Dr.Prathap Haridoss
Department of Metallurgical & Materials Engineering
Indian Institute of Technology, Madras

Lecture No. # 8
Drude Model : Thermal Conductivity

Hello. Welcome to this eighth class in this physics of materials lecture series. So, we have now, through our earlier classes, built a framework for this course and also started building models for the material, for various properties of the materials. More specifically, we have focused on the metallic systems and so, our focus will continue to be metallic systems, where possible, we will generalize to other materials. In the last class, we started developing, in the last couple of classes, in fact, we have started putting together the framework for what is considered as the Drude model for the materials, and specifically, for the conductivity and it is a model that is attributed to Drude and Lorentz.

And, the basic idea is simply that, we take the ideal gas, kinetic theory of gases rules and apply it to electrons in the solid. We treat them as free electrons, which run through the extent of the solid and see, what is the behavior of these electrons that we can predict. Specifically, in the last class, we looked at the electronic conductivity process and we tried to see, what would happen to the electrons in the solid, if you apply potential gradient across it; and how they would respond to it; what are the processes that would then slow the movement of those electrons; in other words, the collisions that they would have with other electrons present within that system; and what is then the equilibrium, that, the kind of static, I mean, sorry, dynamic equilibrium that exists within that system; and then, how the electrons are moving, as a result of this potential gradient that exists. So, they continue to move; they reach some kind of a velocity that, which is the average velocity with which those electrons.

So, this is the kind of system, or the model that we are building, and as I mentioned, this is the Drude model. Today, we will take this model forward; we will try and explain one more of the material properties, which is the thermal behavior of a material, using the same framework, which is, what is being employed for the Drude model. So, that is the basic idea that we will explore through this class today. So, if you look at the thermal property, say essentially conductivity, or heat conduction, if you want to transport heat, then, this is actually a phenomenon that is very important from an engineering perspective. There are a lot of places in engineering, in, and in technology, where heat transfer is a very critical aspect of the functioning of the technology. So, it is a very important aspect of the functioning of the technology, heat transfer and so, a lot of, I mean, it has been studied quite extensively; if you look at literature, it has been studied very extensively..

If you look at heat transfer, in, in general, then, there are three modes by which heat can be transferred from one location to another.

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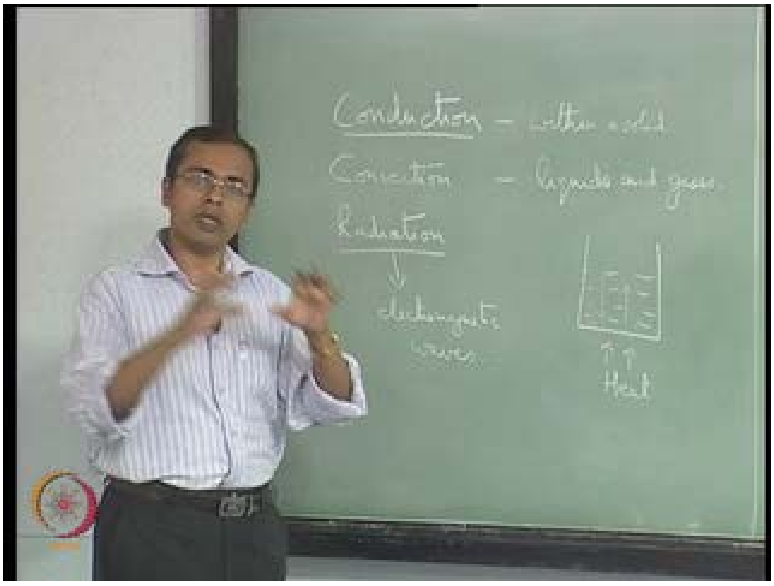

