

Defects in Materials
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Lecture – 19
Stress Field Around Dislocation

Welcome you all back to this course on Defects in Material. Last class we had covered about the different type of dislocations. The screw dislocation, edge dislocation, mix dislocation, flank dislocation, how to define the burger vector of the dislocation and before that we have covered stress and strain field, how they are defined in the tensorial notation all the relationship amongst the components that is what we have covered.

Today's class what we will do is that when a material contains a defect. How do we find out the stress and strain field around the defect? The defect which we are going to consider is a dislocation the simplest type of dislocations which we can consider are a screw and then edge dislocation.

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Philosophy of deriving expression for stress field around a dislocation

Find out solution to dynamic equilibrium equation for the material which contains dislocations

Solution gives atomic displacements in terms of position in the lattice

Derivative strain components from displacement field

From the strain components, stress components can be derived using generalized Hooke's law

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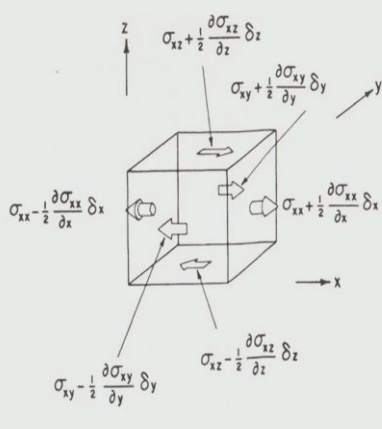
So, what is the methodology which we have to follow to find out information about the stress and strain field around the dislocation? We have seen that whenever stress and strains are around a dislocation, there is some dynamic equilibrium is there that sample. So, we have to find out the equilibrium equation governing that we have looked at it in

the last class. That equation is essentially was written in terms of stress and displacement.

But from what we know about the how the stress can be related to strain and strain can be related to displacement, that equation itself can be written in the form of a displacement. And that is nothing, but a wave equation. Now we have to find a solution to that wave equation that will give you the displacement which the atom is going to atom at a particular lattice point is if a displacement takes place, what are how the displacement should be returned are, what will be the solution to a displacement satisfying this force balance equation. Once we know the displacements then the rate of change of displacement will give the strain and using generalized Hooke's law from this strain, we can calculate the stress field that is essentially the philosophy which is being followed to calculate the stress and strain field around the dislocation.

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Force balance equation



$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}$$

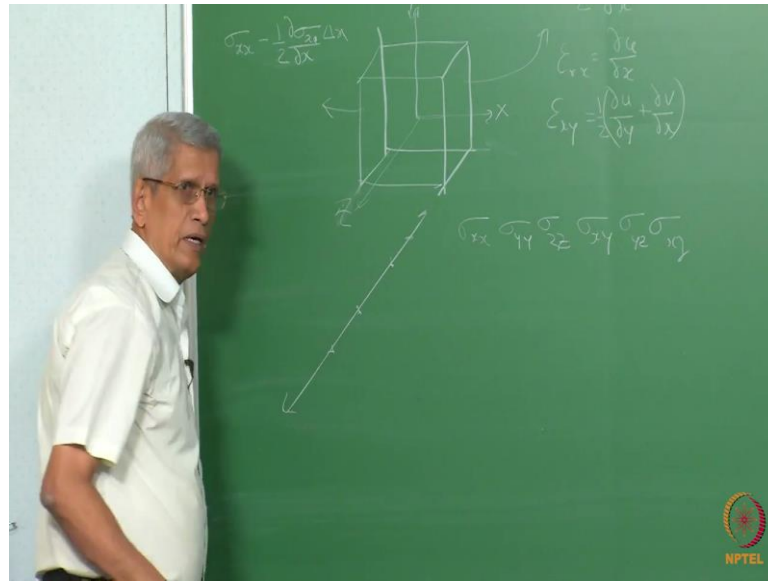
$$\rho \frac{\partial^2 v}{\partial t^2} = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z}$$

$$\rho \frac{\partial^2 w}{\partial t^2} = \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}$$

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So, just to recap, let us go back look at that the force balance equation. The force balance equation what we are trying to we are considering an infinitesimal q , where we assume that the stress is varying as a function of position right. And then we are representing this variation in stress as a derivative that is for σ_{xx} , the derivative is d is dou σ_{xx} by $d x$.

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And if you take a small tube, essentially this is the cube. And the center of the tube system is essentially taken with respect to these 2. And then this will be half the distance. So, from here if you take it how the stretch is going to vary, and on each of the phases we can find out what will be the stress here. If we see x here it is e x y e z; this coordinate system which we follow because it will become clear when we follow this coordinate system.

With respect to this coordinate system at the center if it is sigma x x is contains stress. So, on this surface it will be sigma x x plus into x x into some delta x because this total distance is delta x. So, half of it and similarly on that is what the stress is going to be and the normal stress on the other stress will be sigma x x minus half into delta x right. This is what essentially is being written here. This is one particular direction. Similarly, on the y surface we can see what is going to be the shear stress which is going to act in the x direction. Similarly, on the e z phase also what is going to be the shear stress which is going to act, which we can find out if we multiplied by the area that gives the force which is going to act in this direction. And that is being added up if there is a net force which is going to be there. That force can be using Newton's law; we can express it in the forms of because the force balance has to be there that will be we can express it in terms of rho that density into the rate of change of velocity acceleration.


That is what this force balance has equation essentially is about. So, this equation will be valid, that is when we write the rate of change of velocity means that there should be a moment of that body correct. Other ways it will be in a equilibrium where this when the body is in equilibrium it is static the right hand side will turn out to be left hand side will turn out to be 0.

