Defects in Materials

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Lecture - 13
Tensor – 1

Welcome you all to this course on Defects in Material; in the last class we have talked about coordinate transformation and lattice correspondence, why do we require this? As I mentioned that quite often especially in some of the experiments which we do, we will be measuring properties of the sample with respect to some coordinate system which corresponds to the sample coordinate system, but the property inherently may be decided by a coordinate system which corresponds to that of the materials for example, if you measure trying to find out a texture in a material.

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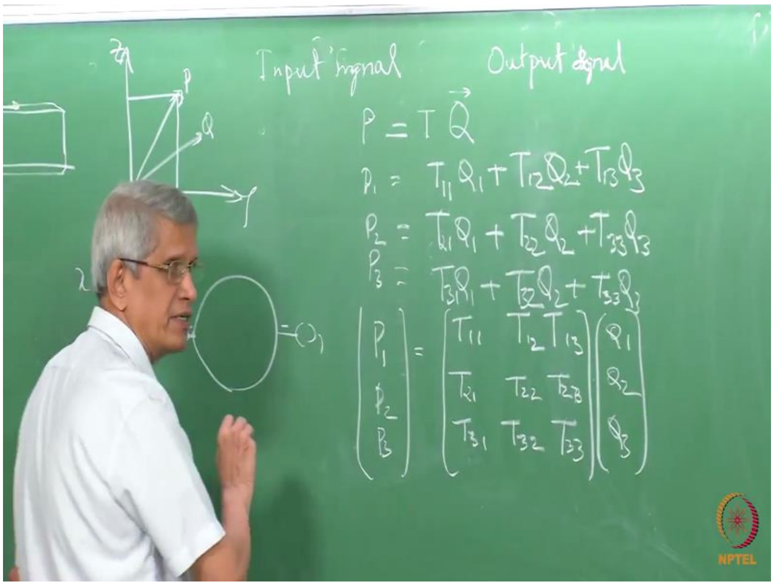

So, when we do a texture what we are trying to do we deform a sample and then we say that with respect to the sample, we decide that these are all the coordinates that is this is the rolling direction, and this is the normal to the rolling direction and then there is the third direction that thickness which comes into the picture. So, this is how we try to say with respect to these 3 directions we define that axis. But Crystallographically if you look at it the orientation could be something else which is different.

So, how the material behaves depends upon the crystallographic orientation. So, often we will have to go from one coordinate system to the other coordinate system, to describe the properties. So, if you see with respect to an external coordinate system the sample coordinate system depending upon the orientation in which the sample has been cut, the properties may be different.

So, if you have to interpret the properties we should understand the property with respect to some coordinate system, which is essentially the crystallographic coil axis of the sample and then we can try to represent the property in terms of the coordinate system which we are measuring, then interpretation becomes easy and we can have a better understanding of the properties of the material, I use the word the property.

So, now the question which comes is that, is the property independent of direction in the materials or could the property also dependent upon different directions. If the property is independent of the material then it is an isotropic system, and most of the material the properties depend on directions. When we say it depends on direction then what is it which is important to represent directions, we have to choose a coordinate system and then tell that how the property changes. So that means, that we have to choose an axis system and with respect to this axis system we should tell how this property is begin correct that is what is very much necessary. That is that information if we have how the property is varying and this property can be we can transform from the system in which Crystallographically it is the crystallographic axis system with respect to it if you define, any other axis system we can find out the transformation of the properties.

So, this property when we describe in terms of this coordinate system, this is called as a tensor or a tensor is a property material. Now let us come back to the actual experiments. So, essentially all these things which we do this mathematical operation of tensors is essentially to get information about the property; when we do any experiment what do we do? We just give some input signal, then we collect the output signal, this could be a simple thing like you apply a input initially a voltage, the output signal which you collect it is a current density then what does this information that relationship between these 2 tell, is what the resistivity or a conductivity of the material that information.

So, conductivity is the property the information which we will get about is the by doing this experiment, giving an input signal and getting an output signal we are getting information about the property. This happens in almost all experiments like for example, when we do an x ray diffraction what do we do? We put essentially incident X-ray in some particular direction, and try to find out the X-ray which is scattered in various directions, that gives information about the scattering behavior of this material that is a property of that sample. Similarly the other experiment is which I will come to shortly is about the in a metallurgy about mechanical behavior, we try to find out the strain in the material. In this input and output signal when we consider with respect to the sample coordinate system this input is may be given in some specific direction, so it could be a vector, we are measuring the output in a another direction so that is also when it is direction specified it becomes a vector. So, the property which we measure is also is the one which relates between 2 vectors.

So, if you measure property between 2 vectors, this property in terms of the coordinate system when we represent it requires some components along different directions the property has to be known. So, how many independent components which will be requiring is which will come to know shortly it is about 9 that is because when we define input signal as a vector, it could be I can just as a general form I can write it as P, the output which I am measuring it is an another vector Q, then this has to be related through some property which I write it as T right. Suppose P has components P 1, P 2, P 3 then Q also has component vector Q 1, Q 2, Q 3 because when we define a vector any vector if we take like this is P and this is Q, this P can be defined in terms of this coordinate system component along the x axis component along the y axis, and the component along the z axis. So, that is what P 1, P 2, P 3 similarly Q also can be defined that way.

So, if this has to be a contents, then this will be in terms of the property if you look at it essentially this can be written in the form of T 1 1 into Q 1, T 1 3 into Q 3. So, essentially what we have done the component of the vector P is measured in terms of Q 1 Q 2 and Q 3 which are components of a, in that specific direction what is the property that property we define as T 1 1, T 1 2, T 1 3 correct, that is with respect to that P 1 similarly we can write it for all. So, this itself can be written sin the form of a matrix; in this way in which we can represent it, but what is important is that Q is a vector and P is also a vector right.

So, let us come to this little bit later now we talk about doing an experiment to measure the property. Suppose we do an experiment for example, we are trying to measure the conductivity of a material, we apply a particular voltage.