And, they are conduction, convection and radiation. So, we have three principal modes by which heat can be transferred from one point to another. And, they are simply conduction, convection and radiation. So, in all engineering, in an engineering sense, anywhere you run into the heat transfer processes, you will see one, or more of these in effect, playing a role. So, when we talk of conduction, it is a process where heat is getting transferred due to direct contact of the material; is with something else. So, atom to atom contact is there; we are not really looking at atomic movement, in the sense of movement of atom from, over any large distance, so, to speak. They are all atoms, which are sort of frozen within their, not frozen,, but they are in fixed positions, relatively speaking, in a lattice, and then, heat is being transferred across through those atoms, or also through the electrons which is present within the system. So, in fact, in highly conducting materials, high thermal conductivity materials, often the heat is being transferred through the electrons that are moving through the system. In less conducting materials, the transfer of heat is occurring through atomic vibrations, where each atom, each atoms’ vibration is then being, partially being transferred to its neighbors and. So, on. In that process, the heat gets, the energy gets transferred from one, from the hot end to the cold end..

So,, but the basic idea being, largely, the atoms are in fixed positions. So, there is…So, this is typically a process that we associate with a solid. So, conduction is a process that we would normally associate as the primary manner in which heat is being transferred from one end of a solid to another end of a solid. So, it is within that solid, whatever is happening, that is the kind of heat transfer that we would, we would refer to as conduction. So, this is within a solid, in general, this is what we are looking at. So, within a solid is a good example of where you are looking at, for conduction. So, conduction, in that, it is in that context that, we most often talk of conduction.

Then, we have convection, which is something that we associate normally with liquids and gases. And here, the transfer of heat is occurring due to a fairly large scale atomic movement. So, in, it is in that sense that, it is distinctly different from the conduction process. So, if you take, for example, a beaker containing some liquid and then, you slowly heat it up. So, you have a beaker here, and some liquid; and, you very slowly heat it up; very slowly heat it up. What will happen is, the temperature of the liquid very close to the bottom of this container, will go up; in general, this also means that, the density of this liquid very close to the bottom of this jar, of this beaker, the density will go down, because the liquid will expand. So, density will go down; there will be colder liquid on top, which is now of higher density; and so, the colder liquid will try to move down and the hotter liquid will try to move up. So, in this process, you will have the liquid getting mixed up and this is going to occur, in a very slow and gradual manner as you, depending on how slow, how slowly you introduce this heat into the system. So, gradually, this is what is going to occur within the system and so, slowly this liquid gets mixed up, simply because, you are providing heat at one location. So, therefore, it gets mixed up. So, such a mixing up of liquid, or a gas, as a result of this thermal differences that exists between one region of this liquid and another region of this liquid, such a process of mixing up, we refer to as convection.

So, so, this is the…So, as I mentioned, largely this is something we associate with liquids and gases. And, to the extent that it occurs in the manner I have described, it is something that we would refer to as natural convection. So, you, if you did nothing else, and you just provided heat from one, on one end of an, of a container containing a liquid or gas, simply because of density differences that will arise within the system, the liquid, or gas, will now start getting mixed up; specifically due to this density difference. Such a process is then, as I mentioned, referred to as natural convection..

In an engineering sense, often, if you want convection in any, as a significant form of heat transfer, if you want to ensure that some heat transfer is occurring, and you want it to occur as quickly as possible, as well as possible and. So, on, or, you want speed up the process, then, normally, in an engineering sense, or a technology sense, you would actually go in for, what we, what is referred to as forced convection. So, therefore, you would have either a fan, or a blower, or something that agitates this liquid system significantly; and in the process, you are deliberately ensuring that, there is a lot of mixing of the liquid. So, whereas, normally, it might have taken several minutes for the temperature at the top of this container to reach whatever is the temperature at the bottom, or very nearly that temperature; by forcibly agitating this liquid, you can ensure that, the top temperature reaches very close to the bottom temperature, almost immediately. So, therefore, in an engineering sense, forced convection is very important and even if you look at a day to day example, so, an air conditioner. So, in principle, it could be putting out a very small amount of cold air very close to the air conditioner's exit and then, over a period of time, the temperature in the room can drop;, but again, as a technological thing, from the perspective of the comfort of the people in the room, you want the temperature of the room come down quickly. So, therefore, you have a fan, or a blower, additionally in the air conditioner system, which then starts forcing the air into the room.

