Fundamentals of electronic materials, devices and fabrication

Dr. S. Parasuraman

Department of Metallurgical and Materials Engineering

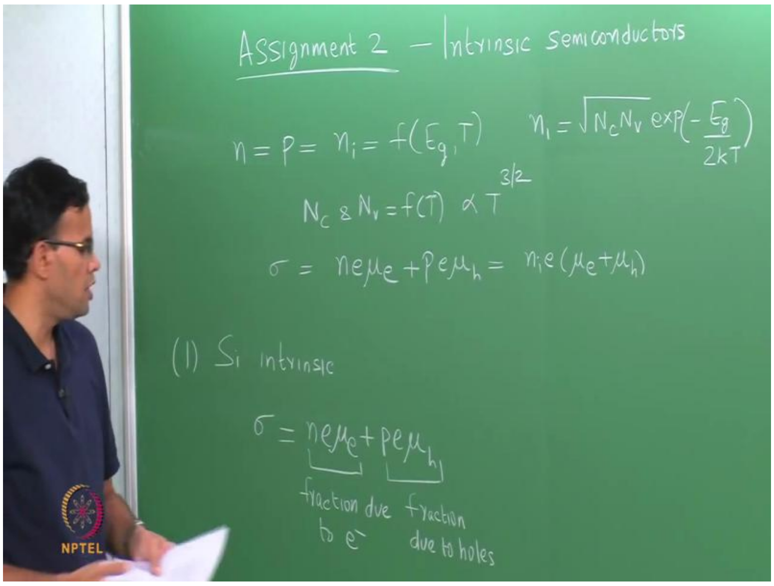
Indian Institute of Technology, Madras

Assignment – 2

Intrinsic semiconductors

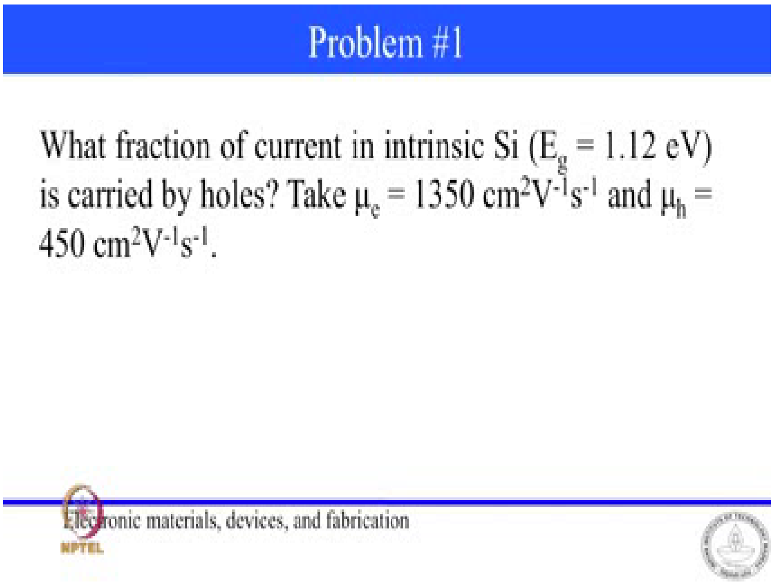
In today's assignment class, we will be looking fully at intrinsic semiconductors. This is assignment-2, and we will be focusing on intrinsic semiconductors. So, before we start looking at the problems, we just do a brief review. So, intrinsic semiconductors or pure semiconductors are essentially single crystals; we say that there are no defects in the semiconductor, because these defects can again create electrons and holes of their own.

(Refer Slide Time: 01:10)



In the case of an intrinsic semiconductor, we say that the electron concentration in the conduction band that is n is equal to the whole concentration in the valance band that is p, and it is equal to something which we denote as n_i, and n_i we call the intrinsic carrier concentration. We also say that n_i is a function of the band gap of the material E_g and also a function of temperature. So, typically n_i is written as $N_cN_cexp\frac{E_g}{2kT}$. So, the intrinsic carrier concentration depends exponentially on the band gap; the temperature term enters in this exponential factor, but N_c and N_v which are the effective density of states at the valance band edge and the conduction band edge are also a function of temperature. So, N_c and N_v are also a function of temperature. Typically they are proportional to $T^{3/2}$, but the exponential term is the one that dominates. We also saw the general equation for conductivity σ is nothing but $ne\mu_e$ and $pe\mu_h$. In the case of an intrinsic semiconductor, this just becomes $n_ie\mu_e + \mu_h$. So, these are just a few points about intrinsic semiconductor; we will be using them today during the course of the assignment.

(Refer Slide Time: 03:29)



