain by simply integrating from all possible values of x and x will vary from 0 to d by 2. So, you integrate to 0 to D by 2 ax dx and we also divide by D by 2 to get the average. Now, this would become 2 by D integral 0 to D by 2 and substitute for a x. So, pi term will come out at we would have D by 2 square minus x square dx, we just need to solve this integral, ok.

Solution to this integral is actually very simple is you do this following substitution; that x is equal to D by 2 cos theta, ok. If I do this substitution in this I will also substitute for dx which his equal to minus D by 2 sine theta d theta change the integral integration limits, now for theta. So what will be the change in the integration limits for this; so you substitute 0 here this will correspond to theta equal to pi by 2. The lower integration limit will become pi by 2 substitutes D by 2 for x here then cos theta should be 1. So, it will be 0. So, the integration limits will change from pi by 2 to 0, ok.

After that when you do this substitution it becomes a very simple integral you will get a sine square theta term inside, which is easy to solve and the final result that you will get out of this integration is simply that the average section area is equal to what you are going to get is let me put this down pi by 6 D square. So, from this you will get average section area as pi by 6 D square, ok.

So, what is going to be beta here? Let us now find that. So, beta is equal to volume divided by a bar to power 3 by 2 substitute this in this write down the expression for volume; what is volume? Volume is pi by 6 D cube upon pi by 6 D square and this whole thing into power 3 by 2. So, you will see that the diameter will cancel out. So, you will get a dimension less number. In fact, you would simply get 6 upon pi to power half, ok.

So, beta turns out to be a very simple expression for sphere. I can write this beta in another in an alternate way and this can be written like this that beta equals let me not cancel out the D’s. If I do not cancel out the D’s I can write this in the following manner that beta is equal to pi by 6 D cube divided by just from here pi by 6 to power 3 by 2 D cube. Let me multiply by D to power minus 3 by 2 and divide by D to power minus 3 by 2. This would become then just a second let me get this a yeah. So, so, this and this will cancel, I can write this as that the top as D to power numerator as D to power 3 by 2 this pi by 6 I cancel with the bottom pi by 6 then at the bottom and left with pi by 6 to power half.

And, here I am left with D cube or D to power 3 by 2 which I can write like this D cube upon pi by 6 D cube to power half, or I am just writing it like this. At this subsequently and sorry, this is also D to power 3 by 2 this I can then write as D cube upon and pi by 6 D cube is volume pi by 6 D cube is volume. So, I write the volume term to power half. So, this is another expression for beta which in general, now diameter D is actually the mean tangent diameter of the sphere. For a general shape I can then write for beta as the mean tangent diameter cubed upon the volume of the shape to power half.

So, this becomes the general expression for any shape now for example, it was a cylinder then I would have been able to write for a cylinder, I know what is the mean tangent diameter I would put this I know for the cylinder what is the volume. So, I would be able to write the shape measure for a cylinder similarly for other shapes. So, now I know how volume I can relate with the section in every section area through this shape measure beta. Now how does that help me, well, the next thing that I can write a few more stereological relationships which I already know. I can write for example for volume fraction if they were numerical density to be N V all particles remember are having the same volume. So, N V times the volume of each particle would be nothing, but simply the volume fraction of particles.

Similarly, I can write what is the area fraction here? Area fraction would be number of particles per unit area here times, what is the average area that gives me area of fraction, ok. Now, take these equations and then let us substitute volume from here on the left hand side and substitute for average area on the right hand side. So, I will get volume fraction upon N V to be equal to beta times substitute for a bar which would be nothing, but area fraction upon N A to power 3 by 2, ok. So, I get now I am getting expressions in terms of measurable parameters, from that 2 dimensional microstructure. So, let us try to rearrange the terms in this, so that I get N V onto one side.