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$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + 2G \frac{\partial^2 u}{\partial y^2} + 2G \frac{\partial^2 u}{\partial z^2} + (\lambda + 2G) \left[\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right] &= \rho \frac{\partial^2 u}{\partial t^2} \\
 2G \frac{\partial^2 v}{\partial x^2} + (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + 2G \frac{\partial^2 v}{\partial z^2} + (\lambda + 2G) \left[\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 w}{\partial y \partial z} \right] &= \rho \frac{\partial^2 v}{\partial t^2} \\
 2G \frac{\partial^2 w}{\partial x^2} + 2G \frac{\partial^2 w}{\partial y^2} + (\lambda + 2G) \frac{\partial^2 w}{\partial z^2} + (\lambda + 2G) \left[\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z} \right] &= \rho \frac{\partial^2 w}{\partial t^2}
 \end{aligned}$$

In the absence of body forces the equilibrium condition is

$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + 2G \frac{\partial^2 u}{\partial y^2} + 2G \frac{\partial^2 u}{\partial z^2} + (\lambda + 2G) \left[\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right] &= 0 \\
 2G \frac{\partial^2 v}{\partial x^2} + (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + 2G \frac{\partial^2 v}{\partial z^2} + (\lambda + 2G) \left[\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 w}{\partial y \partial z} \right] &= 0 \\
 2G \frac{\partial^2 w}{\partial x^2} + 2G \frac{\partial^2 w}{\partial y^2} + (\lambda + 2G) \frac{\partial^2 w}{\partial z^2} + (\lambda + 2G) \left[\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z} \right] &= 0
 \end{aligned}$$


Correct? What have done is that the same equation substituting for σ_{xx} , σ_{yy} , σ_{zz} , σ_{zx} in terms of the generalized Hooke's law which had been given in the earlier class and then substituting for stress ϵ_{xx} , equals $\frac{\partial u}{\partial x}$, ϵ_{xy} will turn out to be $\frac{\partial v}{\partial x}$ using these sort of expressions. If you substitute, then we will be able to derive these 3 equations. In the absence of body forces there are no forces which are acting on the equilibrium condition will be this right hand side will turn out to be 0. This is what essentially the expression which we get it.

Now, we have to find depending upon the situation applying the boundary condition; these nothing, but a wave equation. We have to find out solution to u , v and w which we have to find out, that will change slightly depending upon what sort of boundary condition which we employ. Now let us look at the case of any dislocation.


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Displacement field around dislocation

Consider infinitely long straight dislocation. Z-axis along the dislocation line direction. Since dislocation has infinite length stress do not depend on z-co-ordinate. All positions on the dislocation line are equivalent to each other. Derivatives wrt z is zero

Force balance equation becomes

$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + 2G \frac{\partial^2 u}{\partial y^2} + (\lambda + 2G) \frac{\partial^2 v}{\partial x \partial y} &= \rho \frac{\partial^2 u}{\partial t^2} \\
 2G \frac{\partial^2 v}{\partial x^2} + (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + (\lambda + 2G) \frac{\partial^2 u}{\partial x \partial y} &= \rho \frac{\partial^2 v}{\partial t^2} \\
 2G \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) &= \rho \frac{\partial^2 w}{\partial t^2}
 \end{aligned}$$

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We wanted to find out the displacement field around it. The first thing which we consider is that dislocation is an infinitely long straight line; that means, that in that body it is going across from one line to it is a very long and we consider it to straight dislocation. That is the easiest case which we can consider and it is lying along the e z axis, which is being chosen here. If that is the case then what is essentially going to happen is that, if we consider a point here or a point here or point here any point on this dislocation line, it is assumed to be identical, correct. It is a long dislocation that is say in that case the derivative with respect to e z that is no variation we assumed that with respect to changing from one point to another point in the e z; that means, that in the force balance equation all the ones terms which contain, which are derivative surface term will turn out to be 0. Now this is how the equation will turn out to be.

Now, we can see that derivative with respect to only x and y are present. So, the solution to this equation we have to find out. How do we go about?


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For a stationary dislocations, the equation becomes

$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + 2G \frac{\partial^2 u}{\partial y^2} + (\lambda + 2G) \frac{\partial^2 v}{\partial x \partial y} &= 0 \\
 2G \frac{\partial^2 v}{\partial x^2} + (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + (\lambda + 2G) \frac{\partial^2 u}{\partial x \partial y} &= 0 \\
 2G \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) &= 0
 \end{aligned}$$

The displacement u, v, w must satisfy the above equations and for a complete circuit around dislocations, the displacement from finishing to starting point must differ by Burgers vector. Displacement must be multivalued function of position

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Suppose we assume that the dislocation is stationary in that case the right hand side becomes 0 correct and the next is that u, v, w if you look at it for a dislocation, any point if you consider, if you take one circuit around it from the starting point to the end point when we come back to identical position around the dislocation our displacement of b always has to come, and it should repeat itself; that means, that the solution should be essentially that one full circle if you consider it, it should be dislocation lattice translation vector is a displacement and; that means, that this should be a multivalued function of position. Multivalued function of position can be only trigonometric functions which we can have.