Problem #1

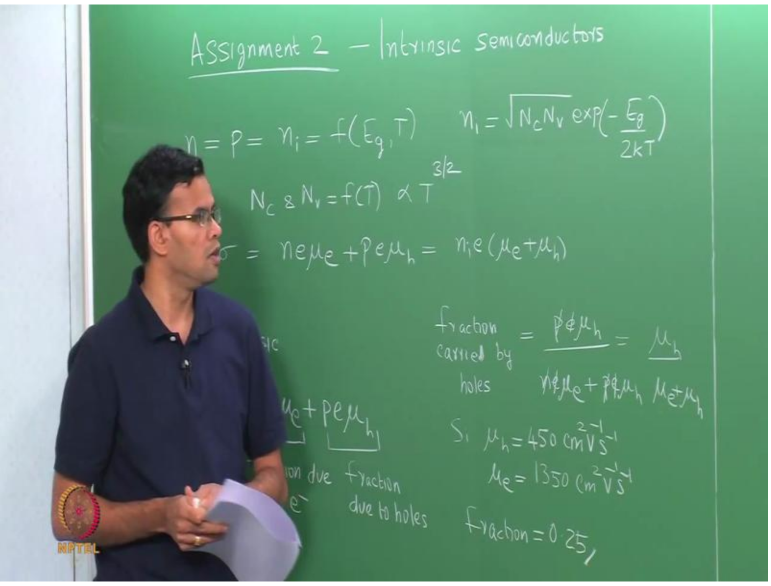
What fraction of current in intrinsic Si ($E_g = 1.12$ eV) is carried by holes? Take $\mu_e = 1350\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$ and $\mu_h = 450\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$.

Electronic materials, devices, and fabrication

NPTEL

So, let us first look at problem one. So, what fraction of current in intrinsic silicon is carried by holes? So, we have silicon and it is intrinsic which means $n = p = n_i$, and the question asked what fraction of current or what fraction of conductivity is defined by the holes? So, if you just say $n = p = n_i$ that means, there is a 50 percent contribution that is a very simplistic answer, the reason is the conductivity not only depends on n, it also depends upon μ_e and μ_h, which is the mobility of the electrons and holes.

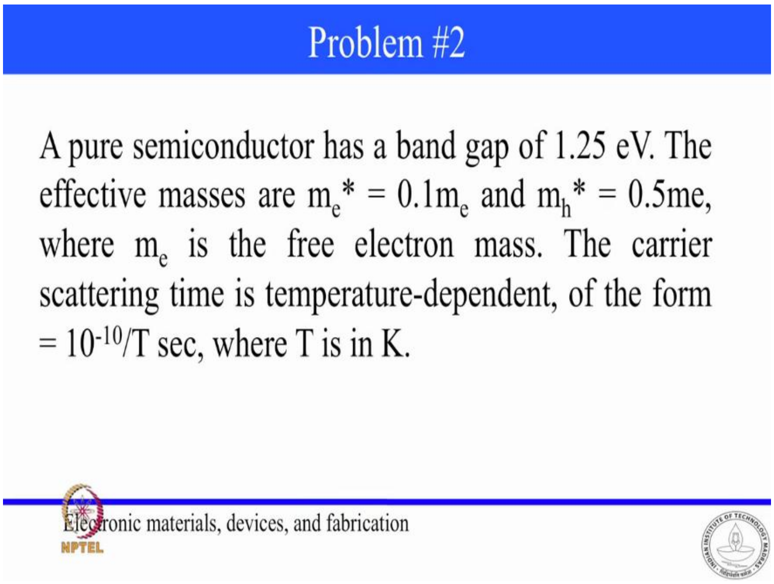
(Refer Slide Time: 04:22)



So, we can write the conductivity equation $n\ e\ \mu_e + p\ e\ \mu_h$; this represent the fraction carried by the electrons - fraction due to electrons; this is the fraction due to holes. So, we can include the numbers from μ_e and μ_h. So, fraction carried by the holes, we can write this in the form of a ratio is nothing but $\frac{pe\mu_h}{ne\mu_e+pe\mu_h}$. So, for an intrinsic semiconductor $n = p = n_i$. So, these terms cancel, e will also cancel. So, this is nothing but $\frac{\mu_h}{\mu_e+\mu_h}$. So, the fraction of current carried by holes is directly proportional to the whole mobility we can plug in the numbers for silicon here. So, for silicon μ_h is 450 cm^2 V^{-1} s^{-1}; μ_e is 1350, so the mobility of the electrons is higher. So, we can plug in these numbers and the fraction is 0.25.

So, even though we have equal concentration of electrons and holes, they do not have the same mobility; and this is because your electrons are moving in the conduction band, and the holes are moving in the valance band. And this ultimately determines what fraction dominates whether the electron conductivity dominates or the whole conductivity dominates. Later when we see an extrinsic semiconductor, we will find that n and p are not the same; one is much higher than the other, and then there one of the term dominates because of the difference in concentration.

(Refer Slide Time: 07:27)



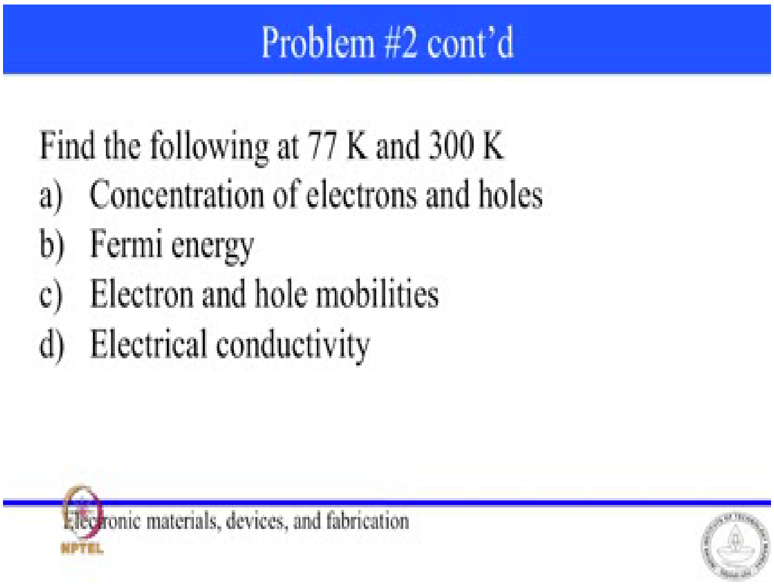
Problem #2

A pure semiconductor has a band gap of 1.25 eV. The effective masses are $m_e^* = 0.1m_e$ and $m_h^* = 0.5me$, where m_e is the free electron mass. The carrier scattering time is temperature-dependent, of the form $= 10^{-10}/T$ sec, where T is in K.

