Defects in Materials

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Lecture - 14
Tensors – 2

Welcome you again to this class on Defects in Materials. In the last class we looked at coordinate transformation and how properties is related through 2 vectors; and that property we call it as a tensor of second rank. Now let us think of an experiment, then how to go about the property transformation from one coordinate to another that is what we have looked at it; what is the mathematical formulation which is necessary to change the second rank tensor from one coordinate system to another coordinate system.

Now, let us think of an experiment which we do to find out this property measurement, one thing which we can think of is between current density and an applied voltage to find out conductivity property of the material. And, as I mentioned these properties can be represented in terms of the principal coordinate system.

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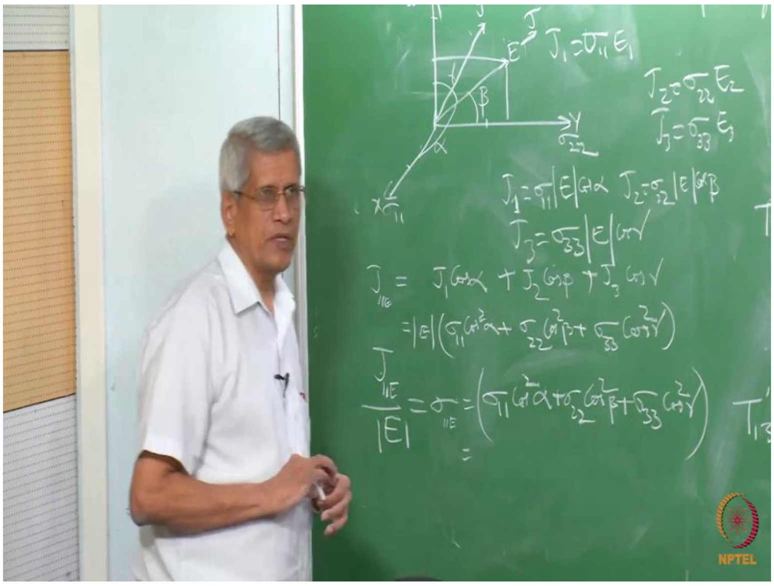

So, if you represent the property in terms of the principal coordinate system choose to represent it, we represent it they are applying a voltage, we apply that voltage is we can represent are this is equal to an electric field. E is the electric field which we applied this electric field will have a component, in this direction there is going to be a component in this direction a component is going to be there in this direction also right, which we can write it as E 1 E 2 E 3 correct.

if these are all the components and since along the principal axis we represent as far as the conductivity is concerned, this will be sigma 1 1 only these 3 remain correct; of diagonals elements are not there. Now we can find out the component of the current along each of these 3 directions, that what it will be. It will be j 1 will be there will be equal to sigma 1 1 into E 1, j 2 will be equal to sigma 2 2 into E 2, j 3 equal sigma 3 3 into E 3. What is E 1 is nothing but if you know the magnitude of E by the angle which it submits with respect to this is the alpha which it submits, with respect to x axis beta with respect to gamma with respect to this axis, then j 1 will be equal to sigma 1 1 into mod E into cos alpha, j 2 equal sigma 2 2 cos beta, and j 3 equal cos gamma correct which is right.

Now when I know these values of j 1; suppose we achieve that j 1 has got some value here, j 2 has got some value here, j 3 has got a value here then I can find out what is going to be the vector which is j; and that vector will have a different direction because that sigma 1 sigma 2 they are sigma 1 1 that conductivity tensor components are different correct? This is how we can write it; normal experiment what do we do we apply a voltage and measure the current in the same direction.

Suppose I wanted to find out the component of this, this what should be in this direction what is going to be in the direction of applied voltage, what is going to be the correct that is what we measure correct. How do we go about to do that? If you wanted to know that now j 1 is there what is the component of j 1 in the direction of E we have to find out; what that will be is equal to j 1 another cos alpha in a what is the value angle between them. The other is in the component of j 2 will be, j 2 cos beta and other is j 3 cos gamma correct. So, that is going to be the component of each of the current in the direction. So, what is going to be the total current I will write it j parallel to E, this is going to be equal to this is how it will turn out to be.