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Stationary screw dislocation

$u=0 ; v=0 ; w=\frac{b}{2\pi} \tan^{-1} \frac{y}{x} = \frac{b}{2\pi} \theta$

$\epsilon_{yz} = \epsilon_{zy} = \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) = \frac{b}{4\pi} \frac{x}{(x^2 + y^2)} = \frac{b \cos \theta}{4\pi r}$

$\epsilon_{xz} = \epsilon_{zx} = \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) = -\frac{b}{4\pi} \frac{y}{(x^2 + y^2)} = -\frac{b \sin \theta}{4\pi r}$

$\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz} = \epsilon_{xy} = 0$

$\sigma_{xz} = \sigma_{zx} = -\frac{Gb}{2\pi} \frac{y}{(x^2 + y^2)} = -\frac{Gb \sin \theta}{2\pi r}$

$\sigma_{yz} = \sigma_{zy} = \frac{Gb}{2\pi} \frac{x}{(x^2 + y^2)} = \frac{Gb \cos \theta}{2\pi r}$

$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yx} = 0$

Cylindrical co-ordinates

$r < 5b, \sigma_{xz} \text{ and } \sigma_{yz} > \text{theoretical strength}$

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Now, let us take the case of a screw dislocation. So, far we have considered that the dislocation is a line, which is a long by z axis so; that means, that this is x y and this is the e z direction. And that is what essentially is being shown in this diagram when we take one circuit around it finally, the results displacement which is burger vector. And the displacement is now only in is z direction right. So, the component of the displacement, if you look at it u and as well as v direction x and y direction that is, either in this direction or in this direction there is no displacement.

The atomic displacement is taking place only in the z direction. So, only the w exists. And what is the nature of w as we move at an angle from going from one position to another, there is going to be a displacement which gradually increases. And as we reach the same point again we find that the displacement of b has to take place. So, that can be expressed in the form of $b \sin \theta$ by $\tan^{-1} y/x$. Or if we express it in other way we can write it as, $\sin^{-1} y/x$ is nothing, but the θ right and this θ is always expressed in radians that one should remember always. You should never use degrees for this.

Now, the strains we have to calculate. As I mentioned about the strains using these expressions ϵ_{xx} by $\partial u/\partial x$ is 0. So, when u is 0 the terms which correspond to ϵ_{xx} ϵ_{yy} those terms will be 0. The only term which is going to be present in this case is ϵ_{yz} as well as ϵ_{zx} . And this is how we define these terms how often it is. How this expression will be right? The component of the strain tensor can be representing it this, what it is now we take the derivative, then it will turn out to be equal to $b/4\pi \times (x^2 + y^2)$. Basically similarly we can find out for ϵ_{xz} also. Now since we know that strength tensor component, now we can find out yeah before going further I should mention that all other terms ϵ_{xx} ϵ_{yy} all other components of the strain they will turn out to be 0. Because only the w is there, the only the term which contains w that can be differentiated with respect to x and y only those terms exist for the strain correct.

So, now if you substitute for and these are all nothing, but shear stresses correct there are no tensile or compressive stresses are there. Only shear stresses are present around the (Refer Time: 14:18) dislocation, that one should always remember. Now if we substitute for this will be into $2G \epsilon_{yz}$. This is from the generalized Hooke's law, when we substitute we will be getting an expression of this form. Similarly, for ϵ_{zx} also we will be getting an expression of a similar form which we are going to get it. Yes.

Student: You have said that sheer stress is around the.

No, we will come to a edge dislocation later. Now in the screw dislocation, the difference between a screw and an edge is that in the case of a screw dislocation, there are only shear stresses. There are no tensile stresses; we cannot hydrostatic stress that is going to be there in an edge the edge will come.

Other than this other stress components like σ_{xx} , σ_{yy} , σ_{zz} in all other components are going to be 0. See we have used a Cartesian coordinate system, but what we could make out intuitively. If we look at it has got the circular symmetry around the dislocation line. So, the dislocation itself can be represented in terms of cylindrical coordinates also right. So, when we use cylindrical coordinates, then we define with respect to if this is a r and this angle θ . So, and this is z the coordinate system which we chose is $r \theta z$. This you might have studied the transformation from one to the other from Cartesian to cylindrical.

Student: (Refer Time: 16:35).

To a spherical coordinate system, I will not go into a detail, but I will just give what are the important significant results which come out of it in the cylindrical coordinate system. If we use then what will happen is that, the value of the stress because in the Cartesian coordinate system value of the stress is given in these expressions. Here it will be $\sigma_{\theta z}$ only will come it will turn out to be $\frac{v}{2\pi} \ln r$. This is what essentially and all other terms like σ_{rr} , $\sigma_{\theta\theta}$, σ_{zz} , $\sigma_{r\theta}$. These terms will all turn out to be just 0.

Corresponding to this we will have a stream 4 (Refer Time: 17:56). What can we make out from this? That is the stress field is essentially radial. That is around the dislocation line that is if suppose the dislocation is like this around this if we consider circle the shear stress is going to be the same at any point r around it the same value of the stress field, which we get it value of the stress correct. What is another information which we can get out of this. The stress decreases as we go away that is set a longer distance from that core of the dislocation, from the dislocation line. Now let us see what happens when we come very close to that dislocation core.

Student: (Refer Time: 18:57).

Then r becomes very small then the value of this stress can become very high. So, high that it can be much above their theoretical shear strength of the material that just cannot happen correct.

Student: (Refer Time: 19:15).

So, what is the problem the problem essentially is that we are applying continuum mechanics to a situation, and using generalized Hooke's law which is applicable for strains which are extremely small values. So, distances of the order of a few atomic layers from the core of the dislocation, the displacements are very high. It is a discrete that is which we have to continuum mechanics cannot be applied. So, that part is called as the core of the dislocation. So, because that the theoretical shear strength generally can be of the order of maybe about something like μ by 30 after this is the order which the theoretical shear strength is which way were derived in the few classes earlier.

