Defects in Materials
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Lecture – 16
Stress

Welcome you all to this course on Defects in Materials. In the last class we covered about the how strains are introduced various components of the strain and shear strain as well as a pure strain all of them are present then how we can transform that into a form where only the principle strains are either pure shear or pure strain.

Student: (Refer Time: 00:50).

Or a pure shear that way how we can transform them and then another aspects also which we considered is that whenever we strain a lattice especially when we shear a sample it is possible that the external form of displacement which we give, apparently could be that pure shear component may be full deformation which has been given need not be pure shear, it could be partly a shear plus a rotation, how to separate this? These are all the aspects which we have covered. What will be the consequence of trying to apply a shear on a material the strain a material?

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Suppose we take a material, pull it in these directions give rise to an elongation, assume that at some particular plane arbitrarily we draw a plane, and assume that there are atoms on either side of it across an atom.

Then when we try to pull that sample apart the body will exert develop and internal stress which will oppose the external stress which we have force which we are applying on it so; that means, that now the atoms on either side of or the separation between them is away from the equilibrium separation. So, they will try to come back. So, there will be a force which is acting from here on this side and a force acting from here on this side on that plane finally, when the stresses are equal it is in that equilibrium position.

Static equilibrium; forces are always going to be there we assume that on an atomistic scale that is looking at distances of the order of atomic separation if the force does not vary that is one assumption which you make then and if it is constant then we can represent this in terms of what we call it as a force per unit area that is what we call it as a stress.

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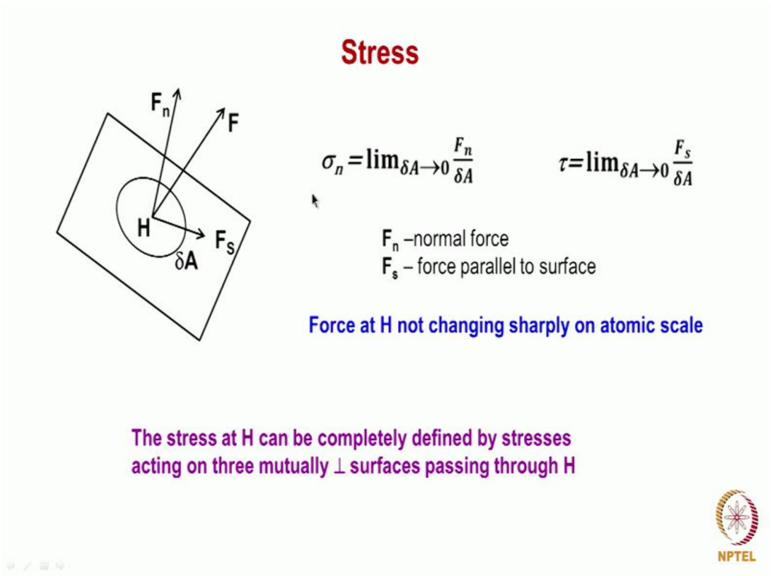

A sigma equals limit that is if you assume that the force is being applied in some particular direction then what we know is essentially across some area of the sample over which this force is being applied; the force as the area becomes tends to 0, what is going to be the force? That is called as the stress. This is how the stress is being defined, is this clear?

This generally when we try to wander to measure a force is a vector, it is always applied in a specific direction, normally when we do experiments, what do we apply? What we know we some load which is being applied, we know what is the area over which this load is being applied from that we can calculate that stress correct that is the only way we can do that and that stress per unit area can be called as relevant into a force; force cannot be directly because what we do is experimental part of it is only the load which is being applied.

The load will be applied over any area from that we have to calculate the force, correct. How do we go about doing it? That divided by the area; we get that correct and multiplied by that area that force will come into the picture.

Student: from load to.

Yeah.

Student: In a (Refer Time: 05:05).

But first should know, what is the area to which we have applied it?

Student: In which (Refer Time: 05:07).

Yes that is; what is stress? Stress is force per unit area.

Student: But when force area.

That is the best way in which we can calculate the force. Now if you apply force on any surface of that sample; small idea if a force is applied in some direction across an area, this is delta A if he considered it, this force can have like we can divide it into 2 components one normal to F which is called as F n, one in the direction; in the plane which we call it as F s; we can call it.

So, the force can be separated into one in the normal to the plane and one in that plane. If we choose a coordinate system and then with respect to that coordinate system this can be further divided into; 2 more components in the direction of suppose x and y are lying on this coordinate system x axis and y axis.

Student: We can also.

We can further separate it into. So, essentially on any surface if you consider this surface now we have 3 forces which are going to be there or 3 stresses we have to consider it one is a stress which is normal to the surface another is 2 stresses which are lying on that plane, correct.

And. since the coordinate system which we are choosing it is x, y and z they are the Cartesian coordinate which we choose any force which is being applied in a particular direction we can split it into components which are lying along component of this force on all these surfaces. So, essentially what we will have is 9 components are going to be there total correct that is if that is at this point when we are trying to measure the force the net force it is applying which is being applied that can be separated into components along 3 surfaces passing through that plane which are perpendicular to each other those 3 surfaces when we consider it is nothing, but equivalent to considering the Cartesian coordinate system and defining the stresses.

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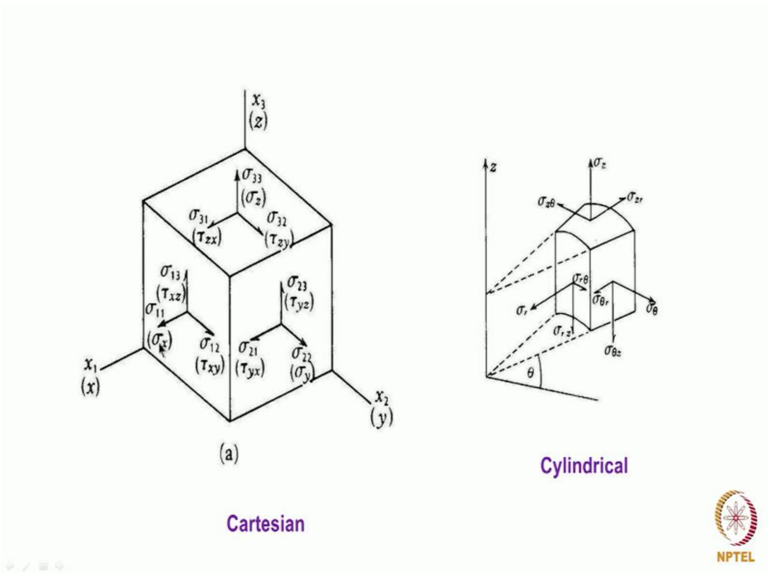

That is what essentially is being shown in this slide if you try to look at it in this; there are 2 notations in which the coordinates also being represented one using 1, 2, 3 to represent the xy and z coordinates, otherwise also writing them xy and z. So, on the positive faces is where these stresses are being shown sigma 11, sigma 1 that is sigma 11 is there stress which is normal to x surface and similarly sigma 1, 2, or shear stress; the 2 xy if I write it is that is this is applied in the y direction, but on the surface x, but how the surface is being defined now normally being defined with respect to the out word surface normal if you look at for these surfaces the outward normal is always in the positive direction.

