Elementary Stereology for Quantitative Microscopy
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Lecture – 12
Basic Stereological Parameters Part - 2

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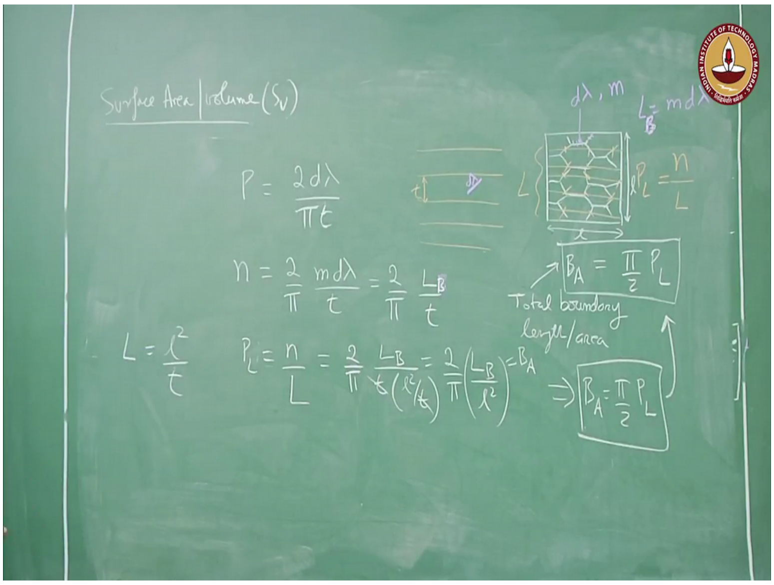

So, after volume fraction, let us look at surface area per unit volume estimation. Let us consider a polycrystalline microstructure something like this and illustrator sketch for a polycrystalline single phase microstructure. I these are grain boundaries the lines and into 3 dimensional structure it is surfaces. Now, if I superimposed on this let me superimpose on this a grid of lines, I count the number of intersections that these lines make with the grain boundaries just like in the case of graphite nodules, you had measured the intersections with the a graphite boundaries.

These are all intersections that you need to count. And if this is n, it is the number of intersection, the total length of the lines of all the lines added together is let us say capital [laughter], then n upon L is gives you the number of intersections per unit length, which you estimated in the graphite nodule microstructure. That relationship was simply written.

Then, the total boundary length of these lines per unit area, ok. So, this is the total boundary length per unit area is given by this relation. Now, we will see how we get this particular relationship. Let us take this line grid and distance between the line grid is let us say t ok. Now, let me do one more thing, divide these grain boundaries in the structure into small infinitesimal elements d lambda, and let there be total of m such elements, then the total length of grain boundaries inside this would be m times d lambda, ok. This is the way I have just subdivide all the entire network of boundaries into the small elements. So, m times d lambda is clearly the total length.

Now, these grain boundaries are completely randomly oriented, this is assumption that we need to make here; which means that each of these element d lambda behaves like a Buffon's needle, ok. Each of these grain boundary segment of infinitesimal length d lambda behaves like a Buffon needle that we have simply thrown on this line grid. What is the probability that a particular segment will intersect one of these lines; that we already seen is 2 times d lambda upon pi t.

Number of expected number of intersection small n that I should get would be this probability multiplied by m needles. So, this would become 2 by pi I will introduce m times d lambda upon t. Now, m times d lambda simply L, ok. This is expected number of intersection.

Now, I want to divide this I should this will get confused here. Let me call this is n L sub B to represent these are the boundaries. So, here also it will be L sub B. Now, what is the total length, what is the total length P L is simply small n upon the total length of these horizontal lines, the line grid. Let me also show here that the dimensions of my microstructures this is l, this is l small l. So, what would be the total length if I have a set of parallel lines space t apart along the length l?

This total length L would be l square upon t, ok l upon t will give you number of lines; each line is of length l, hence it becomes l square upon t, ok. Number of line is l upon t each line is of length l, so, it becomes l square upon t; which means that here also on the right hand side also I divide by the total length. So, 2 by pi L B times t and I divide by l squared upon t.

Now, this t and this t will cancel this will become 2 by pi L B upon L square. What is this quantity L B upon l square? L B is a total length of grain boundaries divided by the area over which it is present so therefore, this is nothing, but boundary length per unit area. So, if I now look at this it is clear that boundary length per unit area is equal to pi by 2 times, we arrive at this relationship, ok. This was a simple derivation to show this one of these stereological relationships.

