Fundamentals of electronic materials, devices and fabrication

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Assignment - 5
pn junctions

In today's assignment, we are going to look at p-n junctions. This is assignment 5. In assignment 4, we looked at metal semiconductor junctions. In assignment 5, we are going to look at junctions form between p and n type.

So, usually the p and n are of the same material, in which case it is a homo junction. You also have seen hetero junctions, where the junction is formed between 2 different materials. In this assignment, we will focus fully on the homo junctions; we will do some calculations on the built-in potential. When a junction is formed the depletion widths, the total depletion widths and also the depletion width on the p and the n-side a p-n junction is essentially a diode, it is a rectifier, so that it conducts current in the forward bias and does not conduct in the reverse bias. So, we will also do some calculations of the forward bias current and the reverse saturation current.

Some of this will be similar to what we did in assignment 4, where we looked at the schottky junction, which is also a rectifier. Later, we can compare the properties of a p-n junction and that of a schottky junction.

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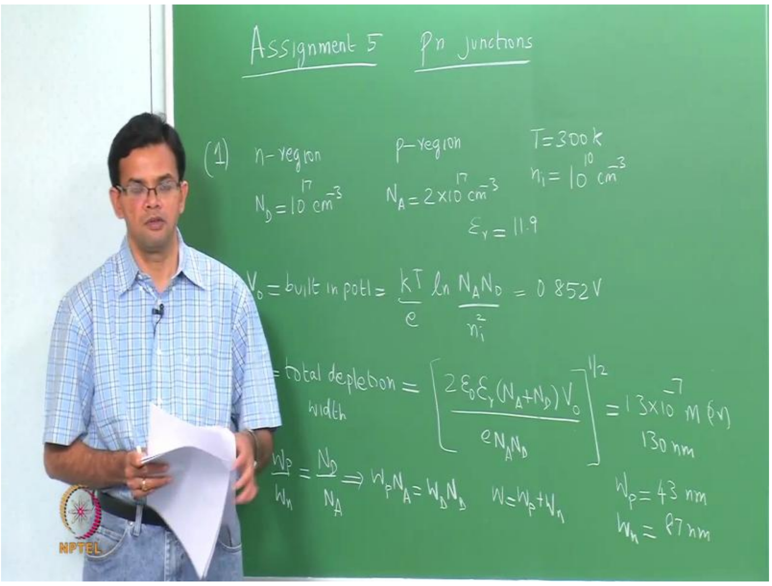

Let me go to problem number 1, we have a silicon p-n junction which has an n-region with 10^{17} donors cm^{-3} n-region. So, N_D is $10^{17} cm^{-3}$ and there is a p-region with acceptor concentration of 2×10^{17} cm^{-3}, N_A the material here is silicon and it is at room temperature. So, temperature T is 300 kelvin. The intrinsic carrier concentration we have seen this so many times in the past; is just 10^{10} cm^{-3}. So, In part-a, we want to calculate the built-in potential of this junction. So, V_o is the built-in potential. So, It is the potential when the junction is equilibrium and this forms because we have electrons from the n-side moving into the p-side. This is diffusion current we have holes from the p-side moving to the n-side, these essentially meet each other and annihilate, so that you have a depletion region.

So, on one side of the depletion region you have a net positive charge this is the n-side, on the other side you have a net negative charge that is your p-side and there is a junction potential that develops. This built-in potential is nothing but $\frac{kT}{e} \ln \frac{N_A N_D}{n_i^2}$. So, This is just a direct substitution of the numbers N_A and N_D are given, n_i^2 is also given, if n_i is not known you can always calculate n_i from the band gap and the effective density of states or the effective mass of the electrons and holes. So, you can just plug in the numbers and the answer is 0.852 volts.

In part 2, we want to calculate the total depletion width, let me call that W that is the total depletion width so, the depletion width again forms because you have electrons and holes moving across the junction and are recombining. So, We have seen this concept of depletion earlier when we look at the schottky diode or a schottky junction. In that case the depletion width is almost entirely on the semiconductor side. So, here the depletion width will be in both the p and the n-side. The total depletion width is again used given by a direct formula substitution. So, $\left[\frac{2\varepsilon_o \varepsilon_r (N_A + N_D) V_o}{e N_A N_D}\right]^{1/2}$ where V naught is your built-in potential. So, ε_o is the permittivity of free space, ε_r is the permittivity of silicon the relative permittivity and ε_r is 11.9, so that is the known value for silicon. So, once again everything here is known we just calculated the built-in potential V_o. So, We can do the substitution and this works out to be 1.3×10^{-7} m or if you want to write in nanometers 130 nanometers.