Electronic materials, devices, and fabrication

NPTEL

(Refer Slide Time: 07:32)



Problem #2 cont'd

Find the following at 77 K and 300 K

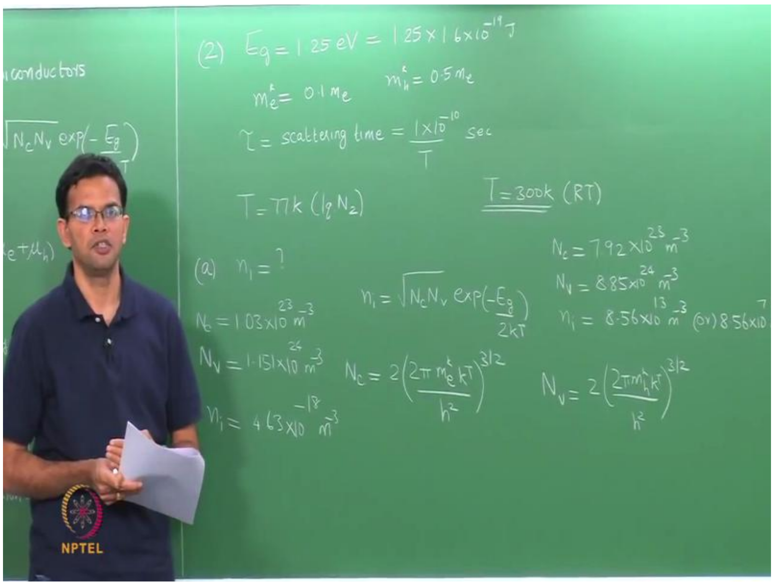
a) Concentration of electrons and holes
b) Fermi energy
c) Electron and hole mobilities
d) Electrical conductivity

Electronic materials, devices, and fabrication

NPTEL

So let us now move to question 2.

(Refer Slide Time: 07:42)



So, we have a pure semiconductor or an intrinsic semiconductor, the band gap is 1.25 eV. So, the effective masses of the electrons and holes are also given. The effective mass of the electron m_e^* is 0.1 times m_e, where m_e is the mass of the electron; and m_h^* is 0.5 m_e, this is the effective mass for the hole. Once again, we have seen the concept of effective mass before. So, effective mass does not mean a change in the actual mass of the electron or the hole it just represents the cumulative of all the forces of the atoms in the lattice that basically acts on the electrons and holes. Once again these numbers are different, because you have electrons that are moving in the conduction band and holes that are moving in the valance band. So, the band gap is given, the effective mass values are given. The carriers scattering time is temperature dependent and that is given of the form, so τ which is your scattering time is a function of temperature, and this is 1 minus 10 to the minus t. So, $\frac{1 X 10^{-10}}{T}$ and the units are seconds.

So, the effective masses are given, the band gap is given, and the temperature dependence of the carriers scattering time is also given. This we will use to calculate the mobility's. So, we want to find the following at 2 temperatures; one is 77 Kelvin and the other is 300 Kelvin. So, 300 Kelvin is room temperature 77 Kelvin is typically your liquid nitrogen boiling point, so that is a low temperature. The first one we want to find is the concentration of electrons or holes, because this is a pure semiconductor. What we want to find is the value of the intrinsic carrier concentration.

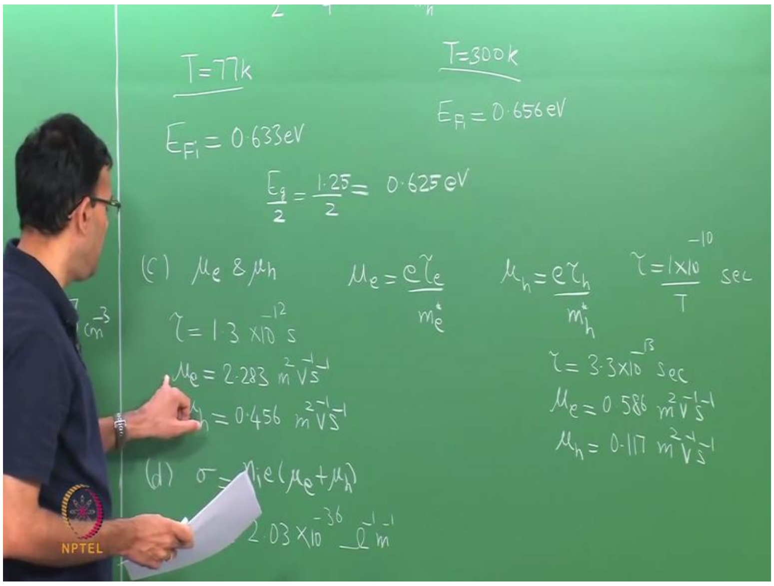
So, we can go back to the equation n_i is nothing but $\sqrt{N_cN_v}exp\frac{E_g}{2kT}$. So, the problem is we do not have the values of N_c and N_v; these are the effective densities of states at the band adjust, but these we can calculate once we know the effective mass. So, N_c which is the density of states at the conduction band edge is nothing but $2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$. N v we can do the same for the valance band edge $2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$. So, we have the values for N_cN_v; again we see both are temperature dependent, they are proportional to t to the power 3 half. So, once we calculate $N_c$and $N_v$for both the temperatures, we can plug in here and calculate the value for n_i the band gap is also known.

