Fundamentals of electronic materials, devices and fabrication
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Assignment - 3
Extrinsic semiconductors

In today's assignment, we will be looking at Extrinsic semiconductors, Assignment 3. In assignment 2, we focused exclusively on intrinsic or pure semiconductors. Today we will be looking purely on extrinsic semiconductors. So, before we look at the numeric problems let me do a brief recap. Extrinsic semiconductors are also called doped semiconductors.

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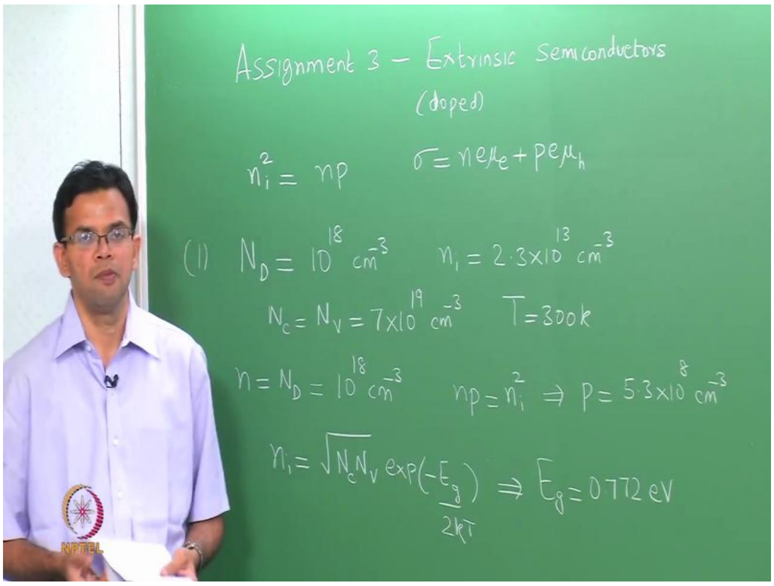

So, The whole process of doping is to selectively increase your carrier concentration. So, either n or p, the law of mass action is something that has to be obeyed. So, n_i^2, where n_i is your intrinsic carrier concentration must be equal to n p. So, If we doped with donor type impurities to increase the concentration of electrons, so if n goes up, p has to go down and similarly, if p goes up by doping with acceptors then n has to go down.

We also saw the conductivity equation last time sigma is $n e \mu_e + p e \mu_h$. In the case of an extrinsic semiconductor, we usually dope such that either n is much greater than p or p is much greater than n either way only one of these terms will usually dominate. So, the conductivity will be either due to the motion of electrons or due to the motion of holes in case of an extrinsic semiconductor once again because, we do not have equal number of electrons in holes. The fermi level will shift from the center of the gap it will shift closer to the conduction band for an n-type semiconductor and closer to the valance band if it is a p-type semiconductor. This will also depend upon the temperature and whether all the donors or acceptors are ionized. These are some of the concepts that we will touch in today's assignment.

Let me look at question one. We have a group 4 semiconductors is doped with donor atoms N_D, the donor atom concentration is 10^{18} cm^{-3}, the intrinsic carrier concentration is given. So, n_i is given 2.3×10^{13} cm^{-3}, the values of N_c and N_v, the effective density of states is also given. So, $N_c = N_v = 7 \times 10^{-10}$ to the 19 cm^{-3}. so, the sample is essentially at room temperature. Temperature T is 300 kelvin. So, the first questions says what is the hole concentration at 300 Kelvin. It is doped with donor ions. So, it is an n-type semiconductor. So, n is equal to N_D at room temperature the impurities are usually fully ionized. So, n is equal to $N_D = 10^{18}$. To calculate the hole concentration we can use the law of mass action. So, n p = n_i^2 the value of n_i is also given, p works out to be 5.3×10^8 cm^{-3}. So, the concentration of holes is much smaller, nearly 10 orders of magnitude smaller than that of the electrons.

What is the band gap of the semiconductor? So, To calculate the band gap we can actually use the intrinsic equation, which we saw in assignment 2. So, n_i is $N_c N_v exp\frac{-E_g}{2kT}$. So, N_c and N_v values are given temperature is known, n_i is known. The only thing that is remaining is E_g and E_g is 0.772 electron volts. We can sought of make a guess that the material is germanium, but germanium usually has a band gap of around 0.67 because in this particular problem we have taken N_c is equal to N_v and that is not true for germanium, but this is the band gap. The band gap is a low value we can see that because n_i is around 10^{13}, for pure silicon the value of n_i at room temperature is 10^{10}. So, you have calculated the hole concentration and also the band gap. Then you want to know the position of the fermi level in the doped semiconductor with respect to the intrinsic firmer fermi level.