In part-c, we want to calculate the depletion width on the p and the n-side. So, the ratio of the depletion widths $\frac{W_p}{W_n}$ is inversely proportional to the concentration of your dopant, this is equal to $\frac{N_D}{N_A}$ and other way of writing this of course, is a $W_p N_A$ is $W_D N_D$ and this comes from the charge neutrality, so that the total positive charge due to your positively charge donors on the n-side must be equal to the total negative charge due to the negatively charged acceptor ions on the p-side and those 2 essentially balanced. We also know the total width W is $W_p + W_n$ and total width W has been calculated to be 130 nanometers. So, we have all the numbers. Again, in case of doing the substitution and the math, So, I will just write down the final values. So, W_p is 43 nanometers and W_n is 87 nanometers. The total width comes out to be 130 nanometers. The acceptor concentration on the p-side is higher so, 2×10^{17}. So, the depletion width on the p-side is smaller.

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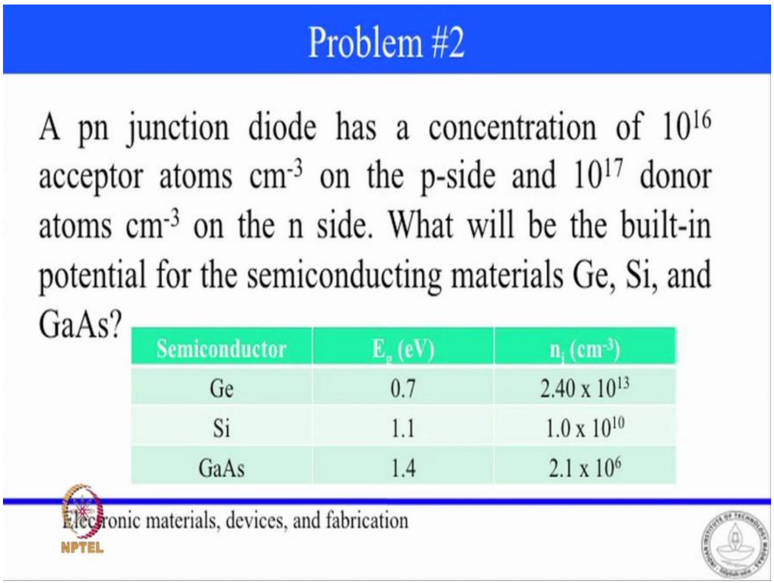

So, Let us now go to problem 2, we have a p-n junction diode with a concentration of 10^{16} acceptor atoms on the p-side and 10^{17} donor atoms on the n-side.

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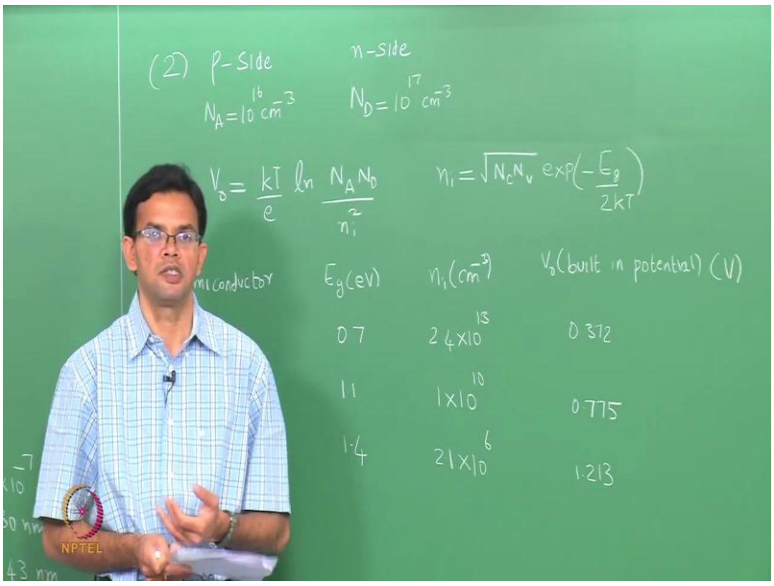

So, once again you have p-side on the n-side. So, N_A is 10^{16} and N_D is $10^{17} cm^{-3}$. So, We need to know the built-in potential, if the material of the semiconductor is different. So, In this particular case, we want the built-in potential for the semiconductor materials germanium, silicon and gallium arsenide. If you go back to the formula for the built-in potential $\frac{kT}{e}\ln\frac{N_A N_D}{n_i^2}$. So, if you change the material and if you keep the dopant concentrations the same when the temperature is also same, typically room temperature. The only thing we are changing is n_i. We have seen earlier that n_i depends upon the band gap $\sqrt{N_c N_v} exp\left(\frac{-E_g}{2kT}\right)$. So, as the value of the band gap increases n_i essentially goes down because it has an exponential with a negative term. If the value of n_i goes down, then the built-in potential will essentially increase.