So, typically you have to keep all of this in SI units. So, you have to convert E_g from electron volts to joules and that we can do by just multiplying by 1.6 x 10^{-19}. K b is also in joules, so it is your Boltzmann constant that has a standard value. So, once we plug in the numbers, I am just going to write the final answers, but you can just go through and check. So, N_c at 77 Kelvin is 1.03 x $10^{23}$$m^{-3}$. So, if you remember the definition of the effective density of states is the total number of states per unit volume that is available for the electron to occupy or the hole to occupy. Similarly N_v is 1.151 x 10^{24} m^{-3}.

We can do the same calculations for 300 Kelvin. I will again just write down the answers. So, N_c is higher 7.92 x 10^{23}; N_v is 8.85 x 10^{24} m^{-3}. So, compared to 300 Kelvin N_c and N_v are higher, this is because we have more density of states available at higher temperature, simply because they are directly proportional to $T^{3/2}$. So, we can substitute these values of N_c and N_v in this expression and calculate the value for n_i.

So, let me just write that down n_i at 77 Kelvin is 4.63 x 10^{-18} m^{-3}, so that is a really small number. n_i at 300 Kelvin is 8.56 x 10^{13} m^{-3}. So, I can also write this in cm^3 or 8.56 x 10^7 cm^3. So, your N_c and N_v values if you look or of by one order of magnitude, simply because you have a rise in temperature, but because your n_i depends exponentially on the band gap, there is a huge variation between 77 Kelvin, which is your liquid nitrogen temperature, and 300 Kelvin which is room temperature. So, here you have a value that is 10^{-18} and at room temperature you have a value for n_i that is close to 10^{30}, so that overall there is a 31 orders of magnitude change as we go from liquid nitrogen to room temperature. This is why we say that out of these 2 terms N_c and N_v and the exponential term the exponential term is the one that dominates in determining the value for n_i. So, this is part-a, where we want to calculate the concentration of electrons and holes. Part-b, we want to calculate the Fermi energy or the location of the Fermi level.

(Refer Slide Time: 16:30)



So part-b, we want the location of E_{Fi}; this is again a standard expression E_{Fi} is $\frac{E_g}{2} - \frac{3}{4}kT\ln\frac{m_e^*}{m_h^*}$. So, if m_e and m_h were equal then E_{Fi} will be exactly at the center of the band. If you remember E_{Fi} is nothing but a representation of the chemical potential or the amount of work that needs to be done in order to remove an electron from a semiconductor. So, even though your electron and hole concentrations are the same, so n = p because you have different effective masses, your E_{Fi} is slightly shifted from the center of the gap. We can once again plug in the numbers, so 77 Kelvin, 300 Kelvin m_e^* and m_h^* values are given temperature is also known. So, E_{Fi} here, if you do the substitution is 0.633 eV; at room temperature E_{Fi} is 0.656. $\frac{E_g}{2}$, if you look at it is just $\frac{1.25}{2}$ which is 0.625 electron volts. So, the values are very close to the center of the band gap, but they are slightly shifted and the shift becomes higher, the higher the temperature. So, this is 0.633 and this is 0.656. So, slightly deviated away from the center of the band gap this is part-b.

In part-c, we want to calculate the electron and hole mobilities. So, we want to calculate the values of μ_e and μ_h. So, μ_e and μ_h are related to the effective mass of the electrons and holes, and they are also related to the scattering time. So, μ_e is nothing but $\frac{e\tau_e}{m_e^*}$ and μ_h is

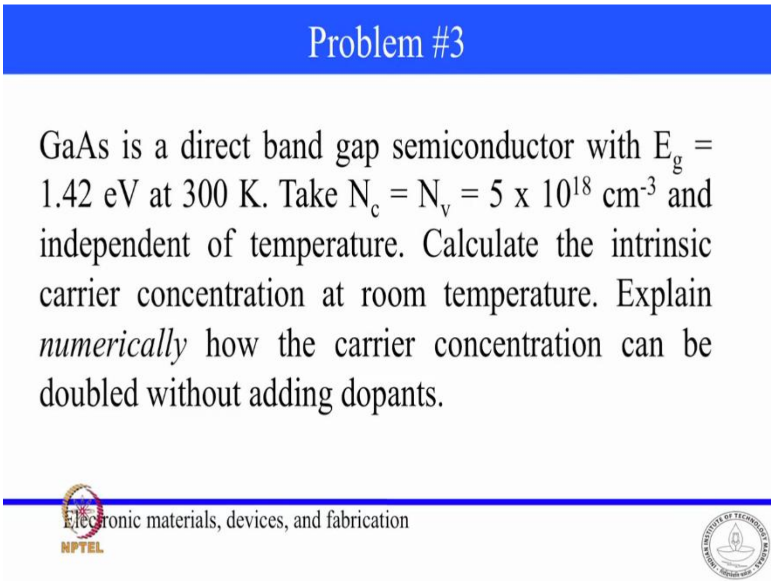
$\frac{e\tau_h}{m_h^*}$. So, m_e^* and m_h^* are given, τ_e and τ_h are your scattering times, and you have said that τ is nothing but $(1 \times 10^{-10})/T$ and the unit is seconds. So, once again we can calculate the values of τ. In this particular question, τ_e and τ_h are both the same, because we do not distinguish between electrons and holes; we only say it depends upon temperature. Once we calculate τ, we can go ahead and calculate μ_e and μ_h and get it for the 2 different temperatures.