Now, when we substitute for j 1 j 2 and j 3 these terms, then what it is going to turn out to be? Sigma 1 1 into cos squared alpha, beta, plus gamma into modulus of E, now we have got that right. Now j 1 1 parallel t E divided by modulus of E is going to be the conductivity in that direction, I will put conductivity in the direction parallel to the elect applied voltage, that will be this clear. If you look at it the property we have expressed in terms of principal coordinates. So, only the components are well defined.

If we apply a voltage in any direction we can find out the current in that direction, and from that conductivity in that direction could be determined. So, in all the direction by varying alpha beta and gamma, we can vary the conductivity correct that is the way. So, this itself can be represented in terms of some expression x squared y squared plus e z squared and the coefficients are this is equal to some constant which the property which we measure, this is something like a equation to a it is a quadratic equation ok. When we will come back to this quadratic equation, the representation of the property, so this is how this equation can be represented. So, we have to do all these.

But, what is the other way in which we can represent it? That is if we are transforming from one coordinate system to another coordinate system, the conductivity has to change. Suppose these are all the conductivity which is defined in terms of the principal coordinate axis, suppose I wanted to find out the conductivity in this particular axis in that direction, what I can do? I can do an another coordinate I can choose, with that axis as the with principal axis with that as sigma 1 1 dash other 2 directions also can be there; if I try to find out what is going to be the value of sigma 1 dash in this direction that should be the same as this one for that.

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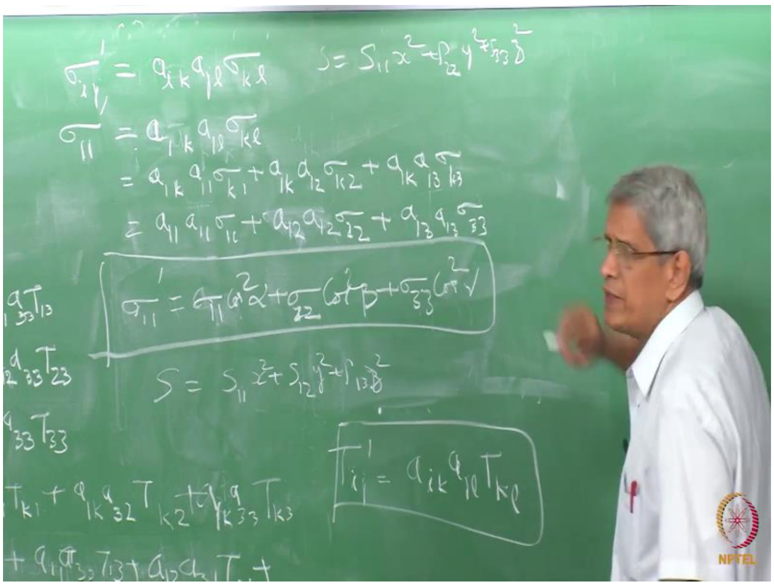

What we should do that is sigma i j dash equals a i k, a j l sigma k l this is how it is going to be correct; this is the general expression which we have written for a tensor transformation.

Student: (Refer Time: 09:27).

Because what we are trying to do is that even if you do not do the measurements you can get it; otherwise also if you know property in one direction, using the coordinate transformation we can find out the component of the property in any direction could be measured this way we can write it. If this is sigma 1 1, we take it, then what this will turn out to be 1 k no j.

Student: (Refer Time: 09:58).

A 1 k.

Student: (Refer Time: 10:00).