So, therefore, you get much better mixing of the air, cold air and whatever is in the room; and, in the process, the temperature comes down. So, convection is something, as I said, we associate with heat transfer occurring in liquids and gases; and, it has lot of technological uses. So, that is what it is. Conduction, as I said, largely, it is the primary mode of transport of heat within a solid. So, that is how conduction is seen; as a, if you want to separate it in…And then, finally, we have radiation, as another means by which heat can be transferred from one position to another; and largely, radiation is something that, where we are associating it with, say electromagnetic radiation, electromagnetic waves. So, in space, for example, the heat that we are receiving from the sun; between sun and earth, we do not have any direct physical contact. So, we do not have conduction between sun and earth. We do not have any, since the atmosphere that we sense around the earth is essentially, just within few, a few 100 kilometers atmost, within the, from the surface of the earth. For most part between earth and the sun, there is no, there is nothing; there is no atmosphere, anything; there is nothing that resembles an atmosphere between earth and the sun, in the distance that is there between the earth and the sun. So, therefore, we cannot really look, in terms of convection as a process by which heat is coming from the sun to earth, to the earth. So, in fact, the heat that does come from the sun to the earth, comes in the form of electromagnetic waves, and therefore, is essentially using radiational processes..

So, radiation is what is being used to send us heat from the sun to the earth. Of course, once it arrives at the earth, to the extent that it heats up the atmosphere and then, gets evenly distributed across this, across the planet; a fair fraction of that is coming from convection, convective processes within the atmosphere. So, a lot of convection is occurring within the atmosphere to evenly distribute this heat across the planet. IN fact, if you look at the difference between, say a spacecraft out in open space, as opposed to any of us standing on earth, the, because of the convective processes and. So, on, which makes the temperature more uniform for us, we do not sense a big difference in temperature between the side of us that faces the sun, and the side of us that is not facing the sun; whereas, the, a similar situation out in space, where you do not have any other medium to evenly distribute the heat, would result in a situation where the side of the spacecraft facing the sun, would actually be significantly hotter, than the side of the space craft which is in the shadow, or essentially facing the dark side of the space. So, therefore… So, that shows you that, there is a, I mean, where these various processes are relative to each other, and how they, and locations and examples where these processes become important to us. So, now, what we are going to do is, I just wanted to put this down, because I want you to get a sense of what it is that we are looking at, and perhaps, what are the other things that are there, which we are not specifically looking at, at the moment.

So, for our purposes of discussion, we will focus more on the conduction process, within the solid. So, conduction is what we are going to look at. So, the Drude model which we extend to the thermal property, specifically to thermal, to heat transfer in, in that sense. We are looking at the conduction process of heat transfer; we are not really extending it to convection or radiation. So, already, we are now taking, looking at a subset of this process. Even here, in the solid, as I mentioned, if you take a typical solid, you can have a good conductor of, a solid which has, say, a lot of free electrons present within it, and is a good conductor of heat. There, a vast fraction of the heat that is being transferred from the hot end to the cold end, is occurring due to the movement of those electrons.

So, or, they are the primary, what to say, mechanism, or vehicle through which the heat is getting transfer, transferred, from the hot end to the cold end. As I also mentioned, if you look at a more insulating type of a material, it is not, I mean, it is, it simply conducts heat in a much slower manner,, but it does still conduct heat, eventually. There the heat conduction is largely occurring due to thermal vibrations, which we would also call phonons;, but that is not something we are looking at immediately. So, the manner of the process of conducting heat through thermal vibrations is, is not something that we are focusing on, at the moment. So, conduction itself, could occur, either through the movement of electrons, or through this lattice vibrations exchanging energy from atomic position to atomic position. Of the two, we are only looking at the former case, which is the electron movement. So, we are not looking at these two modes, and in the conduction mode, we are only looking at the electron transport, transport process for transferring heat. So, I just want you to have that complete picture,. So, that, whenever you read an article, or look at a book, which is looking at the, which is looking at more holistic look at the thermal heat transfer process, you get an idea of what it is, that we have actually focused on here, and what it is, for which the equations that we develop, apply to. So, those equations do not cover everything; they cover a part of conduction, conduction that is relevant with electron movement, the heat, heat conduction using electron movement..