So, because of this reason, this is valid for some value of r , that value of r what we normally choose as the core of the dislocation. The core of the dislocation generally is taken to be up to about something like 5 times the burger vector. Because we should have some unit is with we should have represent it. So, the burger vector is taken as a unit in terms of it, i times the burger vector is where it has been seen in most of the cases that stress does not reach a value close to that of or equal to that, are higher than that of the shear strength. So, we have to choose that value. That will also change from material to material, this value of r which we have to choose it appropriately. So, from this what we can understand is that, what is the essential take home points about screw dislocation. That stress will is has got a radial symmetry and it decreases as one by r as we go away from the dislocation.

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Stationary edge dislocation

$$u = \frac{b}{2\pi} \left[\theta + \frac{\sin 2\theta}{4(1-\nu)} \right] = \frac{b}{2\pi} \left[\text{Arc tg } \frac{y}{x} + \frac{1}{2(1-\nu)} \frac{xy}{x^2+y^2} \right]$$

$$v = -\frac{b}{2\pi} \left[\frac{1-2\nu}{2(1-\nu)} \text{Log } \frac{r}{r_0} + \frac{\cos 2\theta}{8(1-\nu)} \right] = -\frac{b}{8\pi(1-\nu)} \left[2(1-2\nu) \text{Log}(x^2+y^2) + \frac{x^2-y^2}{x^2+y^2} \right]$$

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w = 0

$\epsilon_{xz} = \epsilon_{yz} = \epsilon_{zx} = 0$

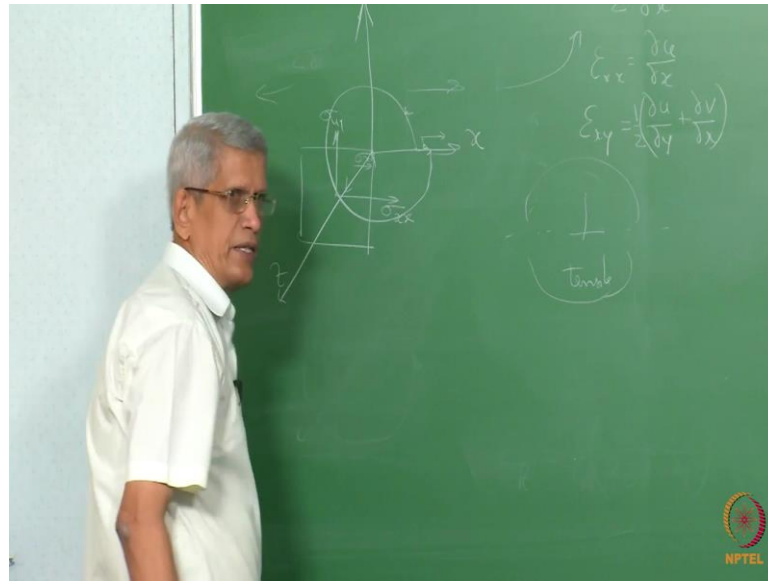
$\epsilon_{xx} = \frac{Gb}{4\pi(1-\nu)} \frac{y(3x^2+y^2)}{(x^2+y^2)^2}$	$\epsilon_{yy} = -\frac{Gb}{4\pi(1-\nu)} \frac{y(x^2-y^2)}{(x^2+y^2)^2}$	$\epsilon_{xy} = \frac{Gb}{4\pi(1-\nu)} \frac{x(x^2-y^2)}{(x^2+y^2)^2}$
$\sigma_{xx} = \frac{Gb}{2\pi(1-\nu)} \frac{y(3x^2+y^2)}{(x^2+y^2)^2}$	$\sigma_{yy} = -\frac{Gb}{2\pi(1-\nu)} \frac{y(x^2-y^2)}{(x^2+y^2)^2}$	$\sigma_{xy} = \frac{Gb}{2\pi(1-\nu)} \frac{x(x^2-y^2)}{(x^2+y^2)^2}$

$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) = -\frac{G\nu b y}{\pi(1-\nu)(x^2+y^2)}$

$\sigma_{xz} = \sigma_{yz} = 0$

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Let us look at the case of a stationary edge dislocation. In this case also the dislocation line is assumed to be along the z direction right. This is the positive sense of the dislocation which we choose it in this direction, and here if we take a burgers circuit around it, there is going to be a displacement b correct. When we take one circuit around it this is the burgers vector and what is the direction in which this displacement is there, this is in the x direction. There in the screw dislocation that burgers vector in the same direction as the line direction of the dislocation. Here it is perpendicular to the line direction and initially looking at it we can immediately make out that the displacements are going to be there only in the x and y directions correct and the z direction no displacement.

So, then you have to find a solution to that the force balance equation. That is much more complex finding out the solution, but the solution which has been derived which is given that most of the textbooks any textbook which you can see it is given and that is what I had given both in terms of θ as well as in terms of x and y also, u and v do exist, that w is always equal to 0. Here you can see that the u essentially if you look at it is $\tan^{-1} y/x$; that means that this takes care of the cyclic behavior. And if we differentiate with respect to x y and the z , then we will find that the strains when we try to calculate by taking the derivative, ϵ_{xz} ϵ_{yz} ϵ_{zz} will be equal to 0. Then ϵ_{xx} will turn out to be this value that is $G b / 4 \pi (1 - \nu)$ this remains a constant term. And then these are all the other terms y^2 / x^3

