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Lecture No. # 09

Drude Model: Successes and Limitations

Hello, welcome to this ninth class in this physics of materials lecture series, where our intent is to build models for materials, so that, we understand from first principles, why materials display the properties that we measure. So, as we have said before, at some level, we spent our time measuring properties and using those measured properties for a variety of purposes. This course attempts to go beyond that; we would like to understand why the material displays that property to begin with, on the basis of whatever it is that we understand of the constituents of the material, how they behave, how they interact with each other, how they interact with their surroundings and so on. So, in this context, we have actually gone over, or we are in the process of discussing the model that we call the Drude model, ok.

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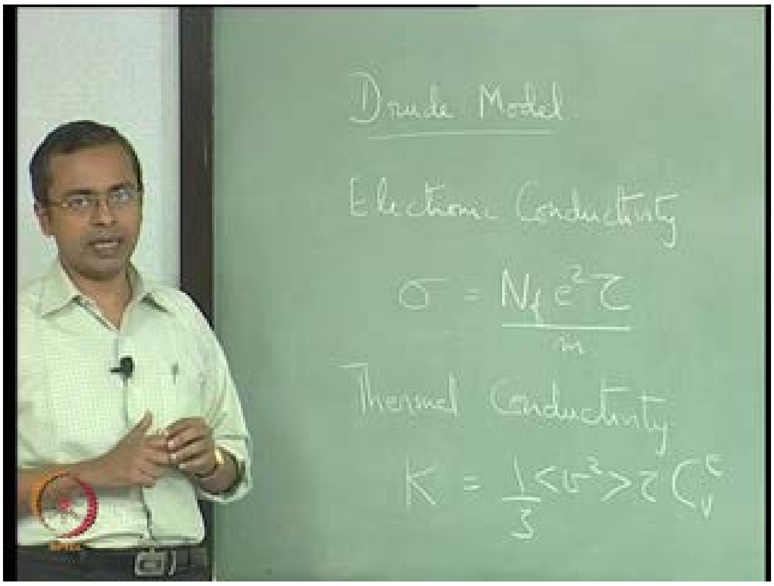

So, the Drude model, this is what we have developed over the last few classes. The basic idea behind the Drude model, is to use the ideas of ideal gases, the kinetic theory of gases and to, merely with minor changes if any, to extrapolate these exact same ideas into the realm of electrons in a solid, with the thinking that, electrons which are free to run around through the extent of the solid, behave like gas molecules. So, on this basis, this Drude model is developed; and, on the basis of this Drude model, we have predicted electronic conductivity. So, we have come up with a description for electronic conductivity. We have looked at how the electron responds to a field that is applied on that sample; how it moves, what are the kinds of restrictions that it faces; and then, within this framework of how it moves and what restrictions it faces, we have been able to come up with, and the number of free electrons that are available per unit volume; on the basis of all of this, we have been able to come up with an expression for the conductivity of, of a metal. So, we got sigma equals N f e square tau by m.

So, this is the kind of expression that we got for electronic conductivity, within the framework of the Drude model. So, this is what we did a couple of classes ago. Then, we continued with this model; we said, we now have a feel for what it does with respect to electronic conductivity; we extended it and tried to see, what it could help us predict, in terms of its thermal conductivity, of a material’s thermal conductivity. So, so, in the last class we looked at thermal conductivity. We wanted to see, if we can take the ideas of the Drude model and again, look at how electrons move and use that to predict thermal conductivity. Here, as we mentioned in the last class, we specifically looked at this, at the general idea, or the framework that, the heat is being transferred on the basis of electrons that are moving within that solid, and the electrons themselves are reaching equilibrium with their surroundings, based on collisions with other electrons. So, electrons at the hot end of the material collide with electrons which are adjacent to them, which are not as hot as them, in, in terms of the average temperature that they have, and then, the heat is then transferred on and on and on, from electron to electron, in terms of the energy that is being attained by each of those electrons; and in that process, the heat is transferred from one end of that solid to the other.

So, on the, on this basis, we came up with an expression for K, the thermal conductivity of that material, and we found that, it was 1 by 3. So, we had the thermal conductivity given as the, the square of the, mean of the squares of the velocities, tau and C v e. So, this is what we ended up getting for the thermal conductivity. So, so, this is the expressions that we got; and, and, as we have seen, the idea is that, we would like to get an understanding; we have actually got a good feel for the, what shall we say, independently of the two properties, the electronic conductivity, as well as the thermal conductivity. And, the basic idea has also been that, the electrons are doing both those processes. So, that is the idea that we have used. So, for example, I also indicated that, you have materials that are relatively insulating in nature. So, there, you even have the atomic vibrations providing the path for heat conduction, so to speak. So, that is not something that we have included in this picture. So, within, taking all that into account, these are the parameters that we find. So, we have got these expressions…So, in terms of the values that they predict…So, for these two, what should I say, experimental, experimentally verifiable, or measurable quantities, we find that, in general, the values that are being predicted are reasonably good, in, in in the sense that, they are reasonably matching with whatever it is, that we find in the literature.