So,, but having cut it down. So, much, I would also like to emphasize that, most of the time, where you are looking at heat conduction, and time to utilize it to serve some purpose, you are actually utilizing materials where there is a lot of free electrons, and the heat transport is going to occur through the mechanisms that we are going to describe. So, therefore, from the perspective of heat conduction, and good conductors of heat, the equations that we develop are quite relevant. So, therefore, it is, even though I am showing you what is the limitation, or the framework within which we are operating, and what are the things we are excluding from it, what we are including, is reasonably significant, and therefore, it is relevant and meaningful to know. So, now, we will go ahead and focus on conduction, from the perspective of what it is, that the electrons provide to this conduction process..

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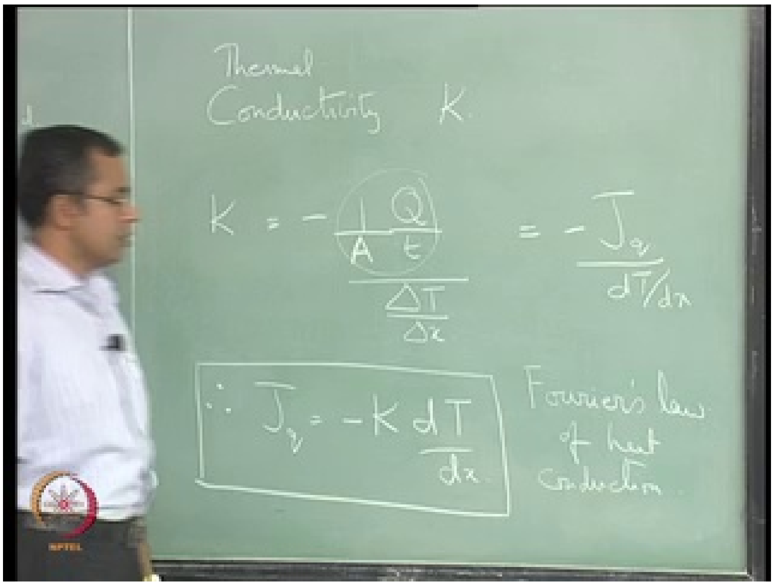

Before we do that, also, we would just additionally add…So, from this, from within this framework, we will define conductivity, or thermal conductivity. It is denoted by a symbol K. So, thermal conductivity is, we will talk of this minus sign later;, but it is basically the transport of heat. So, some heat energy, thermal energy in, say, Joules per unit time per unit cross sectional area and per unit temperature gradient. So, you have some solid; you look at unit cross sectional area of that solid; and in that unit cross sectional area, per unit time, what is the heat that is being transferred and between two points. So, x 1 and x 2, you have; the temperatures between those two points are T 1 and T 2; capital T 1 and T 2. So, you have a delta T and delta x and this is the framework, within what, this is what will give you the thermal conductivity. So, if you look at this, or if you rearrange this, we will get, in differential form, we can write this as, this term here, is simply…So, therefore, this is the heat flux, which is the heat transferred per unit time per unit cross sectional area. So, therefore, we can write J q, J q d T by d x. So, this I have written in a one dimensional form. So, that is why, the, we just have x here; d T by d x is what we have. So, this is one dimensional. This is referred to as the Fourier’s law of heat conduction. So, Fourier’s law of heat conduction is what this is. So, this is the quantity that we are interested, we are interested in K, that is the thermal conductivity. So, this is the quantity that we are trying to explore, and get a value, get a sense for, based on whatever is fundamental, within that material, right. So, so, now, we will focus on, what we will do is, we will start, by actually looking at a one dimensional case; because, this, I mean, it simplifies our analysis. So, we will look at a one dimensional case. So, we will focus only in the x direction; and later, we will generalize if for three dimensions and then see, if we can build first principles model,. So, to speak, or looking, or based on something that is fundamental to the constituents of the material. We will try and build a model, which helps us get a value for thermal conductivity. And, we will do this within the framework of the Drude model, of, of the material,. So, to speak..