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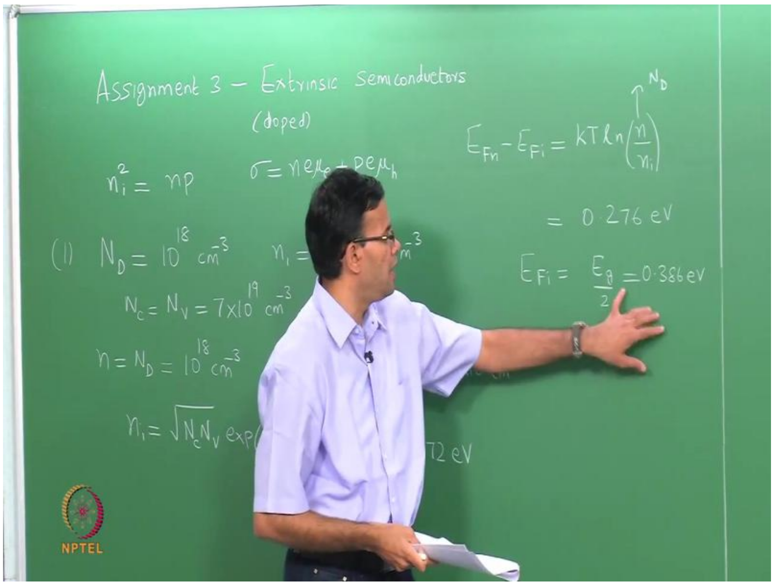

So, E_{Fn} - E_{Fi}, E_{Fn} is a position of the fermi level in the type semiconductor, E_{Fi} is a position in the intrinsic semiconductor is nothing, but $kT \ln \frac{n}{n_i}$. In this particular case n is nothing, but N_D. So, we can substitute all the values, we can use k in the SI units, but then we need to divide by 1.6 x 10^{-19}. So, that we convert it back to electron volts and if you do this E_{Fn} - E_{Fi} is 0.276 electron volts.

In this particular case $N_c = N_v$, we can also calculate and absolute value for E_{Fi} which is nothing, but $\frac{E_g}{2}$, E g is 0.772. This is nothing, but 0.386. So, we can substitute for the value of E_{Fi} here and get the position of the fermi level with respect to the valence band as well.

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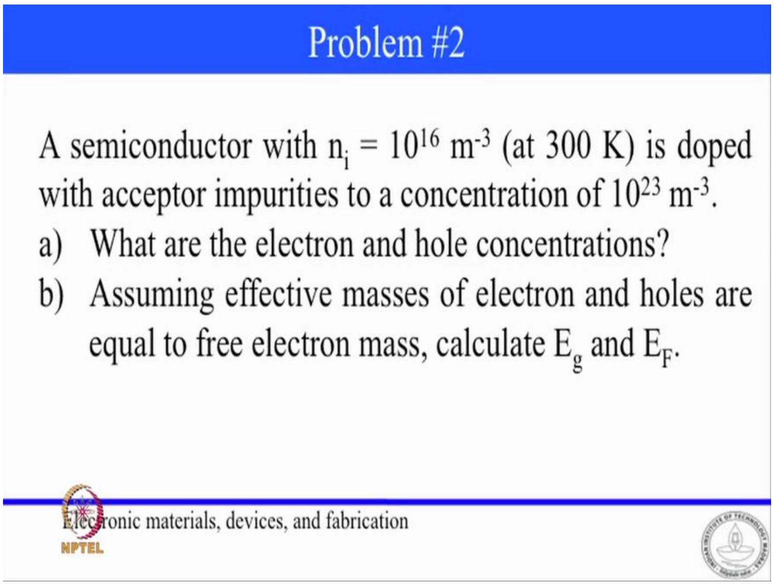

So, let us now move to question number 2. So, we have a semiconductor with the value of n_i.

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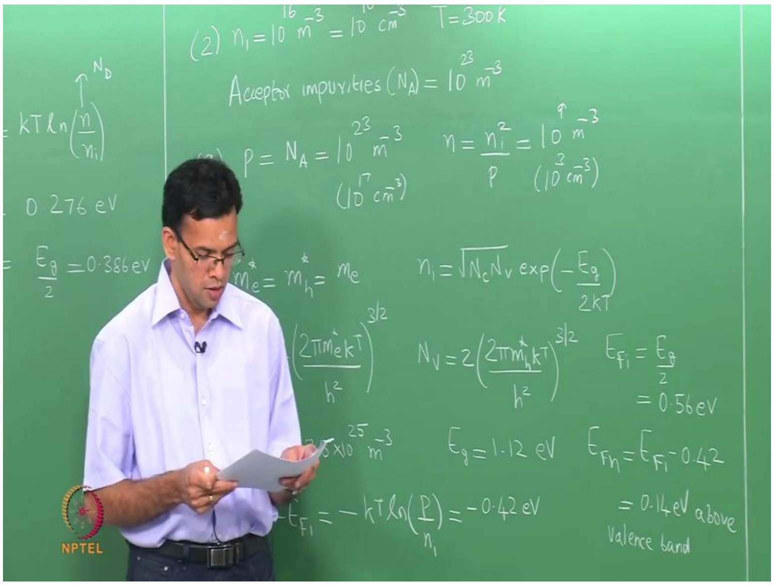

So, n_i is 10^{16} m^{-3}. This we can keep as meter cube or you can convert it to centimeter cube. So, this will be 10^{10} cm^{-3}, but for now we will just work with m^{-3}. A semiconductor with n_i equal to 10^{16} m^{-3} is doped with acceptor impurities. The concentration of acceptor impurities is also given N_A is 10^{23} m^{-3}.