No this is 1 right j is also 1, a 1 l sigma k l correct this is hot what it will be. Now k and l we are going to vary. So now, what it will happen a 1 k suppose just we vary only l, a 1 l can be 1 correct sigma k 1 correct plus a 1, k a 1 2 sigma k 2 correct, plus a 1 k, a 1 3 sigma k 3 correct and this will turn out to be and now you vary 1 from this 1 what it will happen a 1 2 sigma 2 2.

Student: (Refer Time: 11:28).

Sigma 3 3 it will turn out to be correct that is how it will be there. Now if you look at this what all these terms a 1 1, a 1 2, a 1 3 it is nothing but the new axis the angle which it makes they are nothing but cos alpha cos beta.

Student: Squared.

Squared right; so now, this will just turn out to be that is sigma 1 1 dash equals sigma 1 1 cos squared alpha, that is essentially without doing all those steps we can transform from the property by doing a coordinate transformation using this tensor property for the coordinate transformation correct. That is what essentially this is very easy to do, in these components whatever this is a quadratic (Refer Time: 12:47). If we assume that it is from one coordinate to another coordinate system, which is not along the principal coordinates then what will happen each component will have the transform component will have in terms of the old one 9 terms will be there.

Student: (Refer Time: 13:08).

Will have 9 terms will be there, and each of the nine terms if you consider it 3 terms will be which involves square of the angles and the others will be essentially a cross for a the product of like x y, y z, x z this sort of terms will be there.

So essentially if we look at this equation, this equation is each of the component in the transformed one if you look at it, in we are representing it in terms of the old components it is a quadratic type of an equation which we get it. What is it which is changing? This only this angle which is changing correct, if you try to represent it in that way now we can draw the properties in terms of a surface which we can represent it, that will come to how exactly we are going to do that. How that can be done? That is this expression becomes something like equals that is I will because I am just writing it as a symmetric matrix because that is what we are using it here, to represent that along the principal axis then it is x squared, z squared correct this is the way it comes.

Now, to get the property what do we do? We do an experiment in this as we have seen with the last example, you just choose some direction with respect to a principal coordinates which has been chosen, vary the direction in which the voltage is being applied measure the current. Then in the property transformation choose that direction as in the new transformed coordinate as the x axis, component along the x direction. If you do that than using this sort of an expression, what we are trying to do it is in terms of the principal components of the old coordinate system, we will be able to find out what should be the conductivity of the property in each of those directions.

Now if we try to plot that, in terms of a surface then it will be given by an equation of this form. Here this x y and z represents nothing but the direction cosines which we are continuously varying right; that is the one on which because this is what we have done it is that we take these 2 be as one, 1 1 dash. So, that is all that new axis system we choose always. So, by measuring just this component taking it by varying this angle, the coordinates which you have chosen we are able to measure property in the full 2 pi solid angle correct. Complete angle we are making that measurement; if you try to plot that this equation is a quadratic equation. So, quadratic equation can be that if the coefficients become the same, what will this quadratic equation represent?

Student: (Refer Time: 17:03).

It will represent a sphere; if this all the components are positive then it is going to represent an ellipse; if one component becomes negative then it will go to a hyperbole. So, so many types of shapes the surfaces which it can exhibit depending upon what the coefficient of the property in those directions are the standard. So, this expression if you look at it, is a S 3 3 into e z square correct, what is the expression for a ellipsoid we write it x square by a square.

Student: (Refer Time: 17:56).

B squared plus 3 c squared equals 1; if you try to convert this expression to that form.

Student: (Refer Time: 18:07).

Ok, then if you do that what it will turn out to be is that, this derivation one should do it and this is just a simple mathematics algebra.