Now, we go to 3 dimensions because I am interested in the surface area per unit volume, ok. This was an exercise in 2 dimensions let us do an exercise in 3 dimensions.

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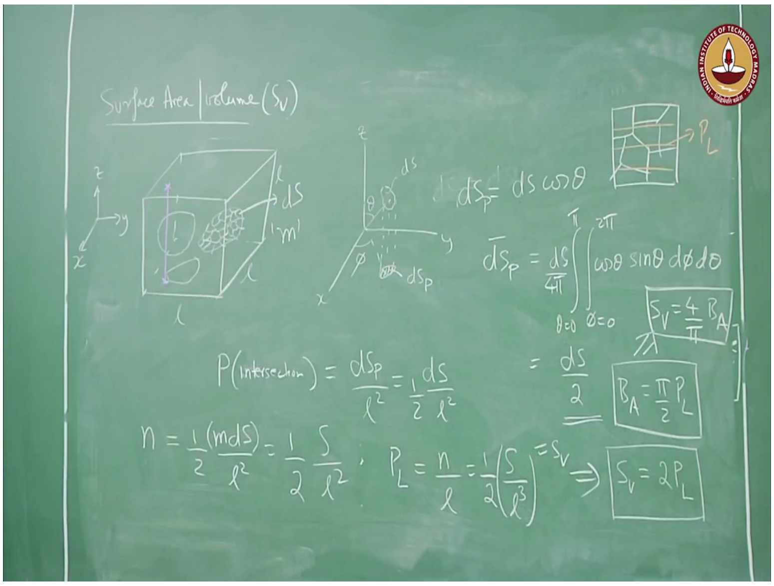

The exercise in 3 dimensions I have now surfaces; surfaces can be in the form of particles, it can be in the form of grains, it does not matter. These are all my surfaces just like I had divided the lines into small infinitesimal elements, I divide the surfaces also into small infinitesimal areas; each of them having an area d S, and these area elements are randomly oriented and randomly distributed in the 3 dimensional structures or within the cubic space.

Now, I introduce a line probe parallel to the z axis, but position randomly. What is the probability that this line probable intersect any one of these the surface elements. Imagine that you have just one such element, which is having some orientation of theta with the z axis and its projection having an angle of phi with the x axis this is my orientation of this one particular anyone element of area d S. It will have it will projected area on the x y plane d S P projected area. This projected area d S P is going to be equal to is going to be some function of theta and d S is going to be.

Now, what is that going to be what would be the projected area of a small infinitesimal element onto the x y plane whose orientations is theta and phi. This is going to be d S P would be equal to d S cosine theta. You can see this when theta is 0, this element is a line perpendicular to the z axis it will projected s d S and cos theta will be 1. If it is theta is 90 degrees then it will project as a line which has 0 area. So, when put theta is equal to phi by 2 it gives you 0 and it is going to be independent of phi. So, you can rotate this you will get the same.

Now, what I need is the average projected area then only I can calculate; what is the probability that a line probe will intersect this element, what is average projected area now? By now we know this and I am go to write this d S will be taken out of the integral 4 pi cos theta and then sine theta is coming from the probability density function of orientations in 3D space and 4 pi is also coming from their sine theta upon 4 pi d phi d theta phi varies from 0 to 2 pi, theta varies from 0 to pi.

Now, I think by this time I do not I am sure you would be able to solve this integral first integrate with respect to phi you will get 2 pi which you cancel with 4 pi; you do not need to integrate all the way from 0 to pi you can integrate from 0 to pi by 2 so factor of 2 will come and you just have to integrate this cos theta sine theta term which is a straight forward integral and the result I am writing directly is d S upon 2, ok, this is the result.

So, probability of intersection that will a call that as P intersection with the line probe is the average projected area d S P upon the area of the face in the x y plane. So, l square or this is simply d S upon l square and there is a factor of half that would be there. If there are total of m such small infinitesimal areas of the surfaces; then the number of intersection small n the expected number is going to be half is going to be P times m the probability times the number of such elements that are there which is m times d S upon l square. Now clearly m times d S is nothing, but the total surface area inside this cubic block. So, let us call that as just S upon l square.

