Electronic Materials, Devices and Fabrication
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Module - 01
Assignment - 02
Intrinsic Semiconductors

In today's assignment class, we will be looking fully at intrinsic semiconductors.

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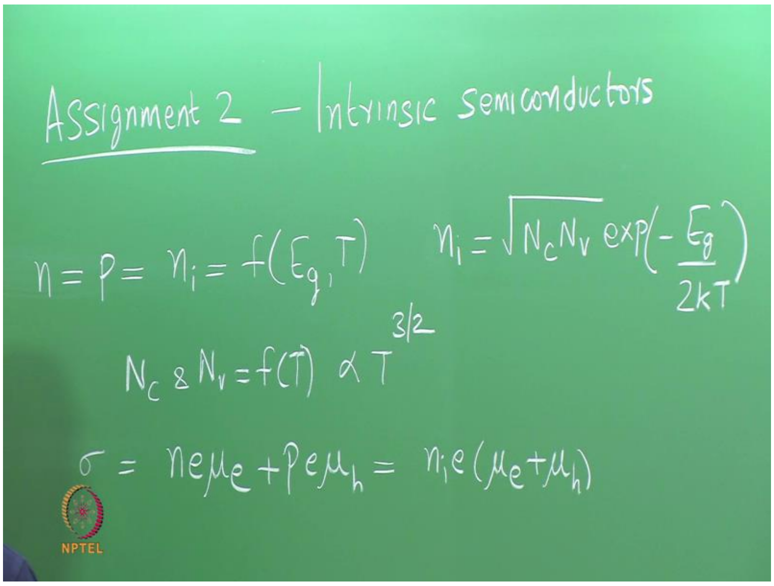

Assignment 2 – Intrinsic semiconductors

$n = p = n_i = f(E_g, T)$ $\quad n_i = \sqrt{N_c N_v}\, exp\left(-\frac{E_g}{2kT}\right)$

$N_c \,\&\, N_v = f(T) \propto T^{3/2}$

$\sigma = ne\mu_e + pe\mu_h = n_i e(\mu_e + \mu_h)$

This is assignment 2 and we will be focusing on intrinsic semiconductors. So, before we start looking at the problems we just do a brief review. So, intrinsic semiconductors or pure semiconductors are essentially single crystals. We say that there are no defects in the semiconductor because these defects can again create electrons and holes of their own in the case of an intrinsic semiconductor. We say that the electron concentration in the conduction band that is n is equal to the whole concentration in the valence band that is p and it is equal to something which we denote as n i and n i we call the intrinsic carry of concentration. We also say that n i is the function of the band gap of the material E g and also a function of temperature.

So, typically n i is written as N c n v exponential minus E g over 2 k t. So, the intrinsic carrier concentration depends exponentially on the band gap the temperature term enters in exponential factor, but N c and N v which are the effective density of states at the valence band h and the conduction band h are also a function of temperature. So, N c and

N v are also function of temperature, typically they are proportional to temperature to the 3 over 2, but the exponential term is the one that dominates.

We also saw the general equation for conductivity sigma is nothing but n e mu e and P e mu h. In the case of an intrinsic semiconductor, this just becomes n i e mu e plus mu h. So, these are just a few points about intrinsic semiconductor we will be using them today during the course of the assignment, so let us first look at problem 1.

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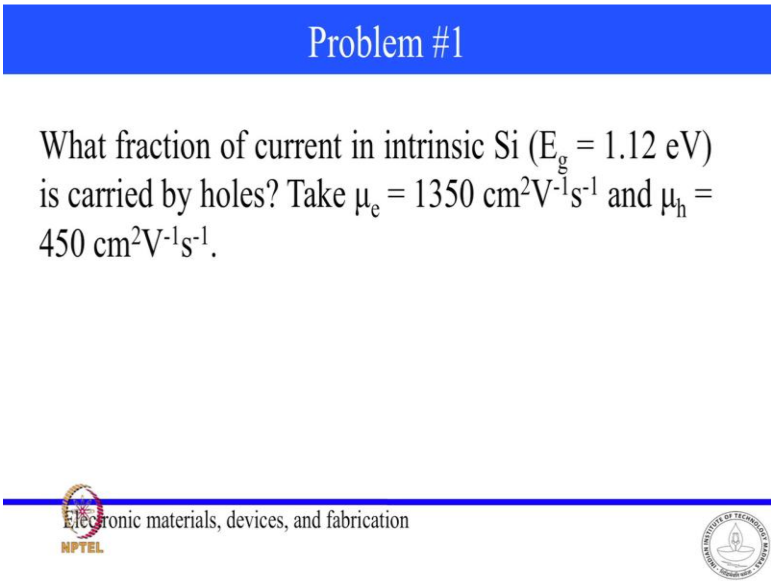

Problem #1

What fraction of current in intrinsic Si ($E_g = 1.12$ eV) is carried by holes? Take $\mu_e = 1350\ cm^2V^{-1}s^{-1}$ and $\mu_h = 450\ cm^2V^{-1}s^{-1}$.

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NPTEL

So, what fraction of current in intrinsic silicon is carried by holes?

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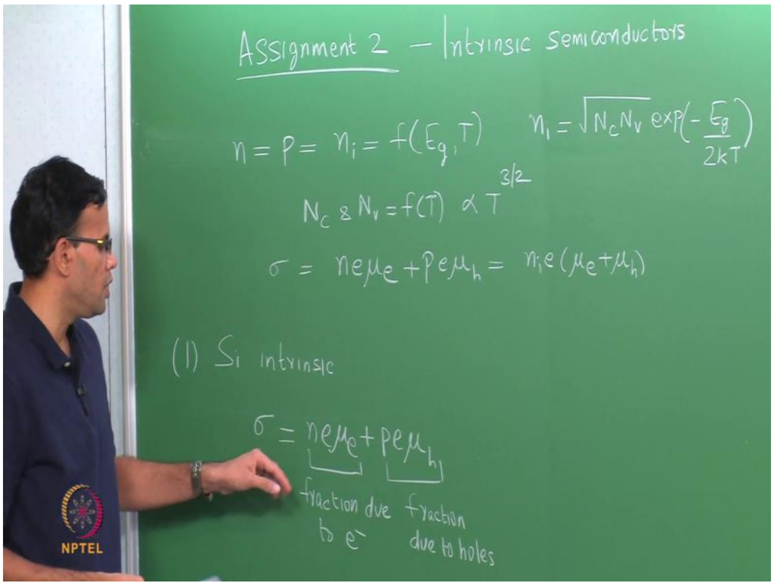

So, we have silicon and it is intrinsic, which means n equal to p equal to n i and the question asked what fraction of current or what fraction of conductivity is defined by the holes. So, if you just say n is equal to p is equal to n i that means there is a 50 percent contribution. That is a very simplistic answer, the reason is the conductivity not only depends on n, it also depends upon mu e and mu h, which is the mobility of the electrons and holes. So, we can write the conductivity equation n e mu e plus P e mu h this represents the fraction carried by the electrons fraction due to electrons this is the fraction due to holes. So, we can include the numbers for mu e and mu h.