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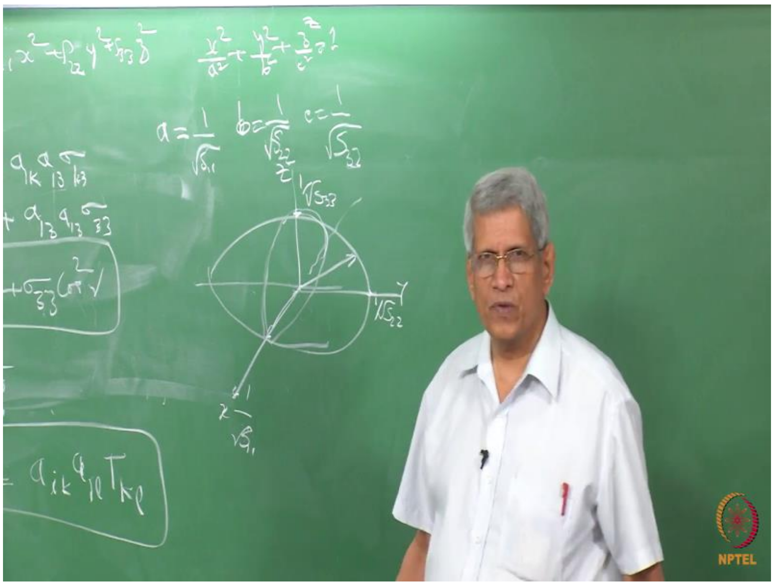

And we will get a will be equal to 1 by root of S 1 1, 1 by root of S 2 2, C will be equal to 1 by root of S 3 3. So, essentially you are getting and ellipsoid of this with x y and z and this is 1 by root S 1 1, this will be S 2 2, set direction is 1 by root S 3 3 correct this is how it is going to be or this is. So, once the surface is there, they show how the property is varying in all the direction. From this surface if you wanted to know the activity, any vector if you draw from here to here, any direction, we take that vector inverse of that vector, the square root of the inverse of that vector will represent the property in that direction right.

In this direction this is 1 by root of this 1 S 1 1 this essentially the property. So, if you take the inverse of that and square it we get the property. So, this vector what we measure it is inverse of that right do square root. So, this is essentially how we can get the property information.

Student: Sir, we have choose the axis along the principle (Refer Time: 20:18).

Axis which is chosen along the principal axis.

Student: And then vary the (Refer Time: 20:21).

Vary the direction only thing that the mathematical operation becomes simpler, but otherwise also this has to be actually; say essentially when you get an ellipsoid you can choose an any other axis also which you can choose it is right is it clear. So, this way we can construct doing a simple experiment of measuring the electrical conductivity or as we started with talking about measuring the strain, we can get information about what is going to be the total strain, which is going to be there in any particular direction. Any direction other than the principal direction if you take it, what will be the strain which will be there that will have a component corresponding to dilatation, plus some of the shear strains component in that direction will add to it that is what is going to be the total strain correct is it clear; that when we talk about we will write some expressions about the strain.

Now one more question which we have to look at it; that is here we have just talked about coordinate transformation from one coordinate to another coordinate, but we never talked about what will be the role of the symmetry, because crystals have got many symmetry which is associated with it right and different directions they exhibit some different symmetry elements, what will be the effect of the symmetry elements on that property, that is also one aspect which you have to look at it.

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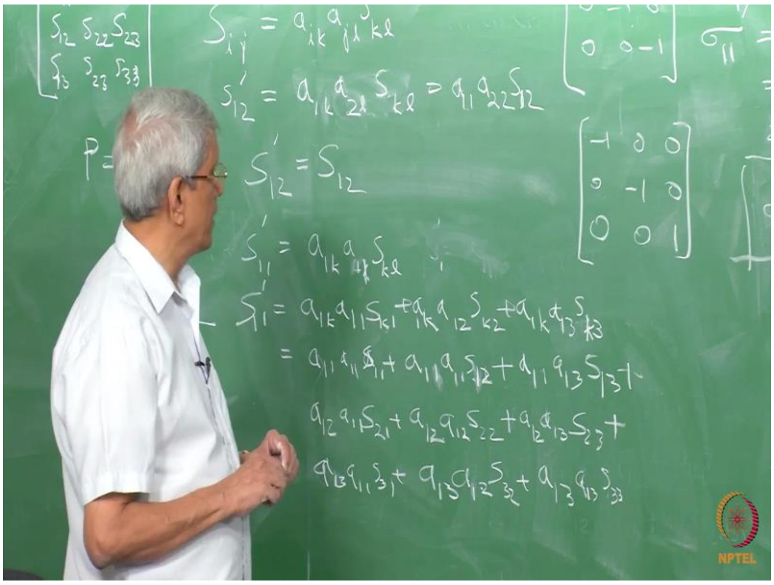