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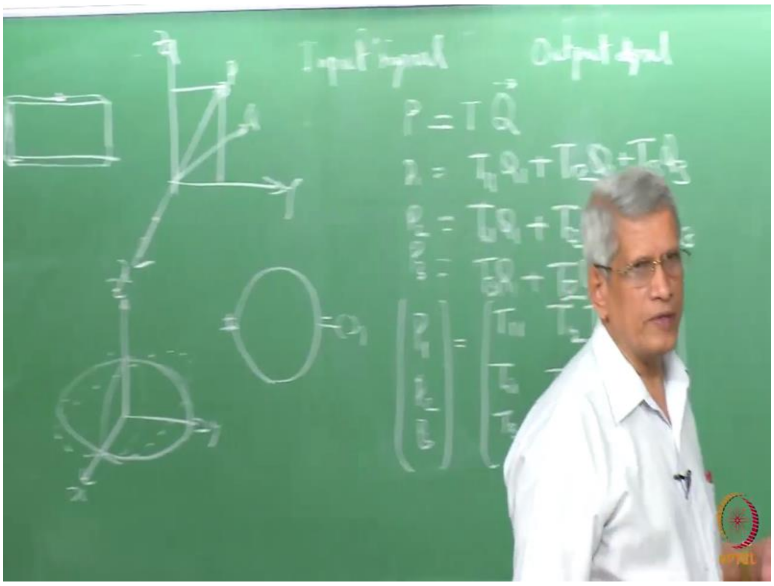

We take a sample which is a spherical sample, you apply a voltage in this direction and measure the current which is flowing through this in this particular one, we have taken a sphere of a material maybe this sample is a if you assume to be their single crystal.

Now, I just fix a coordinate system with 3 axis, and with respect to which I am applying a voltage and measuring the current, and then is just go on changing the direction in which the voltage is being applied; you assumed that I apply the same voltage and measure the current density which is in that direction. So, if I measure it now I can plot and each direction where a voltage is applied I can measure the current.

So, in that direction what is the actual resistance, from that I can find out the conductivity and I can plot it with respect to a coordinate system which I have. If I try to plot this value, there are many options which are there what all the ways in which this can happen and I try to plot it. One this can try to be the conductivity itself can try to be an ellipsoid like this with the maximum value is coming conductivity coming along the axis or it can so happen that it can be an ellipsoid which is in this direction with respect to what because we do not know what the depending upon the results all possibilities are there, these are all the various ways in which the conductivity can come.

A similar experiment which we can think of another is, when we do a mechanical behavior that is deformation of a material. Suppose we take a single crystal of a material and take a wire of that sample of a particular length, and in different direction the particular length in a different direction we have made a material, now we apply the same load to that sample and try to measure what is going to be the elongation in each of the direction, then we will find that in different directions and the load which we are applying so small that the property which we measure in the case of an elastic material it is within the elastic region are in all this property measurement, what we assume is that the property remains, the same does not change by the application of any of the external signal onto the sample. Suppose it happens that if you apply a very large voltage, the raise in temperature can happen then the properties can change, or the material itself can crack there are so many things. So, what we do it is that we apply signal of such strength, that it does not alter the property of the material and that is what we are trying to find out in each of this case.

So, if apply your load which is very small we can find out what is going to be the elongation in each of the direction. And then if we try to when we know that elongation we can either plug displacement, versus the direction with respect to which the load is applied, or we can plot strain with respect to the direction; both are possible that will give rise to some sort of an ellipsoid we will be getting its similar to what we have observed in this particular case.

Now, the whole question is that, what we have measured in this case is essentially an extension which is a displacement, and the position with respect to it we are changing it. So, essentially 2 vectors which we are trying to relate it use and the property is the strain which comes out, that is why we called strain as a tensor and 2 vectors then we call it as a second rank tensor.

That means, that in this one to relate each component we should have 3 components of the independent properties in 3 directions in which the signal is being applied, that is when we apply signal it has 3 components with respect to a coordinate system which you have chosen, with respect to that we should know the properties so; that means, that since the output signal is also a vector it has 3 components, totally 9 components will be there for a second rank tensor; is it clear.

Now, we have just talked about the property of the material, now we mentioned that in many a time we require the coordinate transformation which will be required to measure the properties.

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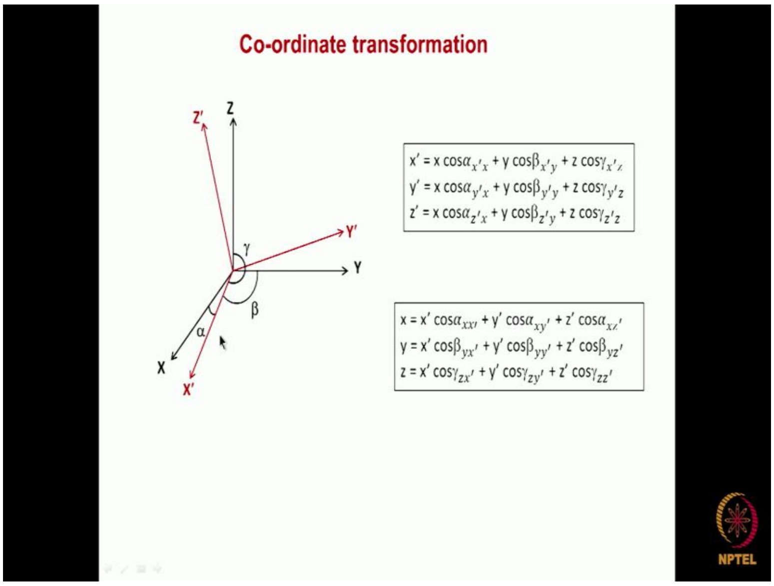

Suppose we assume that this is the sort of a coordinate system which you are choosing it and the black line represents the old coordinate system x y z, and the red line represents the new coordinate system. If then if you wanted to transform from one coordinate system to another, what we should know is that with respect to x dash coordinate if you take it for example, what is going to be the its component essentially its component in the x axis, y axis and the z axis of the old coordinate system if we know that information, then we can represent this coordinate. So, this to represent this is what we require is nothing, but that when we talk about the component of x dash in x, it means that projection of x dash in P x correct. So, we should know the direction cosines which we should know then we can write an expression like this.