So, let me again write down this side is 77 Kelvin, this side is 300 Kelvin. So, τ if you calculate is 1.3×10^{-12} seconds. We can then calculate μ_e, which is 2.283, unit is $\mathrm{m}^2\ \mathrm{V}^{-1}\mathrm{s}^{-1}$, sometimes $\mathrm{cm}^2\ \mathrm{V}^{-1}\mathrm{s}^{-1}$are also there, it depends upon which you want to use. μ_h is $0.456\ \mathrm{cm}^2\ \mathrm{V}^{-1}\mathrm{s}^{-1}$; and μ_h the mobility of the holes is lower, because the whole effective mass is higher than that of the electron. We can do the same for 300 Kelvin; in this case τ is 3.3×10^{-13} seconds. So, higher the temperature, smaller is the scattering time. So, at low temperature, this is minus 12 seconds, this is minus 13 seconds. So, one way to think about this is higher the temperature faster the electrons and holes are moving, because they have higher thermal velocities. So, they can scatter of the atoms quicker. μ_e is $0.586\ \mathrm{m}^2\ \mathrm{V}^{-1}\mathrm{s}^{-1}$; μ_e $0.117\ \mathrm{m}^2\ \mathrm{V}^{-1}\mathrm{s}^{-1}$.

So, the last part of the question, we want to calculate the electrical conductivity - σ is nothing but $n_i\ e\ \mu_e + \mu_h$. So, n_i we got in first part of this question, μ_e and μ_h we just calculated. So, we can just plug in the numbers. So, σ at 77 Kelvin is very small, because if you remember n_i is very small. So, $10^{-36}\ \Omega^{-1}\ \mathrm{m}^{-1}$, you can also have $\Omega^{-1}\ \mathrm{cm}^{-1}$ depending upon what your values the units for n_i and μ_e and μ_h are. So, the same thing we can do at room temperature and σ is $9.6 - 10^{-6}$.

So, by looking at an intrinsic semiconductor at 2 different temperatures, one thing we find is that the carrier concentration increases exponentially with temperature. Similarly, the conductivity will also increase, because the carrier concentration increases. This again is determined by the value of the band gap. So, higher the value of E_g more steeper is this dependence. So, of instead of 1.25, we had done the same problem, which say 2 electron volts, your answers will also be different, but the difference between 77 and 300 Kelvin will also be more pronounced. So, that is something you can always work out you can take the same values, but change the value of E_g to 2 electron volts, and do this question and you can see the difference between 77 and 300 Kelvin.

(Refer Slide Time: 24:25)



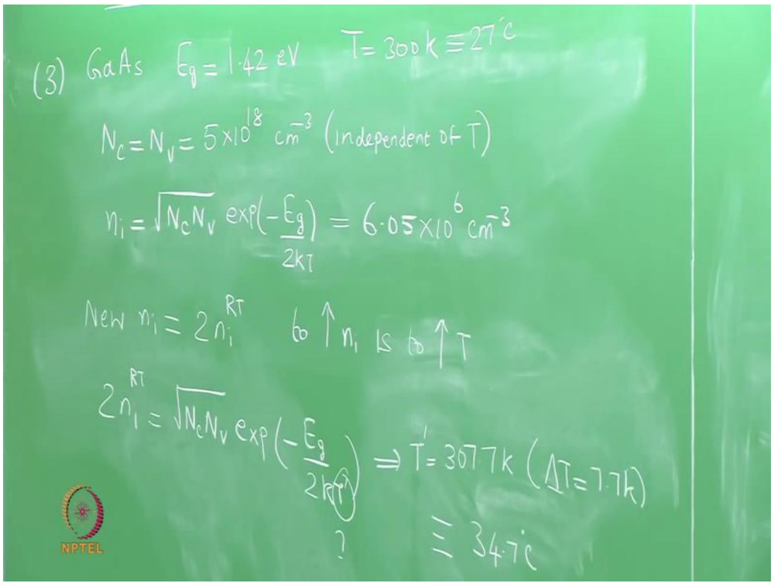
Problem #3

GaAs is a direct band gap semiconductor with E_g = 1.42 eV at 300 K. Take $N_c = N_v = 5 \times 10^{18}$ cm^{-3} and independent of temperature. Calculate the intrinsic carrier concentration at room temperature. Explain *numerically* how the carrier concentration can be doubled without adding dopants.

Electronic materials, devices, and fabrication

Let us now move to question 3.

(Refer Slide Time: 24:43)



(3) GaAs $E_g = 1.42$ eV $T = 300\,k \equiv 27\,^\circ C$

$N_c = N_v = 5 \times 10^{18}\ cm^{-3}$ (independent of T)

$n_i = \sqrt{N_c N_v}\, \exp\left(-\frac{E_g}{2kT}\right) = 6.05 \times 10^{6}\ cm^{-3}$

New $n_i = 2 n_i^{RT}$ to $\uparrow n_i$ is to $\uparrow T$

$2 n_i^{RT} = \sqrt{N_c N_v}\, \exp\left(-\frac{E_g}{2k\boxed{T'}}\right) \Rightarrow T' = 307.7\,k\ (\Delta T = 7.7\,k)$

?