Donor impurities basically produce electrons if we go back to the class; donors are typically group 5 elements. So, If we think about silicon donor impurities are phosphorus arsenic, antimony which donate the extra electron acceptor impurities are elements like boron, aluminum and gallium which accept the extra electron from silicon and create a hole. So, acceptor impurities produce excess holes donor impurities produce excess electrons, so, we have acceptor impurities. So, Once again we want to calculate the electron and hole concentration, So, the temperature is given is 300 Kelvin, So, N_A is fully ionized. So, all the acceptors are ionized. So, p which is the hole concentration is equal to N_A is equal to 10^{23} m^{-3}. We can again use the law of mass action $n = \frac{n_i^2}{p}$. So, we can do the numbers n is 10^9 m^{-3}.

If you want to convert these to cm^3 divide by 10^6. This is 10^3 cm^{-3} and this is 10^{17} cm^{-3} assuming that the effective masses in part b. So, assuming that the effective masses of electrons and holes are equal to the free electron mass, calculate E_g the band gap and E_f, the position of the fermi level, this is part a. In part b, we are given that m_e^*which is the effective mass of the electron which is m_h^* is nothing, but m_e is a rest mass of the electron. So, once again we have to calculate E_g. We know the value of n_i, So, n_i is $\sqrt{N_c N_v} exp\frac{-E_g}{2kT}$. The thing here is we are not given the values of N_c and N_v, but these can be calculated from the mass of the electrons and holes. So, N_c is nothing, but $N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$, N_v is $N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$. So, m_e^* and m_h^*values are given they are equal to the mass of the electron. We can evaluate N_c and N_v. So, $N_c = N_v$ and it is equal to 2.5×10^{25} m^{-3}. So, You can convert this to cm^3 as well you will just have to divide by 10^6.

Now, we know the values of N_c and N_v, n_i is known, n_i is given to be 10 the 16. So, we can go ahead and calculate E_g the value of E_g is 1.12 electron volts. Once again the material that we are talking about here is silicon; we can clearly see that n_i is 10^{10} cm^{-3}, which is what the room temperature intrinsic carrier concentration of silicon is. So, We now have to calculate the position of the fermi level; this is a p-type material because we have acceptors rather than donors. In the case of donors, the fermi level moves closer to the conduction band. In the case of acceptors, the fermi level moves closer to the valence band. So, $E_{Fp} - E_{Fi}$ is nothing, but $-kT \ln\frac{p}{n_i}$. So, p we known, n_i is known, So, we can substitute and this is minus 0.42 eV. So, This is below the intrinsic fermi level and it is -0.42 eV below the fermi level. We can calculate the value of E_{Fi}, E_{Fi} is nothing, but just

$E_g/2$. So, E_g is calculated to be 1.12. So, E_{Fi} is 0.56, so that we can substitute here that gives you E_{Fn} is nothing, but E_{Fi} - 0.42, which is 0.14 eV above the valence band.

So, In a way problem 2, the part b is very similar to what we did in problem 1 except that now we all have acceptor impurities instead of donor impurities. So, in part c questions says donor impurities are now added to a concentration of 5×10^{22}, what are the new values for the 4 quantities calculated above.

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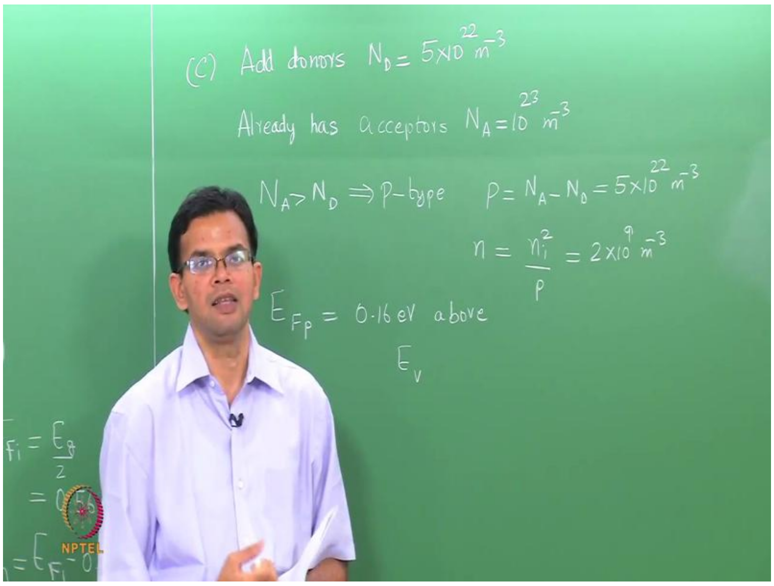

So, We now add donors and the concentration N_D is $5 \times 10^{22}\ m^{-3}$. The sample already has acceptors, acceptor concentration N_A is $10^{23}\ m^{-3}$. So, This is an example of compensation doping where we have both donors and acceptors.