So, in this case we have 3 materials. So, I will write down the table that is given in the problem. We have germanium, silicon, gallium arsenide, the band gap values are given in E_g and these are mainly used just for comparison. We do not need the band gap values as far as this problem is concerned; germanium is 0.7, silicon is 1.1, gallium arsenide is 1.4. What we do need is the values of n_i and n_i is again given cm^{-3}. So, germanium is 2.4×10^{13}, silicon is 1×10^{10} and gallium arsenide is 2.1×10^{6}. So, n_i essentially decrease as the value of the band gap increases. So, We now need to calculate the built-in potential for these 3 materials. We can make use of this formula, just substitute N_A and N_D and then the values of n_i for the different materials. So, we can go through and work out the math. I will just write down the final answer. So, V_o which is your built-in potential is nothing but 0.372 the units is volts for germanium, it is higher for silicon 0.775 and it is even higher for gallium arsenide 1.213. So, as the value of n_i goes down because you have a higher band gap the built-in potential at the junction essentially increases. This information is especially useful when you are trying to built devices with materials apart from silicon because once you know the built-in potential you are also know, what kind of current that it needs to be applied through the circuit for a particular kind of application.

Let us now go to problem 3.

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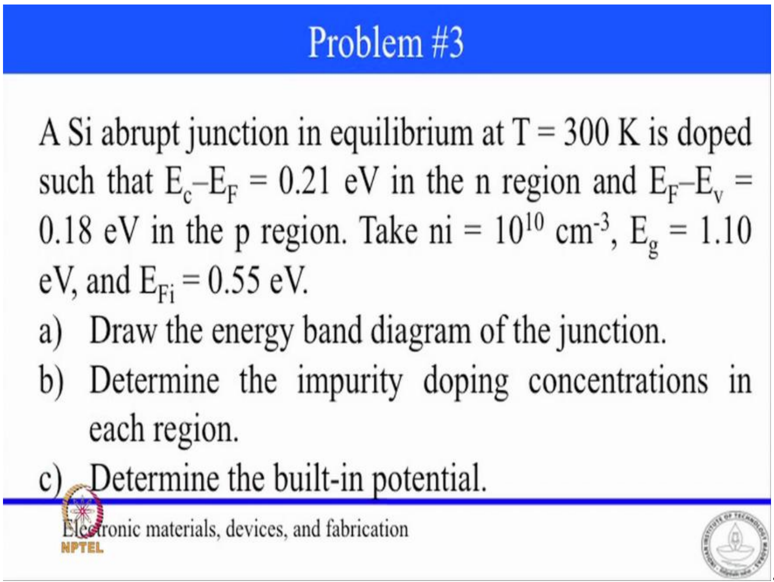

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In problem 3, we have a Silicon abrupt junction which is in thermal equilibrium at room temperature.

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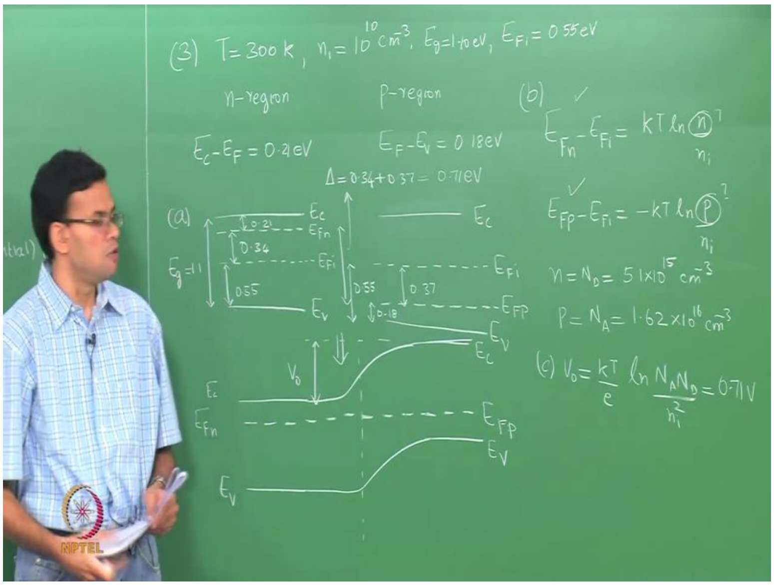

So, temperature T is 300 kelvin, it is doped in such a way such that E_c - E_F is 0.21 electrons volts in the n-region. So, you have an n-region and you have a p-region. So, In this question, the doping concentrations are not given, but the position of the Fermi level is. Here, E_c - E_F is 0.21 electron volts and $E_F - E_v$ is 0.18 electron volts the material is silicon. So, you have somehow with the parameters of silicon n_i which is 10^{10} cm^{-3}. The band gap of silicon E_g is 1.10 electron volts the position of the intrinsic Fermi level E_{Fi} it is not exactly at the center, but it is very close to the center, we can take this 0.55 electron volts. These are some of the parameters of intrinsic silicon that we can use. So, The first part of the question says; draw the energy band diagram for the p-n junction. So, before we do that let me first draw the energy band diagram for the 2 regions separately and then we can put them together to draw the energy band diagram for the junction.