$\equiv 34.7\,^\circ c$

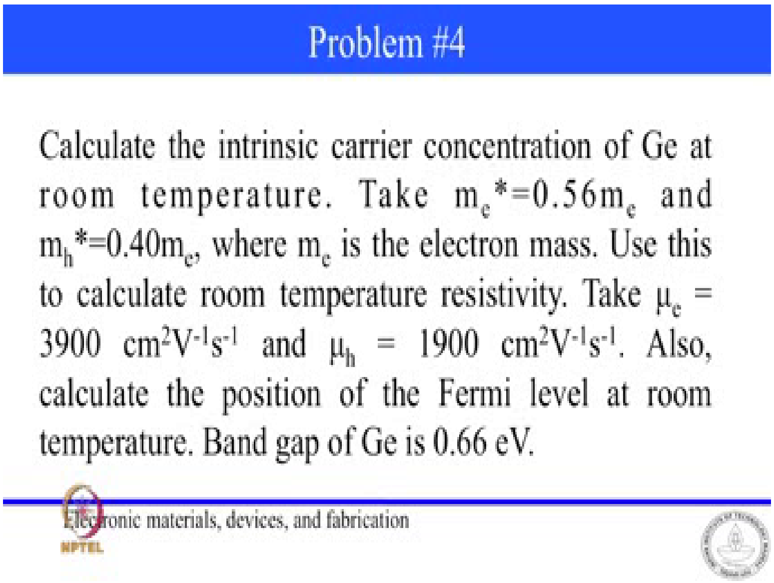
So, question 3, we have gallium arsenide which is a direct band gap semiconductor with E_g of 1.42 electron volts at 300 Kelvin. So, gallium arsenide has a higher band gap than silicon. So, also a direct band gap material, but that is not relevant for this question, it is at room temperature, so temperature is 300 Kelvin. Take $N_c = N_v = 5 \times 10^{18}$ cm^{-3} and also independent of temperature. So, the N_c and N_v values are given. And just for this question, we assuming that both are same and that they are also independent of temperature. Strictly speaking, this is not true, but as you see for this particular question, this is a very valid assumption.

So, first we want to calculate the intrinsic carrier concentration at room temperature, so that is pretty straight forward, you seen the formula before, so $N_c N_v exp\frac{-E_g}{2kT}$. So, we can plug in the numbers the value of E_g is given. So, n_i is 6.05 x 10^6 cm^{-3}. So, this is the intrinsic carrier concentration at room temperature; n_i is a pretty small number for comparison silicon as a value n_i of 10^{10}. So, 2 orders of magnitude higher, but this is because gallium arsenide has a higher band gap.

So, the next part of the questions says explain numerically how the carrier concentration can be doubled without adding dopants. So, we want to keep this semiconductor your pure semiconductor, but at the same time, we want to increase the value of n_i. So, the new value of n_i that we want is double of the n_i value at room temperature if you are not allowed to add dopants, and if you look at this equation the only way to increase n_i is to increase temperature, because N_c and N_v are both temperature dependent terms, n_i also depends on temperature through this exponential term minus $\frac{E_g}{2kT}$. So, increasing temperature will once again increase n_i. So, the only way to increase n_i without adding dopants is to increase temperature. So, we want to know what the new temperature is when your value of n_i is 2 times the n_i at room temperature.

So, we will once again use this expression N_c and N_v are constant. So, it is not a function of temperature; if it were a function of temperature that will also have to be taken into account, but N_c and N_v are constant. We know the new value of n_i. So, it is 2 n_i at room temperature $\sqrt{N_c N_v} exp\frac{-E_g}{2k}$ let me call this temperature T' . So, T' is the only thing we want to know is the only unknown. This is known; these are all known, we can put this and recalculate. And this gives you the value of T' to be 307.7 Kelvin. So, you increase the temperature by 7 degrees. So, ΔT is 7.7 Kelvin, you can double the concentration of n_i. So, temperature equal to 300 is approximately 27 degrees. So, 300 Kelvin is 27 degrees Celsius. So, 307 is 34.7 ºC. So, we find it even for a small increase in the value of n_i. So, you have only doubling the value of n_i you need to increase your temperature by 7 degrees. If you want the really high conductivities that we see in the extrinsic semiconductors can actually calculate that the temperature change must be much higher, this is one of the reason why intrinsic semiconductors are almost never used in the case of devices. Usually doped semiconductors are used, because it is much more easier to control the dopant concentration and then control the carrier concentration and also the conductivity.

(Refer Slide Time: 30:28)