This I mentioned the last class, from this expression we can get a relationship between x dash, y dash, z dash and x y z correct this is what we can get that information.

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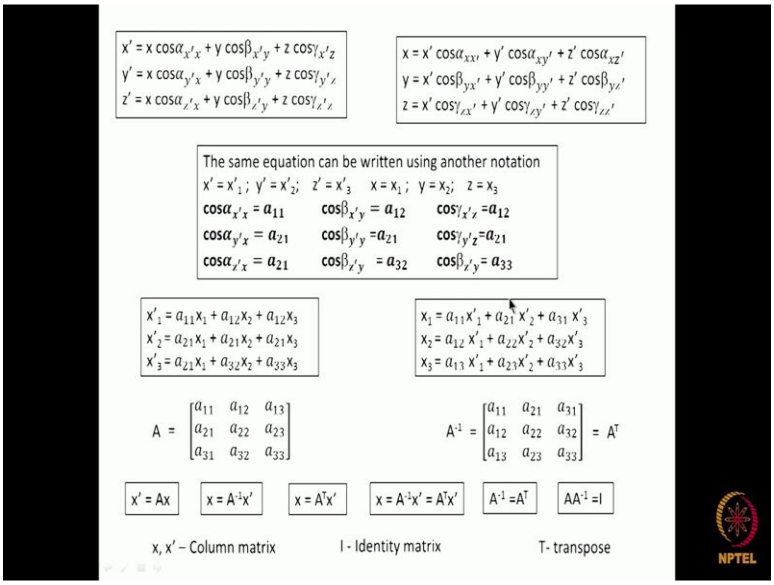

Instead of mentioning these angles as cos alpha it requires a lot of space and we have to remember the notation, another simple method as I mentioned in the last class we can write in terms of a 1 a 2 a 3 right.

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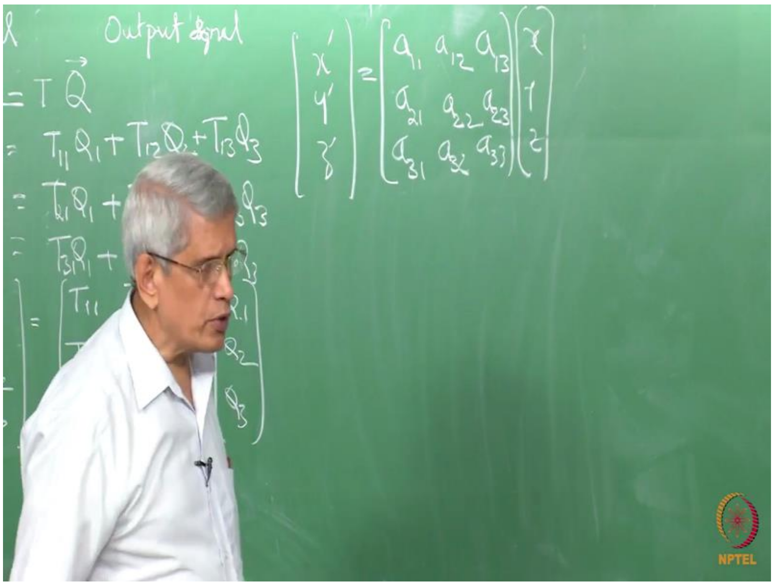

If you do that this z dash can be written in this format. So, you can refer to the last class what I had mentioned about this relationship between the old coordinate and a new coordinate system that way we can represent them.

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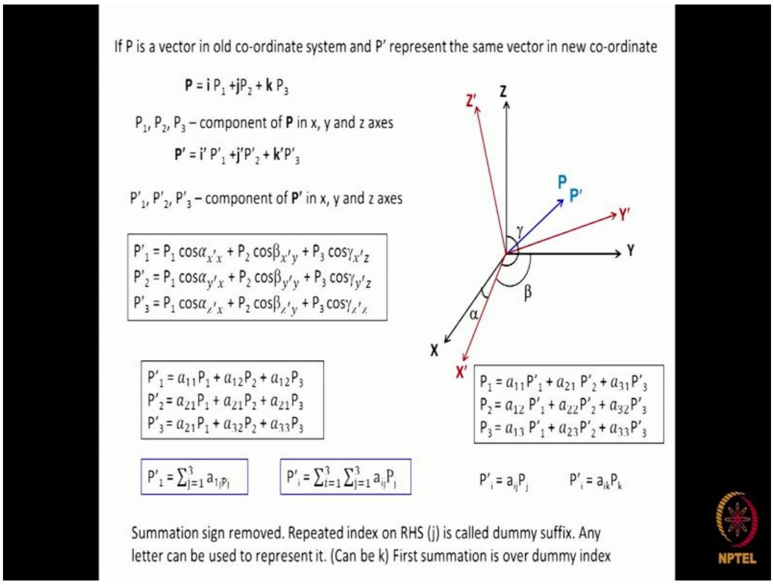

Suppose now we have a vector which we define like this the vector p; that means, that if you do your transformation, the vector P is defined with respect to one coordinate system the input signal. So, when we are trying to do with respect on another coordinate system then not only the properties, but the vectors also will change their components because the magnitude of the vector will remain the same because this only a rotation which is being implemented. But what is essentially is going to happen is that the components of the vector P with respect to old and the new coordinate system will change.