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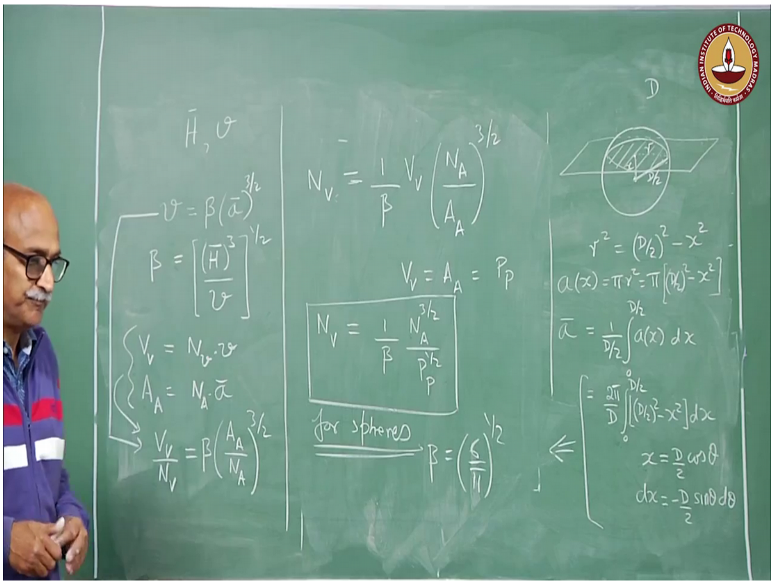

So, I will get N V to be equal to the numerical density to V equal to 1 upon beta volume fraction times N A divided by area fraction to power 3 by 2. Now, you see if you look at area fraction and volume fraction I can replace both of them by the measure point fraction. Volume fraction comes from area fraction and this can be measured using point fraction, then N V would become 1 upon beta N A to power 3 by 2 divided by point fraction to power half.

So, all I have to do if I can assume spherical particles, I just need to measure number of particles per unit area on the 2 dimensional section, get the point fraction and get beta; which we have already shown for spheres if I assume them to be spheres beta is 6 by pi to power half just a number, ok. So, this is equal which is very practical in determining the numerical density.

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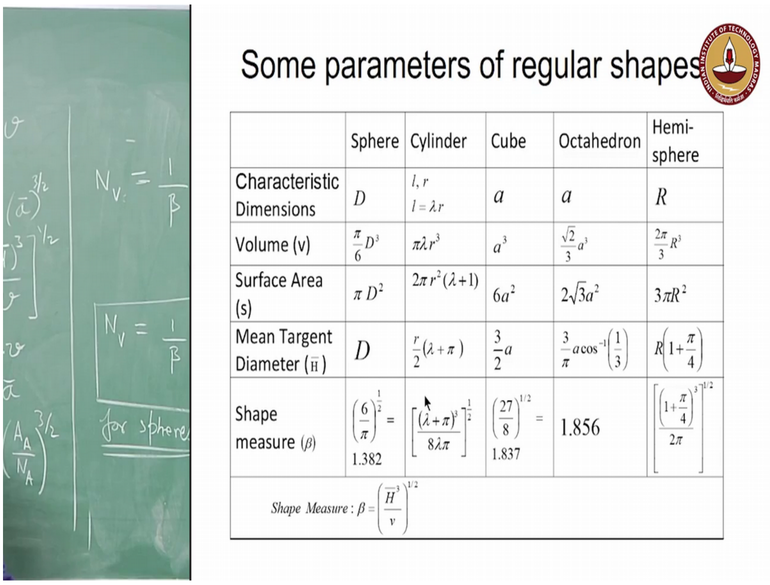

So, this was one of the derived parameters and if I look at some shapes, if you look at some of the shapes I have listed down in this table; let us say for a sphere for each shape I am also listing down a characteristic dimension, ok. So which completely defines the shape so, for a sphere it is diameter D, volume is pi by 6 D cube, surface area is pi D square, mean tangent diameter is D and the shape measure beta is 6 upon pi to power half which is 1.382.

If I look at a cylinder now cylinder will require two parameters; length and radius, ok. So, this is not one single shape just by saying cylinder is not enough you will have to either say that length is somehow related to radius, through some shape coefficient lambda, then you can say l is lambda, r then in terms of all of this you will get your volume surface area mean tangent diameter in terms of r and lambda and then the shape measure in terms of in terms of simply lambda. So, it is a dimensionless number.

Here, you would use the expression for beta as the mean tangent diameter and the volume of the cylinder. Then, for cube you will go through this, octahedron shape is here if you had a hemispherical particles then these are the expressions. So, these are the terms and shape measured shape measured is listed here as this. So, this is how the shape measure is calculated for all of these quantities, ok. So, this was one parameter numerical density how we can tackle we will have to make some assumptions about shape. Let us look at some other derived parameters for grain size. If I have to understand grain size of let us say first single phase polycrystalline materials.