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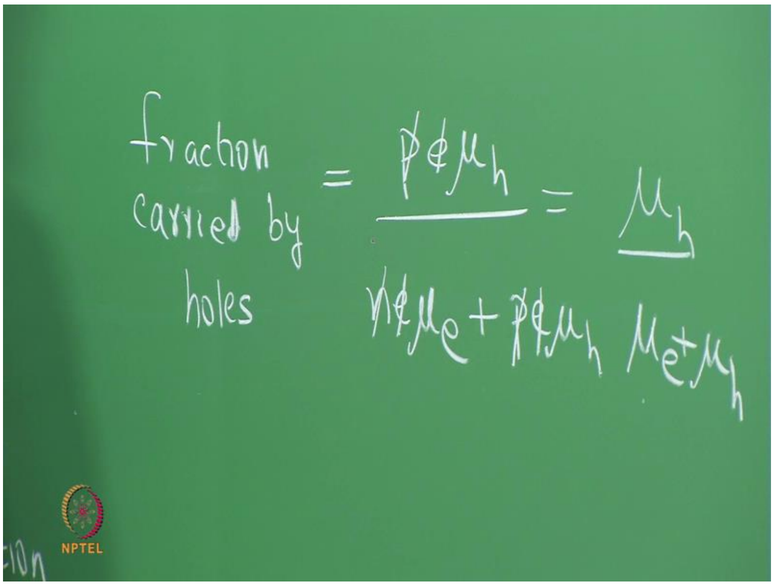

fraction carried by holes

$$= \frac{\cancel{p}\cancel{e}\mu_h}{\cancel{n}\cancel{e}\mu_e + \cancel{p}\cancel{e}\mu_h} = \frac{\mu_h}{\mu_e+\mu_h}$$

So, fraction carried by the holes we can write this in the form of a ratio it is nothing but P e mu h divided by the total that is ne mu e plus P e mu h. So, for an intrinsic conductor semiconductor n is equal to p is equal to n I, so these terms cancel, e will also cancel. So, this is nothing but mu h over mu e plus mu h. So, the fraction of current carried by holes is directly proportional to the whole mobility, we can plug in the numbers for silicon here.

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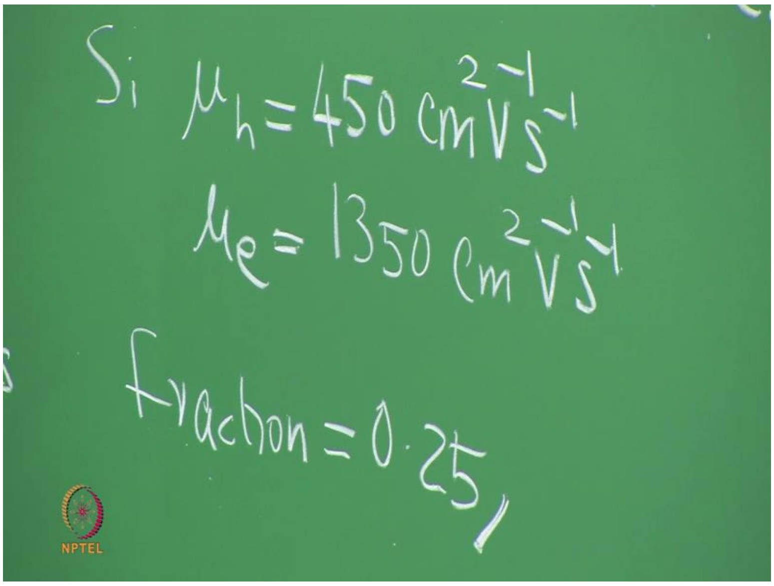

Si $\mu_h = 450\ cm^2 V^{-1} s^{-1}$

$\mu_e = 1350\ cm^2 V^{-1} s^{-1}$

Fraction = 0.25

So, for silicon, mu h is 450 centimeter square per volt per second, mu e is 1350. So, the mobility of the electrons is higher, so we can plug in these numbers and the fraction is 0.25. So, even though we have equal concentration of electrons and holes they do not have the same mobility and this is because your electrons are moving in the conduction band and the holes are moving in the valence band. This ultimately determines what fraction dominates whether the electron conductivity dominates or the whole conductivity dominates. Later, when we see extrinsic semiconductors, we will find that n and p are not the same one is much higher than the other and then their 1 of the term dominates because of the difference in concentration, so let us now move to question 2.

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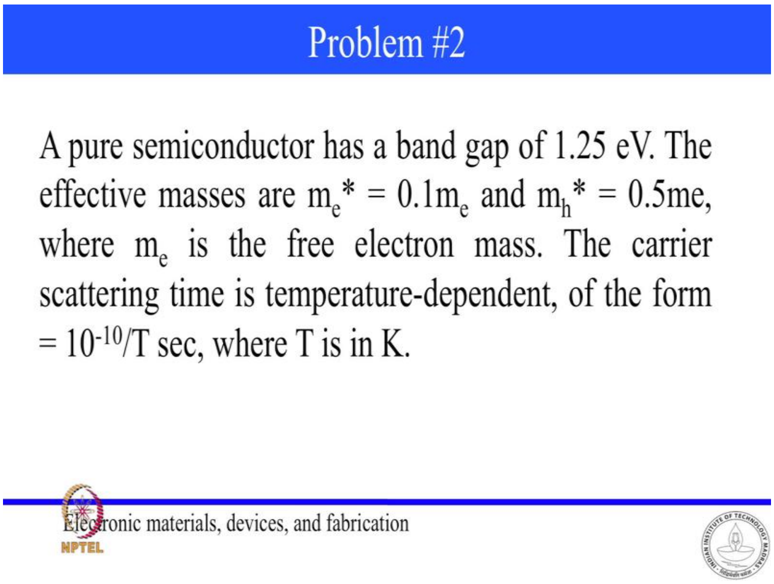

Problem #2

A pure semiconductor has a band gap of 1.25 eV. The effective masses are $m_e^* = 0.1m_e$ and $m_h^* = 0.5me$, where m_e is the free electron mass. The carrier scattering time is temperature-dependent, of the form $= 10^{-10}/T$ sec, where T is in K.