So, this is good. So, that is how we define whether it is going to be a positive or a negative value that is being decided by the direction in which the load is being applied and outward normal together that defines the stress is positive or negative that is a way and one should be very careful that these symbols which are being used. The other directions the magnitude of the direction which we use here it appears they are equal correct we should not be fooled by that because these values could be it is only arbitrarily it is being shown, it can have values which can be quite different depending upon the components of the force in those directions similar to Cartesian coordinates.

We can use cylindrical coordinates also, but in many cases especially when we are looking at defects like dislocations the Cartesian coordinates instead of Cartesian coordinate one can use the cylindrical coordinate because most of the time the dislocation is taken to be the line direction of the dislocation to be lying along the z axis and most of the displacements which are taking place which gives rise to stresses are in the xy plane where we can define that in terms of cylindrical coordinate itself and another aspect which one has to be very careful about it is that I have taken from 2 books how these stresses are defined this one has to be very careful about it because different books follow different notation.

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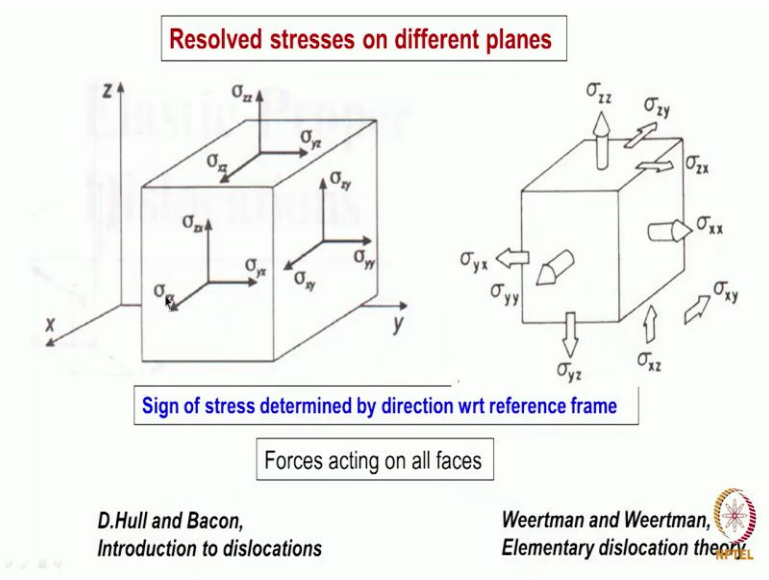

In this particular one, the first letter represents the direction of the stress; the second letter essentially is representing the plane normal that is that tells; which is the plane in which it is being done. So, here if you look at it that direction in which it is being applied sigma it is y first represents in the y direction you look here.

In the next diagram if you look at it here with respect to a coordinate system which is chosen this is the y face on which this is the y normal which is the first letter and x represents the direction in which it is being applied.

Student: The first one is normal.

The first one is no x represents the direction in which the stress is applied sigma x and here sigma y is there y direction it is being applied that when sigma first letter z is in the z direction is being applied and the second suffix later suffix represents; what is the plane normal? That is common because it is applied on this surface x here it has just changed.

Student: (Refer Time: 12:27).

Different books follow different notations. So, one should first see that what is the notation which is being followed and one should use consistently one particular notation and because of this quite often we find that some signs will change using calculations and one should not be fooled by looking at these information that one book has given something else other because given an exactly of course, that is not the case because always all the magnitudes other value not only the magnitudes y that direction whether it is positive or negative because this can make a lot of difference suppose you calculate some value on the basis of the sigma x which is given.

If you do not follow a consistent value for stress we use one notation strain you use another notation then we will have a lot of problem with mean attractive and repulsive forces the lot of confusion will come if that should not happen we should always follow one notation only and we should know what is the notation with which these stresses and strains are described. So, far what we have defined is 9 components.

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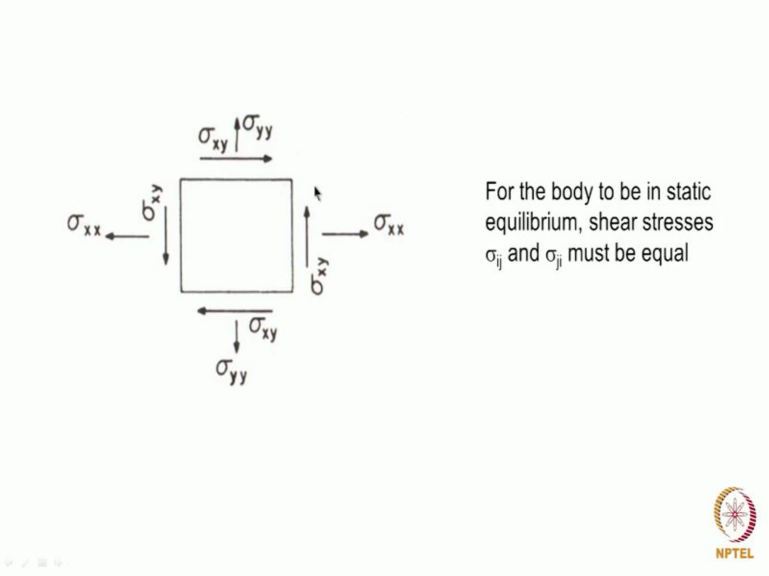

Especially their shear stresses when they are being sigma xy which is shown in the parallel which is being applied in this direction and sigma xy this been applied in this direction if you look at it if their magnitudes are not the same there can be a net moment and that can be a rotation which it can come normally in this case when the stress are small the body is in equilibrium it is static equilibrium.

So, the moment has to be 0 because of which sigma xy will be equal to sigma yx or this we can write it as a generalized form sigma i j equal sigma j i a so; that means, that we will have 6 components of the stresses are going to be in the previous figure we have defined the stresses with respect to surface which are perpendicular to either x, y or z directions this is how all the components are defined.

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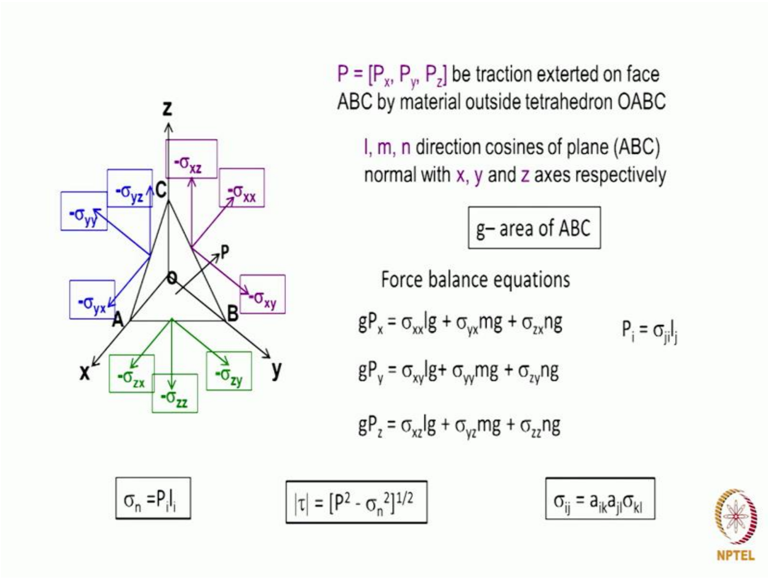

Now, suppose we wanted to find out stress along that is the force which is being applied arbitrarily in a any particular plane that is plane could be in any plane it need not be along x, y and z.