In this particular case, N_A is greater than N_D, so, ultimately the material becomes p-type and the concentration of holes is just N_A - N_D. So, p is $5 \times 10^{22}\ m^{-3}$ n. We can calculate as before again using the law of mass action $\frac{n_i^2}{p}$, which is $2 \times 10^9\ m^{-3}$. We can go ahead and calculate the position of the fermi level. So, E_{Fp} we can repeat the calculation that we did before, except using the new values of p. So, E_{Fp} if we calculate becomes 0.16 electron volts above ev. The band gap will not change because the band gap is calculated based on the intrinsic values, but because the values of p and n change the position of the fermi level will change.

So, let us now move to question 3.

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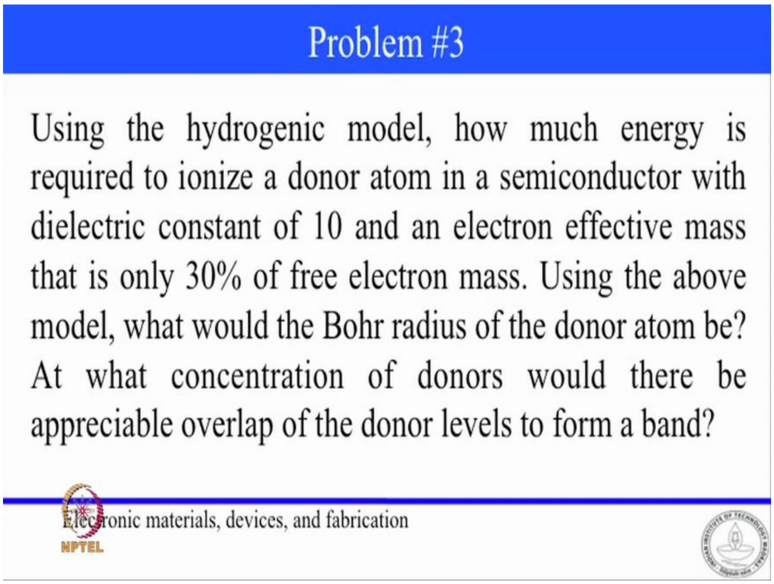

Question 3: Using the hydrogenic model, how much energy is required to ionize a donor atom in a semiconductor with a dielectric constant of 10 and an electron effective mass that is only 30 percent of the free electron mass? So, we look at the ionization energy of a donor or an acceptor earlier, we found that these donor levels are close to the conduction band and the acceptor level is close to the valence band and typical ionization energies are of the order of tens of milli electron volts. This is why these levels are usually fully ionized at room temperature.

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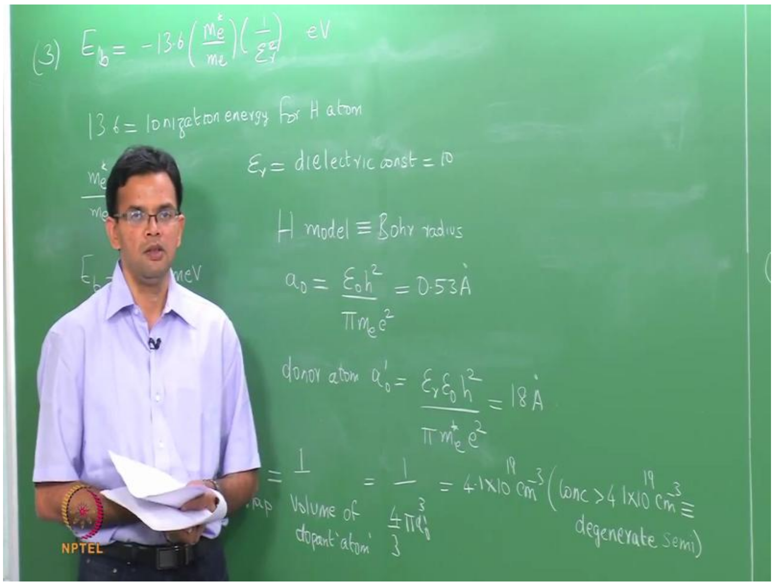

So, to look at the ionization energy E_b, we typically used a hydrogenic model, but we modify the mass of the electron and also the dielectric constant. So, the equation for that is something similar to what we worked out in class is $-13.6 \frac{m_e^*}{m_e} \frac{1}{\varepsilon_r^2}$, this answer will be in electron volts. So, 13.6 is the ionization energy for the hydrogen atom this value is in electron volts $\frac{m_e^*}{m_e}$ is nothing, but the electron effective mass. In this particular problem, it says the electron effective mass is 30 percent of the free electron mass. So, $\frac{m_e^*}{m_e}$ is 0.3 ε_{or} is the relative permittivity of the semiconductor, So, again for this particular problem the permittivity or the dielectric constant is given as 10. So, we have all the values that we need we can substitute that and evaluate E_b and E_b. If we calculate comes out to be -41 milli electron volts, So, this is the ionization energy of the donor atom. So, This is the energy that is required to remove the electron from the donor atom and take it to the conduction band for comparison the thermal energy of an electron at room temperature is 25 milli-electron volts. So, at room temperature it is possible to easily ionize the donors and take the electron to the conduction band.