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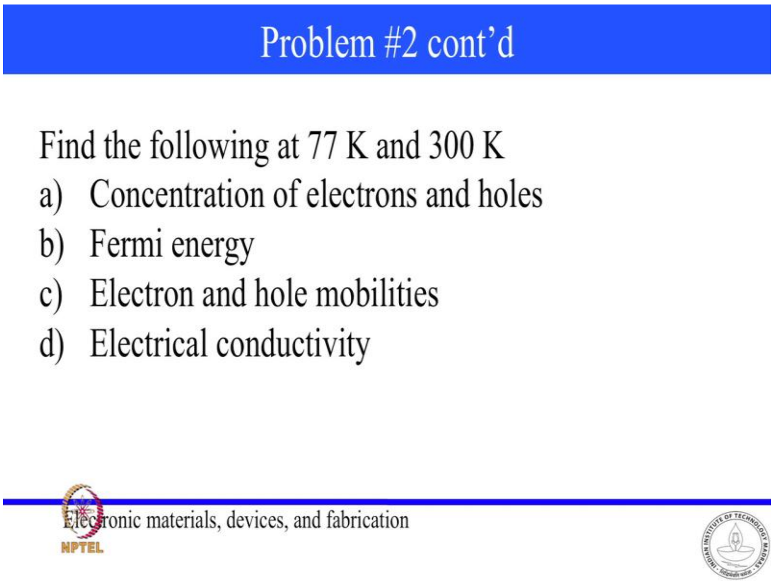

Problem #2 cont'd

Find the following at 77 K and 300 K

a) Concentration of electrons and holes
b) Fermi energy
c) Electron and hole mobilities
d) Electrical conductivity

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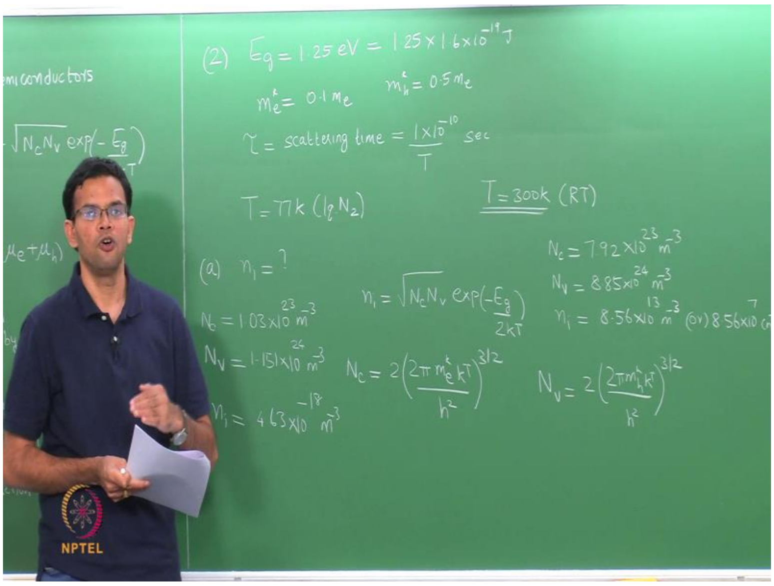

So, we have a pure semiconductor or an intrinsic semiconductor the band gap is 1.25 eV. So, the effective masses of the electron and holes are also given the effect to mass of the electron me star is 0.1 times m e, where m e is the mass of the electron and m h star is 0.5 m e. This is the effective mass for the hole, once again we have seen the concept of effective mass before effective mass does not mean a change in the actual mass of the electron or the hole. It just represents the cumulative of all the forces of the atoms in the lattice that basically acts on the electrons and holes.

Once again, this number are different because you have electrons that are moving in the conduction band and holes that are moving in the valence pat. So, the band gap is given the effective mass values are given the carriers scattering time is temperature dependent and that is given of the form. So, tau which is your scattering time is a function of temperature and this is 1 minus 10 to the minus t. So, 1 times 10 to the minus 10 divided by temperature and the units are sec’s. So, the effect to masses are given the band gap is given and the temperature dependence of the carriers scattering time is also given this we will use to calculate the nobilities.

So, we want to find the following at 2 temperature 1 is 77 Kelvin and the other is 300 Kelvin. So, 300 Kelvin is room temperature 77 Kelvin is typically your liquid nitrogen boiling point. So, that is the low temperature, the first one we want to find is the concentration of electrons or holes because this is a pure semiconductor.

What we want to find is the value of the intrinsic carrier concentration. So, we can go back to the equation n i is nothing but square root of N c and N v exponential minus E g over to k t. So, the problem is we do not values of N c and N v these are the effective of states or the band edges, but these we can calculate once we know the effective mass. So, N c which is the density of states at the conduction band edge it is nothing but 2 pi m e star k t over h square whole power 3 over 2 N v. We can do the same for the valence band edge 2 pi m h star over k t by h square whole power 3 over 2. So, we have the values for N c and N v again we see both are temperature dependent, they are proportional to t to the power 3 half.

So, once we calculate N c and N v for both the temperature s, we can plug in here and calculate the value for n i the band gap is also known. So, typically you have to keep all of this in SI units. So, you have to convert E g from electron holes to joules in that we can do by just multiplying by 1.6 times 10 to the power minus 19 kb is also in joules. So, it is your Boltzmann constant that has its standard value. So, once we plug in the numbers, I am just going to write the final answers, but you can just go through and check. So, N c at 77 Kelvin is 1.03 times 10 to the 23 per meter cube.