What is the information which we have as far as we know is that what is the force which is going the stresses which are acting and all the specific planes x, y and z surfaces that information is available if that information is available can we find out the force we acting in any particular direction on any plane that is hold the whole issue z; that means, that suppose I assumed that there is a force which P is acting on this plane.

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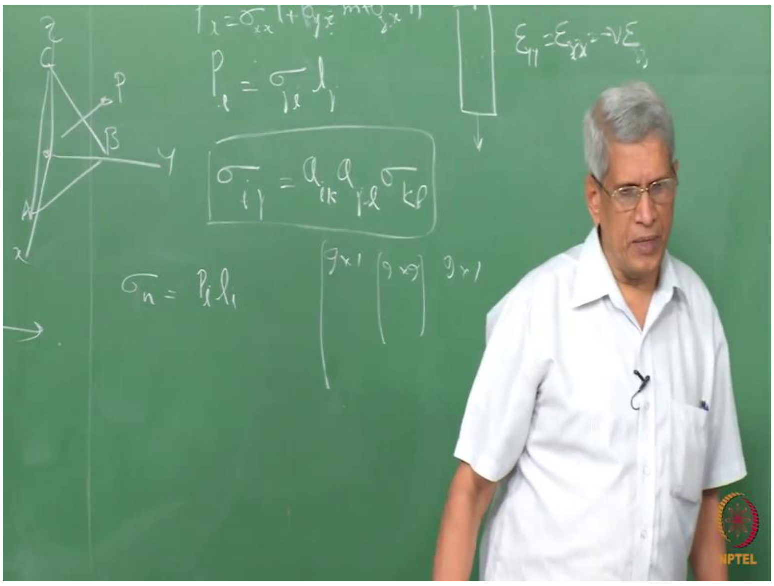

That is then arbitrary directs what we wanted to know, what is the magnitude of P? The P can have components that spin since P is a vector it can have component P 1, P x, P y, P z, in the x, y and z direction, correct. Now that means, this is the component of the force in the xy and z directions if we get the component of this one in terms of forces which are acting on the x surface, y surface and z surface then we can get complete information about all the forces therefore, then the complete informed the force acting on any direction across this plane a, b, c, we can get it, correct.

So, for which what we have done is that if the plane normal is defined by a vector and that makes l m n then are the direction cosines of that plane normal with respect to xy and z axis, if he has got that a information, this is a tetrahedron, correct.

In this tetrahedron.

Student: When is the.

This is the plane which is perpendicular to y, is a o c, correct, that is the equal the triangle the area on which the stress is acting and b o c:

Student: (Refer Time: 17:59).

Is the area of the triangle over which that x direction force acting and a o b is the plane over which look at that surface over which these stresses are acting in the z direction. So, if we have this information what is going to be the force if he is acting in each of this direction suppose the component is P x, what is going to be the; what is g? Essentially I represented it the area of this with respect to a the g into P x is the area over which the component is of the force is going to be there correct and this is acting on this P x, we can find out to be suppose with respect to x direction we wanted to find out the force which is acting in this sigma xx, correct normal to the surface in the same direction.

If you look at it sigma yx will be the force which is acting in that direction then sigma z x is the force which is going to act on that surface in that same direction. So, if we take the total sum total of those forces we get the component P x correct that is what essentially is being given by this formula.

So, this way using this diagram we can find out the forces which are the components of the forces in the x, y and z direction if you know that just by using the vector notation then immediately we can find out the direction in which the force is going to we present on the sample on the surface a, b, c, which we have considered this same force which is being applied on this surface now we can find out the component of it which is one normal to the surface and another one which is in that plane normal to the surface is the one which gives a tensile are the compressive stress the stress which is lying on the plane gives the shear stresses.

So, these are all the expressions which are being given for how to find out these stresses. So, what essentially has been done this equation itself here when g is going to be there if you removed from this expression g then this becomes P x equal sigma xx into l z x into n where sigma xx is the stress normal to the this is the sigma yx is the stress which is acting on the y surface, but in the direction of this one. So, if we multiplied by the area we are going to get the force. So, here sigma z x is essentially on the z surface, but the force which is being applied in the x direction. So, we are taking the sum of sum total of it this itself can be;

Student: total on the (Refer Time: 21:44).

Written in the form of P i equals sigma j i into we can write it as l j, this is a Einstein summation noting notation which you have learnt earlier.

Another aspect which we have to see it is that here we have calculated the components of the force directions of the surfaces the stresses which are acting under phases which are parallel to x, y and z axis this is with respect to a coordinate system which we have chosen, we can choose another coordinate system which 3 other surfaces with respect to applied normal. If you take it that may be making some angle with respect to the old coordinate system like what we have learned earlier in the case of coordinate transformation in such a case, we can find out what is going to be the stresses along different directions with respect to that coordinate system in terms of the old coordinate system and that will be given by this formula.

So, this one has learned this way we can find out with respect to a new coordinate system, this will aspect this sort of transformation, we will require quite often because the dislocations when we wanted to calculate stresses and strains around dislocation we defined them with respect to a coordinate axis which is with respect to a dislocation, but that axis does not match is not the parallel to the crystallographic axis which we choose to describe the atom positions in the unit cell, is it clear? So, this way we can represent the different components of the stress and strain what we have written this equation this equation is nothing, but a force balance equation.

There is when we write this expression with respect to g this is essentially a force which is being balanced.

Student: (Refer time: 24:32).

This one, here what we have trying to calculate; if you wanted to calculate the shear stress, if you know the normal stress, how do we calculate this stress which you have done? If we wanted to find out the normal stress, we know the components of P x, P y and P z in the normal direction we have to find out normal to this plane then we will get sigma n that is why a put P i into l i that is l i is the.

Student: I.

Represents the direction cosines which their different components that is P x, P y, P z make with respect to the plain normal. So, then we can find out the component of the x, y and z; P x, P y, P z in the plane normal direction when we add them together we get the normal stress correct that is essentially what is being written the tau is that if we know the stress which is acting or the force which is acting in the direction of P that we have the total 1.

Student: Sir.

And that magnitude if you take it this is nothing, but constructing a using the Pythagoras theorem, we can find out sigma n we know that take the whole square and then square root will give you what is going to be the magnitude of the shear stress this only the magnitude which we are calculating with respect to a coordinate system which we have defined x, y and z we know the strain that components of the strain along these directions both the pure tensile or compressive strain dilatation as well as the shear strain similarly the stresses which are also there in these directions what will be the response stress which is going to be generated because of the application of this strain.

Student: Strain.