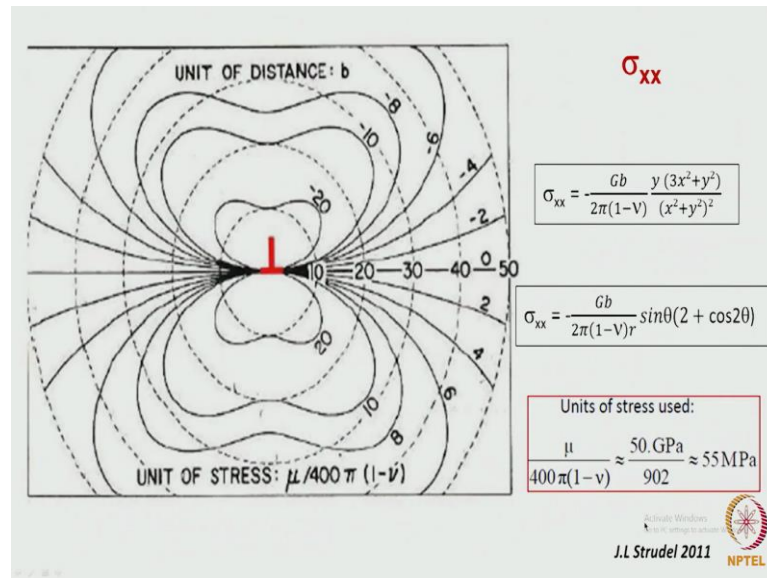
plus y^2 by x^2 plus y^2 the whole squared. These are all simple algebraic derivation only one will be able to get these expressions, one does not have to remember this expression, but one should know that these are all the expressions which one can see in any book their value.

If we have this value, from this using generalized Hooke's law, we can find out the stress. That is what essentially has been given and the calculated value of stress are the derived value of stress is given here. And one should note that though the strain is 0 σ_{xz} this comes from the pythons this one that is when there is a stress. That is clear. And only σ_{xz} and σ_{yz} is not there. σ_{xy} is there. Shear stress only in the xy plane correct whereas, in the y plane as well as in the x plane there are tensile compressive stresses are present then. In the z direction also there is a stress which is present. These are all the stresses which are present.

Now, what we will do it is that, try to just find out the distribution of each of these stresses each of this component of the stress σ_{xx} . Let us see how it is distributed around the dislocation σ_{xx} is nothing, but around the dislocation what is going to be that is stress it could be the tensile or a compressive stress, that is around the full dislocation we are trying to find out what is sort of intuitively we know that with respect to our dislocation when it is present if this is the slip plane, on the top half of it, if it is going to be compressive stress.

And the bottom half of it, it is going to be a tensile stress correct, but this intuition tells only that either compressive or tensile, but it does not give the magnitude correct. When you have to do any quantification, we should know what is going to be the magnitude of it as a function of different distances around the dislocation correct. That is what these expressions help us to get that information. So, for any quantification we require these values now let us look at it.

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Here σ_{xx} is plotted. This is x and this is y and it is plotted at a distance from the center as these dashed circles which are there are in terms of burgers vector the distances are given from the center of that core of the dislocation which is 10 20 30 40 like this 50. And then we are trying to find out what is going to be the stress which is calculated. When all the stress is being given these are all tensile compressive stress is given as say equivocator plot. So, different values what we have done. We can normalize this stress by dividing it by $G b$ by 2π into 1 minus μ . That is what essentially is being taken some value. And then it will be just a numerical factor of this terms will turn out to be and that could be either positive or negative.

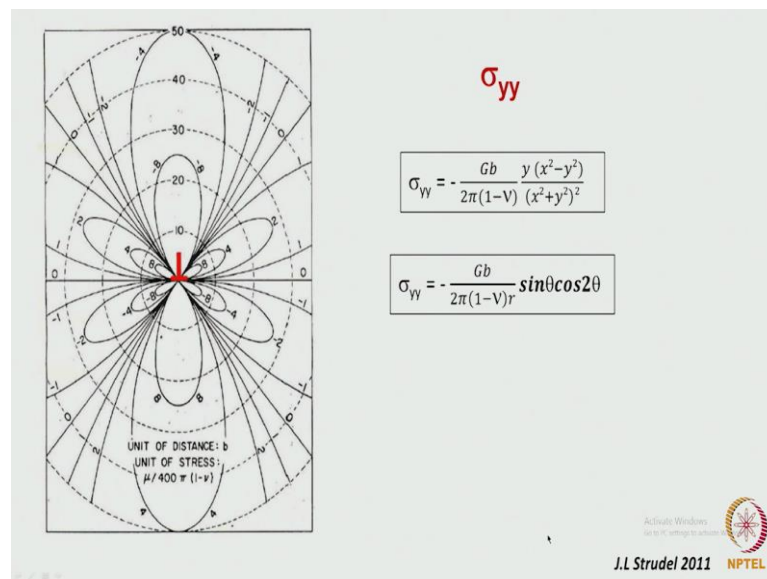
When it is negative this value then the stress is going to be compressive, and this side is that is on the positive side of y the stress is essentially compressive stress. Negative side of y that is this is the negative side of y , the stress is essentially one side and this is that a, and it changes from the positive to negative. So, the line corresponding to a y equals 0 that is what corresponds to a 0 stress. And another aspect also which we have to see it is that, the radial symmetry has been lost. If I try to calculate stress around it in the case of a screw dislocation the value remains the same.

Student: Shear stress.

Shear stress here now you see that, at this particular point if we consider. This is for a particular distance the stress turns out to be some factor of 20 minus and here also just

above it also that same value you get it, but in between at an angle at a larger distance you get this 20 r the same value of r, if we consider here, the values of stress is going to be much higher. So, around this that stress is not the same, it is going to be from this what we are able to make out is that quantitatively, we have got some express plot quantitative plot the map, which shows how the stress is varying as a function of r as well as theta correct. But the essential general point which we can take it is below the slip plane. It is tensile and above the slip plane it is compressive, correct.