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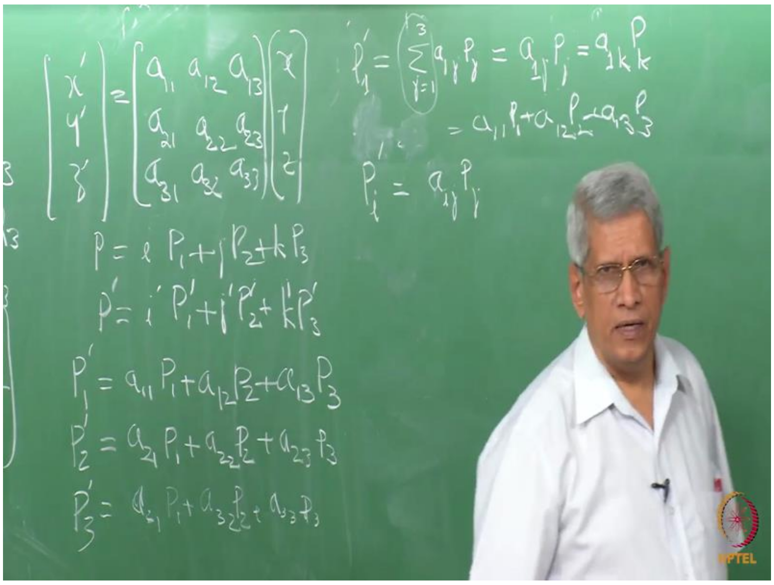

If you represent this in terms of the new and the old coordinate system, suppose we represent P as is equal to I into j into P 2, plus k into P 3 and with respect to a new coordinate system i dash P 1 dash, j dash, P 2 dash plus k dash, P 3 dash correct this is how the same vector is being represented. What will be the relationship between these 2 coordinate system; using this transformation of matrices we can get that, I will just write only the this cell that is P 1 dash will be equal to a 1 1, plus a 1 3 into P 3, P 2 dash equals a 2 1 into P 1, P 3 dash equals a 3 1 into P 1, a 3 2 into P 2, plus a 3 3 into P 3 this can be written in a matrix format also.

What is the way in which this can be represented? One is that this equation itself is we have to write all the components right; is it what is the short form in which we can write it? P 1 dash will be written as equal to sigma j equals 1 to 3, a 1 j P j, I can write it this form correct; this clear in this form also it can be written. If I write it in this one, suppose all the 3 come here the component P 1 P 2 P 3 it is varying if I have to represent this also, this I use an another suffix i. So, then I will write P i dash. I have to write it that will be equal to no just before that let us see that, it can be written in this form correct. What is the Einstein convention he did was that, when we have to write expressions in this particular format what is the method in which we can simplify it further.

So, he said that whichever is the suffix which is repeating, that you call it as the dummy suffix. This itself is inherently we can write it as assumption that this itself is equal to a 1 j P j, that is when these 2 suffixes we put it and when that suffix repeat in a term, it implies that there is a summation over that. So, we do not have to put this summation symbol may not be mentioned at all, we do not have to explicitly give put that symbol write that symbol. Then another is this summation symbol is nothing, but the 1 which only shows that something which is getting repeated.

So, there is no need that you can do it any symbol this itself can be written as instead of it, I can write as well as a 1 k P k also I write can write it; because when I expand it what it happens k will be from 1 2 3. So, this will become finally, a 1 1, P 1 plus a 1 2, P 2 plus a 1 3, P 3 right whether I use j or k does not matter. So, that is why we called this as a dummy suffix, is this clear? And then this expression can be written as the; this also the suffix is changing.

So, we will write it as P i dash can be written as a i j P j, this form we can write it right? Then the expression for the components of the vector is completely different. So, in a very short form 3 equations involving 9 components can be described using this one single equation correct? This is essentially what is being followed in many of this tensor mathematics. If you understand this then most of the work involving tensor algebra becomes much simpler; is this clear correct. If this is clear now let us come back to the properties with respect to experiment which we are doing it, what I will do it is now I will just clear this. So, that we have space for writing or P or this can be you can write it as j does not matter.

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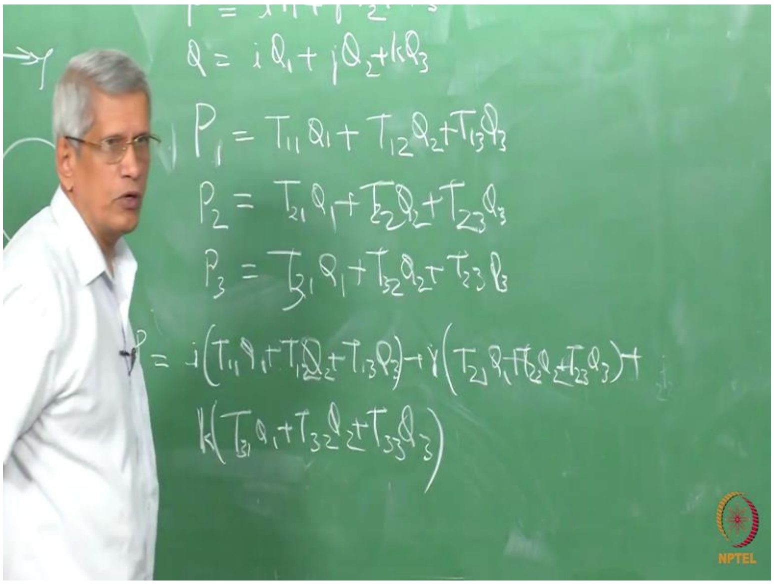

So, these are all the components correct Q equals i into Q 1, plus j into Q 2, plus k into Q 3 correct. So, what does P 1 represent essentially the component in the direction i. This now how will we write this in the form of a vector? So, we have to write it in from top with the components like this way we can write it so; that means, that if we substitute this then what it will happen? The vector P will be equal to I into T 1 1 into Q 1, Q 2 plus T 1 3 into Q 3 correct plus j into the P 2's component T 2 1 into Q 1, plus T 2 2 into Q 2 plus T 2 3 into Q 3, plus k into T 3 1 into Q 1, plus T 3 2 into Q 2 plus T 3 3 into Q 3 correct this is how it will turn out to be.