So, from that perspective, already we see that, there is some validation for this model; that, as I mentioned in one of our early classes, all the, any model that we put together may give you very nice expressions, fancy looking expressions and so on. But, at the end of it all, it has to match experimental data. So, in that sense, the hierarchy of truth, so to speak, is the experimental data; the experimental data comes right on top; any theory that we use, should actually match the experimental data. Of course, in, in the, the landscape of science, we could also go the other way round; we can have theories that predicts certain experimental results; and then, if you can actually show that experimental result which may not have been shown before, then, that also validates the theory; but again, in, in both these circumstances, that experimental result is sacrosanct. So, you have to actually ensure that the experimental results are matched. In terms of the experimental result for these two properties, we find that, there is a good match. So, therefore, on that basis we are on reasonably good grounds to proceed with looking at this. Now, in this regard, I wish to point out a couple of things.

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This is an object, that you may not be very familiar with, depending on what work you do and so on. What you actually see, is that, there is a tube consisting of copper; there is a tube consisting of copper around which there is a, another tube of a smaller diameter that is wound all around it. So, that tube also consists of copper, or is made of copper. So, we have actually, we can say, a cylinder of copper in the middle and all around that cylinder we have a tube of copper that is wound around it, alright. So, this is actually a piece of an equipment that you may, depending on your familiarity with experimental equipment, in some experimental setup, you will see something that looks very similar to this. The basic idea of this setup here is that, you can actually flow some fluid through these pipes that are wound around this, wound around this central pipes. So, through this pipe, you can flow some liquid; it can enter in one, from one location, goes circularly around this central pipe and then, come out from the other side. The basic purpose for which this is used, is to provide cooling.

So, this is a cooling system. So, in many places, where you are generating a lot of heat, so, for example, you could have, may be an arc or something, which is of, which is generate, which generates very high temperatures and that could be in the middle. So, you would position this around that system, around the location of heat. So, that heat would now have to be absorbed by this, will be taken up by this copper, the sheet of, copper tube of large diameter, then transferred on to those copper pipes of smaller diameter and then, the liquid that is flowing through those copper pipes will actually take the heat out . So, you call these various things, I mean, in, you could call this heat exchanger, or basically, just a cooling system; so, cooling coils it would be. So, in many experimental systems, in, in an engineering sense, you would find something like this, which is, which is present; but for our purposes, the, the concept of, the information of interest is that, this is a, this is made out of copper; this is actually, this is a piece, that is actually in use. So, it, it does not have a very shiny finish to it; but I wanted to show you something, that looks realistic. So, I have brought, brought this to you. So, if you went and bought a brand new piece and you will had it nicely polished, you could see the nice gleaming copper all around it, but this is something that is directly in use.

So, that is why it looks, the way it does. So, the information of interest here, as I mentioned is that, this is a material that is made, I mean, the material that, that is used to make this object is copper. And, you will find that, in many systems where you want to provide cooling, you will find the material copper appearing, from some, I mean, in, in the engineering that has gone into that equipment. In fact, even the cooling system that are often used, say, inside a computer, in a laptop especially, where you would like to get the heat out quickly to some other location where you can then dissipate the heat, heat from the processor for example; then, many of those tubes would also have some, would often be, one possibility at least is that, you could make it out of copper. So, wherever you have a heat source, and one purpose in your experiment is to protect other things around it from the heat, you will find copper based cooling systems. So, this is something that is commonly seen. Various experimental equipment have this kind of (( )) and in many places, you will find this kind of a system. So, but the common idea being that, copper would be very likely in use, alright.

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Now, I would like you to take a look at this. What I have in my hand, is a copper coil. So, this is just a copper wire; this is wound, in this case, it is hand wound; it is wound around some piece of metal, here and it is copper wire. So, this is just an example that I am bringing to you; this is actually, again as I said, hand wound; but, you will find many other places, where it is machine wound, so to speak; and, a good example for you to find, good places where you could find something like this is, in a fan. So, if any fan that you take, any ceiling fan that you take, if you actually are able to, you know, an old fan, if you are able to safely open it, you will find that, you will find lot of copper available inside it and all that copper is actually wound in the, in the form of windings present within that, with, within that fan. What is the purpose of copper there? Actually, the purpose of copper there is actually to carry electricity. So, it carries a lot of electricity inside that, inside that fan and so, copper is being used there.

Actually, there is more, there is a little more to it than just that, but the point is, you now have two situations, both of which are easily seen by you, at, in various locations, in, in your lab, or in your home, where you would find copper in use. In one case, the copper is being used to conduct heat; in the other case, the copper is being used to conduct electricity. So, we, we find that, it is easy for us to find the material, in, in common use, which appears to serve both as a good conductor of electricity, and a good conductor of heat. So, you may already have known this intuitively, or, or through reading it in various places, but I wanted you to have a good look directly at, at something that you can go ahead and actually see. So…So, we, what we find is, we are able to find such materials which are good conductors of heat and good conductors of electricity. Copper is just one example. You could, you could also look at silver; you could look at gold and so on. Of course, from the perspective of an engineering use, or a technological use, cost would then become the other consideration.