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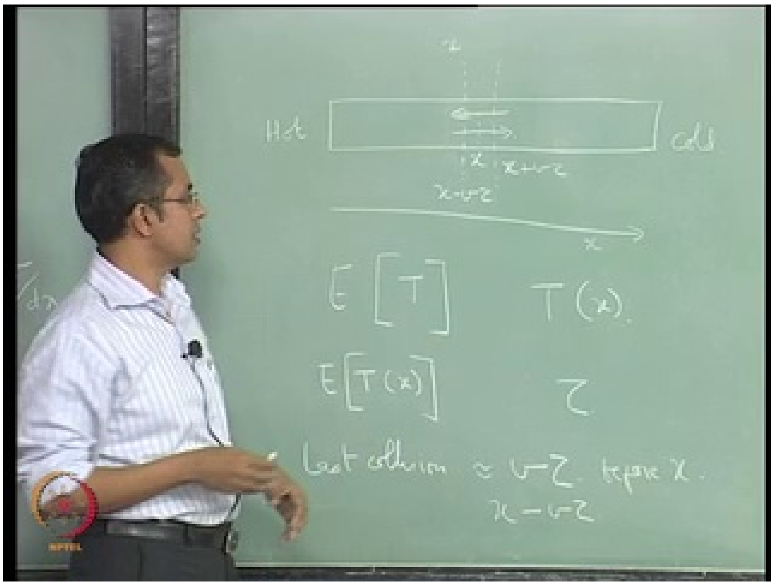

So, now, what we will do is, we will actually look at this particular experimental case. So, we now have a one dimensional solid, an example of a one dimensional solid. We have just put something down here and we will say that, this end of it is hot and this end of it is cold. So, heat has to move from the hot end to the cold end. So, this is the x direction; fine. So, this is the x direction. So, now, we would like to get a sense of how this process is going to occur, and what are the parameters that are going to impact its occurrence. So, the first thing we have is, we see that, there is a difference in energy, from one end to the other. So, we have a hot end and a cold end.

So, what we will say is, and we will assume everything is linear. So, what we see is, the energy is actually a function of, is effectively a function of temperature. So, we are talking of thermal energy here. So, the energy is a function of the, of that, of the, of whatever is the location. So, at any given location, if you want to know what the energy is, you have to have a sense of what is the temperature at that location; then, you have a sense of what is the energy. So, energy is actually a function of temperature. So, we will denote it like this. Now, the temperature itself, because there is a hot end and a cold end, the temperature itself is a function of x. So, T is a function of x. So, energy is a function of temperature; temperature is a function of x. So, therefore, as a combined notation, we will simply write it as E T x. So, this is basic. So, this is all what we mean; we have just combined all this information into this one notation. Energy is a function of temperature, and the temperature itself is a function of x, the position, in this one dimensional sense, fine..

So, so, this is what we have. Now, we would like to see, how the heat is moving from the hot end to the cold end. We will just take some arbitrary location x. So, this is the x direction, of which we are taking the particular value of x; and, what we will do is, we will look at heat that is moving from the hot side to the cold side, across x; we will also look at the heat that is moving from the cold side to the hot side across x. Essentially, what happens in all these cases is, if you look at it from an atomic perspective, all these transfers that are occurring, the atoms as such do not know which is hot, which is the hot side, which is the cold side and. So, on. So, even though we are putting down, you know, a hot side and a cold side and. So, on, in any process, whether it is heat transfer, whether it is diffusion, and. So, on, in all these cases, when you go down to the constituents of the material, there is no reason to believe that, any of those constituents know exactly which is the hot side, which is the cold side. So, what is actually occurring is that, at any given location, movement is occurring in all directions. Things are moving from the hot side to the cold side; things are also moving from the cold side to the hot side. What we perceive from an experimental sense as heat transfer, is now then the difference between the two processes that are occurring. So, we, something occurs from hot to cold; something occurs from cold to hot; you subtract the two; whatever is the net that is occurring, is then the heat transfer that is occurring..