So, on the n-side this is my conduction band, this is my valence band, this gap is nothing but E_g which is 1.1 E_{Fi} is at the center of the gap so that is 0.55 and E_g is 1.1. This question says E_c - E_F is 0.21. So, the Fermi level on the n-side E_{Fn} is 0.21 electron volts. So, This height which is nothing but 0.55 - 0.21 is 0.34, So, all the energy are in electron volts. I am just not writing the units, but everything is in eV, we can now do the same for the p-region. So, the material is the same. The band gap is the same you just draw this slightly better, So, it is the same silicon. So, E_c and E_v are located in the same place, E_{Fi} will also be located in the center. E_{Fi} in this particular problem E_F - E_v is given to be 0.18 electron volts. So, E_{Fp}, this is 0.18. So, E_{Fi} to E_v is 0.55, so that this height is nothing, but

0.37. So, this is 0.55, this is 0.18, this is 0.37.

So, We have the energy band diagrams of the n and the p-region separately, we can put them together and draw the energy band diagram of the p-n junction, but before we do that I will like to calculate the concentration of electrons and holes on the n and the p-side, so that we can do a basically using the formula E_{Fn} - E_{Fi} is $kT \ln \frac{n}{n_i} (E_{Fp} - E_{Fi}) = kT \ln \frac{p}{n_i}$. So, The position of the Fermi level is related to the concentration of the majority carriers on the n-side, it is your donors on the p-side it is the acceptors. So, Here this term is known and this is the unknown, same way here this is known and this is the unknown. So, E_{Fn} - E_{Fi} is 0.34 and E_{Fp} - E_{Fi} is -0.37. So, we can substitute in the values, so that we get $n = N_D$ just equal to 5.1×10^{15} cm^3 p is nothing but N_A is slightly higher $1.62 \times 10^{16} cm^{-3}$. So, even without doing the numbers we could have predicted that N_A will be higher than N_D simply because the Fermi level on the p-side is located much is closer to the valence band it is only 0.18 compare to the Fermi level on the n-side which is 0.21 electron volts below the conduction band.

So, you have drawn the energy band diagrams separately. We also have the concentration of the electrons and holes. So, Let me draw the energy band diagram, when the junction is formed to do that we know that the Fermi levels must essentially line up at equilibrium. So, this is E_{Fn} this is E_{Fp} far away from the junction you still have an n-type semiconductor and you still have a p-type semiconductor. Let me just arbitrarily mark and interface between these 2 and we can show the bands bending, so that these 2 join. This is E_v, this is E_c, this is E_c this is E_v. So, you have the Fermi levels lining up, and there is a built-in potential. This is a straight line and there is a built-in potential V_o formed at the junction.

So, part-b we need to determine the concentration of the impurities, we actually just did that. So, This is essentially part-b just by looking at the shift in the Fermi levels we can calculate the concentration of the impurities. Part-c, we want to calculate the built-in potential. So, V_o is nothing, but $\frac{kT}{e} \ln \frac{N_A N_D}{n_i^2}$, we can do all the substitution and the numbers. This works out to be 0.71 volts. We can also calculate the built-in potential by looking at the energy band diagram. So, In this particular case, the distance between E_{Fn} and E_{Fp} is essentially $0.34 + 0.37$. So, this distance delta is $0.34 + 0.37$ which is 0.71 electron volts. So, When the junction is formed, we know that the Fermi levels have to line up. So, we can think of as either the n-side shifting completely by 0.71 or the p-side shifting up by the same 0.71, so that they line up. So, the built-in potential or the built-in voltage is nothing, but the difference between the Fermi level positions. So, This is 0.71 electron volts; if you divide by e, it is 0.71 volts. So, instead of using the formula you can also calculate the built-in potential by just looking at the energy band diagram.

So, let us now go to problem 4.

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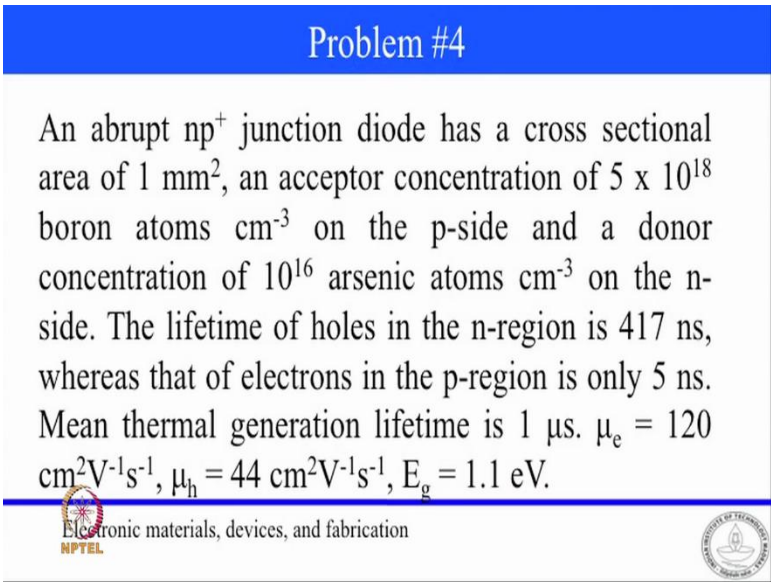

So, In problems 1, 2, 3 we looked at the p-n junction in equilibrium, so that there was external potential applied and no current was flowing through the junction. In problem 4, which is slightly a long problem, we are going to look at a p-n junction and this essentially biased. And we are going to calculate some values for the current in the forward and the reverse bias. So, problem 4 you have an abrupt pn^+ junction.