So, given all the factors that could play into this kind of a situation, often copper is the material of choice; where, it is still a relatively expensive material, but in the grand scheme of comparing with silver or gold, it is much cheaper. So, therefore, you find many situations, where copper is used. So, now, we have a interesting idea that we are able to identify that, a good conductor of heat is also a good conductor of electricity. So, this goes back to our idea that we had with the Drude model, which is that, the Drude model, actually in terms of both the electronic conductivity that it is identifying, and in terms of the thermal conductivity that it is trying to predict, both these predictions that it makes, in both cases, it is using the concept that electrons that are moving within the system are actually doing both these processes. So, in, in a sense, this is consistent with the experimental finding that we see, that the thermal conductivity, when you see good thermal conductivity, you also happen to see good electrical conductivity. So, given that, these two are going hand in hand, it is reasonable to expect that, the mechanism that enables electronic conductivity, is closely related, if not the same, as the mechanism that enables good thermal conductivity, ok.

So, given that the properties are going in hand in hand, the cause for the properties are likely to be similar, or the same, is a reasonable expectation. Therefore, also, when you look at the model which has actually done something along those lines, that it has taken the same concept of free electrons to come up with a, an expression for electronic conductivity, it has also looked at the same free electrons and come up with an expression for thermal conductivity. There is reason to expect that, this is actually, likely to be a good approach, because that is consistent with some experimental findings that we have, alright. So, we will just leave these two expressions here, for the moment. What we would like to do is, I have just said this in a descriptive sense, that something that has good thermal conductivity also has good electronic conductivity; from experience, we find this to be true. But, there are people who have actually done, who have studied this idea in a more systematic manner, and looked at variety of materials, especially metals, to see if, in fact, this is generally true, and if so, is there some constant that we can associate it with, associate this process.

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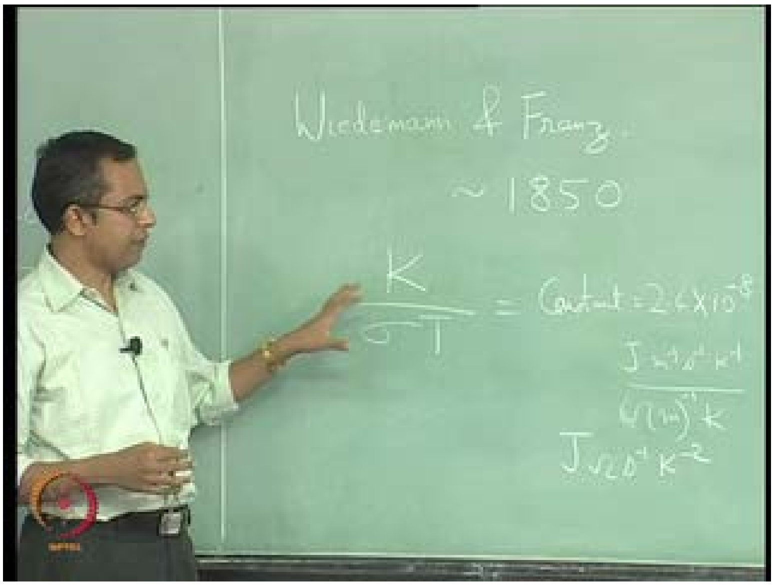

So, the people who have studied this in great detail are Wiedemann and Franz. Wiedemann and Franz are the people who have studied this in detail and there is a...So, they have come up with an empirical relationship. So, they just, empirical meaning, this was done in around 1850s. So, approximately the year 1850, around that time, around that time frame, they actually looked at a lot of metallic systems and, and found that, at room temperature, or at the same temperature, for all of them, the ratio of thermal conductivity to electrical conductivity, or thermal conductivity to electronic conductivity worked out to be the same, for a variety of materials. So, they actually found that, K by sigma T equals constant. So, this is, works out to about 2.4 into 10 power minus 8. So, we can put the units down here. So, this would be Joules per meter per second per Kelvin, that is Kelvin here, and this is ohm meter minus 1. So, this would work out to Joules ohm second minus 1 Kelvin minus 2. So, this is the units that you would get.

So, watts, so, if you want watts, ohm Kelvin minus 2 is the unit you would have, and 2.4 times 10 power minus 8 is the value we are looking at. So…So, this is, this is the thing that we have and for a variety of materials, this is found to be true. So, this was experimentally determined and to the experiment, to the extent that this was experimentally determined in 1850s, we would consider this an empirical, sort of an empirical law. So, because there is no immediate scientific understanding which leads us to this constant, it is empirically found to be the case that, this works out to be this constant. Actually, there is a slight range in this, but this is roughly the value that we are looking at. So, this is the Wiedemann Franz law and it sort of, systematically puts down all of these thermal conductivities and electronic conductivities, and gives you this ratio, fine. So, what we would like to do, is to see, if our understanding of the Drude model, or the way in which we have built the Drude model, then enables us to actually also make, come up with this kind of a constant for the Wiedemann Franz law; to see if the Drude model is consistent with the findings of the Wiedemann Franz law. So, this is something that we are interested in.