Now, we know that Q is also a vector right, suppose we wanted to write it in terms of that then what is the way in which this could be written.

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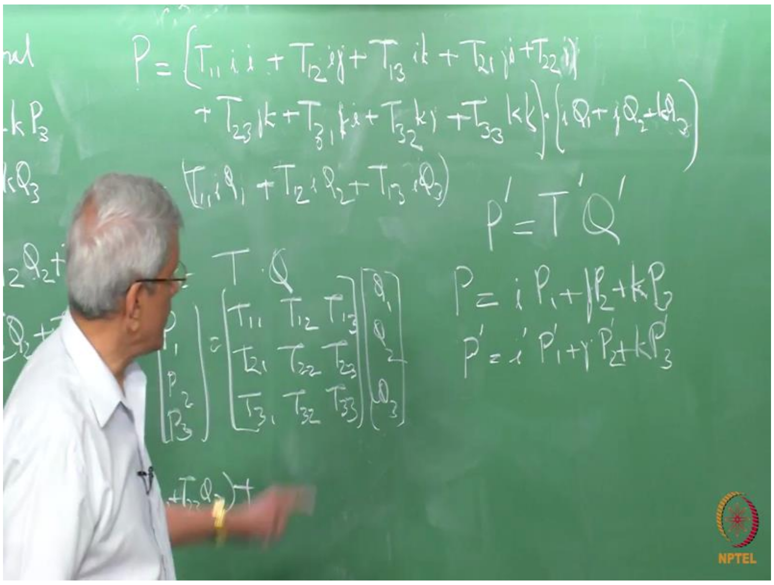

One way in which it could be written is equal to T 1 2 into i j, plus T 1 3 into i k plus T 2 1 into j i, plus T 2 2 into j j here, plus T 2 3 into j k, plus T 3 1 into k i, T 3 2 into k j plus T 3 3 into k k into a dot I can write I into Q 1 plus I into j into Q 2, plus k into Q 3 this form I can write it. Here instead if you take the dot product like a normal way to take these 2 i dot i what it will turn out to be 1. So, this will become T 1 1 into i into Q 1 similarly when we write it finally, we will be ending up in a expression of this type; see you see that i into Q 1 right, T into this into i into Q 2 correct T into i into.

Q 3; so if you take i out this becomes like a term like this.

Student: (Refer Time: 31:56).

Which one? No this is P is a vector, P has a component P 1, P 2, P 3 these are all the components of P 1, P 2, P 3 in terms of Q. Now I am just P can be written as a vector I into we can substitute it that is what we have done that, so the vector P is defined. But Q 1, Q 2, Q 3 is also a vector suppose we wanted to write in terms of the relationship between the 2 vectors how do we write? So, what I have taken it is that here we are just putting it that this is 2 unit vectors I into I, then if you write it in terms of that and then put a dot then I dot I will turn out to be 1. So, what we will be ending up with will be the first term will be like this. So, if we take I out it become similar to this one correct.

So, what we have now done which we have written this as an operator, that is P is equal to an operator T it is like a dot product, this is between an operator and this will Q. This way writing it is a laborious and tedious what is the way in which we can write it that is this itself can be written in terms of the component, that is why we write it in the form that if it is P 1, P 2, P 3; this is only for the components which we are writing it right. This is that actual detailed way in which if the vector has to be written we have to write it this way. Suppose we find that there is a transformation which we have to undergo to correspond it with respect to a property which we measure in a new coordinate system, then how will we represent this?

We will represent that is if the vector P becomes P dash in the new coordinate system, and the vector Q becomes Q dash, and the vector T becomes T dash, but still this relationship has should be maintained correct. Now we know the relationship between P dash between the old and the new coordinate system we know, between the Q we know that between the old and the new coordinate system, and how is it being done right at the beginning when we wrote about this, we mentioned that the vectors also i j k also become a corresponding vector correct, that is a change which is going to be there I dash j dash and k dash in the new system.

Between the old and the new coordinate system, this is how we can represent the unit vectors also can be represented using a transformation matrix correct, this is from the old to new this is from new to old. Suppose in this equation itself we wanted to do the transformation, then what it will happen? This P what we have written it here is nothing, but I into P 1 right plus j into P 2 plus k into P 3, then we have to transform this to the new coordinate system, essentially how many components which we will have when it is transformed we require essentially 9 terms will be there in that transformation.

Is it not because P equals i into P 1, plus j into P 2, plus k into P 3; and if we take in P dash it becomes i dash into P dash 1, plus j dash into P dash 2, plus k into P dash 3 right. We can substitute we have to substitute for if you wanted to transform into a old coordinate system what we should do it? We can substitute for this i dash and P dash. So, when we do that we end up with a terms which are going to be there a transformation finally, we will be getting the P 1, P 2, P 3 we are going to get it the number of terms.

Now, let us look at similarly it will happen with respect to Q also when we undergo a transformation from one to the other correct? Here this is the property this is i i, i i can be written in terms of i dash and j dash, let us just look at only what is going to happen with respect to a components. In terms of the new coordinate system it will be in terms of I should be written in terms of i dash and j dash. So, each one of them if you substitute there will be 3 into 3. So, essentially 9, so we have already 9 terms are there, so totally the number of things going out to be essentially 81.

So this way of these very exact you get the correct answer, but the number of terms becomes very large. So, is there any way in which this could be simplified and written. So in fact, in this one if you try to look at any specific coefficient let us just look at that.