In the next problem, In problem 4, we will look at those calculations explicitly. In part 2 of problem 3, we also need to calculate the Bohr radius of the donor atom. So, once again we are using the hydrogen model and you want to calculate the Bohr radius, the hydrogen model the Bohr radius a_o is given by the expression $\frac{\varepsilon_o h^2}{\pi m_e e^2}$. This is again based on calculations and the Bohr radius works out to be 0.53 A°.

So, In this particular case, we can use the same expression, at ε_o. So, for the donor atom, the Bohr radius, I am going to call it $a_o^{'}$ is nothing but $\varepsilon_r \varepsilon_o h^2$. So, we replace ε_o by ε_r times $\varepsilon_o$and m_e is replaced by m_e^*. So, The expression is the same except that we are adding the dielectric constant and also the electron effective mass. So, We can plug in these numbers and this gives you a Bohr radius of around 18 A° the way to think about this is that this represents the influence of the donor electron. So, it represents a size of the influence of the donor electron.

So, In the case of an extrinsic semiconductor we usually treat these donor levels as individual atomic levels. So, The concentration is usually of the order of part per million or parts per billion, so that we treat them as individual atomic levels, but if we keep on increasing the concentration of the donors then these atomic levels will come together and when 2 donors see each other, which means when the distance between them comes to be less than 18 A°, then they will start to interact and they will form a band. Usually, this high concentrations donor formed instead of single atomic levels a donor energy band which can then overlap with the conduction band. So, these types of semiconductors are called Degenerate Semiconductors.

So, to calculate the number of dopants or the dopant concentration when we have degenerate semiconductors, So, I will call it donor overlap, it is nothing, but 1 over the volume of the dopant atom. I will put the word atom here with in parenthesis to actually talk about this sphere of influence of your donor. So, this is nothing, but $\frac{1}{\frac{4}{3}\pi {a'_o}^3}$. This particular value works out to be 4.1 x 10^{19} cm^{-3}.

So, At this concentration and at higher values of concentration your donor atoms are essentially too close, so that the atomic levels mingle and we have a donor band. So, This determines the conditions performing a degenerate semiconductor. So, when the concentration is greater than 4.1 x 10^{19} cm^{-3}, we get a degenerate semiconductor. So, This is a simple back of the envelop calculation where we use the effective Bohr radius of the donor atom to calculate the concentration, where we get a degenerate semiconductor.

Let me now go to problem 4.

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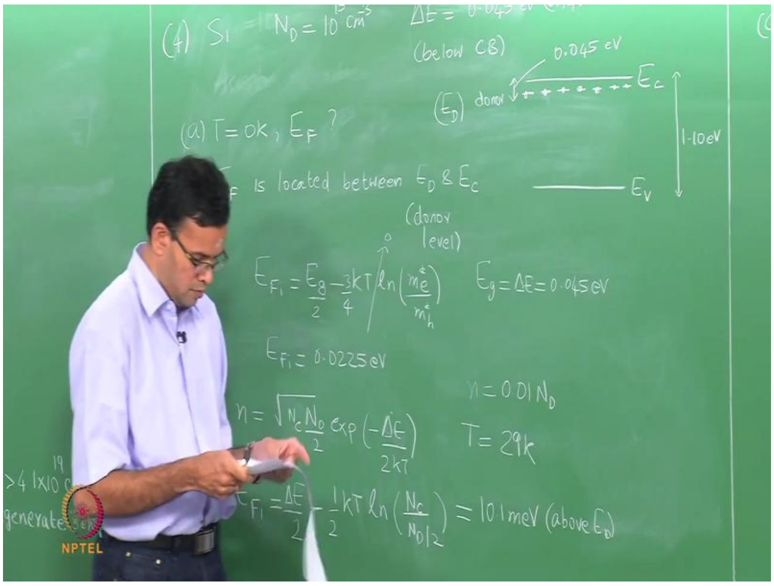

With 10^{15} phosphorus atoms, phosphorus is a donor. So, N_D is 10^{15} cm^{-3}. The donor energy level for phosphorus in silicon is 0.045 ev below the conduction band edge. So, ΔE, which is your donor ionization energy, is 0.045 ev or 45 milli ev and this is below the conduction band. So, CB is your conduction band.

So, Where is the fermi level located at 0 Kelvin? SoT = 0 Kelvin. We want to know the position of the fermi level. So, at low temperatures, if you think about the model so, let me draw schematic of the band diagram. This is E_c, this is E_v the material is silicon. So, your band gap is typically 1.10 electron volts. We have a donor level there is very close to the conduction band edge, so, this is your donor level and this energy is 0.045 ev. So, the diagram is not to scale, but this just shows you that the donor level is very close to the conduction band. So, At 0 Kelvin the donor level is not ionized, so, we basically have electrons. Here the valence band is at a much lower energy level. So, we can sought of ignore the valence band and we can treat this as an intrinsic semiconductor with the donor level being the valence band and the conduction level being the conduction band of silicon.

So, In this particular case E_F will be located between the donor level E_D and the conduction level E_c. This is nothing, but your donor level. So, let me mark that here as E_D and the temperature is 0 Kelvin. So, If you use this expression, E_{Fi} is $\frac{E_g}{2} - kT\ln$. So, $\frac{3}{4} kT \ln \frac{m_e^*}{m_h^*}$ temperature is 0 Kelvin. This term goes to 0 E_g is nothing, but ΔE which is 0.045 electron volts, so that E_{Fi} is half of that. So, 0.225 electron volts right in the middle of the donor level and the conduction band.