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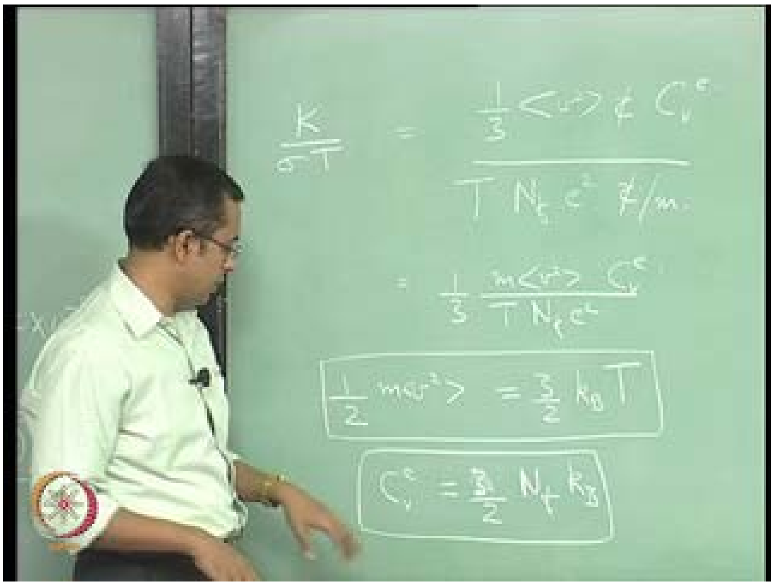

So, now, we will look at this same expression K by sigma T, and we will see, what it is that the Drude model has given us, put those numbers down, and see if that gives us a value that is very close to what is being predicted here. So, if you see, we can put this down, this is 1 by 3. So, this is the, and times T. So, K by sigma T is what it is. K is what is on top, T is the first term here and sigma is all the rest of it. So, this is what we have; if you simplify this, we would like to see, if we are getting values which are of interest; what is the value that comes, and how consistent that is with the finding of the Wiedemann Franz law. So, let us just simplify it. So, we lose this tau here; this m will go to the top. So, we would have…Now, a couple of classes ago, we actually looked at some relationships associated with the kinetic theory of gases and specifically, I pointed out that, there are a couple of relationships that will show up there, as a result of the derivation of the kinetic theory of gases; and that, those couple of relationships are something that we will use a little later. So, now is when we are going to use them. We had actually the average translational kinetic energy associated with, with an electron; the average translational kinetic energy associated with an electron. We got this to be 3 by 2 k B T.

So, this was something that we derived as part of our derivation for the kinetic theory of gases. So, this is a relationship that we had and we also said C v e, C v e is 3 by 2 N f k B. In both these cases, k B is the Boltzmann's, Boltzmann's constant and T is, of course the temperature; N f is the number of free electrons per unit volume. So, this is the, these are the parameters that go into this. And so, we started, in that case with a kinetic theory of gases, which was for the molecules of the gas; and then, the results that we got, we associated, instead of molecules of gas, we associated it with the electrons. So, the m here would then be the mass of the electron, so to speak. So, mass of the electron and this N f would then become the number of free electrons per unit volume. So, otherwise, this relationship could be extended to the molecules. So, this is the idea here. So, we will take these two; we see C v here, the constant, the specific heat at constant volume associated with electrons; so, or the electronic contribution to the specific heat at constant volume. This C v e is available here and m v square term is here, m v square term is here. So, we can substitute this back. So, we will just do that substitution now.

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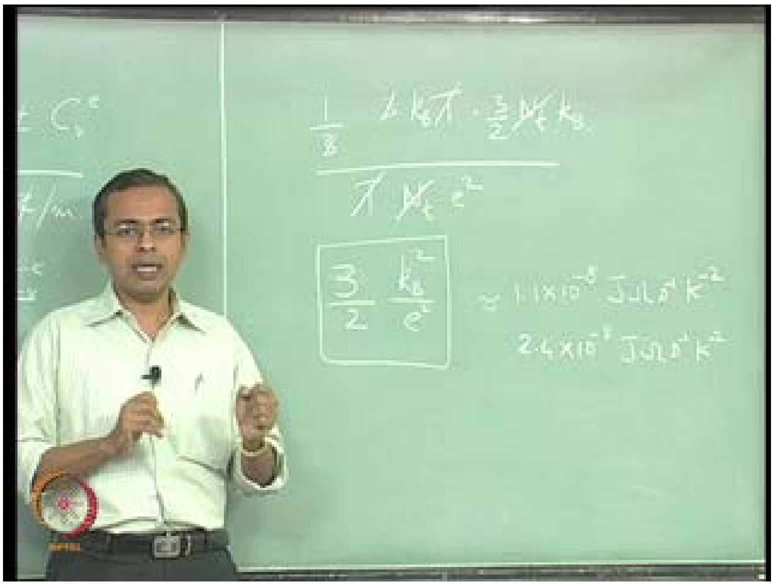

So, we have, instead of m v square, we will put 3 k B T, and instead of C v e, we will put 3 by 2 N f k B divided by T N f e square. So, we have just taken the expressions associated with the electronic conductivity given to us through the Drude model and the expression given for thermal conductivity from the Drude model; both of these expressions we have just put the ratio down of those two; and, we have used some relationships that we found in the kinetic theory of gases, which give us other expressions for specific parameters that are there in the thermal conductivity and the electronic conductivity. So, there and therefore, we get this expression here.