So, generally your symmetric tensor when we represent it; so essentially that is 6 independent components are going to be there for a symmetric tensor, correct suppose there is no symmetry which is associated with the material, then what is going to happen if there all the components are going to exist for the properties correct. Now the question which comes is that, suppose in some direction there is some symmetry element that the simplest which we can think of is that suppose there is a center of symmetry is existing in that, what will be the effect of the center of symmetry existing in the material? Whenever the center of symmetry is existing in that material from one direction to other direction if you look at it, the property has to be there correct it has to be the same.

Student: (Refer Time: 23:33).

Same it has to be there, but in actual crystal systems in many systems if you look at it, many crystals are there without center of symmetry, but when we measure property always property shows a center of symmetry. Then how this expression; that means, that what we have to do it is that, if you look at the that transformation from one coordinate to an another coordinate, from new coordinates to a old coordinates or from the old coordinate to a new coordinate when we do this sort of a transformation, how is the property going to change that is what we have to see it.

This can be written in terms of S k l correct; whenever a transformation takes place when we say that coordinate transformation there is a center of symmetry of a property is going to be there, what will be the coordinates which will come for that the transformation matrix. The transformation matrix is essentially will be essentially, this is for the transformation matrix will be to go from one coordinate to the other right so; that means, that a x y z will be changed to x dash y dash is a which is negative.

This what is going to happen with respect to the vector also, is that the vector will be changed from that is we generally write this property, that is P equals t into Q if it is symmetric we can write it S into Q correct. Now your P changes to minus Q also will be changing with the transfer minus 1 that remains the same. Now what is essentially is that what happens to yes. So, we can take that to be another, this one coordinate and find out S dash. Now what will be the S dash which will turn out to be, suppose we take it to be 1 and 2, a 1 1 a 2 2 this will be S 1 2 right no let us let just write it.

Student: A 1 k.

A 1 k.

Student: (Refer Time: 26:20).

A 2 l, S k l, k and l becomes 1 and 2. So, this will be a 1 1 right a 2 2, yes.

Student: 1 2.

1 2 correct; now what is the value of a 1 1 and a 2 2 both are turning out to be minus 1 and minus 1, if you put that substitute it then it will happen that S 1 2 dash equals S 1 2; that means, that the property remains the same. By doing a symmetry operation the property cannot change the property has to be that same.

Student: (Refer Time: 27:31).

The property remains, this is with respect to a center of symmetry because all properties have an inversion symmetry this was proposed may 1 (Refer Time: 27:44) this called a Normans principle for second rank tensors. So, essentially will have the property will exhibit the center of symmetry. So, now, we have considered it.

Now let us consider a crystal where it has along one axis, let us along the e z axis we consider that there is a 2 fold symmetry is there; then what it will happen.

Student: 1 0.

Minus 1 0 0.

Student: 0.

0, no; that is around the e z axis we are doing a rotation. So, x and y will change, correct 2 fold rotation is around this axis if minus 1 0, 0 0 1. In this also now you should try to apply the same principle and find out the coordinates, suppose we found out S 1 dash what it will become.

Student: (Refer Time: 29:10).

A 1 k, a 1 j then a 1 l k l correct k, a 1, l is 1 right plus k 1 plus a, k 1 a, 1 k a 1 2, S k 2 plus a 1 k, a 1 3, S k 3 correct. Now k we vary from this what it will become 1 2 right S 1 1 plus a 1 1, a 1 3, S 1 3 it will be right this is going to be k 3, then plus a 1 2, a 1 1 right.