This is part a in part b, at what temperature is the donor 1 percent ionized and where is the fermi level located at this temperature? So, we start at 0 Kelvin, where we have a donor level that is completely that is not ionized at all and the conduction band that is completely empty we then start to increase the temperature, so that electrons from the donor level start to move and occupy the conduction band. So, We can again treat this as an intrinsic semiconductor. So, concentration of electrons is nothing N_c $N_D/2$ and the reason for the 0 is because these are individual atomic states. So, they can only take 1 electron $-E_D$ or –$\Delta E/2kT$. So, We are using the expression for an intrinsic semiconductor except that instead of writing N_v, we write $N_D/2$ and ΔE is the ionization energy n the question says is 1 percent ionized 0.01 N_D. So, everything else is known except for the temperature. So, This we can substitute in, so temperature we can calculate to be 29 kelvin. All we have done is to take the expression for an intrinsic semiconductor and then modify it. We can calculate the position of the Fermi level, So, E_{Fi} once again we can use the expression for an intrinsic semiconductor. So, it is $\frac{\Delta E}{2} - \frac{1}{2}$ and we will use the effective density of states, so, $ln\frac{N_c}{\frac{N_D}{2}}$. So, Once again if you plug in the numbers this works out to be 10.1 milli electron volts and this is above E_D.

So, Let us look at, part c. At what temperature does the fermi level lie in the donor energy level? The fermi level is complete; when the donor is completely ionized, E_F will be at E_D. So, E_F will be equal to E_D when the donors are completely ionized.

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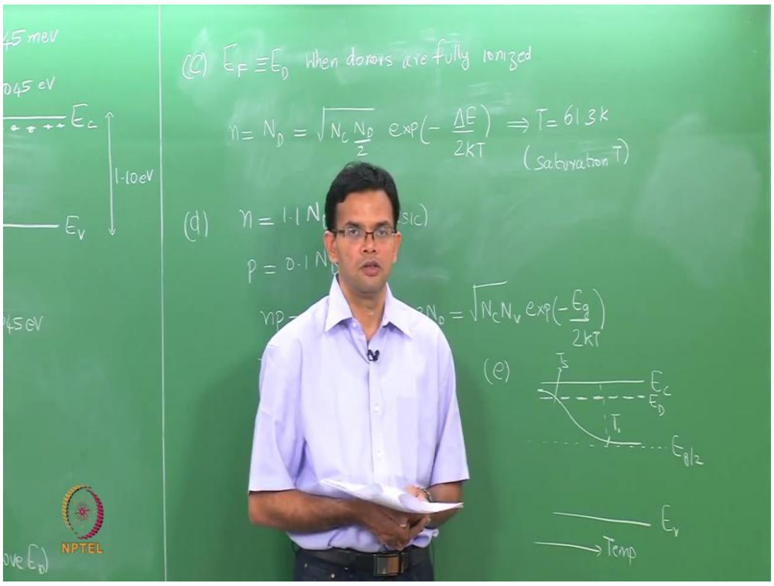

So, Once again n is equal to N_D is a $\frac{\sqrt{N_c N_D}}{2} exp\frac{-\Delta E_g}{2kT}$. So, Everything else is known except for the temperature. So, the temperature gives calculation gives you a temperature of 61.3 kelvin. So, at a relatively low temperature of around 60 kelvin, you get the donors to be completely ionized. So, estimate the temperature when the sample behaves as if it is intrinsic when the samples behaves like an intrinsic semiconductor n is around 1.1 N_D. So, if we go back to the nodes we say that the sample is intrinsic when the electron concentration is 10 percent more than the donor concentration.

So, We have a regime between saturation and this is your saturation temperature and the intrinsic temperature where the concentration of electrons is within 10 percent of the donor concentration. So, n is 1.1 N_D, p is nothing but 0.1 N_D, this is something we can get by just doing a charge balance, so that the net positive charge must be equal to the net negative charge. $n\ p = n_i^2$ which means n_i is 0.33 N_D, this is your intrinsic carrier concentration. So, this is now N_c N_v and these are the conduction and the valence band of the silicon $\frac{-\Delta E_g}{2kT}$.

We can substitute the numbers; temperature T is around 618 kelvin. So, the question also gives you the density of states. Now the value of N_c and N_v E_g is known, so, the only thing that is unknown is temperature. So, This temperature T is called your intrinsic temperature. This is called your saturation temperature, so that within the between the saturation and intrinsic your concentration of electrons which is your majority concentration is within 10 percent of the donor concentration

Part e, you are asked to sketch a schematic. So, In part e sketches schematically the change in fermi level with temperature. Let me just draw it here. So, this is your conduction band E_c, this is the valence E_v, these are my donor levels E_D. So, just for schematic let me take this to be a temperature access and this is the center of the band gap $E_g/2$. So, at zero kelvin your fermi level start to be here as a temperature raises electrons start to go from the donor level to the conduction band the fermi level drops at some particular temperature which is T_i or T_s the saturation temperature the donors are completely ionized and then the fermi level start to fall down and at really high temperatures it becomes equal to $E_g/2$. So, this temperature is T_i and within this particular regime your co electron concentration is almost a constant.