So, let us just simplify it. You lose the T here; this 3 and 3 will go; the N f and N f will go. So, therefore, they all cancel out and you would end up with 3 by 2 k B square by e square. So, 3 by 2 k B square by e square, alright. So, just looking at the Drude model and extending all its findings, and just using its findings, if you do, come up with, if we write down the expression for K by sigma T, then, this is the expression that we come up with. If you evaluate this, this will work out to 1.1 times 10 to the power minus 8 and the units should be the same, which would be Joules ohm second minus 1 Kelvin minus 2, right. So, this is what you would get; unit should be the same and this is what you will get. As I said, the experimental finding is about 2.4, right. So, the experimental finding is 2.4 and this, what the theory is predicting is, 1.1. So, so. So, this is what we find, for the two values that we have here. So, we find that, as, as I mentioned, one of the things is that, at some level, in all these theories, when we compare it with experimental data; our intent is, to see how well it predicts within an order of magnitude, so to speak. So, from that perspective, from the perspective of predicting an experimental result, within an order of magnitude, this actually works out to be very accurate; I mean, it is pretty good; I mean, you are coming well within an order of magnitude, in terms of the prediction that is being made.

So, therefore, the Drude model not only predicts independently the thermal conductivity, and independently the electronic conductivity of the material, it is also able to predict, or give, give us a result that is consistent with the Wiedemann Franz law. So, the Wiedemann Franz law is being predicted reasonably well by the Drude model. So, these three are then considered as successes for the model; the model actually does a good job in predicting experimental data, in all of these three categories. So, if you step back and see, at, at first glance, when we started out, I cautioned you that, we are actually taking something that belongs to gases, and pushing it, taking a theory that belongs to gases and pushing it onto something that exists within a solid; and, we also said that, there is reason to be cautious about this because, the number density of the particles is three orders of magnitude higher, in the solid. So, given that, you needed to be cautious and therefore, this was something that we needed to be careful about. Despite this reason for caution, we find that, three major results are actually pretty good, in, in terms of the match between what the theory is predicting, and what the experiments are showing, ok.

So, therefore, there is something good about this theory, even though, at first glance, it seems to be a very simplistic theory. So, therefore, these are considered successes. Now, however, there are reasons to be cautious about it and which is what we will discuss now. While we are happy that the Drude model actually has done some accurate predictions, and this is something that, then enables us, the whole purpose of all these exercise is that, if you can come up with such a theory, then, you can make other predictions; you may be able to predict other behavior of materials, which may, or may not have been experimentally seen; and that is how you push the frontier of science. So, that is, that is the whole idea of this entire exercise. So, given this, if you examine all the things that it has done correctly, and if you find that there are things that it may not be doing correctly, then, it gives us, at least, it tells us something about the boundaries of this kind of an approach, fine.

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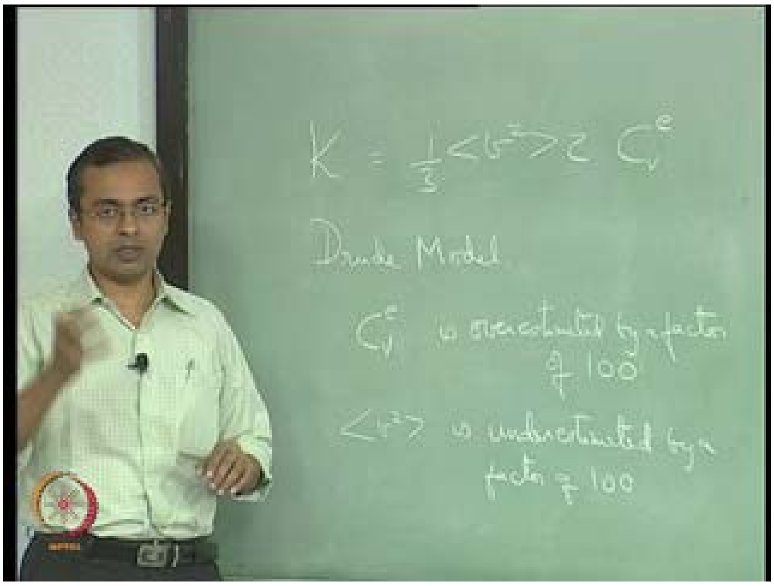

So, now, it turns out that, if, if you actually look at the expression that comes up for the thermal conductivity, which is K equals 1 by 3…So, this is the expression that the Drude model gives us for the thermal conductivity, right. Now, it turns out that, if you actually look at the…We said that, you know, this value actually works out fine; the value for thermal conductivity that we get, actually works out fine, in, in terms of how well it matches the experimental data; but on closer examination, we find that, this match with the experimental data has occurred due to a couple of errors in this, in this prediction; and, it is just a matter of, it, it just, it is just a matter of chance, or luck, that it turned out that, the errors were such that, the prediction ended up still being correct. What were the errors? Basically, the electronic contribution to specific heat C v e, is over estimated.

It turns out that, if you actually, people have been able to do other experiments and you can, you can measure specific heat at lower and lower temperatures, where the contribution to this specific heat is mostly due to the electrons present; and, it will turn out that, the electronic contribution to the specific heat, as estimated through this theory, works out to be 100 times more than what it actually is. So, whatever is the value, it is predicting it 100 times more. It also so happens that, the mean square velocity, mean square velocity, as predicted through this theory, when it is applied to the electronic system inside a solid, this mean square velocity is under predicted, is underestimated. It so turns out that, the mean square velocity as predicted through this theory is underestimated by a factor of 100. So, we have one parameter in this equation, which is overestimated by a factor of 100, and another parameter in this equation, which is underestimated by a factor of 100; and, since they are all getting multiplied here, they cancel out; and this, this occurrence is just a matter of chance. It just so happens that, the expression ends up such that, the value that is underestimated by a factor of 100 and the value that is over estimated by a factor of 100, simply multiply with each other in, in that equation, in that expression; and, as a result, we are, we end up with values of K, which are actually quiet consistent with what is available in the literature, which is consistent with what is available for, what is known for various metallic systems, for various materials. So, it is sort of, a matter of luck, that this is, that, this happened.