So, if you remember the definition of the effective density of states is the total number of states per unit value that is available for the electron to occupy or the hole to occupy. Similarly, N v is 1.151 times 10 to the 24 per meter; we can do the same calculations for 300 Kelvin and again just write down the answers. So, N c is higher 7.92 times 10 to the 23 N v is 8.85 times 10 to the 24 per meter cube. So, compared to 300 Kelvin N c and N v is higher this is because we have more density of states available at higher temperature, simply because their directly proportional t to the 3 over 2.

So, we can substitute this values of N c and N v in this expression and calculate the value for n i. So, let me just write that down n i at 77 Kelvin is 4.63 times 10 to the minus 18 meter cube. So, that is a really small number n i at 300 Kelvin is 8.56 times 10 to the 13 per meter cube. So, I can also write this in centimeter cube or 8.56 times 10 to the power 7 centimeter cube. So, your N c and N v values if you look or of by 1 order of magnitude simply because you have rise in temperature, but because your n i depends exponentially on band gap. There is a huge variation between 77 Kelvin, which is your liquid nitrogen temperature and 300 Kelvin which is room temperature.

So, here you have a value that is 10 to the minus 18 and a room temperature you have value for n i that is close to 10 to the power 30 so that overall there is a 31 orders of magnitude change as you go from liquid nitrogen to room temperature. This is why we say that out of these 2 terms N c and N v and the exponential term, the exponential term is the one that dominates in determining the value for n i. So, this is part a, where we want to calculate the concentration of electrons and holes part b, we want to calculate the firm e energy or the location of the firm e level.

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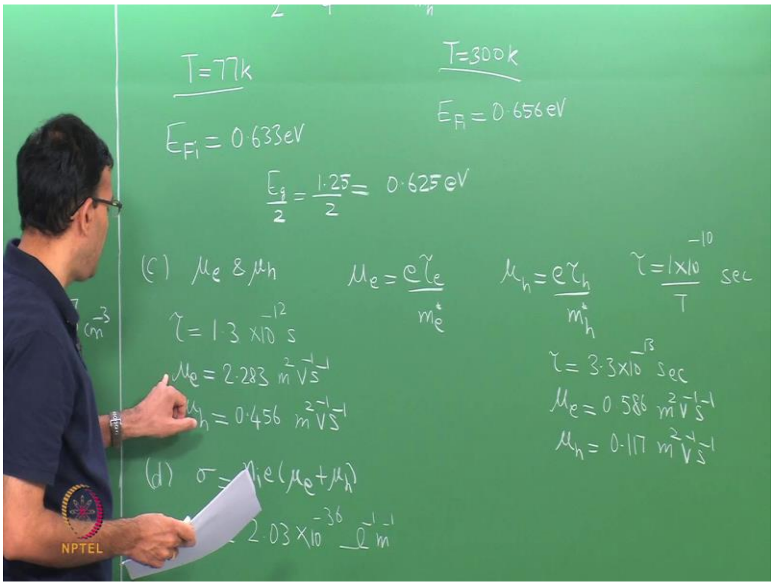

So, part b we want Val the location of e f i this is again a standard expression e f i this E g over 2 minus 3 4th k t lone of me star over m h star. So, if m e and m h were equal then EF i will be a exactly the center of the band, if you remember EF i is nothing but a representation of the chemical potential or the amount of work that needs to be done in order to remove an electron from a semiconductor. So, even though your electron and whole concentration are the same, so n is equal to p because you have different effect to masses your EF i is slightly shifted from the center of the gap we can once again plug in the number. So, 77 Kelvin 300 Kelvin m e star and m h star values are given temperature is also known.

So, EF i here if you do the substitution 0.633 electron works at room temperature EF i is 0.656 E g over to if you look at it is just 2.36 over 3, so 0.625 electron volts. So, the values are very close to the center of the band gap by they are slightly shifted.

The shift becomes higher, the higher the temperature. So, this is 0.633 and this is 0.656. So, slightly deviated away from the center of the band gap this is part b in part c we want to calculate the electron and whole nobilities. So, we want to calculate the values of mu e and mu h. So, mu e and mu h are related to the effective mass of the electrons and holes and they are also related to the scattering time. So, mu e is nothing but e tau e over me star and mu h this e tau h over m h star some star and m h star are given tau e and tau h or your scattering times and you said that tau is nothing but 1 times 10 to the minus 10 over temperature and the unit is seconds.

So, once again we can calculate the values of tau, in this particular question tau e and tau h are both the same because we do not distinguish between electrons and holes. We only say it depends upon temperature. Once we calculate tau, we can go ahead and calculate mu e and mu h and get it for the 2 different temperatures. So, let us again write down this side is 77 Kelvin this side is 300 Kelvin. So, tau if you calculate is 1.3 time 10 to the minus 12 seconds, you can then calculate mu e which is 2.283 unit i e meter square per volts per second sometimes centimeter square per holes per second were also there.

It depends upon which you want to use mu h is 0.456 meter square holes per second and mu h the mobility of the holes is lower because the hole effect to mass is higher that of the electron. We can do the same for 300 Kelvin in this case tau is 3 point 3 times 10 to the minus 13 seconds. So, higher the temperature smaller is the scattering time, so at lower temperature this is minus 12 seconds this is minus 13 seconds. So, one way to think about this is higher the temperature faster the electrons and holes are moving because they have higher thermal validities. So, they can scatter of the atom quicker mu e 0.586 meter square per volt per second mu h 0.117 meter square per volt per second.

So, the last part of the question we want to calculate the electrical conductivity sigma is nothing but n i e mu e plus mu h. So, n i we got in the first part of this question mu e and mu h just calculated, so we can just plug in the numbers. So, sigma at 77 Kelvin is very small because if you remember n i is very small. So, 10 to the minus 36 inverse meter inverse you can also have o m inverse and centimeter inverse depending upon what your values the units for n i and mu e and mu hr.