So the response stress depends upon the property of this material, the response stress in any particular direction.

In general will depend upon all the shear stresses which we are trying to apply, correct the simplest way in which we can consider it is that suppose I take a sample, I try to along it in this direction, this will assume that the strain is going to be some epsilon xx in the x axis direction or it can be taken to z because if we normally choose this is the z direction if you choose this. This will generate a stress, this we know that this stress sigma equals into correct.

Student: (Refer time: 27:52).

But this stream generates a corresponding compression in the other 2 surfaces, correct. So that is given by where nu is a Poisson ratio, correct. Now this can be written in terms of that is we find that the sigma yy and sigma zz and sigma xx depends upon the stress which is there in the z direction, correct; that means, that a stress in a particular direction depends upon the strain which is being applied in all other directions it will have a component in that direction. So, if we write in terms of those components.

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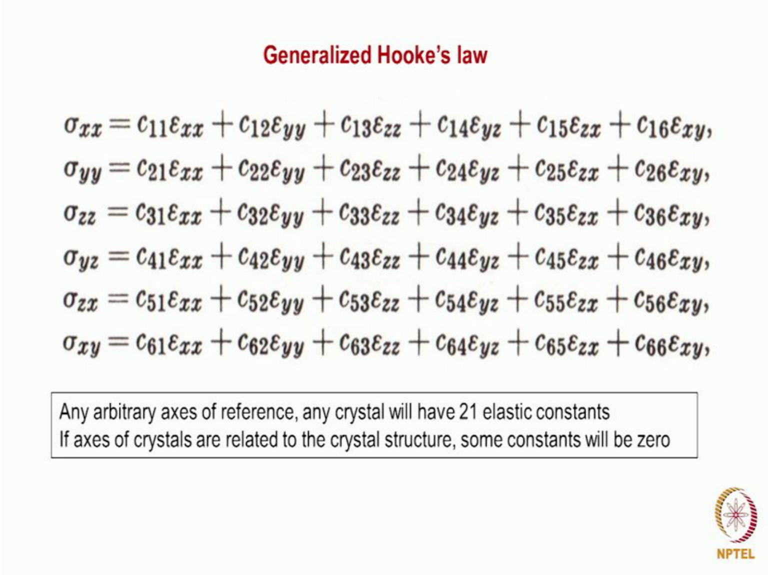

Then we will be getting a relationship between stress and strain in this form, what are these components? C11 is essentially that contribution to stress from strain which is being applied in the x direction, this is from strain which is being applied in the y direction, this is the strain which is being applied in the z direction, these are all the contributions from the 3 shear strains which are being applied this is going to be the total valley correct is correct.

So, now if we look we have essentially 6 stresses and there are 6 strains are there, components of the strain and the stress is also a tensor of second rank and strain is also a tensor of second rank when the relationship between them when we try to write them in a matrix notation essentially there should be in general case, we have 9 components of stress. So, 9 by 1 matrix and here another 9 by 1 matrix in an equation form, if we write it each equations will have 9 terms.

So, essentially this will be a 9 by 9 matrix which will be required that components of the property will be there is 9 by 9 means 81 components are going to be there when these stress and strains and they are infinitesimal and the body is in static equilibrium we have derived earlier that in a symmetric case, it will be only 6 components then this reduces to a 6 by 6 matrix. So, 36 components which has to be there, correct.

These are all the; this is the equation which gives that relationship between stress and strain which we call it as the generalized Hooke's law. So, if you look here there are thirty 6 components are going to be there. How does it change to 21? If I have to change it to 21, there should be some components which should be equal, how does that happen?

Student: (Refer Time: 31:48).

Before we go in to a now what we have got it is that we have got an expression which tells the relationship between stress and strain, how to relate them? This is expression we have and we know the stress which is being applied under different directions and you know the corresponding strange also if you know the stress and strain we can calculate what is going to be the total energy increase in energy of the system because of the application of the stress and strain correct. So, the totals energy will be given by a formula of this type with eight plus sigma zz, epsilon yz, sigma xz and epsilon xz plus sigma xy into epsilon xy this is nothing, but the total energy which the system is filling, correct.

This is right, this way this expression is given in terms of stress as well as strain, we can write stress in terms of strain. So, that this expression will become only in terms of stress strain we can write strain in terms of stress then this expression can be written in terms of only stress or this itself can be finally, substituting for a strain in terms of displacements because the strain we defined with respect to displacement of that atom from the lattice the normal position that way we can write this in terms of nothing, but the displacements itself.

So all the 3 forms; it could be written down, I would expect all of you to work out these formulations derive the expression for energy in terms of stress alone strain alone as well as displacements that will give you a fair idea of what sort of relationship it exists between them.

And another important aspect which we have to consider it is that suppose I am deforming, a sample I first deform x it in that s direction and then n deform in the y direction afterwards. So, some energy is pinned in deforming in x direction. It is being held in that then I try to deform in that x direction.

Student: (Refer Time: 35:28).

The total energy is path independent; that means, that suppose I deform now instead of doing it deform first in the x direction i give a deformation in the y direction first.

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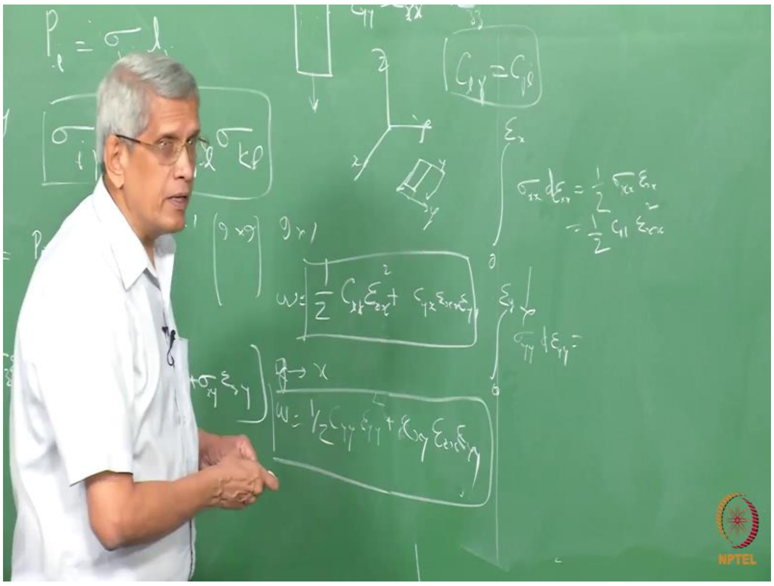

That is suppose with respect to this coordinate system x, y and z, deform the sample initially in the x direction and holding it at this position when its elongated deform it in the y direction are I can deform it initially from this position in the y direction the same amount of deformation hold it and deform it in the x direction the total energy remains that same, correct.

So, if we apply this principle then we will see that the terms like C i j will be equal to C j i this is what it will turn out to be;

Student: (Refer Time: 36:44).

Yeah.

Student: Sir no metal and further oriental (Refer Time: 36:55).