So, this is the same process. This is the same kind of a situation, that you would also have, if you were talking of diffusion. You can have actually, randomly, atoms moving in all directions; there is a net movement of atoms in some directions, then, you have a diffusional process. So, the same thing we will look at here, in the heat transfer process. So, we would like to see, what is the heat that is going from this side to this side; we would also like to know, what is the heat is going from the cold side to the hot side. So, to do that, what we want to know is, to see what is the energy that is crossing over x, this location x, from the hot side, we want to know, what is, what is the energy of that set of electrons, which are now arriving at x. So, that then determines, what is that energy, that is crossing the x direction, or this particular interface, this particular location..

We discussed in our last class, within the framework of this model that, you have a certain mean time between the collisions, for the various electrons. So, that mean time between collisions, we designated as tau, right. So, this mean time between collisions is of significance in our discussions right now, which we will see in a moment. The other thing we also mentioned within the framework of this model is that, the electrons exchange energy with each other, only by means of collisions. So, that is the primary and only, in fact, the only mechanism by which they are exchanging energy with each other. So, when I say that, energy is moving from the hot end to the cold end, it is occurring by a series of collisions, where electrons which are close to the hot end, have an energy corresponding to that higher temperature that is there at the hot end; they collide with the electrons which are little further away, which had actually a lower temperature; they convey some of their energies to that electrons, which are now at a lower temperature, which pick up this energy as a result of that collision, and then that process continues. And, that is how the heat gets transferred from the hot end to the cold end. So, if you want to know what is the energy of the electrons which are arriving at this interface, at this surface, or this particular location, from the hot side, actually what it is that we are looking for is, where was the last collision that the electrons had, before they came to this location. So, if you, since the mean time between collisions is tau, any electron arriving at x, on average, has had a collision at x minus, has had a collision tau seconds earlier. And, given that, its velocity is v, v in the x direction, let us say, its last collision at v tau. So, an electron arriving at x from the hot side, has had its last collision at x minus v tau; last collision was v tau away from this location. It occurred at tau seconds before that collision, and so, the last collision v tau before x. In other words, the last collision that that electron had, was at a position x minus v tau. The collision occurred tau seconds earlier; at which point, it was v tau distance away from x, in this direction; and, so, the actual position, the coordinate of that position in the x direction is x minus v tau. Now, if we look at our location, the energy corresponding, the temperature corresponding to that location, would be T of x minus v tau and therefore, the energy corresponding to that would be, E of T of x minus v tau, fine.

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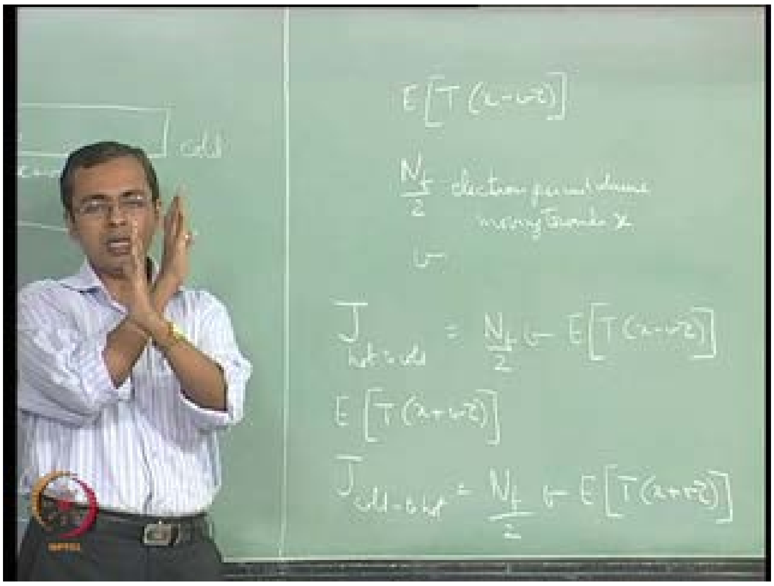