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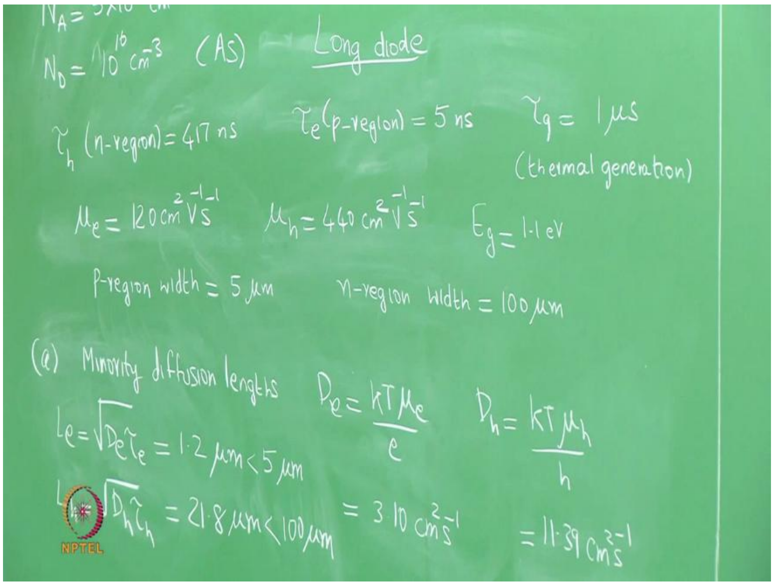

When we say a pn^+ or np^+, the plus essentially denotes that this is heavily doped. So, when one of the carriers or when one of the sides of a p-n junction is heavily doped then the depletion region lies almost entirely on the other side. So, one way to see that is to go back to this equation. So, $N_A W_p = N_D W_n$, So, when N_D is much greater than N_A, this implies W_n is much smaller than W_p, so that the depletion width is almost entirely on the p-side. I will also just write the reverse, when N_A is much greater than N_D then you have W_p much smaller than W_n and the depletion is almost entirely on the n-side. So, We have an abrupt p-n junction, the cross sectional area A is $1\ mm^2$, we will use the cross sectional area to calculate the current the acceptor concentration of 5×10^{18} boron atoms on the p-side. So, N_A is $5 \times 10^{18} cm^{-3}$ and there is a donor concern. This is boron, there is a donor concentration N_D is $10^{16} cm^{-3}$ and this is arsenic.

In this problem N_A is much higher than N_D. So, this should actually read p^+n not p and + it is my mistake because N_A is much greater than N_D. So, we have 5×10^{18} boron and 10^{16} arsenic atoms on the n-side the whole life time values are also given. So, the life time of the whole τ_h in the n-region. So, These are your minority carriers; this is equal to 417 nanoseconds. Similarly, the life time of the electrons in the p-region is only 5 nanoseconds and this difference is because of the difference in concentration of the dopants, the thermal generation life time is also given. So, τ_g is 1 micro second. Some other values are also given for this problem, μ_e which is the mobility of the electrons. So, $120\ cm^2\ V^{-1}\ s^{-1}$ μ_h is 440 $cm^2\ V^{-1}\ s^{-1}$ and E_g is 1.1 ev. The length of the p and n-regions are also given. So, you have a p-region with is 5 micro meters and the n-region with this 100 micro meters. These are a whole set of data there is given about the silicon p-n junction.

So, the first thing we need to calculate is the minority diffusion length and to determine what type of diode this is. So, We want to calculate the minority diffusion lengths. So, In the case of a p-n junction under equilibrium we have a dynamic equilibrium, so that electrons and holes are moving across the junction and constantly get annihilated, when we apply a forward bias the p-side is connected to the positive the n-side is connected to the negative the Fermi levels no longer line up, but essentially get shifted and when this happens the barrier comes down. So, V naught is the built-in potential or the barrier during equilibrium when you apply an external potential the barrier is V_o - $V_{external}$ when the barrier goes down it basically have minority carriers moving across the junction. So, we have electrons from the n-side moving to the p-side where they are minority carriers. We also have holes from the p-side moving to the n-side and there they are the minority carriers.