That does not matter because that energy is going to be the same. So, this we can take an example also suppose we deform it in that x direction a little bit how will we calculate that energy that is integral 0 to some value the epsilon x, what is the stress which is going to be applied in this direction xx into d epsilon xx, correct, this will be half into sigma xx into epsilon xx correct and this we can write it in terms of sigma xx will be what? C 11 epsilon xx squared because when we deform in this direction apply this strain we make an assumption that there is no strain which is being applied in any other direction. That means all other strain terms are going to be 0; that means, that in this expression if we look at it if you see in this expression only this term is going to be there all other strain terms are going to be 0 that is why we can substitute this expression.

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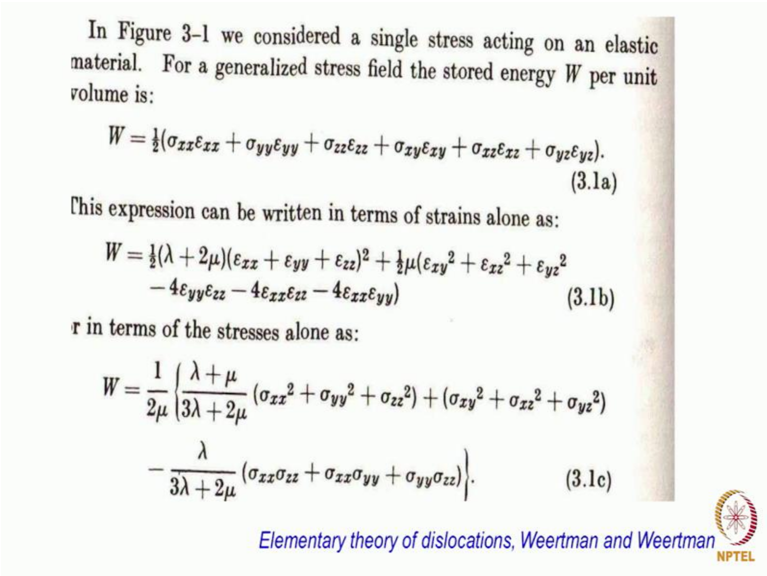
In Figure 3–1 we considered a single stress acting on an elastic material. For a generalized stress field the stored energy W per unit volume is:

$$W = \tfrac{1}{2}(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{zz}\varepsilon_{zz} + \sigma_{xy}\varepsilon_{xy} + \sigma_{xz}\varepsilon_{xz} + \sigma_{yz}\varepsilon_{yz}). \quad (3.1a)$$

This expression can be written in terms of strains alone as:

$$W = \tfrac{1}{2}(\lambda + 2\mu)(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})^2 + \tfrac{1}{2}\mu({\varepsilon_{xy}}^2 + {\varepsilon_{xz}}^2 + {\varepsilon_{yz}}^2 - 4\varepsilon_{yy}\varepsilon_{zz} - 4\varepsilon_{xx}\varepsilon_{zz} - 4\varepsilon_{xx}\varepsilon_{yy}) \quad (3.1b)$$

r in terms of the stresses alone as:

$$W = \frac{1}{2\mu}\left\{\frac{\lambda+\mu}{3\lambda+2\mu}({\sigma_{xx}}^2 + {\sigma_{yy}}^2 + {\sigma_{zz}}^2) + ({\sigma_{xy}}^2 + {\sigma_{xz}}^2 + {\sigma_{yz}}^2) - \frac{\lambda}{3\lambda+2\mu}(\sigma_{xx}\sigma_{zz} + \sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz})\right\}. \quad (3.1c)$$

Elementary theory of dislocations, Weertman and Weertman

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Similarly, if we now keep it in this position and we apply the next strain then what it will happen this will be from 0 to epsilon y, sigma yy into the epsilon yy, correct, and this; what it will turn out to be?

Student: (Refer Time: 38:48).

This if you look at this expression, now it has xx already has been applied source and y is also going to be there. So, now, sigma yy becomes C 21 into x then xx plus.

Student: C 2 into;

C 2 into epsilon yy. So, this we have to substitute it now into this term and then do the integration if you do this the answer will be C 2 epsilon xx into epsilon yy will be a costume will come, this is how this energy term will turn out to do, if you do it the other way round; this is the total energy w, if you follow the path from 2 first applied in the y direction elongation then in the x direction then this will turn out to be half into C yy plus C xy into epsilon xx epsilon yy. This is how it will turn out to be correct; that means that if the energy has to be that same this part also has to be there.

That means that C yx should be equal to C xy this is what it reduces now the number of components to 21 components will be there. In this one, the terminologies which we have used it C 11, it actually means that it is essentially C xx then xx, this is 1 equal to C 11.

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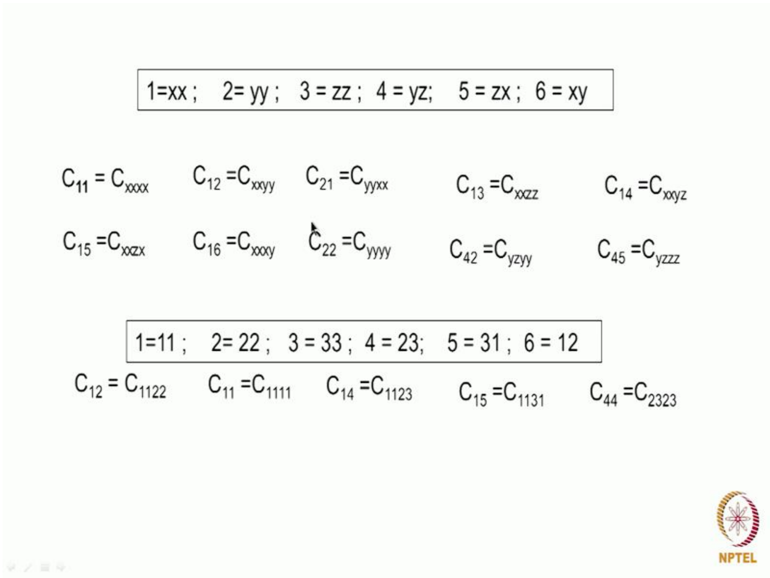

Suppose we use C 12, this will be C xx yy, this is how these definitions are already given in this table; in this slide it is being given one can go through this and get one self familiarized with it. Is it clear?

But so far we talked about stresses which we consider as uniform throughout the sample, but that need not be the case always before we come into this let us look at one more aspect of it is that the elastic constants now the number of independent constants reduces to 21. That means, that if we consider the case like a triclinic where there is no symmetry which is associated with it then the number of independent components of the stiffness coefficients which will be there is 21.

Now, crystal systems other crystals monoclinic crystal system if you consider around one axis, we have a 2 fold rotation is going to be there what does essentially the 2 fold rotation means that if we rotate the crystal around that.

Student: (Refer Time: 43:18).

Axis take it the stress and strain values will be that same; that means, that the property which relates that in those direction also has to be that same. So, applying this norm you can find out in which directions the property is going to be same and which of the directions in which the sum of the components the property may turn out to be 0 values some of the components that is it will turn out to be when we apply the symmetric rotation some for some directions the permutation combination when we take it, it will be a product of plus and minus in some of them that wherever that product turns out to be positive like what we did it for a electric field vector and current density vector to determine the conductivity tensor I explained how it should be done to find out how many independent components will be there.