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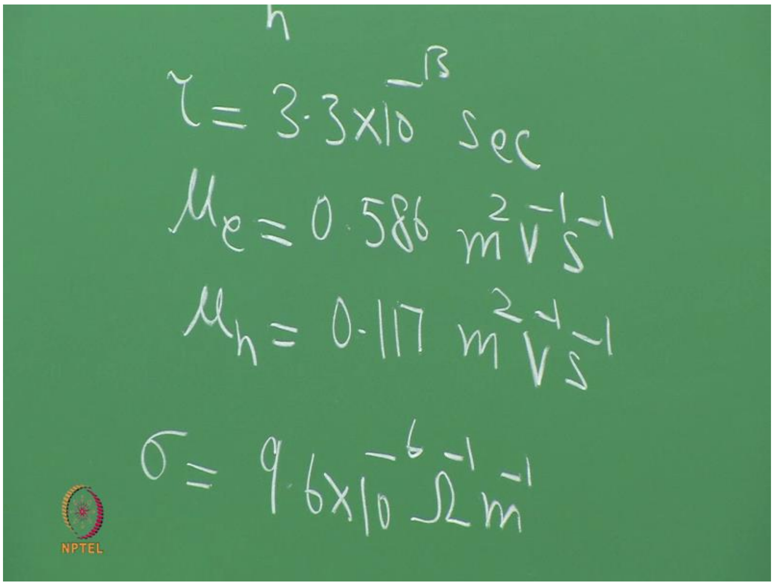

n

$$\tau = 3.3\times10^{-13}\ \text{sec}$$

$$\mu_e = 0.586\ m^2 V^{-1} s^{-1}$$

$$\mu_h = 0.117\ m^2 V^{-1} s^{-1}$$

$$\sigma = 9.6\times10^{-6}\ \Omega^{-1} m^{-1}$$

So, the same thing we can do at room temperature and sigma is 9.6 minus 10 to the minus x. So, by looking at an intrinsic semiconductor at two different temperatures, one thing we find is that the carrier concentration increases exponentially with temperature. Similarly, the conductivity will also increase because the carrier concentration increases this again is determined by the value of the band gap. So, higher the value of E g steeper is this dependence. So, instead of 1.25, we had done the same problem will say two electron volts, your answer will also been different, but the difference between 77 and 300 Kelvin will also be more pronounced.

So, that is something you can always work out you can take the same values, but change the value of E g 2 electron holes and do this question and you can see the difference between 77 and 300 Kelvin, so let us now move to question 3.

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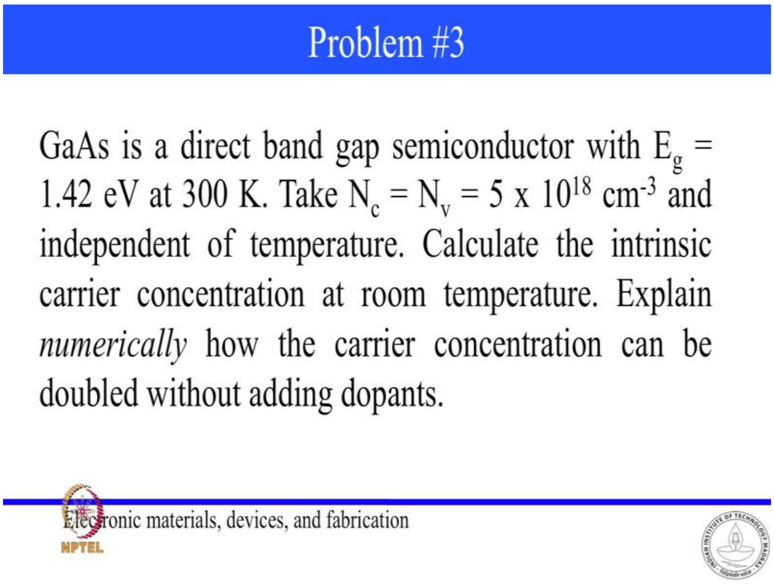

Problem #3

GaAs is a direct band gap semiconductor with E_g = 1.42 eV at 300 K. Take $N_c = N_v = 5 \times 10^{18}$ cm^{-3} and independent of temperature. Calculate the intrinsic carrier concentration at room temperature. Explain *numerically* how the carrier concentration can be doubled without adding dopants.

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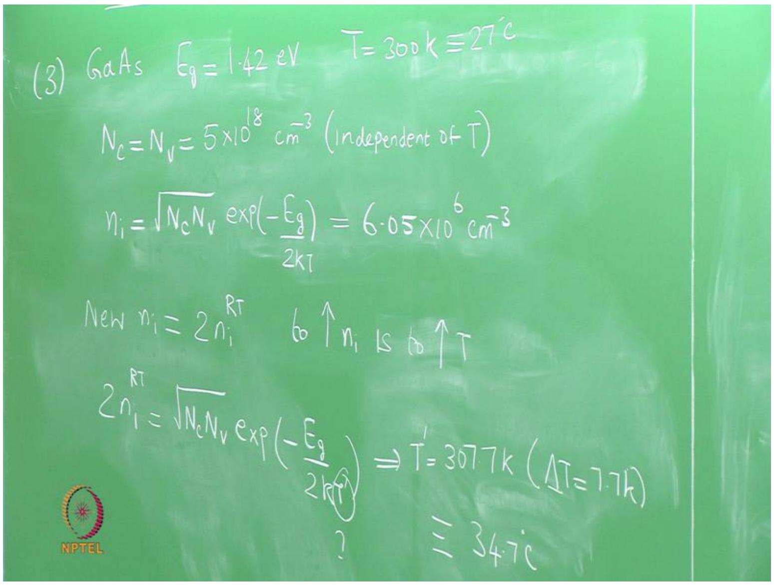

(3) GaAs $E_g = 1.42$ eV $T = 300k \equiv 27^\circ c$

$N_c = N_v = 5 \times 10^{18}$ cm^{-3} (independent of T)

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right) = 6.05 \times 10^{6} \ cm^{-3}$$

New $n_i = 2n_i^{RT}$ to ↑ n_i is to ↑ T

$$2n_i^{RT} = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2k\boxed{T'}}\right) \Rightarrow T' = 307.7k \ (\Delta T = 7.7k)$$

? $\equiv 34.7^\circ c$

So, question 3, we have Gallium arsenide which is a direct band gap semiconductor with E g of 1.42 electron holes at 300 Kelvin. So, Gallium arsenide it has a higher band gap than silicon. So, also a direct band gap material, but that is not relevant for this question it is a room temperature. So, temperature 300 Kelvin take N c equal to N v equal to 5 times 10 to the 18 per centimeter q and also independent of temperature. So, the N c and N v values are given and just for this question we assuming that both are same and that they are also independent of temperature. Strictly speaking, this is not true, but as you see for this particular question, this is very valid assumption.