Let us now look at the 5th problem.

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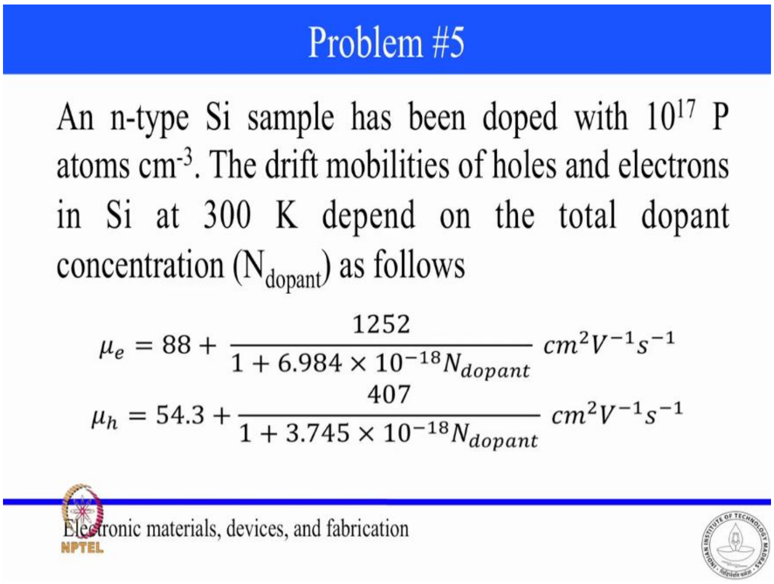

So, We have an n-type silicon, which is doped with phosphorus atoms.

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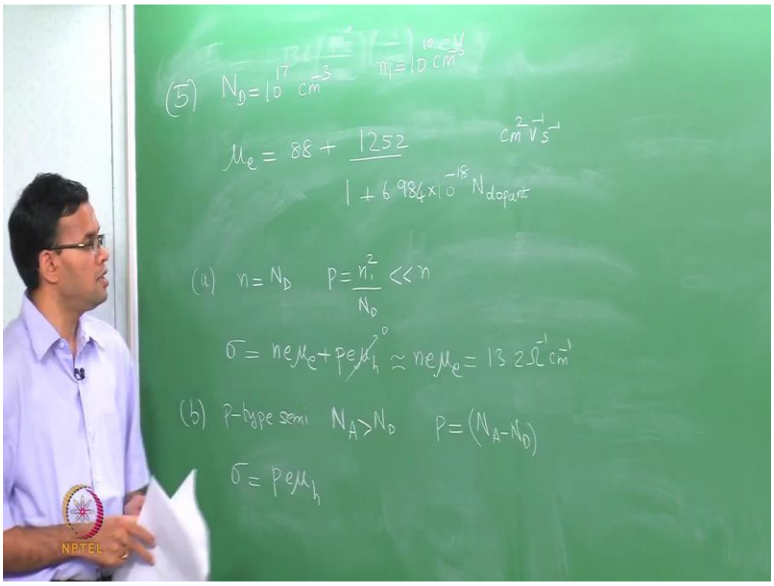

So, N_D is 10^{17} cm^{-3}, the drift mobilities of electrons and holes in silicon depend upon the total concentration of the dopant and the expression is also given. Usually, we take the drift mobility to be a constant, but actually with increase in doping especially in extrinsic semiconductors the mobility actually decreases. You have seen this in class, this is because we have the electron or the hole being scattered by the ionized donor or the acceptor. So, In this particular case, μ_e is given to be $88 + \frac{1252}{1+6.984 \, x \, 10^{-18} N_{dopant}}$ the units are cm^2 V^{-1} s^{-1}.

So, In the first part, we want to calculate the room temperature conductivity. So, we know n is equal to N_D it is an n-type semiconductor. So, p is n_i^2/N_D is usually much smaller than n. If it is silicon, n_i is 10^{10}. So, you can actually work it out p comes to be 10^3. So, In this particular case, conductivity σ is n e μ_e + p e μ_h p is much smaller than n. So, this term goes off. So, This is nothing, but n e μ_e, we can plug in the numbers to calculate μ_e we plug in the dopant concentration which is 10^{17} and you get the value of μ_e. So, If you do that, the value of σ comes to be 13.2 Ω^{-1} cm^{-1}.

In part b, we want to do compensation doping in this system. So, we are going to add acceptors to make this sample p-type with having the same conductivity value. So, Now, we have a p-type semiconductor and we do this by adding acceptors N_A and this must be greater than the donor concentration N_D. So, p in this case is nothing, but N_A - N_D and conductivity σ is p e μ_h. So, There is a similar expression for μ for the p-type or similar expression for the mobility for the holes μ is 54.3 + 4071 + 3.745 18 N_{dopant}.