Exactly the same logic has to be applied here then we will find out that some of these components C i j is will turn out to be 0 and because of that for the monoclinic system we will have only 13 independent components will be there, this one can do it for other crystal systems also taking the symmetry elements which are associated with it and find out how many independent components are going to be there finally, most of the crystal systems which we use are in the case of cubic system where it turns out to be that we request essentially it 3 independents that is C 11, C 12 and C 44, these are all the only components of the stiffness coefficient matrix which will remain.

Student: Matrix, try to is more symmetry most material less number.

The more symmetry that symmetry element the number of independent components will get reduced because in those directions because of the symmetry, they have to be that same. So, the number of independent components gets reduced.

In some other directions these symmetry consideration says that the value has to be 0 that is what it will turn out to be. So, that decides the number of component some components become 0 some components only remain this aspect of it, I will talk about their symmetry and the number of components on the crystal system separately. So, essentially one should understand that depending upon the symmetry associated with the crystal structure for the stiffness coefficients property tensor when we consider it which is a 4 tensor of 4th order tensor or 4th rank tensor.

The number of independent components will change for a triclinic system we will have 21 components monoclinic it is come down to 13 and finally, to cubic it turns out to be there are only 3 independent components which are required, I think one has to remember only this much at this stage. So, for a cubic 3 components are going to be there if the crystal is isotropic property is it exhibit then the number of components reduces to only 2.

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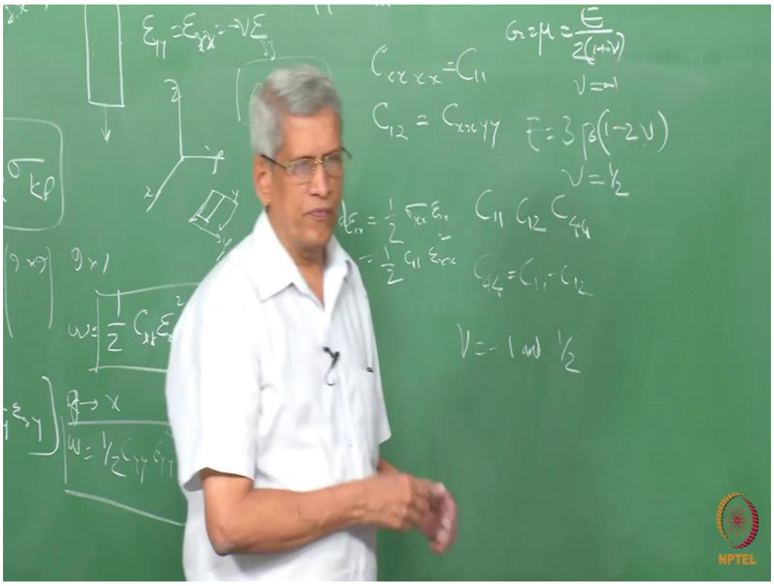

So, essentially it will be C 11, C 12, C 44 and in an isotropic case C 44 becomes C 11 minus C 12 and this basis for an isotropic case we can find out the relationship between stress and strain using generalized Hooke’s law.

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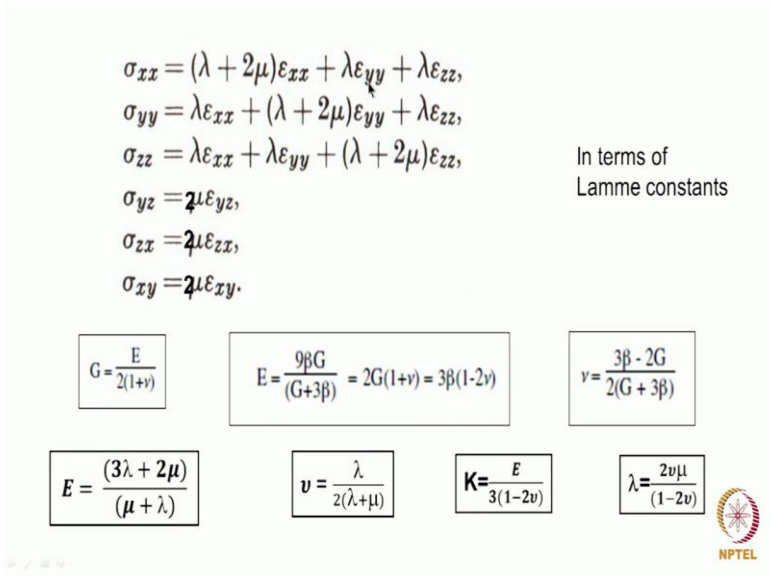

So, if you try to represent them this is the form in which the expressions turn out to be and now we can see that the sigma yz is given by 2 mu, mu is the shear modulus twice mu it will come.

Because epsilon yz is giving nothing, but the shear tensor actual shear stress is going to be twice that of that is why these 2 comes in that picture and the lambda is called a lamiae constant essentially from these expressions we have a lot of expressions which can be derived one is between shear modulus and the youngs modulus. So, shear modulus equals because there are 2 ways in which shear modulus is being represented using one using letter g in many books some let books you will find that they use the mu to represent the shear modulus i had given both the expressions here. So, g equals mu e by 2 into one plus nu which is being given correct what is nu? Nu is nothing, but the poissons ratio this expression itself tells you what is the value of the poissons ratio which one can have that is suppose this nu becomes equal to minus one this term will become 0.

What by consider shear modulus infinity that does not happen. So, upper limit is it is one minus one similarly another expression which can be derived is between e that is Young’s modulus. This is a bulk modulus 1 minus 2 nu from this expression, if you see if nu equals half the Young’s modulus will turn out to be 0 not possible. So, the value of nu has to be between minus 1 and half, correct, this is a maximum value you can have physical meaning of nu is that if we apply any strain to the sample in any particular direction when we apply elastic strain there is going to be some component of that strain which it is going to introduce in the other directions that is nothing, but the ratio of the strain which have applied in the longitudinal direction the ratio of the strain which is being applied it that is if you try to a lattice which kinds of atoms which are going to be there in different directions which are all in equilibrium in 3 dimension if I try to pull in one direction. Stress, what is it going to be? It is effect on the other directions in the how far it is going to be disturbed from the equilibrium position.

Student: In direct make equally try to derive it equilibrium (Refer Time: 51:26).

Yeah.

Student: There is a there is a limit up to which we can derived it.

No, when we try to along it, it is being pulled away from the equilibrium position normally when we try to draw.