So, It is this minority carrier diffusion that essentially causes current to flow in a p-n junction. So, the first thing you want to calculate is the diffusion lengths to know the diffusion lengths we need to know the diffusion coefficients. So, D_e which is the diffusion coefficient of electrons is nothing, but $\frac{kT\mu_e}{e}$. So, it depends upon the mobility and D_h is $\frac{kT\mu_h}{h}$. So, the values of μ_e are given μ_h is given everything else is a constant. So, we can plug it in, so that D_e is 3.10 cm^2 s^{-1}, D_h is 11.39 cm^2 s^{-1}. So, D_h is higher than D_e because μ_h is higher than μ_e and this is because the holes are diffusing on the n-side and the concentration on the n-side is two orders of magnitude less than the p-side. So, because you have less concentration of your dopants the diffusion coefficients are higher from the diffusion coefficients we can calculate the length. So, l is nothing but $\sqrt{D\tau}$. So, L_e is D_e τ_e L_h is D_h and τ_h. So, once again D_e and D_h we have calculated τ_e and τ_h are given. So, we can substitute the numbers. So, L_e works out to be 1.2 micro meters, I am not doing the math so, all your units are in centimeters. So, your answer will also be in centimeters, you can just convert them into micro meters. So, L_e is 1.2 and L_h is larger it is 21.8 micro meters.

If you looked at the length of the diodes on the p and the n-side on the n-side the diode length is 100 on the p-side the diode length is 5 micro meters. So, L_e is smaller than the 5 micro meters which is the length on the p-side. L h is smaller than 100 micro meters which is the length on the n-side, so that this is essentially a long diode. So, a long diode is 1 in which the diffusion lengths are smaller than the physical lengths of the p and n-region. So, This is part-a, now let us go to part-b.

In part b, we want to calculate the built-in potential across the junction. This is the potential when the junction is in equilibrium.

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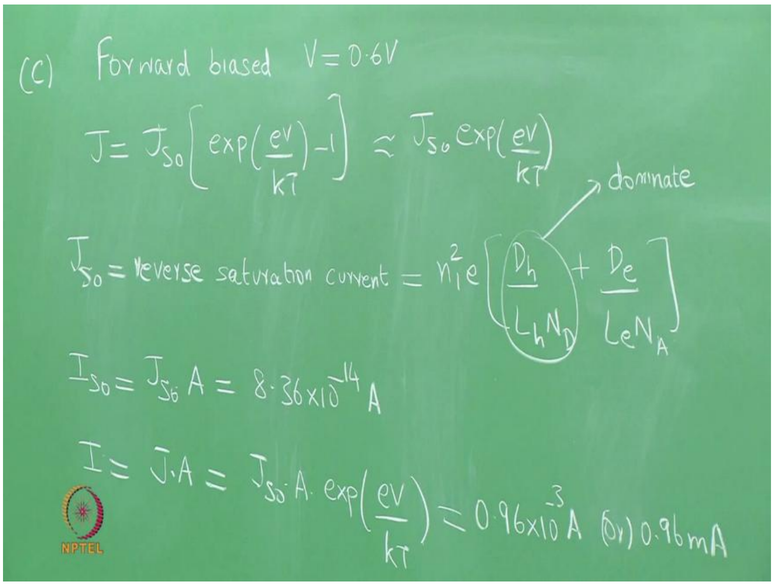

This is fairly straight forward. So, just $\frac{kT}{e}\ln\frac{N_A N_D}{n_i^2}$. So, we have all the numbers we just need to substitute that this works out to be 0.877 volts. So, this particular problem does not ask you to calculate the depletion widths, but you can go ahead and do the calculation and you will find that the depletion width is almost entirely on the n-side and that there is a very small depletion width on the p-side.

Part-c, what is the current when there is a forward biased of 0.6 volts across the diode. So, Now, you have the diode to be forward biased the external potential V is 0.6 volts. So, when we apply an external potential the barrier height is lowered. So, that there is an increase in current due to the minority carriers diffusing across the junction in this particular case the current density is given by a constant $J_{so} exp\left[\frac{eV}{kT} - 1\right]$ usually the exponential term is much higher than 1, so that this can be return has $J_{so} exp\left[\frac{eV}{kT}\right]$. J_{so} is your reverse saturation current and this is given by $n_i^2 e\left[\frac{D_h}{L_h N_D}+\frac{D_e}{L_e N_A}\right]$.