Student: S 2.

S 2 1 plus a.

Student: A 1 2.

1 2, a 1 plus.

Student: A 1.

1.

Student: 2.

Student: A 1.

A 1.

Student: S 2 3.

S 2 3.

Student: A 1 (Refer Time: 31:19) S 3.

S 3 1 plus a 1 3.

Student: A 1 2.

2 a 2 1 what is.

Student: (Refer Time: 31:33).

A 1 2 correct, here also it should be a 1 2; yes.

Student: (Refer Time: 31:42).

3 2 plus.

Student: A 1.

A 1.

Student: 3.

3.

Student: A 1 3.

A 1 3 in this is what are terms which are going to be there.

This 1 is going to be there correct, then if we then any other term is going to be there.

Student: (Refer Time: 32:24).

All other have you kept an a 2 2 here.

Student: No.

So, only this term is the one which is going to remain here correct out of this term.

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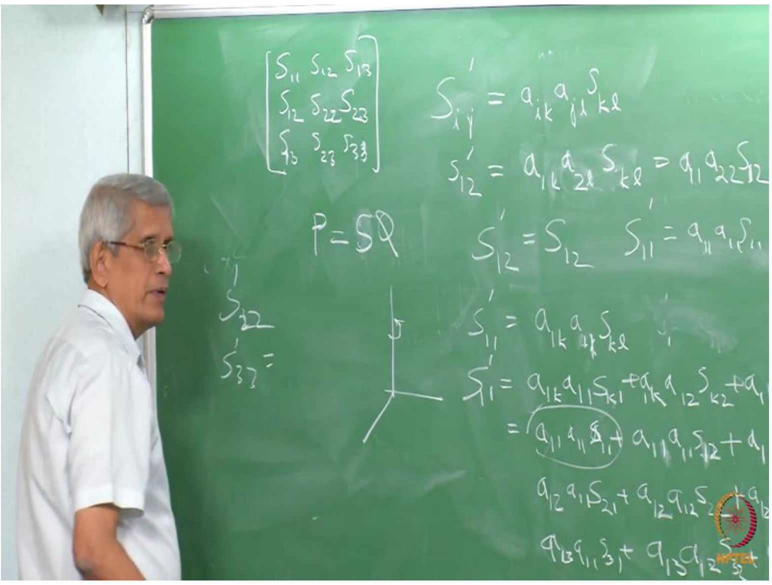

So, S 1 1 dash equals a 1 1, a 1 1, S 1 1 this what it turns out to be correct. In this when we substitute what it will turn out to be again 1 so it is symmetric it is possible correct. If you write for other term like S dash 2 2, and similarly S dash 3 3, we will be getting similar type of a result which will be coming correct.

Student: (Refer Time: 33:29).

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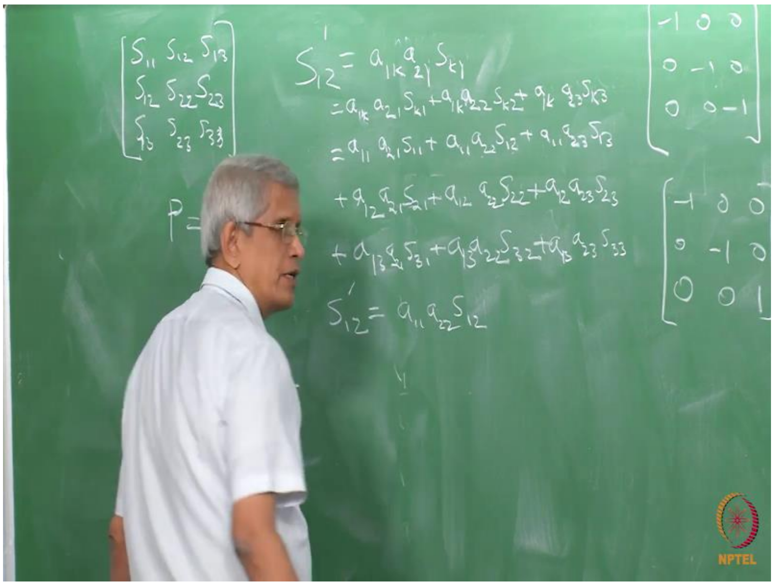