Now, divide both sides by L if I divide the left hand side by L I will get P L number of intersections per unit length. So, P L is equal to small l upon L; which is equal to half times S divided by L cube both sides I have divided S upon L cube is nothing, but surface area per unit volume. Hence surface area per unit volume would be 2 times P L ok. So, we have now a relationship that if I take a microstructure; let us say a polycrystalline microstructure or particle microstructure does not matter I get P L out of this 2 times that is simply an estimate or the surface area or those grain boundaries in a unit volume and remember that boundary length per unit area of this was pi by 2 P L this we had already seen, which means that I can relate S V and B A as well and I can write S V to be equal to 4 upon pi, B A.

That if I have so many boundary lengths in a unit area multiply this by a factor of 4 upon pi I will get so much of surface density in the material, ok. So, we have done surface area per unit volume.

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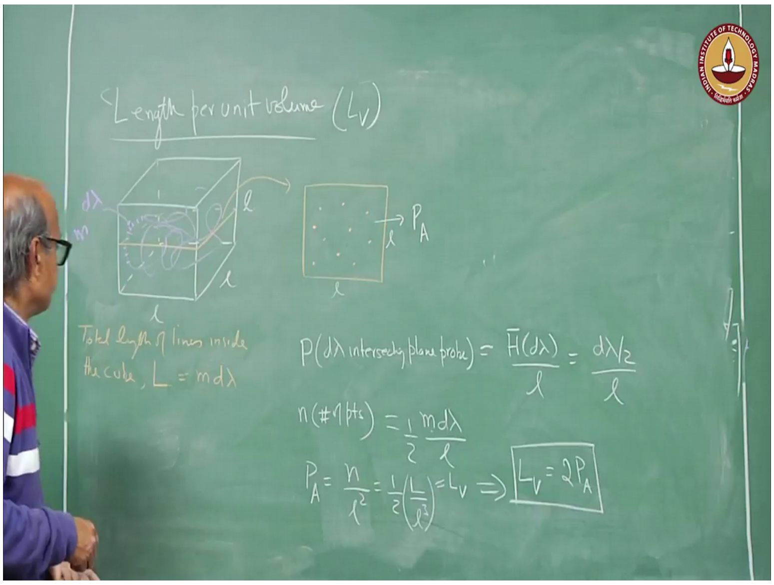

Now, I want to do length density or length per unit volume or the parameter L sub V. So, that the third parameter that we want to get a stereological relationship for and this you will see is going to be very easy and we will following the same line of reasoning. Again, we just restrict ourselves to cubic space. So, I have I have full of linear elements inside my 3 dimensional structure all randomly oriented. If I take a section through this; then on this section I should be able to see points. So, I should be in my microstructure be able to isolate points in the structure.

And, let us see what I can do with this points and how looking at the structure I get it length per unit volume. We will do exactly the same thing just divide these all of these line elements into small small infinitesimal element; again let me call that is d lambda only and there are m such elements and hence the total length of lines inside the cube let that be capital L is nothing, but m times d lambda. So, we start with the same thing, that this section will cut some of these small d lambda elements what is the probability of a d lambda element intersecting a plane probe, probability of d lambda element intersecting a plane probe. It should be the mean tangent diameter of the d lambda of the one element divided by L simply.

We have already seen this; that plane probe intersects a feature that one is look at the mean tangent diameter; what is the mean tangent diameter of such an element that we saw in the last lecture; we derived a relationship for a cylinder and then we let the radius of the cylinder go to 0, then it becomes a line and hence the mean tangent diameter for that was would be the length of that element divided by 2.

So, this is simply d lambda by 2 divided by l. Number of points; so, n number of points expected in the structure in the 2 dimensional structure would be there are total of m such elements and hence m times this probability should be the expected value of the number of intersections. So, this would be m d lambda upon l and this is a factor of half.

Now, what I measure remember I said that we measure basically ratios of two quantities. So, I actually would measure number of points per unit area P sub A. So, P sub A would be n divided by area of my section and area of my section is l square. So, this is half m times d lambda is the total length of the elements and I also divided I have to divide by L squares. So, this will become L cube. So, this capital L upon L cube is a total length divide by the volume of the cube. Hence this is nothing but length per unit volume. So, from here length per unit volume is 2 times the number of points per unit area. So now I have related P A here, points per unit area on the slide you can see is related to length per unit volume.