So, we saw the derivation for this during the course notes, but this is your reverse saturation current and this is something with plugs in here. If you remember the assignment from the schottky junctions, the metal semiconductor junctions we had a similar expression to this except that the constant out front had a different value which depended upon the thermionic emission, but now here we have a p-n junction. So, the constant here is your reverse saturation current. In this particular problem N_A is much higher than N_D and since they are in the denominator this term will essentially dominate over the other term. So, This is the reverse saturation current density, to calculate the current we just need to multiply this by the area. So, all the numbers are here we calculated D_h and L_h. In part-a N_A and N_D are known n_i is also known, so that J_{so}, instead of J_{so}, I will directly write I_{so} which is J_{so} times the area. So, this is 8.36×10^{-14}A. So, the reverse saturation current is essentially a really small value. Once you know I_{so} or J_{so} can calculate the current during forward bias. So, I is nothing but J times which is J_{so} A $\exp\left(\frac{eV}{kT}\right)$, V is 0.6 that is given. So, the current works out to be 0.96×10^{-3} A or 0.96 milli amperes. So in the current in the forward biased is 0.96 milli amperes, so that is nearly 10^{11} orders of magnitude or 10^{10} orders of magnitude higher than the reverse saturation current. This is why we essentially call a p-n junction to be a rectifier because it conducts very well during forward bias and the reverse biased current is very small.

So, let us now go to part-d. In part-d, we want to estimate the forward current at 100 degrees. So, the temperature is now increased.

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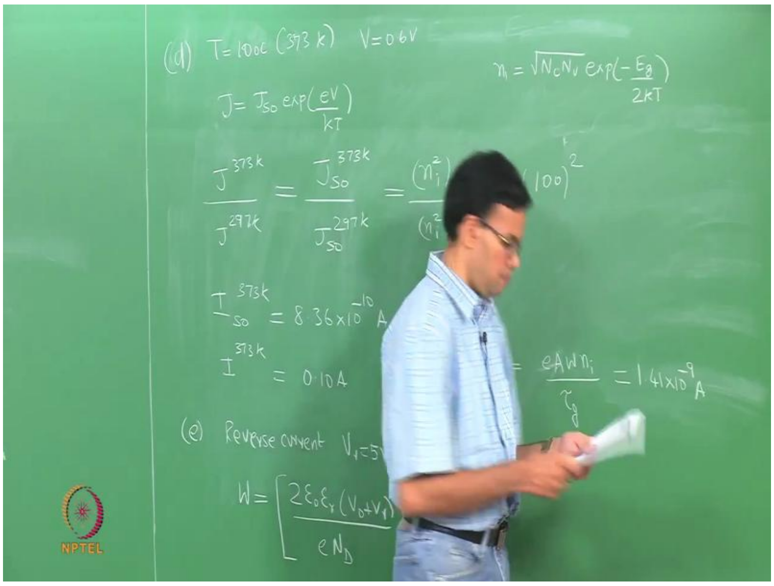

You can write this in Kelvin, so that is 373 kelvin the voltage is the same. So, V is 0.6 volts, the question also says that assume the temperature dependence of n_i dominates over everything else. So, over the diffusion lengths the diffusion coefficients itself also the mobilities, if only take n_i into consideration. So, we can see that the current or if you write this down J is J_{so} $\exp\left(\frac{eV}{kT}\right)$. So, the ratio of J at 373 kelvin to that a 273 or 293 kelvin, this should be 373. So, 373 by 293 kelvin or 297 kelvin which is room temperature.

Let me just draw this right, this is 297 is nothing but J'_{so} or J_{so} at 373 kelvin divided by J_{so} 297 kelvin. So, the potential is the same. So, the ratio of the currents or the ratio of the current densities is nothing but the ratio of the reverse saturation currents this is directly proportional to n_i^2, so that this is n_i^2 at 373 kelvin divided by n_i^2 297 kelvin n_i^2 is nothing or n_i $\sqrt{N_c N_v} exp\left(\frac{-E_g}{2kT}\right)$. So, for this problem we can take N_c and N_v to be independent of temperature. So, the ratio of n_i is just given by the exponential term. So, if you do this ratio works out to be approximately 100 so that the reverse saturation current is increased by 100, when we go from room temperature to 100°C. So, The new values of I_{so} 100 square so, the new values of I_{so} at 373 kelvin. Just write down the final answer is 8.36×10^{-10} and current I at 373 kelvin 0.10 amps. So, that the current essentially increases by 4 orders of magnitude.

In part-e, we want to calculate the reverse current when you have a voltage of 5 volts.

So, we want to know the reverse current, when V_r is 5 volts. To calculate the reverse current, we first need to calculate the new width when you apply a reverse bias. So, the width W is $2\varepsilon_o\varepsilon_r (V_o + V_r)$ and we said that the depletion region lies almost entirely on the n-side. So, I only have e N_D whole to the half. So, This formula is something we used before to calculate the width of the depletion region. When you have a p-n junction under equilibrium, So, you only modified it to add the reverse biased voltage and we also removed the contribution due to N_A because N_D is much smaller than N_A. So, we can plug in the numbers; the new depletion width comes out to be 0.88 micro meters and most of this is in the n-side. When you have a reverse biased, you have thermal generation of carriers within this depletion region and this thermal generation of carriers is responsible for your reverse current. So, I gen which is the current due to thermal generation of carriers is e times the cross sectional area times the depletion width times $\frac{n_i}{\tau_g}$ where τ_g is the thermal lifetime of the carriers in that value is also given. So, everything here is known, we can substitute the numbers and I gen works out to be 1.41×10^{-9} A. This number is again much smaller than your forward biased current which is of the order of milli amps. So, once again even if you take thermal generation of carriers into account, we essentially have a rectifying action in a p-n junction. So, Let us now look in the last question.