So, first we want to calculate the intrinsic carrier concentration at room temperature. So, that is pretty straight forward if you see in the formula before. So, N c n v exponential minus E g over 2 k t, so we can plug in the numbers the value of E g is given. So, n i is 6.05 times 10 to the power 6 per centimeter cube. So, this is the intrinsic carrier concentration at room temperature n i is a pretty small number for comparison silicon has a value of n i of 10 to the 10. So, for order of magnitude higher, but this is because Gallium arsenide has a higher band gap.

So, the next part of the question says explain numerically how the carrier concentration can be doubled without adding dopes. So, we want to keep this semiconductor your pure semiconductor, but at the same time we want to increase the value of n i. So, the new value of n i, we want is double of the n i value at room temperature if you not allow to add dopes. If you look at this equation the only way to increase n i is to increase temperature because N c and N v are both temperature dependent terms n i also depends on temperature through this exponential term minus E g over 2 k t. So, increasing temperature will once again increase n I, so the only way to increase n i without adding dopes is to increase temperature.

So, we want to know what the new temperature is when a value of n i is 2 times the n i at room temperature. So, we will once again use this expression N c and N v is constant. So, it is not a function of temperature if it were a function of temperature that will also have to be taken in to accounts, but N c and N v are constant we know the new value of n i. So, it is 2 n i at room temperature square root of N c N v exponential minus E g over 2 k let me call this temperature t prime. So, the t prime is the only thing we want to know is the only unknown this is known these are all known we can put this and re calculate and this gives you the value of t prime to be 300 and 7.7 Kelvin. So, you increase the temperature by 7 degrees.

So, delta t is 7 point 7 Kelvin you can double the concentration of n I, so temperature equal to 300 is approximately 27 degrees, so 300 Kelvin is 27 degree Celsius. So, 307 is 34.7 degree Celsius. So, you find that even for a small increase in the value of n i. So, we are only doubling the value of n i you need to increase your temperature for 7 degrees if you want really the high conductivities that we see in the extrinsic semiconductors. You can actually calculate that the temperature change must be much higher this is one of the reason where intrinsic semiconductors are almost never used.

In the case of devices usually dope semiconductors are used because it is much easier to control the dopant concentration and then control the carrier concentration and also the conductivity, so let us now go to problem 4, so problem 4.

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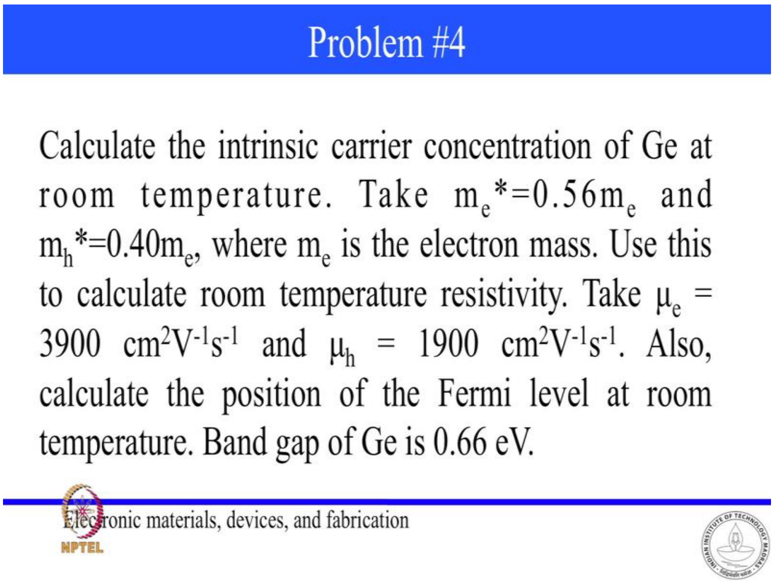

Problem #4

Calculate the intrinsic carrier concentration of Ge at room temperature. Take $m_e^*=0.56m_e$ and $m_h^*=0.40m_e$, where m_e is the electron mass. Use this to calculate room temperature resistivity. Take μ_e = 3900 $cm^2V^{-1}s^{-1}$ and μ_h = 1900 $cm^2V^{-1}s^{-1}$. Also, calculate the position of the Fermi level at room temperature. Band gap of Ge is 0.66 eV.

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We want to calculate the intrinsic carrier concentration germanium.

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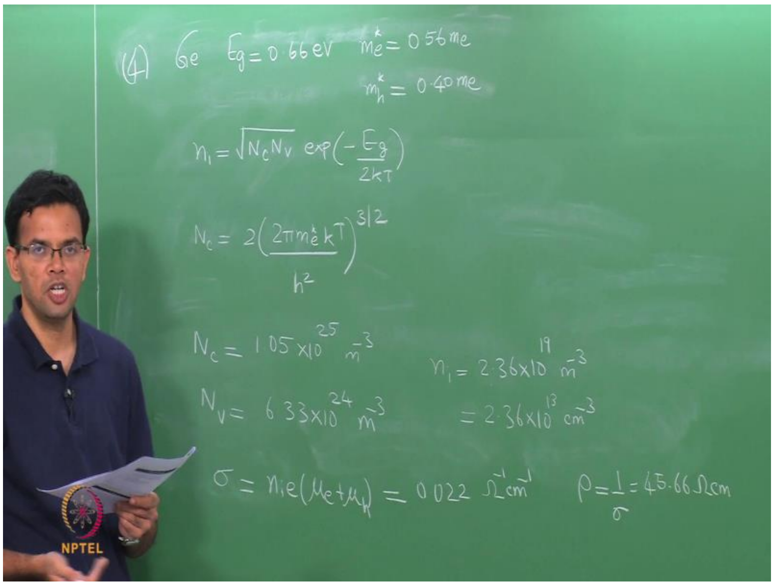

④ Ge $E_g = 0.66 eV$ $m_e^* = 0.56 m_e$

$m_h^* = 0.40 m_e$

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right)$$

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

$N_c = 1.05 \times 10^{25} m^{-3}$

$N_v = 6.33 \times 10^{24} m^{-3}$

$n_i = 2.36 \times 10^{19} m^{-3}$

$= 2.36 \times 10^{13} cm^{-3}$

$\sigma = n_i e(\mu_e + \mu_h) = 0.022 \Omega^{-1} cm^{-1}$ $\rho = \frac{1}{\sigma} = 45.66 \Omega cm$

So, we have germanium with the band gap E g of 0.6 electron volts, this is lower than that of silicon. In fact, germanium was the first semiconductor that was used m e star is 0.56 m e and m h star is 0.40 m e, so we want to calculate n i.