Now, I have the last fundamental relationship left in stereology; which is number per unit volume. I have so many particles in a unit volume how can I estimate that?

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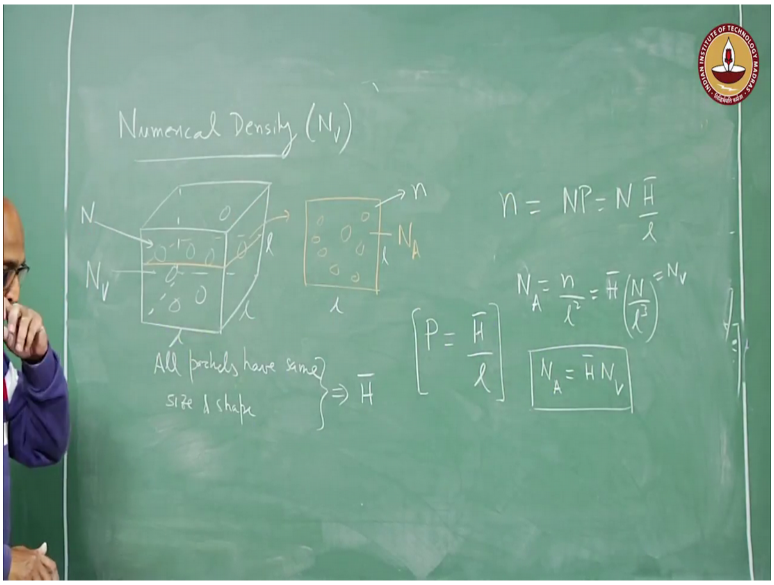

So, numerical density N V number of particles per unit volume. So, I have again particles of precipitates inside and there is some number per unit volume of such particles that I need to estimate. When I take a section through this; I get this section profiles of this particles and here I can measure N A number of particles per unit area. What is the relationship between N A and N V? Let us assume that all my particles in this so, all particles here have same size and shape which means that they will all have same value of the mean tangent diameter, which means that this section the probability of intersecting a particle by the 2 dimensional probe would be H bar upon L well L again is my dimension of the cubic space.

In this space let us say I have capital N number of particles and out here I have small n number of section profiles then the relationship between small n and capital N would clearly be related through this probability that a capital N times the probability would give me an estimate of small n. So, n would become capital N times the probability which is N times H bar upon L. To get points per unit area not point, to get number for unit area on my 2 dimensional microstructure.

I have to divide small n by the area of my image which is l square. So n upon l square here also I need to divide. So, this would become H bar times n upon L cube. Now, n upon L cube is nothing, but number per unit volume. So, this is equal to number per unit volume. Hence N A is equal to H bar times N V, ok. So, this was the fourth fundamental relationship that we have derived here. If you look at just to summarize what we have done.

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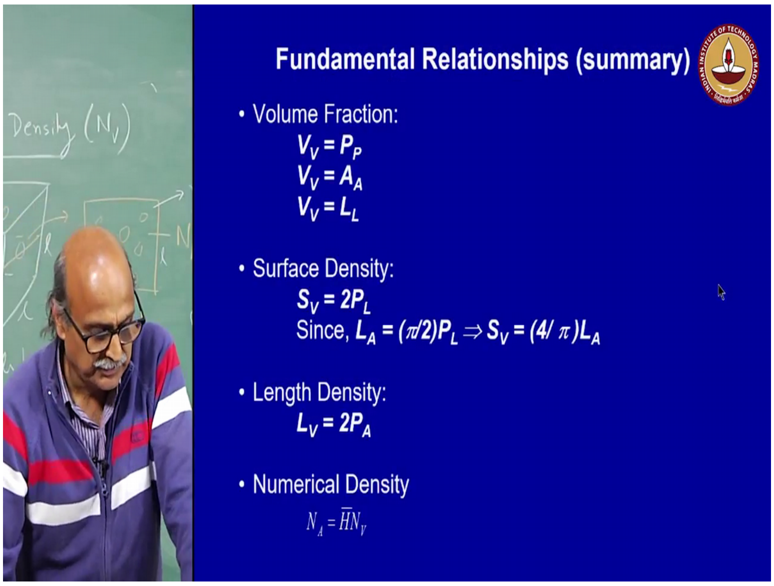

This is what you have done. This is just a summary of 3 ways through which we can measure volume fraction, then relationship between surface area per unit volume here I have written L sub A, on the board I had written B sub A boundary per unit area both of the same quantity. So, L sub A, B sub A are the same here length density L sub V equal 2 times P A, N numerical density, which is N A is equal to H bar N V which we just now derived.