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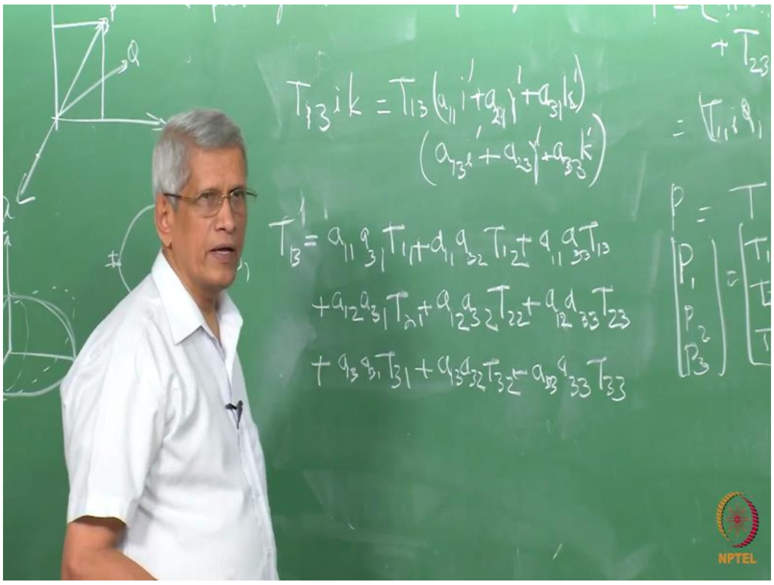

Suppose we look at T 1 3 it is i k correct, and i n k if we substitute for that in terms of these coefficients, how many it is going to be a 1 1.

Student: (Refer Time: 39:43).

I dash plus a 1, a 2 1, a 2 1 j dash, plus a 3 1 k dash correct.

Student: (Refer Time: 40:03).

A 1 3 I dash plus.

Student: (Refer Time: 40:07).

A 2 3 j dash, plus a 3 3 k dash, correct all these operations when we do. So, now itself you can remember that T 1 1 2 these are all the components which are going to be there correct? If you have to relate between this and the old 1 what we have to take finally, this is T 1 3 we are taking it is the component, if you wanted to take T dash 1 3 then we have to take with respect to components which correspond to i dash and k dash only has to be taken correct. So, here i dash and k dash the components are there; this if you try to do it what it will turn out to be I will just write that expression only a 3 1 T 1 1 because here we are taking it this is the products when we take it that some of the terms will remain the other terms will go. So, we are only taking that i k we are leaving it out only the

Student: (Refer Time: 41:58).

The.

Student: (Refer Time: 42:00).

Ok 1 2 a 3 2, T 2, plus a 1 3 into a 1 2, a 3 3 into T 2 3, plus a 1 3 into a 3 1 into T 3 1, plus a 1 3 into a 3 2 into T 3 2, plus a 1 3 into a, 3 3 into T 3 3. So, essentially we have a with respect to a old one for each of the new component of the tensor we have 9 which is going to be there. Not only that you can see that from the coordinate transformation, transformation matrix which we use this is getting repeated correct is it correct this way it can be done. So, for all other 9 components we can write it we will have around.

Student: (Refer Time: 43:52).

Essentially 81 this is going to be a very laborious way of doing it. So, if we use the Einstein notation, then it the job becomes much simpler right. So, how can we go about and do it.

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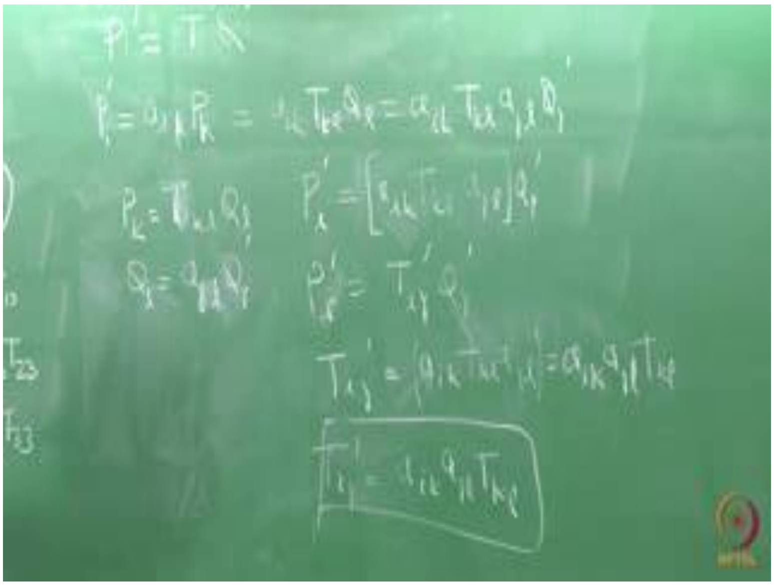

So, the expression which we write it is that P dash equals T dash into Q dash right, how can you do this transformation? First P dash let us look at this P dash itself, it can be written in terms of the transformation matrix right if we take a component one into, I can write it in this form right. Now what is the way in which P k can be written, P k is the old coordinate system. So, we can write this component P k is equal to a k.

Student: (Refer Time: 45:30).

l I can write it as.

Student: (Refer Time: 45:35).

Q l I can write it because this is only a dummy suffix correct; no this will be T this will be T k l correct this is how this relation, because that is P dash we have transformed it into a old coordinate, and then it is written in terms of P, P is related to Q in the old transformation using this vector P equals T into Q. So, we can write it in terms of this one, this represents vector. Now, this will become equal to a i k right T k l into Q l is it clear? Now how can you write Q l that is Q l is now we transform it from this to the new coordinate system? So, then Q l becomes a l j, I think Q j dash correct no this will be a j l this is because that when we change that index will change correct, because one is the transpose of the other. So, Q l becomes a j l into Q j dash this.

Now, we substitute for this then this expression becomes a i k, T k l, a j l Q j dash right. Now if we look at it this is P i dash equals i k, T k l into Q j dash it has become is it not now so; that means, that if we write the relationship between this vector and this vector how accurately this relationship is going to come with respect to a coordinate system which we have considered here, this will be P i dash equals T i j dash Q j dash this clear.