So, the equation is the same n i is N c N v exponential minus E g over 2 k t, so N c and N v are again related to m e star and n h star. So, N c is 2 pi m e star k t over h square whole power 3 over 2. We can write a similar equation for m h star we saw that earlier during question 2. So, once again we can calculate N c and N v plugged back in and get the value of n i. So, will do the numbers you can write down the final answers N c is 1.05 times 10 to the 25 per meter cube N v which is the same equation except m e star is replaced by m h star n e is 6.33 times 10 to the 24 per meter cube.

So, N c and N v are known we can calculate n i n i if you do is 2.36 times 10 to the 19 per meter cube, I am just writing down the final answers the math can always be worked out or 2.36 times 10 to the 13 per centimeter cube. So, we saw the gallium arsenide has a value of n i that is 4 times or 4 orders lower than that of silicon. Germanium on the other hand has a value of n i that is nearly 3 orders of magnitude higher then silicon. Once again, the differences are all related to the band gap values, we can then calculate sigma, sigma is n i e m e and m h the value of mu e and mu h are given m e is 3900 ad m h is 1900 centimeter square per volt per second.

So, that sigma is nothing but 0.022 inverse and centimeter inverse, if you also want to calculate the resistivity row is 1 over sigma, which is equal to 45.66 per centimeter. The question also asks to calculate the position of the Fermi level at room temperature.

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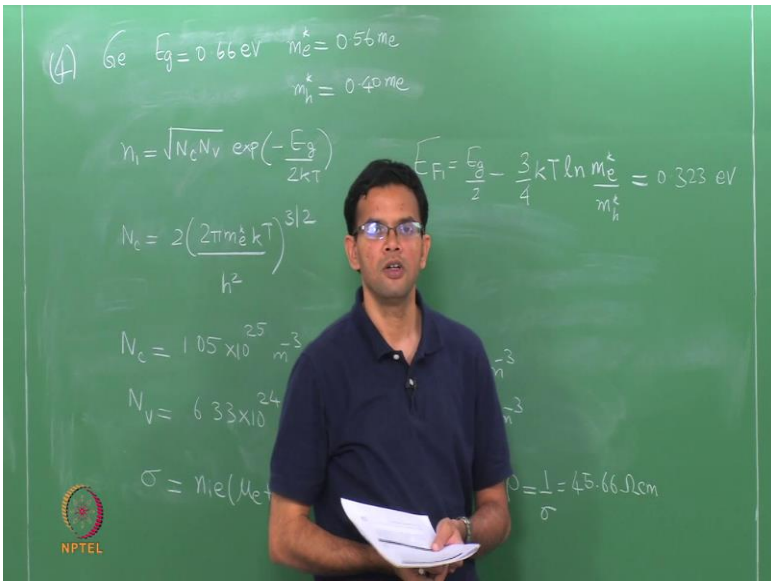

So, that is again an application of the formula e f i is E g over 2 minus three-fourths k t lawn of m e star over m h star. So, this again very close, but it is not exactly at the center of the band gap, so let us now look at problem 5.

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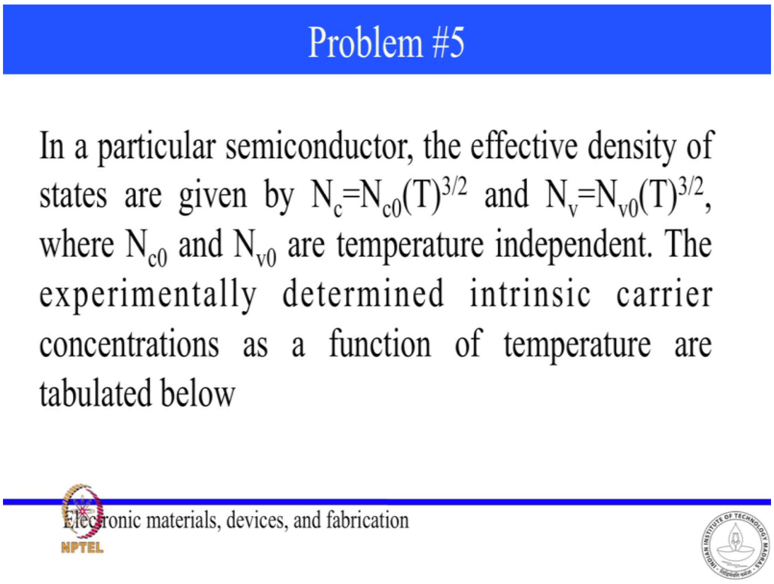

Problem #5

In a particular semiconductor, the effective density of states are given by $N_c = N_{c0}(T)^{3/2}$ and $N_v = N_{v0}(T)^{3/2}$, where N_{c0} and N_{v0} are temperature independent. The experimentally determined intrinsic carrier concentrations as a function of temperature are tabulated below

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There is a particular semiconductor.

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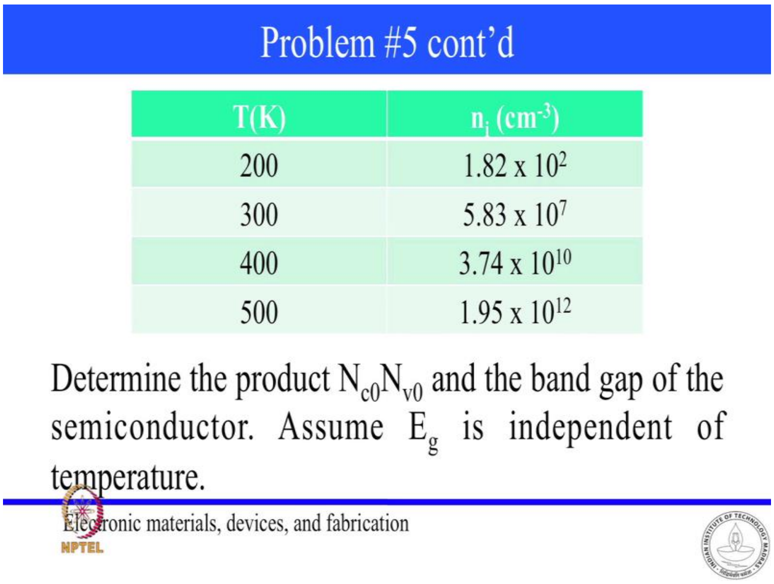

Problem #5 cont’d

T(K)	n_i (cm^{-3})
200	1.82×10^{2}
300	5.83×10^{7}
400	3.74×10^{10}
500	1.95×10^{12}

Determine the product $N_{c0}N_{v0}$ and the band gap of the semiconductor. Assume E_g is independent of temperature.