Very quickly we just look and summarize this as well.

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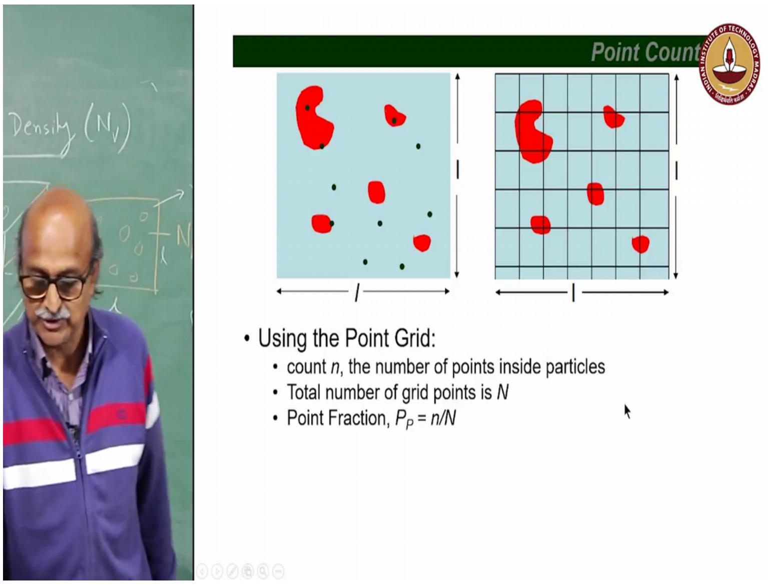

That if I to get volume fraction get a point count by putting a grid and get a point fraction and that is your estimate of the volume fraction that we have done repeatedly I do not have to stay with it.

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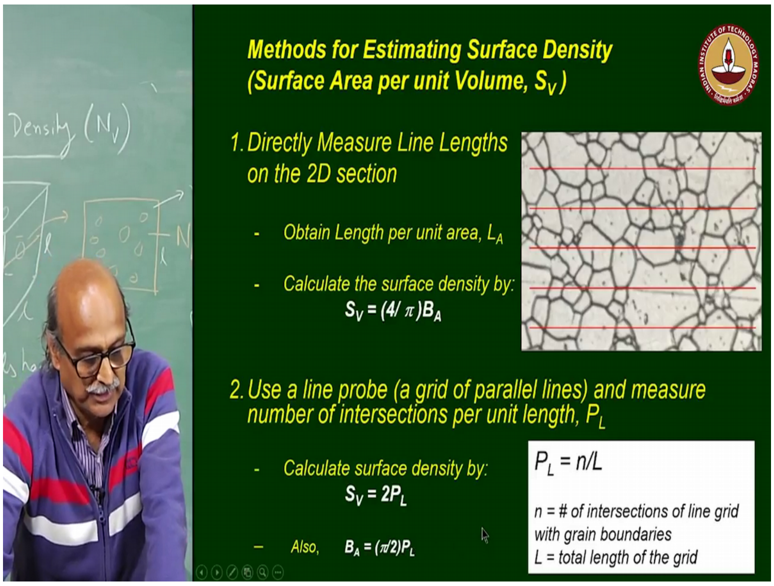

Now, I have to get surface area per unit volume of this polycrystalline microstructure; one is directly measure all the lengths and get boundary length per unit area multiply that by 4 upon pi and that gives you surface area per unit volume which you just now saw or put a grid of lines count the number of intersections with the boundary get number of intersections per unit length you have seen how we can do that, where L is the capital L is the total length of the grid n is a number of intersections of the line grid with the grain boundaries and you can calculate the surface density S V as 2 times P L and also we have seen that boundary length per unit area is pi by 2 times P L.