Problem #4

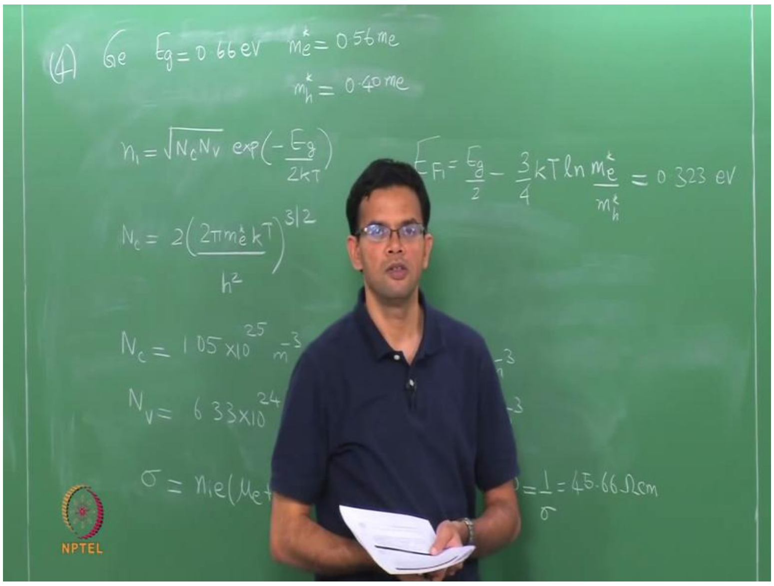
Calculate the intrinsic carrier concentration of Ge at room temperature. Take $m_e^*=0.56m_e$ and $m_h^*=0.40m_e$, where m_e is the electron mass. Use this to calculate room temperature resistivity. Take μ_e = 3900 $cm^2V^{-1}s^{-1}$ and μ_h = 1900 $cm^2V^{-1}s^{-1}$. Also, calculate the position of the Fermi level at room temperature. Band gap of Ge is 0.66 eV.

Electronic materials, devices, and fabrication

NPTEL

So, let us now to go to problem 4. So, problem 4 we want to calculate the intrinsic carrier concentration of germanium.

(Refer Slide Time: 30:36)

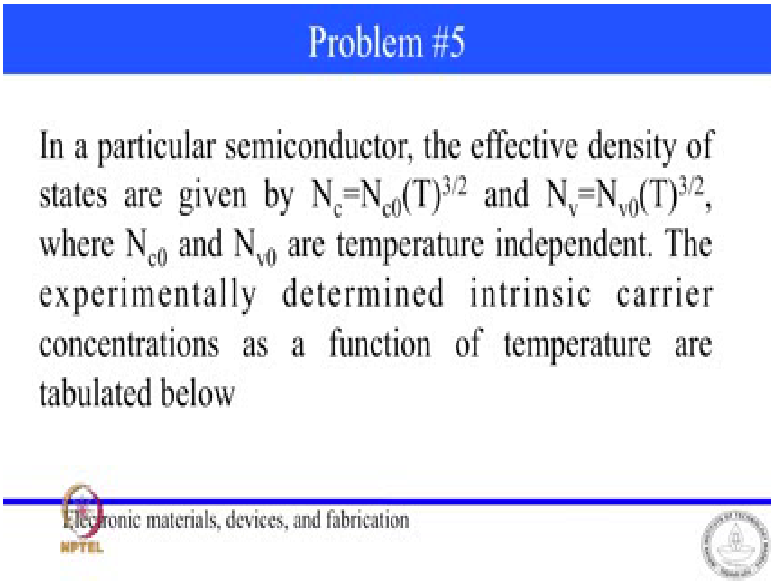


So, we have germanium with the band gap E_g of 0.66 electron volts; this is lower than that of silicon. In fact, germanium was the first semiconductor that was used. m_e^* is 0.56

m_e and m_h^* is 0.40 m_e. So, we want to calculate n_i. So, the equation is the same n_i is $N_c N_v exp\frac{-E_g}{2kT}$. So, N_c and N_v are again related to m_e^* and m_h^*. So, N_c is $2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$. We can write a similar equation for m_h^*; we saw that earlier during question 2. So, once again we can calculate N_c and N_v plug it back in and get the value of n_i. So, I will do the numbers you can write down the final answers N_c is 1.05×10^{25} m^{-3}. N_v, which is the same equation except m_e^* is replaced by m_h^*; N_v is 6.33×10^{24} m^{-3}. So, N_c and N_v are known, we can calculate n_i. n_i, if you do it is 2.36×10^{19} m^{-3}. I am just writing down the final answer, the math can always be worked out are 2.36×10^{13} m^{-3}.

So, we saw that gallium arsenide has a value of n_i that is 4 times or 4 orders lower than that of silicon. Germanium, on the other hand has a value of n_i that is nearly 3 orders of magnitude higher than silicon. Once again the differences are all related to the band gap values. We can then calculate σ, σ is n_i e μ_e and μ_h. The value of μ_e and μ_h are given; μ_e is 3900 and μ_h is 1900 cm^2 V^{-1} s^{-1}, so that σ is nothing but 0.022 Ω^{-1} and cm^{-1}. If you also want to calculate the resistivity, ρ is $\frac{1}{\sigma}$ which is equal to 45.66 Ω cm. The question also asked to calculate the position of the Fermi level at room temperature. So, that is again an application of the formula E_{Fi} is $\frac{E_g}{2} - \frac{3}{4} kT \ln\frac{m_e^*}{m_h^*}$. So, this again is very close, but it is not exactly at the center of the band gap.