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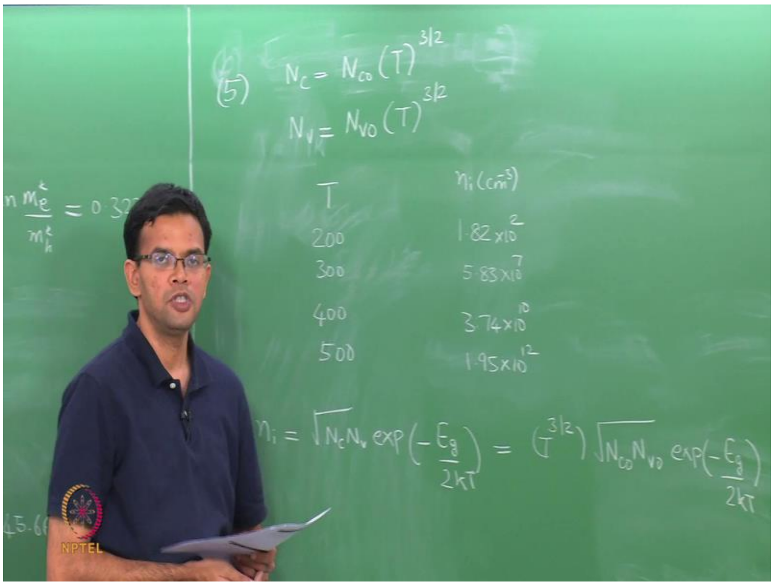

It says that the effective density of states is a constant N c naught times temperature to the power 3 over 2 and same way N v is N v not times temperature to the power 3 over 2. So, the experimental values of n i at different temperature s are given, so we have temperature values of n i in centimeter cube. So, 200, 300, 400 and 500 the values of n i 10 to the 7, so we can see that with increasing in temperature the value of n i is also increases. So, the question ask us to determine this product n v c not times N v not and also the band gap. So, both E g not known and these 2 numbers are not known you can go back to the original equation n i is N c N v exponential minus E g over 2 k t N c and N v we can substitute these x terms.

So, that this simplifies to t to the 3 over 2 square root of N c naught N v naught, which is at temperature independent term times exponential minus E g over 2 k t. So, we can chose any 2 temperatures, so we have 4, we can take any 2 temperature s and take the ration of n i at these 2 temperature s.

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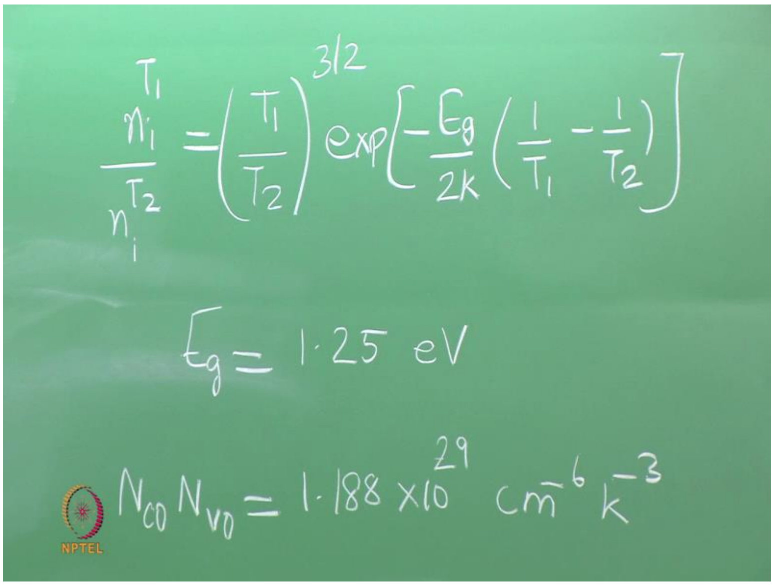

$$\frac{n_i^{T_1}}{n_i^{T_2}} = \left(\frac{T_1}{T_2}\right)^{3/2} \exp\left[-\frac{E_g}{2k}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)\right]$$

$$E_g = 1.25 \text{ eV}$$

$$N_{C0} N_{V0} = 1.188 \times 10^{29} \text{ cm}^{-6} \text{ K}^{-3}$$

So, n I some temperature t 1 n i at another temperature t 2 it is nothing but taking this ratio which is here this is a temperature independent term. So, this becomes t 1 over t 2 whole to the power 3 over 2 exponential minus E g over 2 k 1 over t 1 minus 1 over t 2. So, t 1 and t 2 values are known n i values are known for example, your t 1 could be 200 Kelvin t 2 could be 300 Kelvin in which case the n i values are tabulated the only unknown here is e g. So, I did this calculation taking 200 and 300 you can take it with the any of the other temperature s and you also done and checked if you substitute the values E g works out to be 1.25 electron volts. So, this is the value of E g, which is the band gap of the material.

Once you know E g, you can substitute E g for any of the temperature s and evaluate N c not and N v not this is a temperature independent term if you do that N c not times N v not is nothing but 1.188 times 10 to the power 29. The units here are crucial N c and N v the square root of that should have the units of centimeter per centimeter cube or per meter cube. In this particular question, this also depends on t to the 3 over 2.

So, the units of this product is centimeter to the power minus 6 Kelvin to the power minus 3 that way when we substitute for square root units work out in the right way. So, today we have looked at various problems related to intrinsic semiconductors, the important thing to remember is that the intrinsic carrier concentration is a function of temperature and depend upon the band gap of the material.