So, as a result, even though when you examine this...So, when you examine this theory, you find that, at a more fundamental level, there is an error. So, there are other parameters that the theory is predicting; the C v e itself, is something that the theory is predicting; the C v e itself is wrong. So, so, while the thermal conductivity works out correctly, the electronic conductivity works out correctly, and the Wiedemann, the prediction of the Wiedemann Franz law also works out correctly; while all of these things work out correctly, it turns out, the electronic contribution of specific heat is wrong, but not by a small margin; it is wrong by a factor of 100. So, that is two orders of magnitude off; and, as I mentioned, one of the things that we are looking at, is the order of magnitude; that is the parameter of interest, so to speak; and, we are off by two orders of magnitude. The mean square velocity which would then translate to the, would then be, would then give us the translational kinetic energy, is also off by a factor of 100.

So, key parameters to something that is fundamental to that material, are off by two orders of magnitude; and by luck, it so, happens that, many other things are falling in place. So, therefore, we are able to see, experimentally, we are able to identify that, there are, there is something significantly wrong with the manner in which this equation, this model is calculating out it is parameter. So, these two are identified as, sort of the failures of this model, if you want to call it. So, there are three grand successes that it has got; it has got a couple of failures here. There is one more thing, that it does not predict very well and that is called the Hall effect, ok.

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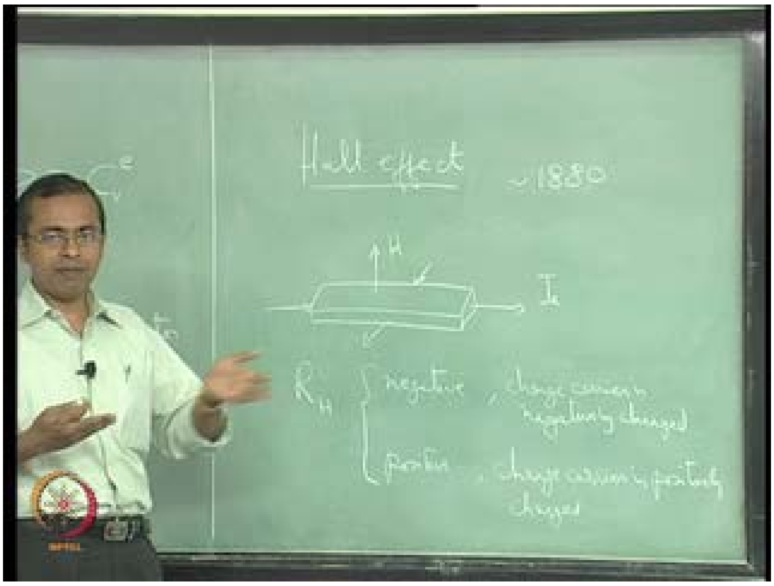

The model actually predicts the Hall effect. It is not that it does it wrong, but we would discuss this in a moment. This was also an effect that was discovered around the year

1870,1880; so, around 1880 and the basic idea in, in this effect was that, if you have, if you have a conductor that is carrying current, some, some current density J x and perpendicular to it, you apply a magnetic field; you apply a magnetic field perpendicular to it. What would happen is, the, since the charges are, charges are moving through this material, they would be deflected to one direction, based on this, based on their interaction with this magnetic field. And, and therefore, you would get a potential that you can measure, in this direction. So, you will get a potential in a direction perpendicular to the direction of flow of current, alright. So, and what, basically, what is happening is, it is deflecting the charge carriers to one direction. So, there is a buildup of charge on one side; on each side, there is a, there is a buildup of opposite charges and that can go on only for a short period of time, because then the potential gradient that the buildup of charges is putting together in that location, would then oppose the force with which the charge carriers are being deflected. And so, you then reach some kind of an equilibrium.

So, therefore, you will get a potential across this sample, in a direction perpendicular to the direction of flow of current. Now, while we, it is not of immediate interest for us to go in into great detail of this Hall effect and how it works out, a bottom line is that, depending on the sign of the charge carrier, whether it is a positive charge carrier, or a negative charge carrier, your voltage would then work out to be positive or negative. So, and therefore, we have something called a Hall coefficient, R h it is called. So, that would work out to be negative, if charge carrier is negative, is negatively charged; and, it would work out to be positive, if charge carrier is positively charged. So, the basic information that this gives you…So, this Hall coefficient is that, you are able to identify the sign, the, the sign of the charge that is present on the charge carrier. So, normally, otherwise, you would just measure a current; you would not know, and since we are conventionally giving the direction of positive current as the conventional current, so to speak, you would, regardless of which charge carrier is moving, that is the current that you will know, you will measure. So, just measuring the current will not tell you, what is the charge carrier that is participating in the process. So, this Hall coefficient now enables you to find out whether positive charge carriers are taking the current, or negative charge carriers are taking the current. And, so, you could have…So, for example, whether it is electrons or holes that are supporting the current; that is something the Hall coefficient gives us.