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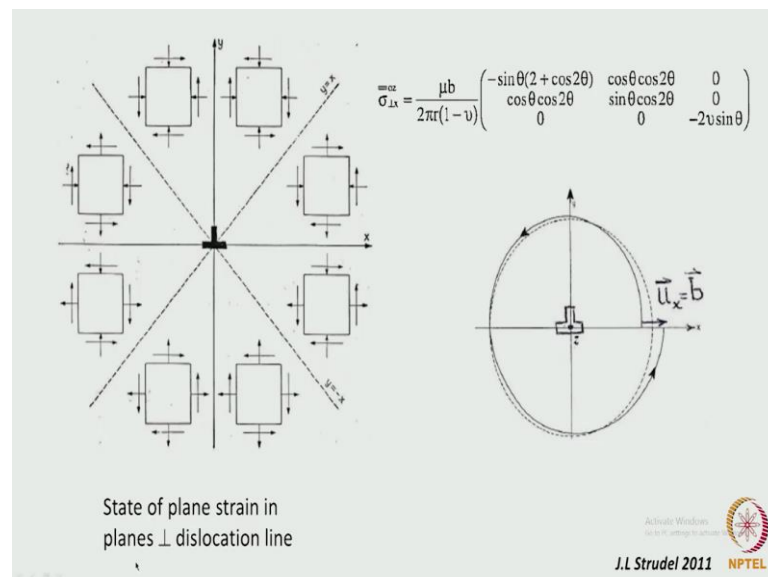
Now, this is with respect to sigma x x. Let us take with respect to sigma y y, with raised to sigma y y, if you try to see how the stress is varying. Here, one can see that, because this is given in terms of cylindrical as well as the both the Cartesian coordinate, above the slip plane and below the slip plane, if we consider it that is just at the slip plane the value of stress is 0. Across the slip plane where y equals 0, and that is because here y if you put it this here a value 0 this stress is going to be a 0 value correct from this expression we get it, but since this as x squared minus y squared is there, for all values of x equals y also this term will turn out to be 0. So, that is what this line and this line represent. So, not only at the corresponding to a slip plane are on the x axis, but at angle making 45 degrees with respect to x axis there also the stress has become 0.

In between if you see in this quadrant, it is essentially the values are positive. When the values are positive what is it going to be? It is going to be a tensile stress. Correct? And then from here to in this region in this particular in this arc if we consider here it is all.

Student: (Refer Time: 32:09).

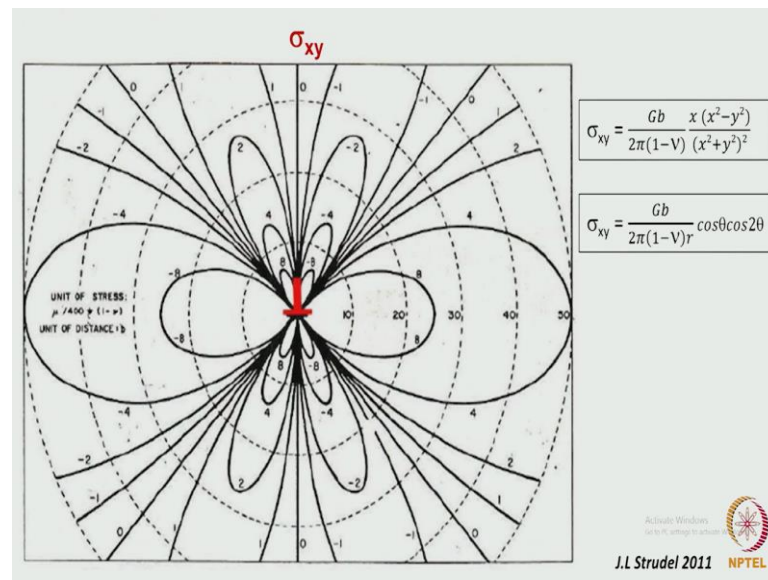
Compressive stress again a tensile stress here again a compressive stress and in this region tensile stress. This is how it is merging. So, here what we have plotted only just sigma y y correct. That is though here our displacement is only in the x direction, if we consider anywhere this means that there is going to be a small displacement in the x, and there is going to be a small displacement in the y direction also. Correct that is all it essentially means.

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You just look at this a curved figure for a moment; in this figure you can make out that this is one circular symmetry. And if you take the burger circuit is like this, you can see that it is at any particular point with respect to a point if you consider it. There is a displacement both x and y which is going to be there.

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We will come back to it later. Now we just look at σ_{yy} . So, depending upon the value of θ , even on the same size, we can see that there are compressive as well as tensile stresses are present. Because this is also in the plane which is the y plane the plane perpendicular to y direction.

Now, let us look at the case in the $x-y$ plane what is going to be there. So, that is essentially the shear stress which we are considering σ_{xy} . Here again if you look at the expressions, this expression and the expression for shear stress except for this term the x becomes y otherwise they are identical expression. So, a similar behavior is expected. So, here again if we see only thing which we can see now is that the along the y direction.

Student: (Refer Time: 34:15).

The σ_{yy} σ_{xy} the shear stress becomes 0. Then with 45 degrees with respect to x axis it becomes 0.

And in between depending upon between here between plus 45 to minus 45, the stress is always going to be shear stress is always positive.

Student: (Refer Time: 34:44).

Then between 45 plus 45 into 90 it is going to be negative like this, we can make out that the stress is going to change shear stress is going to change correct. So, now, we have considered only a shear stress component. Now let us look at this case over all. So, what we have looked at it σ_{xx} that is perpendicular to the z direction. That is if you consider a plane which is perpendicular to z direction. We have find out the stress in the y direction σ_{yy} , stress in this direction σ_{xx} , we have looked at it and also a shear stress which is going to be σ_{xy} , we have looked at this correct this is all with respect to any point perpendicular is z direction. This plot just is a combination of all of them. There we had given some values here what have we done. Here we have divided into 8 quadrants, around the dislocation core. Suppose we assume a small region around the dislocation core in this quadrant a small q .