(Refer Slide Time: 34:47)



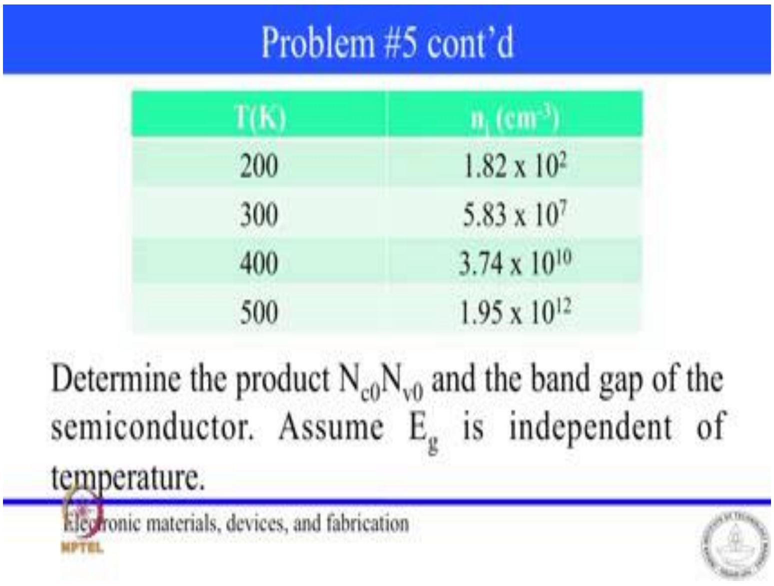
Problem #5

In a particular semiconductor, the effective density of states are given by $N_c = N_{c0}(T)^{3/2}$ and $N_v = N_{v0}(T)^{3/2}$, where N_{c0} and N_{v0} are temperature independent. The experimentally determined intrinsic carrier concentrations as a function of temperature are tabulated below

Electronic materials, devices, and fabrication

NPTEL

(Refer Slide Time: 34:52)



Problem #5 cont'd

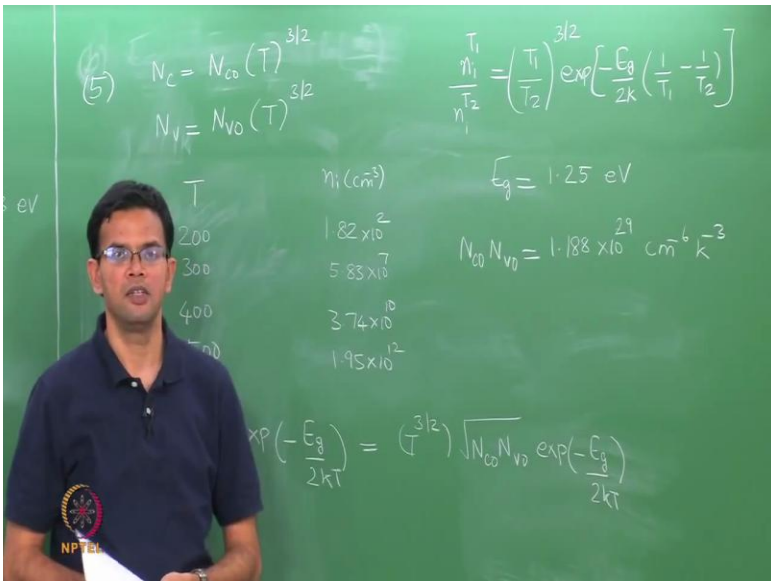
T(K)	n_i (cm^{-3})
200	1.82×10^2
300	5.83×10^7
400	3.74×10^{10}
500	1.95×10^{12}

Determine the product $N_{c0}N_{v0}$ and the band gap of the semiconductor. Assume E_g is independent of temperature.

Electronic materials, devices, and fabrication

So, let us now look at problem 5.

(Refer Slide Time: 34:57)



There is a particular semiconductor and it says that the effective density of states is a constant N_{co} $T^{3/2}$. In same way, N_v is N_{vo} $T^{3/2}$. So, the experimental values of n_i at different temperatures are given. So, we have temperature values of n_i in cm^3, so 200, 300, 400 and 500 and the values of n_i 10^7. So, we can see that with increasing in temperature, the value of n_i also increases. So, the question asked us to determine this product N_{co} N_{vo} and also the band gap. So, both E_g is not known, and these 2 numbers are not known.

You can go back to the original equation n_i is $N_cN_vexp\frac{-E_g}{2kT}$; N_c and N_v we can substitute these x terms. So, that this simplifies to $T^{3/2}$ $\sqrt{N_{co}N_{vo}}$ which is a temperature independent term times $exp\frac{-E_g}{2kT}$. So, we can choose any 2 temperatures. So, we have 4, we can take any 2 temperatures, and take the ratio of n_i at these 2 temperatures. So, n_i sum temperature T_1, n_i at another temperature T_2; it is nothing but taking this ratio which is here this is a temperature independent term. So, this becomes $\left(\frac{T_1}{T_2}\right)^{3/2} exp\left(\frac{-E_g}{2k}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)\right)$. So, T_1 and T_2 values are known, n_i values are known, for example, your T_1 could be 200 Kelvin, T_2 could be 300 Kelvin in which case the n_i values are tabulated we only unknown here is E_g. So, I did these calculations taking 200 and 300. You can take it with any of the other temperatures and also done and checked.

If you substitute the values E_g works out to be 1.25 electron volts. So, this is the value of E_g, which is the band gap of the material. Once you know E_g you can substitute E_g for any of the temperatures, and evaluate N_{co} and N_{vo}. This is a temperature independent term. If you do that $N_{co}N_{vo}$naught is nothing but 1.188×10^{29}. The units here are crucial N_c and N_v the square root of that should have the units of cm^{-3} or m^{-3}. In this particular question, this also depends on $T^{3/2}$. So, the unit of this product is cm^{-6} K^{-3} that way when we substitute for square root, units work out in the right way.

So, today we have looked at various problems related to intrinsic semiconductors. The important thing to remember is that the intrinsic carrier concentration is a function of temperature and depends upon the band gap of the material.