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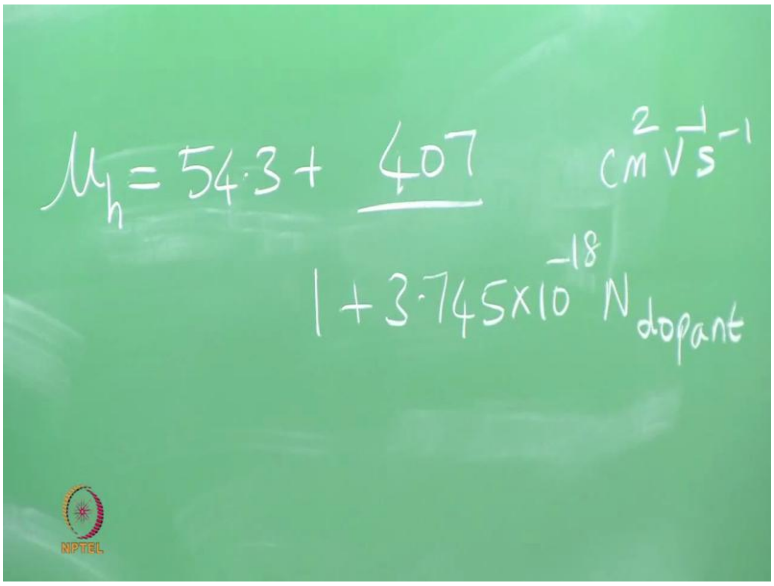

So, The important thing to remember here is that, this is the case where we have acceptor and donor impurities. So, the total dopant concentration in part b is equal to N_A + N_D. So, We use the same equation σ; it is now p-type. So, it is p e μ_h p is nothing, but N_A -N_D and

N_D we know mu h we can use this expression except that n dopant will be $N_A + N_D$. So, What we have is an equation where everything is known except for n a. We can simplify this expression, I won't go through the math, but this is essentially a quadratic equation which you can solve.

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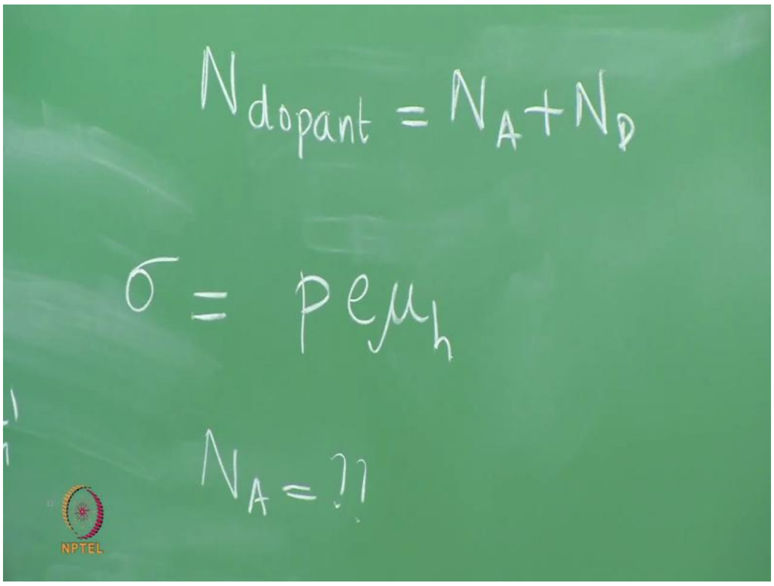

And when you solve you get the final value of N_A and the value of N_A is 3.04×10^{17}. So, This is greater than the value of N_D, which is 10^{17}. So, what you have is essentially a p-type semiconductor, where you have done compensation doping by adding excess amount of acceptors.

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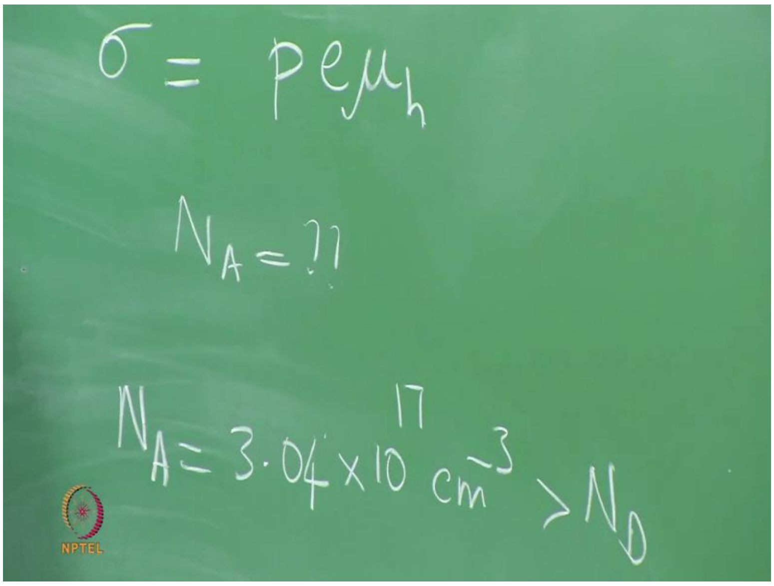

So, the main point here is that when you have whether an n-type or a p-type, the conductivity is essentially determined by the majority charge carriers.