So, now T I j dash equals.

Student: (Refer Time: 49:10).

a i k, T k l, a j l actually here we can write it in the order does not matter. So, we can write it as a i k, a j l into T k l. It is the form in which we can write it. So now, we have got a handle to represent it. So, in this term what are the indices which are repeating k and l no?

Student: k (Refer Time: 50:08).

K and l are the dummy set, now if you try to expand this with respect to let us take 1 3.

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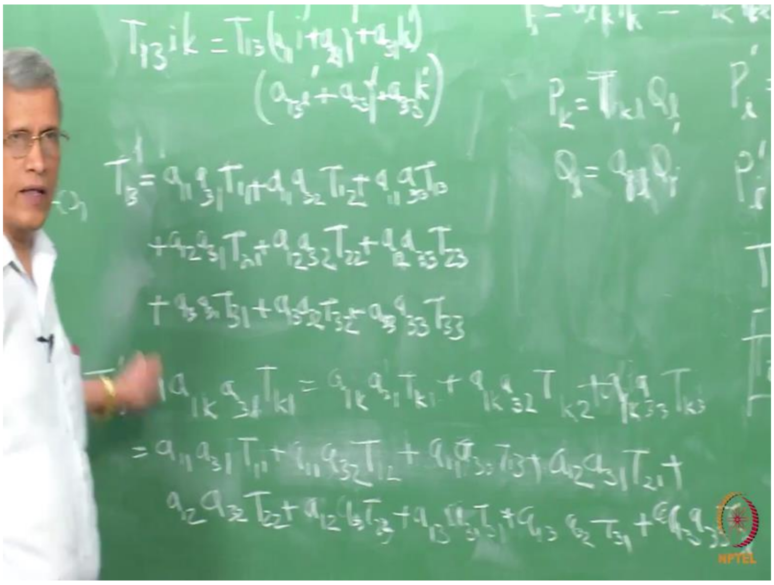

Let us see what it happens; that means, that i and j turns out to be 1 and 3 right a i k.

Student: (Refer Time: 50:47).

A 3 l, T k l correct because if we take.

Student: (Refer Time: 51:02).

For a specific this will be equal to suppose first let us take it with respect to l, then what it will happen a 1 k a 3 1, T k this will be 1, here we are taking l changing from 1 to 3, plus a 1 k, a 3 2.

Student: (Refer Time: 51:52).

T k is from 2 correct plus a 1 k a 3 3.

Student: (Refer Time: 52:08).

T k 3, now let us do for the indices k then what is going to happen a 1 1.

Student: A 3 1.

A 3 1.

Student: T 1 1.

1 1.

Student: Plus a 1 1.

1 1.

Student: A 3 2.

A 3 2.

Student: T 1 2.

T 1 2.

Student: Plus a 1 1, a 3 3.

A 3 3.

Student: T 1 3.

T 1 3 plus.

Student: Plus a 1 2.

Yeah.

Student: A 3 1, T 2 1.

Yeah.

Student: Plus a 1 2, a 3 2, T 2 2.

Yeah.

Student: A 1 2, a 3 3, T 2 3.

Yeah.

Student: Plus.

2 3.

Student: A 1 3, a 3 1, T 3 1.

Yeah.

Student: Plus a 1 3, a 3 2, T 3 2.

Ok.

Student: 1 1 (Refer Time: 52:19) plus a 1 3, a 3 3, T 3 3.

T 3 3; so now if you look at it this and this is essentially identical, but in this simple one expression we can write it right.

Student: (Refer Time: 53:39).

From this we can make out that this is the easy way in which we can do transformation from one component to another component.

So, in this class so far what we have looked at it is a coordinate transformation what is the relationship between the vectors which is governed by the property which decides the connection between the vectors, then if you represent the vectors in terms of some coordinate system then the property also is component should be there in terms in those directions. Those components are essentially 9 which is required, because otherwise if you should take a matrix 3 by 1 matrix, and the another 3 by 1 matrix if you have to lead by a matrix it has to be a 3 by 3 matrix which is a property matrix. This rule we can do it if it is the going to be a 4 by 1 matrix, if a vector we represent it in some 4 dimensional space, the other vector is going to be 4 by 1, then the number of components of the tern the tensor will have will be 16 will be there correct is it clear.

Student: (Refer Time: 54:52).

So now, what we have done it is that we have written these vectors correct. So, this is the form in which this vector could be written these 9 components of this vector; is that is it that this vector has to be symmetric? No there need not be, all the 9 components could have different values it may not be a symmetric vector, but know this tensor need not be vector components of this tensor operator this matrix need not be a symmetric tensor, but a way tensor which is not symmetric we can write it in as a symmetric tensor as a sum of symmetric and an anti symmetric tensor that can very easily be done.

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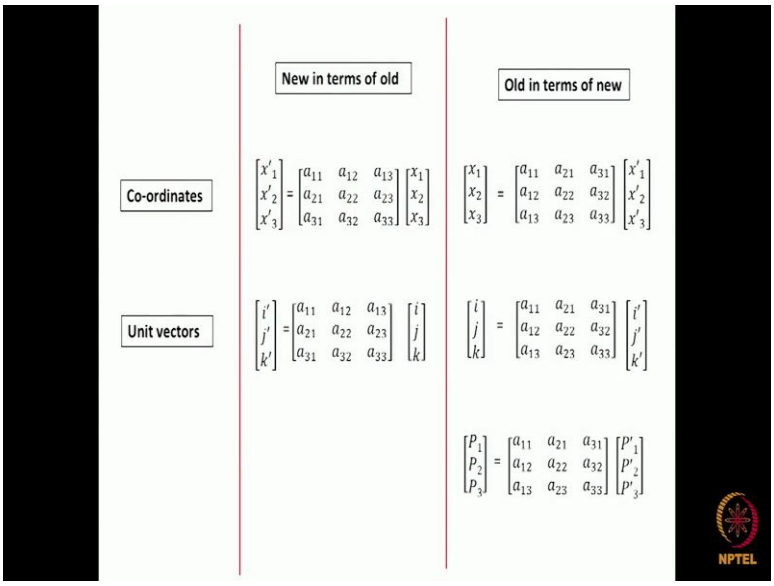