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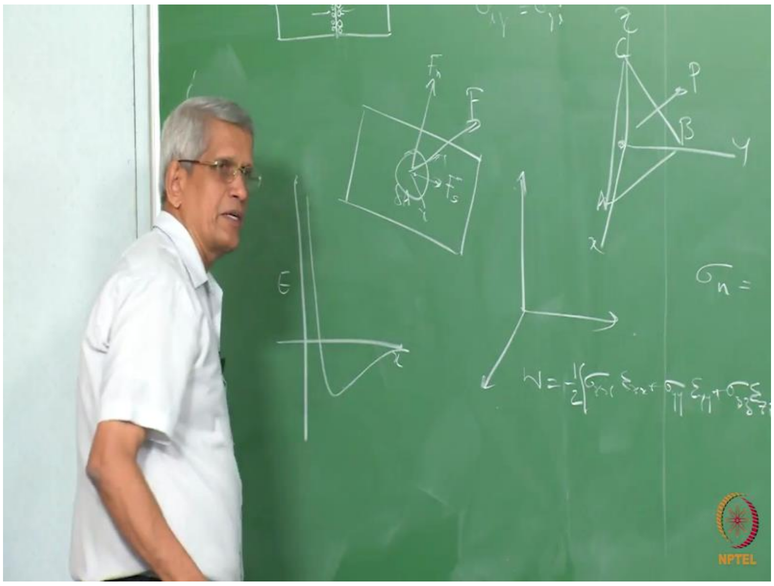

Potential or energy versus distance between atoms we are essentially drawing a graph like this, this is a non dimensional plot now if you look at the 3 dimensional plot that will have an effect on the crystal structure also into this now this if you try to do any separation in one direction.

Student: In the effect of them other.

In the other directions also which is going to be there how far it is going to be affected the how far the bonding is going; that means, bonding is going to be changed, if we take the secondary derive of q of this bonding, this is going to give you essentially this stiffness constants, what it is going to be and how much it is going to be affected in terms of just that contraction that is what the new represents new represents nothing, but equals suppose it is a epsilon yy by epsilon in that is if it is that x direction we are trying to explain expand that sample what is going to be the corresponding contraction that ratio is what we are defining it as new that is e itself could be represented in terms of lambda and mu and how lambda is defined in terms of mu and nu all the types of expressions are there.

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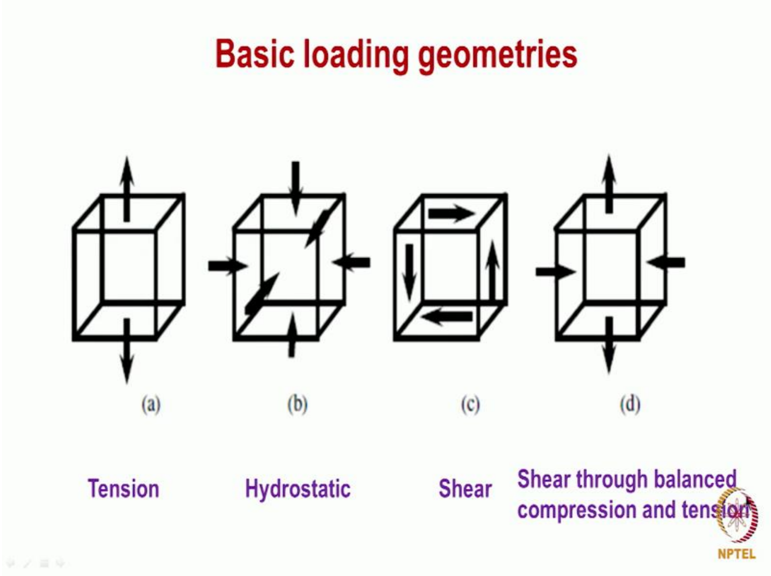

What are the basic loading geometries in which we can load the sample either in tension we can load by applying tension in one direction or we can apply a hydrostatic pressure and load the sample or we can apply just shear and do it or we can have a balanced strain that is we can have a shear by applying a balanced compression and tension that is what we showed in the last class when we discussed about that how we can do a diagonalization if a pure shear strain is being applied how that can be converted into principles strains along axis which are in plane with respect to it.

So, far we considered that the stress which is being applied is uniform throughout the sample homogenous stress, but that is not a will always take case stress can vary, but it does not vary independently because some forces which are acting here what we are trying to do an assumption is that what is the rate at which the stress varies depends upon the position that is we can take a derivative of it and find out how much is a variation per unit like that is how it is being defined.

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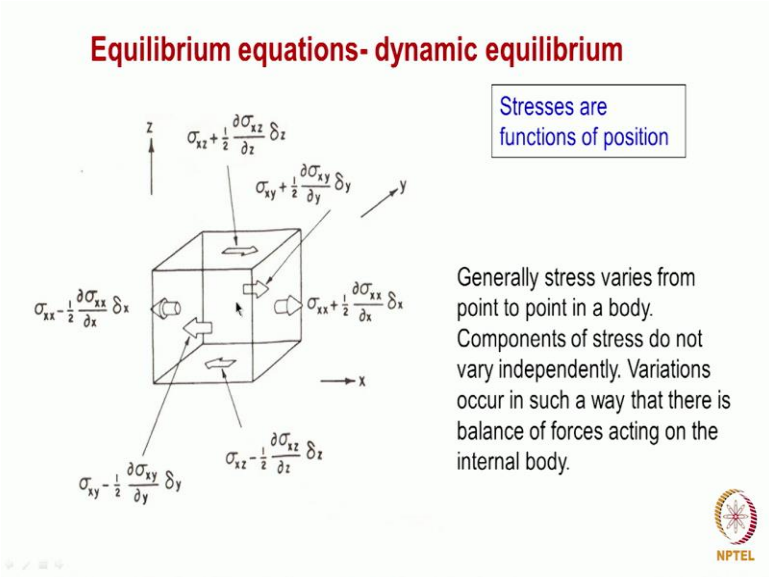

So, in this what is essentially is being done the origin is at the center of this cube if sigma xx is the stress in the x direction positive x direction then on this surface it is going to be half into and the dimensions of this cube is delta x delta y and delta z in xy in a direction then this is equal to half into delta x into dou sigma xx by dou xx, correct.

Similarly, on this one it is going to be a negative value that is sigma xx is one which is going to be there on the negative direction 2 b minus of x similar to that we can write it for the shear strains as well that is what essentially is being done in this diagram. So, if you have got these values in any particular direction what is it going to be the force which is going to act because finally, the force balance equation we have to look at it in this x direction this [diff /difference] difference this one will give you the what is the force going to be their normal force which is going to act in this interface, what is the area of this surface which is going to be there.

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$$\left[\sigma_{xx} + \frac{1}{2}\frac{\partial \sigma_{xx}}{\partial x}\delta x\right]\delta y \delta z - \left[\sigma_{xx} - \frac{1}{2}\frac{\partial \sigma_{xx}}{\partial x}\delta x\right]\delta y \delta z$$

$$+ \left[\sigma_{xy} + \frac{1}{2}\frac{\partial \sigma_{xy}}{\partial y}\delta y\right]\delta x \delta z - \left[\sigma_{xy} - \frac{1}{2}\frac{\partial \sigma_{xy}}{\partial y}\delta y\right]\delta x \delta z$$

$$+ \left[\sigma_{xz} + \frac{1}{2}\frac{\partial \sigma_{xz}}{\partial z}\delta z\right]\delta x \delta y - \left[\sigma_{xz} - \frac{1}{2}\frac{\partial \sigma_{xz}}{\partial z}\delta z\right]\delta x \delta y$$

$$= \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}\right)\delta x \delta y \delta z.$$

Total force in x -direction $= \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}\right)\delta x \delta y \delta z$

Is equal to this into delta y into delta z this area over which it is acting right because this is the x direction this is y and this is z. So, this stress this stress is going to act on an area which is delta y into delta z. So, that is what it is being done if you take the difference we will be getting the force which is going to act on the x direction from that is component of the force acting in the x direction from the normal stress similarly shear stresses are also going to act the shear stress is and this y surface if you consider it we have say stress which is going to act in this direction right similarly these the stress which is going to act in this particular direction.