So, this is the general idea, and so, therefore, this is considered a very useful kind of an experiment to do. Now, what happens is, in some systems, you can find that, this work, you can get a Hall coefficient that is positive and therefore, indicating that you have positive charge carriers present in the system. The Drude model, it turns out, is actually, it is able to predict certain things that the Hall coefficient is telling us, but it is able to do so, only for the negatively, for the negative Hall coefficient. When it shows a positive Hall coefficient, the Drude model is unable to give us results that are consistent with this Hall effect, so to speak. So, it, it has some issues handling the Hall effect, which is a very fundamental experiment, in terms of giving us some idea, or some insight into how this, how the, how the current is being carried, within a material. So, we find that, there are some drawbacks. So, we find that, there is an over estimation of the electronic contribution to specific heat. There is an under estimation of the, the mean square velocity that we associate with the electrons and there is an important experiment like Hall effect, which the theory is not able to predict. So…So, taken, taken, if you take all of these things together, we find that, there are successes for this model; there are also failures associated with this model. So, therefore, there is reason to…First of all, the fact that there are some failures, immediately implies that, this places some boundaries on the, on this model, which means, you cannot just like that, use this model; you have to be careful of the circumstances, under which you are using this model, to specifically be conscious of the fact that, if you are using this model to predict specific parameters, you could be wrong.

We could have easily had a situation, where we have, when we have this expression for the thermal conductivity we could have easily had a situation, if, if the parameters had worked out such that, instead of cancelling out, it could have been off by four orders of magnitude, right. So, all these things could happen; you could be off in a big way, in a very big way, with respect to the predictions. It so happens that, they are cancelling out. So, therefore, there is reason to be cautious about it. There are, as I mentioned, there are, there are lot of other things also about materials, that we would like to know. So, for example, since then, we have learnt a lot about, if you look at, you know, the semiconducting industry and so on, the whole concept of semiconductivity, the, the idea that we have of systems such as, you know, insulators, semiconductors and metals, as being three classes of materials with respect to the, to their electronic properties, we would like to know, if our theories that we put together for materials are able to predict all these things, are able to show us, why within a system, such processes occur, or such phenomena manifest themselves. So, so, in, in terms of a band, or whatever it is, that we are calling a band. So, this is something that, also we wish to examine. In general, we have heard of bands, but never really looked at where they come from, so to speak.

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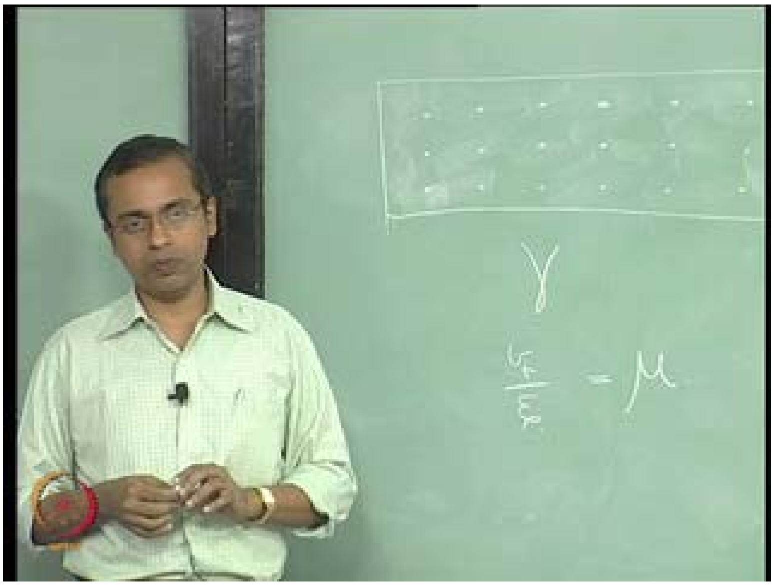

And, if you also look at our original picture for the material, we basically said that, in the Drude model we just had ionic cores; and then, we have the free electron gas running all around it. So, we have ionic cores and a free electron gas. So, this is the model that we use, and we have largely neglected the ionic cores. We have no, we are, in no specific manner are we including the impact of the presence of those ionic cores, on the behavior of the electrons. It is only a very general resistive term that we used, like gamma that we use, that we came up with, which we use. So, so, in fact, the resistive term that…This is actually a very general term, which is actually averaging out all the behavior of the ionic cores and the interaction with the other electrons. So, but the point is, if the crystal structure were to change, or for example, in different materials, we find that, there is also anisotropy. So, we…

When we have anisotropy, that is something that we are conscious of, or we have heard of and so on. It means that, simply knowing the number of free electrons per unit volume is itself not going to give us, is itself not a complete information; because, even if you have anisotropy, I mean, if you have an anisotropy in a sample, and you can definitely find samples which show you anisotropy. If you take some reasonably sized unit volume, the number of free electrons within it is the same. So, the number of free electrons within it is the same. So, therefore, if you simply use that as a parameter, as your starting parameter, or as a primary parameter, based on which you are able to say, what is the extent of conductivity, then, we, we have a conflict. So, because the, it, it should not, the direction should not matter. So, direction is something that comes in terms of, the direction is also something that is defined only with, if there is some framework, within which you can define it. So, in a gas, for example, if it were really a gas, I mean, if you take a ideal gas for example, in that ideal gas, in the container that contains the ideal gas, there is no immediate relevance to up or down, front or back, left or right and so on. So, there is no, there is no specific preferred direction; the directions are all equally probable for any molecule; so, the velocities it possesses are equally probable in all these directions.