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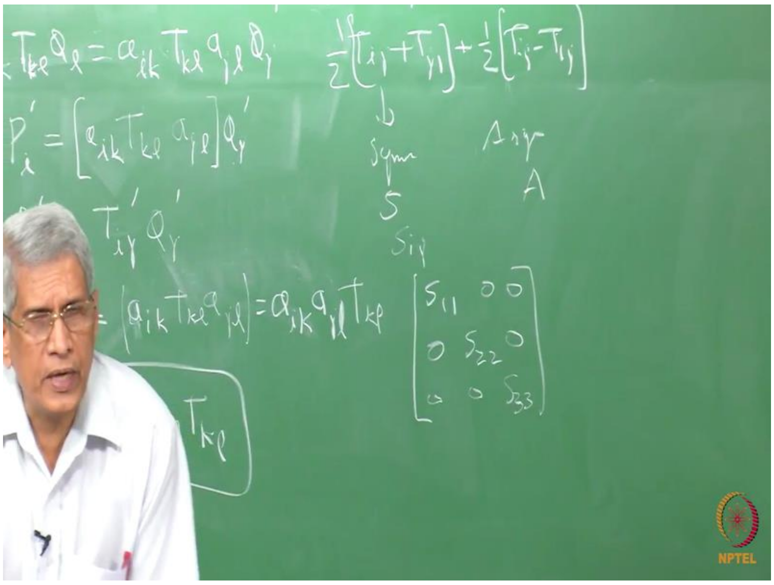

Then if you do it how will it turn out to be this vector, that is if a tensor we write it as T with components T i j representing this component, then we can write it that T i j half into T i j plus T j i, plus half into a minus T this form if you write it this is a symmetric tensor this is a.

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Asymmetric tensor this form we can write it; and the symmetric part of the tensor if we represent it that symmetric part can be written in terms of because it involves again 9 elements are there, diagonal elements and off diagonal elements are there. Like for example, if we consider this to be a strain matrix, so the diagonal elements represent essentially the principle, pure strain that is dilatational, and the others represent their shear strain right. So, any matrix in this form can be represented in another way in a diagonal matrix we can represent it. So, that if you represent this matrix symmetric matrixes S if you use the notation to do that, a symmetric matrix as A then these components can be written as S i j correct.

So, if you write it in terms of a diagram it will become S 1 1, that is if we are looking for the property of this material there is the tensor, in some direction this tensor itself can be written in terms of principle components, what do we understand? Let us take an example with respect to a strain itself, suppose we take as a strain as a matrix.

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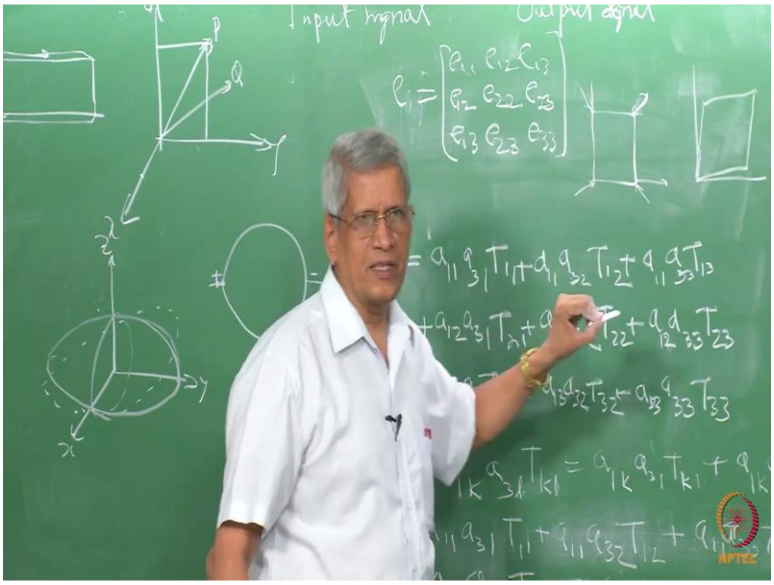

And we represent it in terms of e i j that strain can be written as e 2 3, this is a symmetric tensor these are all the shear components this is the dilatation correct. If you take a sample which contains all these component; that means, that what it means if you take a cube, and if the shear stress and all the stresses are being applied it changes into a parallelepiped. The shape change is going to be there and there is going to be a volume change also. When we say in terms of a principal component what we means is that, by applying only the dilatational that is either a compression or extension along some directions, the same volume and the shape change could be obtained without involving a shear strain itself to a sample which we can do it, how can we think of that.

Let us take a just a square, this square if I apply a shear strain, this is how the square will be turned into correct. The same thing can be obtained by applying just compression in this direction and a tension in this direction, we can make it into this shape; that means, that along some axis we are applying only applying dilatational we can obtain the same shape change; that means, that the same shape which has been obtained with respect to a a strain tensor, which contains dilatation and shear strain, in some direction we can apply dilatation and shear components and do that the same shape change could be shape change with the same volume change could be obtained.

So that is what essentially means that now these are all the diagonal way we can represent it. Now this itself can be represented to represent the property in this, which is that is we can choose this as the axis one axis principle axis another, these are all axis which are perpendicular to each other. So, if you represent the property choosing this principle axis, then the other components are going to be 0.

So, far now we have covered, how to relationship that is a coordinate transformation relationship between different vectors which are connected by property, how to transform from one coordinate system to the other, and the symmetric tensor we a understood that it can be written in terms of a principle components, this way we can write it.

Now, next we will take an example and how this information which you have gained could be used to explain what we started with a property measurement, how it could be done in the next class.