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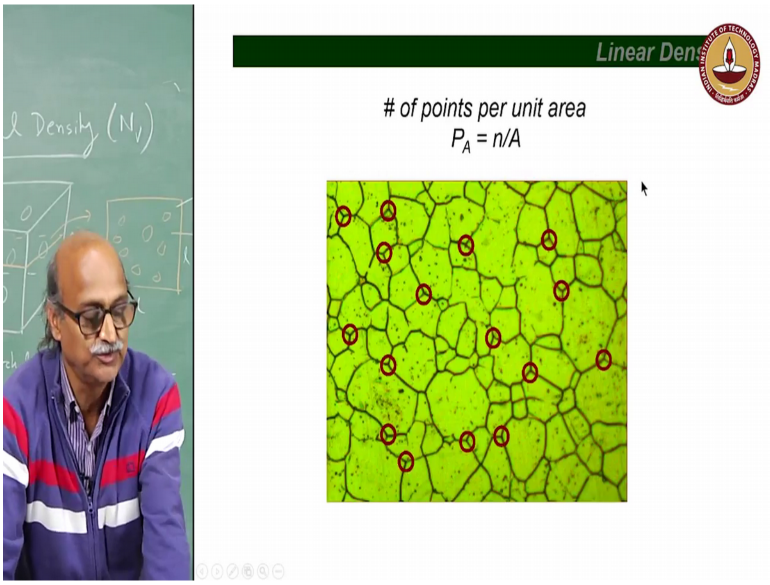

Now, this is a microstructure where I want to estimate a length per unit volume, but where are the points in this. If you look at the triple points these are all triple points, in 3 dimensional structure these are triple edges. So, you would count each triple point, count all exhaustively you have to count all the triple points in the structure, I have circled only a few, but you would count all of them in this given area of the microstructure or you can put a smaller frame in a in it and count the triple points.

So, which will you be shown as demonstration as exercises in the next lecture as to how to do this get P A. So, small n is the number of counts of the triple points divide by the area of the microstructure for area of the frame inside which have done the counting and get P A number of points per unit area and the linear density then is given by 2 times P A.

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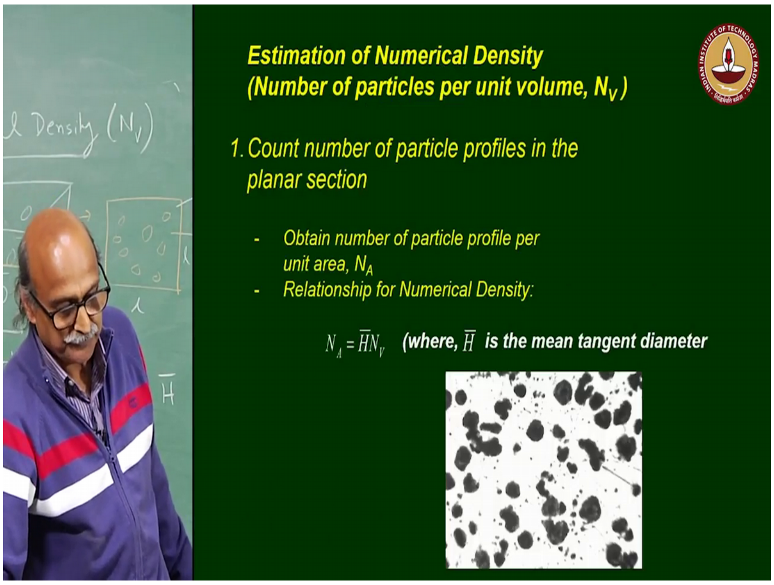

Estimation of numerical density well here is the again a very simple relationship. So, if you want to count this particles you will get N A, but there is a catch in this relationship all of these relationships. The catches we do not know H bar the mean tangent diameter. So, we have two unknowns and we are only able to make one measurement. So, it becomes that here in order to get numerical density number per unit volume; one would have to make some assumptions regarding shape. Once if you can assume that then we can try to solve this problem. So, out of all those relationships if it once again look at the summary all of these relationships have been derived independent of the shape, right.

But, the last relationship if you need to solve it then you would require a knowledge of shape, but all other relationships you do not require knowledge of shape. I am go to talk about a little bit more on the numerical density, but I think this particular lecture I will stop here and then we will look at, what are called as derived relationships, where out of this we will derive some of the relationships like the particle diameter.

If you assume a certain shape then what kind of numerical density how what is the relationship for the numerical density we can obtain. Then we also talk about for polycrystalline structure. How we can define grain size and even particle size and those are the kind of things, we will look at the next lecture.