Now let us consider S 1 2, S 1 2 dash will be equal to a 1 k, a 1 a 2 j correct S k j this will be equal to a 1 k, a 2 1, S k this will be 1 correct plus a 1 k.

Student: (Refer Time: 34:15).

2 S k 2 plus.

Student: (Refer Time: 34:20).

K a.

Student: 3.

2 3 k 3 now this is if we substitute k equals 1 right.

Student: (Refer Time: 34:35).

A 1 1, a 2 1, S 1 1 plus a 1 1, a 2 2, S 1 2 correct plus a 1 1, a 2 3, S 1 3 plus 1 2, a 2 1, S 2 1 plus.

Student: (Refer Time: 35:02).

A 1 2 right.

Student: A 2 2.

A 2 2, S 2 2 plus a.

Student: 1 2.

1 2, a 2 3, S 2 3, a 1 3, a 2 1.

Student: S 3 1.

S 3 1 plus a 2 2.

Student: A 1 3.

A 2 3.

Student: S 3.

S 3 3; in this if you see what all the terms which are going to remain; these teams contain a 1 1, a 2 2, S 1 2 is there what about the other terms.

Student: (Refer Time: 36:02).

All other terms are cross product terms 1 2 or 1 3 they will become 0 correct. So, S 1 2 dash becomes.

Student: A 1 1.

A 2 2, S 1 2 this if you take the product this becomes symmetry S 1 dash equals S 1 2, so this possible.

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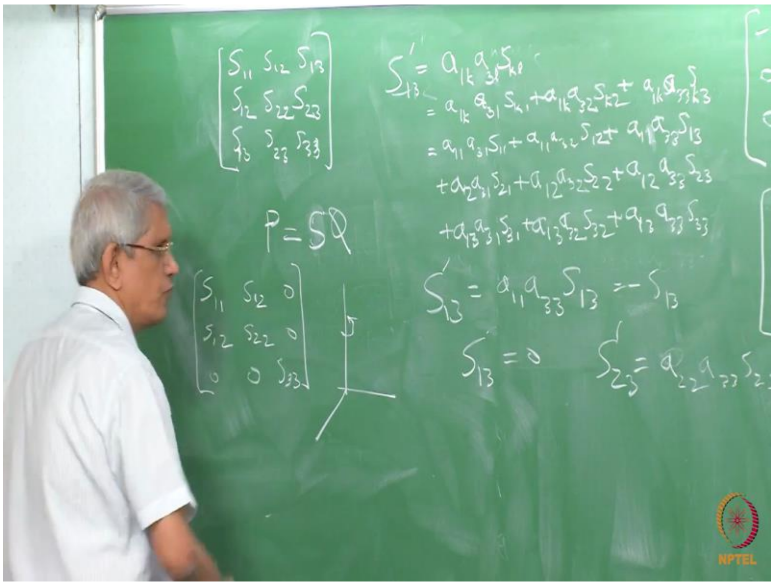

Now, let us look at the case S 1 3 dash what will happen? A1 k, a 3 l, S k l correct this will be a 1 k, a 3 1, S k 1, a 1 k, a 3 2, S k 2, a 1 k, a 3 3, S k 3 correct.

Student: (Refer Time: 37:26).

A.

Student: (Refer Time: 37:28).

1 1.

Student: A 3 1.

1.

Student: S 1.

1 1.

Student: (Refer Time: 37:39).