So, in our derivations, in fact, that is why the v square, the v x square, the average of the velocities in the x direction, the square of the velocities in the x direction was treated as one third of the average velocity; because, the v x is just as probable as v y, just as probable as v z. So, in that sense, it is equally random, truly random in, in a ideal gas; but when we put that kind of a electron gas inside a, free electron gas inside a solid, and you actually have a framework, where you have this ionic core sitting in some kind of frame, there is some sense in, in terms of directionality; there is, to the extent that the ionic cores are able to manifest themselves, manifest their presence; their presence gets manifested in the form of the directionality that they hold. They are not randomly distributed. They are distributed in a typical crystalline solid, in a very ordered manner. So, if it is in fact true that, the ionic cores completely do not participate in this process, in the process of conduction, in the process of any electronic property that we are measuring, or the thermal properties that we are measuring; if they do not participate in any manner, then, it should not make any difference; it really should make no difference, whether they are there, or they are not there. On the other hand,

if they do participate in, in any manner, in this process, either constructively, or destructively, either they are assisting the process, or they are actually slowing down the electrons in some manner, or completely preventing them in some manner from carrying out the conduction process; regardless of what manner it is, either constructive, or destructive, if the ionic cores are actually participating in the process in some manner, then, at some level, the ordered manner in which they are present within that solid, should also impact the extent, or the manner in which they are able to effect, affect the movement of the electrons. So, it, what I am saying is, even, even as, as it is now, even though we have developed this model, and we have actually, directly seen some successes and failures of this model, if you step back and look at this overall picture of what it is that we have to tried to model and what is the level of detail we have gone into so far, in trying to model the system, we immediately start seeing that, there are reasons that we need to reexamine our approach, to actually see, if we can at least refine our approach, to see if we can come up with a better approach, which, which accounts for more interactions, more, more of the final details of the material, so that, presumably if we again rebuilt this picture using all those final details, the kind of prediction it will make, will now start accounting for more of the details that will, will actually at least get rid of some of the failures of the existing model.

So, this is something that we need to look at. So, so, we have this picture, where we have had this electron cloud, so to speak, and how it moves. In, in fact, when we did this, we actually had a parameter v f by e which comes into this picture. So, this actually is referred to as mobility, mu. So, when we actually did this electronic conductivity, we came up with this parameter, mobility, mu; and, this actually represents the ease, or difficulty with which the electron moves within the system. This, this parameter, it is the velocity per unit field, so to speak, velocity attained by the species per unit driving force. So, this is something actually which is more general parameter, which actually we would see even in other places like diffusion and so on. There also, where you have a gradient and there is a diffusion species, there is a driving force and there is a diffusion, diffusing species, there would be a mobility associated with the diffusion, diffusing species. So, we have such parameters. In, in our approach, we have just used, in terms of the resistive force and so on, we have lumped many things together and then, come up with this resistive parameter, or therefore, the mobility mu. So, this is what we have done; but as I mentioned, we now need to refine this process more; we need to put in together more of the details of the material and evolve a better model. So, if you look back, to summarize now, we have actually done the, developed the Drude model to a reasonable degree, and looked at two major predictions that it makes, the electronic conductivity, as well as the thermal conductivity.

There are other predictions also it makes, but this is, these are the two that we have looked at so far - the thermal conductivity and electronic conductivity. We find both predictions are good. We looked at the ratio of thermal to electronic conductivity; we find that, that ratio is also good; but on further analysis, we find prediction of the electronic contribution to specific heat is wrong, is off by two orders of magnitude; prediction of the mean square velocities of the electrons is also off by two orders of magnitude. And, we also find that, we are also now aware that, this Hall coefficient is something, that it is able to explain partly, but not completely; in parts, it is able to explain. So, this is the summary of what we have done so far.

As we proceed forward, we will examine a little bit more on, what is that thing that is fundamental about this Drude model, about the assumptions; we have used some assumptions; we have developed some equations and so on. We will explore that a little bit more; we will look at our assumptions a little, a little bit more carefully, to identify what is that specific aspects, or what are those few specific aspects, that are things that we need to change. So, when, when I say we have to refine the model, we cannot arbitrarily refine the model. We will look at this assumptions carefully over the next class also, and see, what is it that we need to pin down, which now needs to be changed and why it needs to be changed. So, so, first, we will identify what needs to be changed, or at least see, where is it that we could have possibly gone wrong; then, we will see what is the change that would be the most appropriate change to put together; and then see, if now, with this new change put in, put in place, does the model, does it rectify the major problems that we find in this model. So, so, this is the approach we will take and it will help us refine this model and come up with a better model. So, but for today, we will halt with this, and (( )) take up our refinement, we will revisit some of these equations and assumptions and proceed forward. Thank you.