So, that net that is going to be a force which is going to act on this y surface, but in the x direction similarly on that z surface also on the x direction there is going to be a net force which is going to act the sum total will give you the total force which is going to act, correct. This is being given by this expression.

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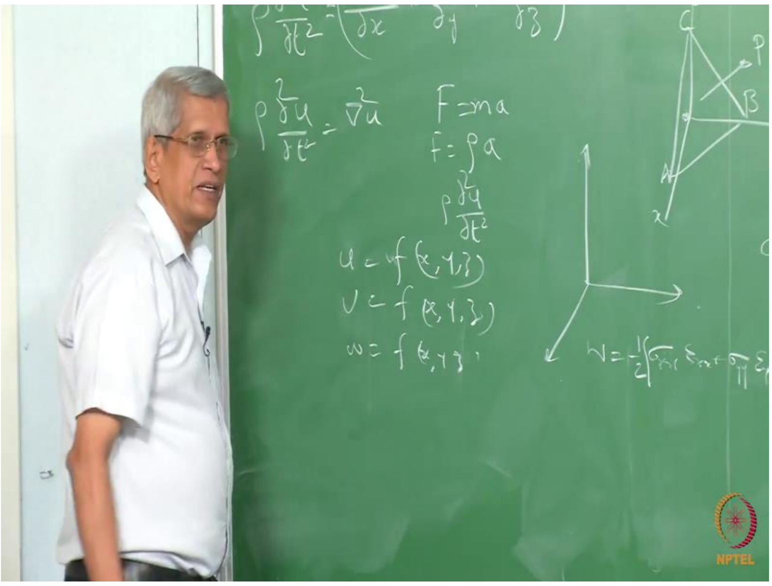

So, the total force which is acting in the x direction correct the delta z is the volume of this a q which we have considered this is equal to force which is acting on that surface in the x direction, correct. And if we divided by that volume that tells you what is their force? How do we represent it? Now; that means, that suppose this force is acting on a sample net force is existing.

Then how can we describe of force where Newton's law, we can describe this force is equal to mass into acceleration and if per unit density if you wanted to find out we can write this in terms of that unit volume then this will be rho into a, correct. So, here it will be total force (Refer Time: 59:30) when we divided force per unit volume in becomes the; so, this will become rho into in the x direction d squared d u by if the displacement is u.

That is this force acting gives out a displacement of u then we can find out what is the force write this expression in terms of rho into d square d u by d t square and this term will disappear. So, now, this is what an expression which we get it, correct.

This is nothing but a force balance equation that is displacement in the rate of change of displacement in the x direction depends upon the rate of change of stress. In the x direction suppose because we are considering a Cartesian coordinate, now the displacement in x direction we have got a formula.

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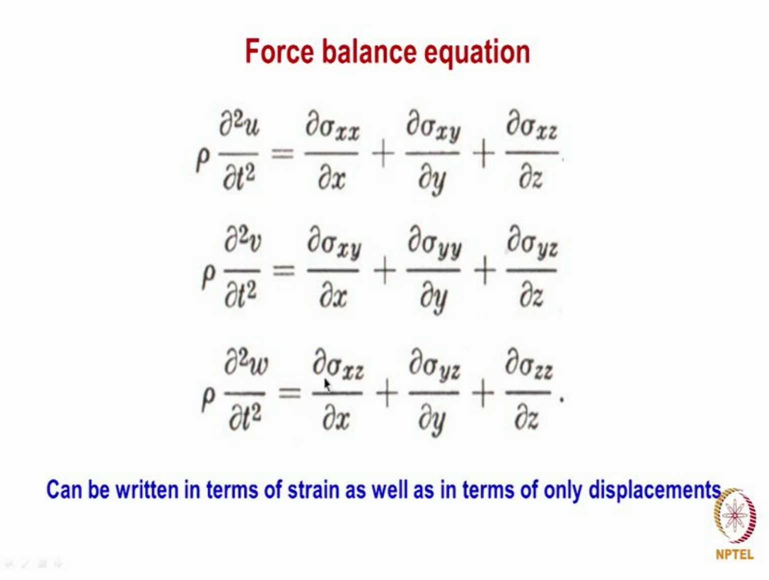

Similarly, we can write an expression for displacement in the y direction displacement in the z direction we can have a formula. So, this is most important formula this is the force balance equation what it will turn out to be is that we know that sigma xx can be returned in terms of sigma xx can be turn a find it in terms of strain.

Student: Strain.

And the strain can be written in terms of displacement.

Student: (Refer Time: 61:01).

Finally, this equation will actually turn out to be equals to u; this is how it will turn out to be; what is this equation? This is nothing, but a wave equation.

Student: (Refer Time: 61:16).

So, the solution to this equation this is for sound wave propagation. So, all the tress strain calculations within the body can be done by first finding out what is the displacement which is taking place because of some defect which has been introduced into the sample.

Then whatever is the displacement which we get it should satisfy this equation that find is the formula for the displacement in terms of the x and y satisfying this equation then from that expression we can find out that is it will be essentially nothing, but you will be a equals function of x, y and z similarly once this expression has got it we differentiate it we get the strain then if we then using the generalized Hooke's law, we can find out the stress. So, about the stress and strain distribution in that sample everything other the stress and strains could be calculated at every point and the displacement should satisfy this wave equation this is the most fundamental about calculating stress and strains in the material.

If one understands this then the rest of the job of finding out interaction between dislocations the stresses and strains around dislocations about various types of defects it can easily be calculated i have given this expression only in terms of stresses, but i would expect all of you to derive it in terms of displacement which is a most fundamental equation.

So, we have considered whatever the essential important thing that is the different components of that stress using these relations between stress and strain we can calculate the energy that if you try to deform a sample how what is the relationship between the relationship of energy with respect to stress and strain and from that we got information about how the number of independent components reduces from 36 to 21 and then the force balance equation which we considered and this is the one which is most important in this expression what is essentially important is that suppose the body is in a static equilibrium nothing is moving.

Student: Then moving.

Then the left hand side will become 0 then del squared u will become 0 that becomes the expression which one has to use it. So, we will stop it here now whatever is the basic mathematics which is required for deriving stress and strain field around dislocations around defects and energy of the defects that part of it we have completed now we will in that from the next class onwards we will start talking about because. So far we are not considered about the point defects we have considered about the line defects we have not started, we will have the class on line defects.

We will stop here now.