A 3 2, S 1 2 plus a 1 3, a 3 3.

Student: S 1 3.

A.

Student: 2 1 a 3 2 S 2.

2 correct.

Student: A 1 2.

3.

Student: S 2 3.

2 3, 1 3 a 3 1, S 3 1 plus a 1 3, a.

Student: 3 2.

3 2.

Student: S 3 2 S (Refer Time: 38:26).

3 2 plus a 1 3, a 3 3, S 3 3; now this term is essentially a 1 1 correct is the only term.

Student: (Refer Time: 38:40).

So S 1 3 dash turns out to be all other terms are 0, a 1 1 into a 3 3 into S 1 3. If you take this product this turns out to be minus S 1 3, that is we have measured the property by taking a crystal along 1 direction S 1 3 direction, we have given a 2 fold rotation and measuring the property, but by symmetry we means that the property has to be symmetric, but what we get it is that the property turns out to be negative, can the property become negative.

Student: (Refer Time: 39:32).

Right that is we are measuring a property in one direction and then another direction which is 180 degree, where a symmetric related we measure the property the property cannot become negative right property has to be the same. What is the condition under which this sort of a thing can happen only if property in that direction that component turns out to be 0, there is only condition where this equation will be satisfied so; that means, that S 1 3 becomes 0.

Similarly S 2 3 if you take it, that will also turn out to be a similarly here the same S 2 3 what it will happen S 2 3 dash will be equal to a 2 2, a 3 3, S 2 3 it will turn out to be. This will also just cannot exceed so; that means, that a crystal which has got a 2 fold symmetry if it exists then the property. So, then what will be the number of independent components now what it will turn out to be S 1 1, S 1 2.

Student: (Refer Time: 40:57).

Three 4 independent components correct are only required to represent this property, is it clear. So, this way they are applying the symmetric consideration what exactly we are trying to do it. We are taking a crystal measuring a property along a direction along chosen the with the crystallographic direction, when then when it has a 2 fold symmetry around that either you rotate the crystal, are you just a rotate the axis that is you measure the longer direction which is 180 degree, both is equivalent correct you do that, but then you find that the property has to become negative that just cannot happen; that means, that a component of the property has to be 0 in that direction.

So the number of independent components which you require gets reduced. Now we can do it for a fourfold symmetry what will happen, we can do it for a 6 fold symmetry or a threefold symmetry like this. This I would like you people to work as an exercise and find out how the number of independent components will change. In the case of cubic crystals essentially what will happen is that all the components only one in component will be required this is all for a second rank tensor, the number of independent components which is required for the property is essentially one and that is that same throughout it will exhibit symmetric properties for a cubic material.

Student: (Refer Time: 42:42) something like.

That is why when you look for electrical conductivity and all this property of a material which is a cube you do not need, but all other materials which have less symmetry there the property will change with respect to direction, that information we can get it directly from this analysis. So, essentially what all we have covered is a coordinate transformation, how especially for second rank tensor or where the properties which we measure that is the input and the output signals are represented in terms of vectors, and these all the vectors and then they are related by a property when the property is a second rank tensor, how the second rank tensor itself what is the transformation which will take place the coordinate transformation associated with it.

That easy way in which we can write that component are in that transform matrix, and then what we have looked at it is that if it is a symmetric tensor, and if we represent it along the principle axis, then how this property surface will look like and then if symmetry is associated with the system, how the symmetry affects the number of independent components which are required to represent the property in the crystal.

And since we are dealing with defects in material especially in metallic materials now we look at it, we have to look at with respect to the properties which relate stress and strain. Stress and strain as you know they are all tensors of second rank so; that means, that each of them can have components nine components are going to be there, then we will require a property matrix which will contain essentially 81 components, but that looks a large number of components and whether the properties are symmetric or not, all brings down to the number of independent components to only 21.

In the case where there is no symmetry element exists in the crystal. We will just briefly look at this in the next class we will stop here.