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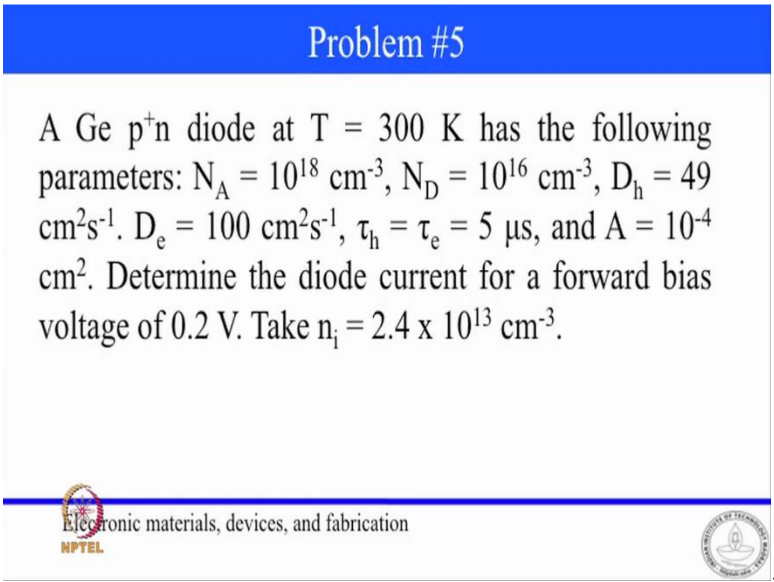

So, problem 5 you have a germanium p-n junction diode.

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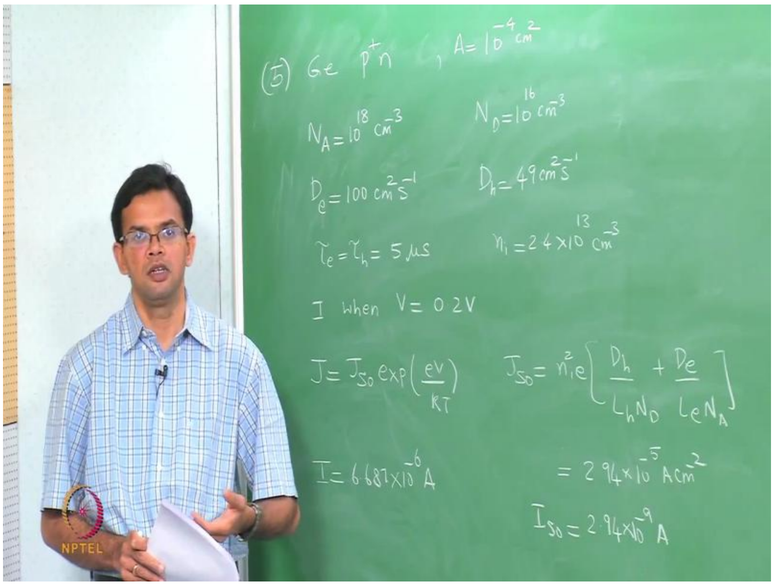

So, it is germanium p^+n the values are given. So, N_A is $10^{18}cm^{-3}$ N_D is 10^{16} D_h. So, D_h; I will write on this side. So, D_h is 49 cm^2 s^{-1} .D_e because we are look at minority carriers is 100 cm^2 s^{-1}; $\tau_e = \tau_h$ which is equal to 5 microseconds in the cross sectional area is $10^{-4}cm^{-2}$. So, we want to calculate the diode current, we want to calculate I when we have a forward bias V of 0.2 volts n_i is 2.4 x 10^{13} cm^{-3}.

This is very similar to the previous question, so, J is J_{so} $\exp\left(\frac{eV}{kT}\right)$ and then J_{so} is $n_i^2 e\left[\frac{D_h}{L_h N_D} + \frac{D_e}{L_e N_A}\right]$. So, all the numbers are given. So, J_{so}; I will just substitute and write the final answer, but you can directly do the substitution J_{so} is 2.94 x 10^{-5} A cm^{-2}. So, the current I_{so} is 2.94 x $10^{-11.9}$ A. Once you know I_{so}, we can substitute here and we can get the current the current I is nothing but 6.687 x 10^{-6}.

So, In this particular case, the difference between the current and I_{so} is not as high as in the case of silicon. One particular reason is because your applied voltage is very small, it is only 0.2 volts and another difference is that the material is germanium, so that the band gap is smaller. The built-in potential is also smaller at the same time n_i is larger, so that J_{so} is also larger. So, these are some of the factors you have to take into account and choosing materials performing p-n junction.