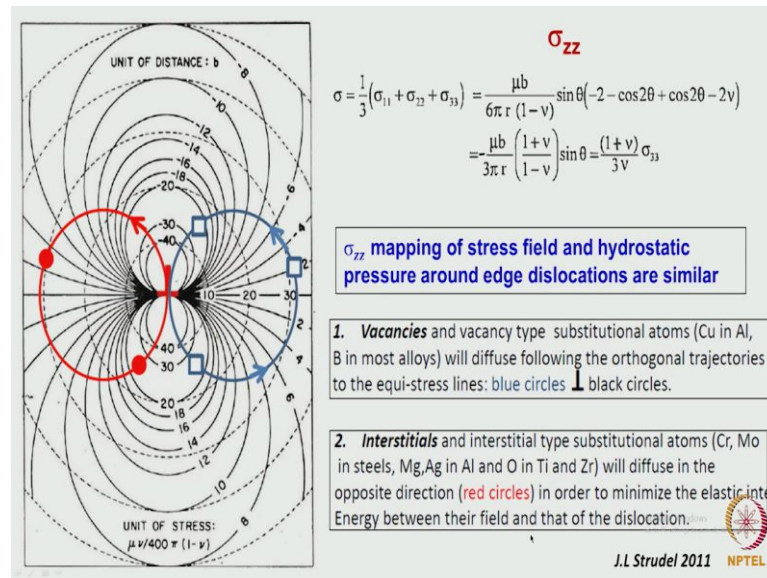
What are the types of stresses which it will be facing? If you look at it, on this phase perpendicular to that x direction it is a feeling a compressive stress.

Perpendicular to the y direction it is a tensile stress, and then in this direction in the positive direction it is a shear stress which is positive, like because on the positive half it is positive. So, it is positive in this quadrant if we look at it the stress is essentially now the shear stresses towards it is negative and we look at the y direction, it is as again compressive. In the along the x direction again it is compressive. So, these stresses which they feel overall all the stress components of the stress is they change direction. Different quadrants this is a different type of stresses which different regions will be filling.

This is a qualitative way in which we can easily understand. The same each of the component in the previous slides, we have just plotted each of them separately correct. This is the sum of the this is something like a plane strain condition where along is z direction any direction along the dislocation, we look at it perpendicular to that plane, what all the in the plane which is perpendicular to it on that plane what are stresses which are going to act, that is what we have tried to calculate. And this is essentially to show what is the sort of a displacement, this is a burgers circuit around the perfect crystal.

Now, from the displacement equation for the different values of x and y , we can calculate what will be a value of the position after point over from the original point how it is displace that is what essentially is being plotted.

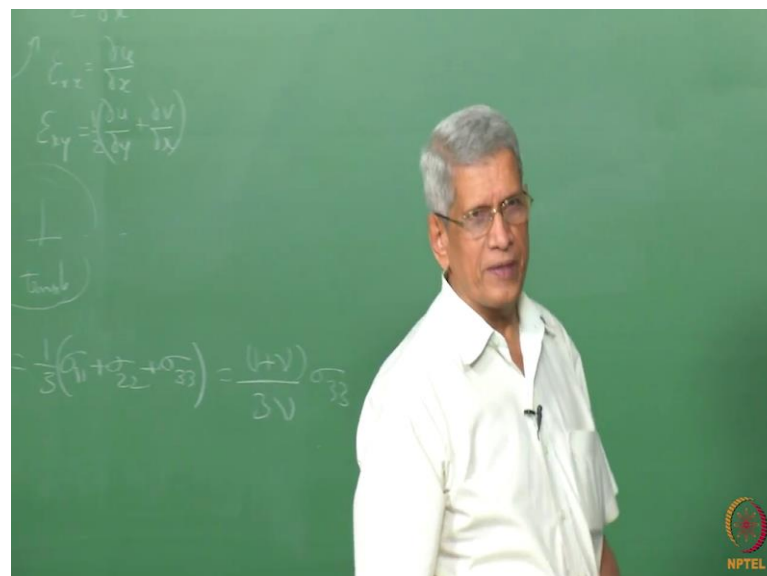
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Now, let us look at the case of because so far we considered all the stresses which are in the perpendicular to research.

Let us look at the stress σ_{zz} which is going to act on that sample. This stress can be considered as similar to something like a hydrostatic stress. It builds a relationship between them. That is mapping σ_{zz} and the hydrostatic pressure around the edge dislocation are similar. And we know that the hydrostatic stress is given by $\frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33})$, correct.

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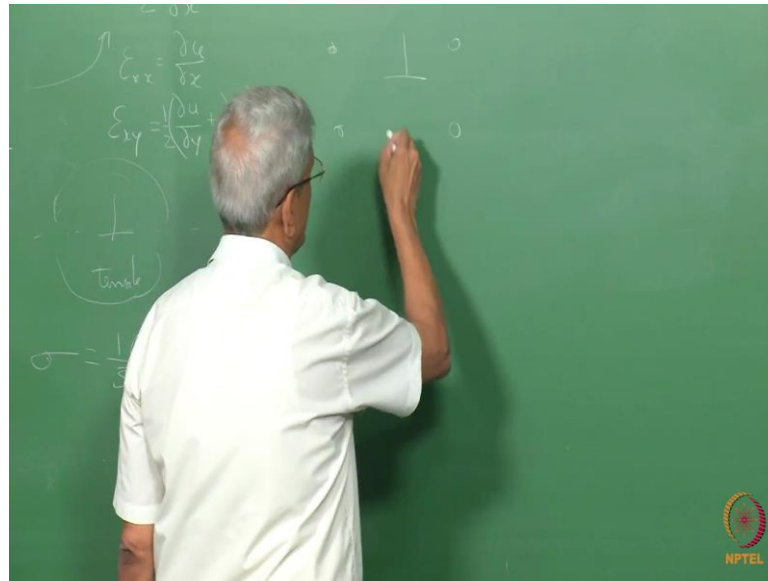


We can substitute for this, and to try to solve this equation. Then this σ_{zz} will turn out to be $1 + \nu$ by these expressions which is very important. Where ν is the Poisson ratio, where this stress itself can be represented in σ_{xx} or σ_{yy} itself can be represented in terms of a hydrostatic stress and the hydrostatic stress is nothing, but one third of the sum of the stresses acting in σ_{xx} , σ_{yy} and σ_{zz} direction correct.

This also if you try to plot it, the way this stress will turn out to be as the plot which we can see it, is it is essentially it is negative it value it shows it shows that this will be a compressive stress which is going to be here, yeah. Both the slip plane and tensile stress, below that σ_{zz} . The only difference is that when we calculated σ_{xx} the plot looks like that qualitatively it is very similar to what we see for a tensile stress σ_{xx} , but only difference which is happening is that if you look at the equip stress contour plots these are all circles where is tangent at the dislocation line there what is going to happen is that it is not a circle it has got different shapes.

Correct the values are different. So, that is the only difference between this stress what is the other thing is important in this case. Then suppose this sort of a hydrostatic stress is present under sample, we assume that generally we know in most of this material, when dislocation is there in addition to it the other type of defects which are present are in the sample some interstitials could be there, which could be impurities or we can have point defects which are distributed in there sample right.

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You are on the dislocation core there may be some vacancies here, and just showing it as vacancy. Let us just consider the case of a vacancy type of a defect is being present. And if your vacancy is being present here when a compressive stress is going to be there is a energy minimization, but what we can see that if the stress becomes much smaller and smaller if it can come close to the as it reaches the core of the dislocation this energy minimization will be maximum it will try to reach that path.

Now, the question comes is that what path it will choose? Is it that it will just like that it will come here? Or from here if it is there if you try to reach here, that is what this essentially important. This plot looking at it we can get an answer to it that is a very moment which it takes from here to here there should be a reduction in the energy has to take place. So, that are the stress has to get reduced. That is precisely what it is suppose there is a one which is present here; it is with respect to a stress which is minus 2.

From here if it moves in this direction it reaches minus 4, right.

And this is the way; it will just try to reach this core of this dislocation. That is, we can consider the case the dislocation is present when we do the aging treatment, some defects are present like a substitutional defect which is like a vacancy type vacancy type, we mean that the atom which is there, produces the strain which is essentially a like after strain which a vacancy produces. And we know that when we do aging at that temperature, these defects come to the dislocation core and form atmosphere around it

and in some cases they form a precipitates also around it. These we call it as a heterogeneous precipitation.

But what is the path which it will take to reach that core is given by this plots; that means, that it is taking a plot which is almost perpendicular to this i q counted plots right.

This is a way to take place similarly let us consider the case where it is an interstitial type of a defect. That is, it is I will just put it as a plus sign to indicate that it introduces sum strain positive strain in the lattice. So, if it is introducing a positive strain. And if it is going to be there on this side, where essentially a compressive strain is there already this will add to that strain correct.

Student: Yes.

But if that atom can be there anywhere around the lower half the strain is going to be minimum reducing the strain or it means that, when one with a positive strain comes in to a tensile region then strain energy which it feels going to be reduced and if it moves further and further and comes closest to it.

That is where the tensile stress is going to be maximum. Then they follow more reduction in free energy where more reduction in energy will be there strain energy will be there. So, it will try to reach here, but the path it will take these, this sort of a path something which is there on the compressive strain. It will not just like that reach here. It has to move along this contour like this. And it will be trying to reach here. Suppose they assume that it is being present here, then it will move along this part and try to come close to the core of the dislocation. That is straight away from here it cannot reach this point. It has to move around the path diffuse around the path and then reach the core of the dislocation to form.

So, the diffusion lengths are not like a profile distance and it is going along as circle. Where that the direction is normal to the e q contour plot. So, this is what essentially the information which we can get, additional information we can get from this plot. So, like that many situations which we can think of where this sort of plot tells a lot of information about how different types of processes occur because of the presence of defects like dislocations in this particular case when defects have to migrate to a dislocation that whether it is in dynamics strain aging or a static strain aging phenomena

which is taking place, they are what sort of path defects distributed around the dislocation will take and reach the core of the dislocation are close to the core to form precipitates or atmospheres around them that information we can get it from this plot.

So, essentially what we have considered so far is the displacement field around screw under net stationary dislocations. The strain field around them stress field around them. What are the implications of this stress field around the screw and the edge dislocations where happened? Here also we have to understand that close to the core because in all the expressions which we have derived here as the r becomes very small. This value of stress can become very high because r is always in the negative, right.

Because of that beyond the particular r which that region of the dislocation we call of the core of the dislocation, where the displacements are very high. That the energy calculation cannot be handled by a conventional continuum mechanics, and there we have to do some molecular dynamics simulations to get energy value, some computational calculations have to be done that is who those values are obtained.

Now, we have only we have got the expression for stress and strain. And their distributions we have looked at it some implications we have looked at it. What will be the effect of this stress and strain and the total energy of the system that is in a sample where no defect is present and if we introduced this defect how much the energy increase is going to or decrease is going to take place. This aspect of it we will cover in the next class. We will stop here now.