So, energy of the last location where this collision occurred, is simply, E of T of x minus v tau, right. So, for a given electron which arrives at this position x from the hot end, the position where it got its last collision was at x minus v tau; and therefore, the temperature there was T of x minus v tau, and therefore, there is an energy corresponding to that, fine. Now, we have this, arbitrarily I will just mark this here, x minus v tau. We also, within the framework…So, we are now using the parameters that are relevant to this model, which is, for example, in this case, v and this is tau; the other parameter that is of interest was, is the number of free electrons per unit volume, which is N f.

So, since we have a certain number of free electrons per unit volume, that is the same number of free electrons per unit volume across this entire length. So, now, at this location, if you consider a unit volume around this location, you have enough free electrons available per unit volume. Of these, and all of them are having this energy corresponding to this location. And, since this movement is random, half of them could continue moving towards the hot side, and half of them would continue moving towards the cold side. So, we will have N f by 2 electrons per unit volume, moving towards the hot, moving towards x, towards the position x..

And, their velocity is v. We said that, the velocity is independent of position; again, something, which is the, within the framework of this model. So, therefore, the flux of heat J q from hot to cold, equals…So, we have these many electrons per unit volume, moving with this velocity and having this energy. So, that is the number of electrons having this velocity and with this energy. But then, if you take the product, then, you are looking at the energy being transferred from the hot end to the cold end, per unit area. So, this is the flux being transferred across this, sort of, across this system. So, so, this is what we have. So, if you want to check…

So, this is number of electrons per unit volume per meter cube. So, this is meters per second and this is Joules. So, Joules per meter square per second is what you have. So, therefore, dimensionally, it works out appropriately. So, now, this same thing, we can actually, look at this image here, and we will see, what is the energy, that is being transferred from the cold side to the hot side. In other words, what is the energy of the electrons which are arriving at x,, but from the cold side. Those are the electrons, which will now cross it from the cold side. So, for them, the exact same energy will, will, same set of reasonings will hold, except that, their collisions would have occurred at tau seconds before, away from this interface. So, in other words, at x, at x plus v tau..

So, the electrons arriving at this location x, from the cold side, would have had their last set of collisions at x plus v tau. So, so, that is where their collisions would have occurred. In a very similar and analogous way, we will say that, they are again arriving here with a velocity v, and we have a number of free electrons per unit volume is again N f; it does not really matter, whether you are at the hot side, or the cold side; that is again fixed for the system. So, we have a number of free electrons is N f; again, from this location, half of those free electrons can continue moving towards the cold side; half of them could continue moving towards the hot side. The point of this half is that, it is a random process, right. So, given that, they are all sitting there with that energy, they could either go left, or they could go right; there is nothing..

We are not giving any preferential treatment that, it has to go right, or it has to go left. It is a random process; it could go, either it which way, it could go. And, it is a one dimensional sense, and so, we have two options available to us, right or left. And, so, we just say that, for the sake of keeping it random, we say half of them move that way, half of them move this way. So, again, the same reasoning would hold true. We would have the energy of the previous collision, of where the previous collision occurred, would be E of T of x plus v tau; and therefore, the J, cold to hot; so, energy being transferred from the cold side to the hot side through that interface x. We are again, we are now bringing all this down to that interface x; at that interface x, what is moving from hot to cold, what is moving from cold to hot ..

What is moving from hot to cold is simply, what arrived from that hot end to that interface; and whatever is moving from cold to hot, is simply what arrived from the cold end to that interface. And, what arrived came from the immediate previous collision. So, that is the body of information that is being contained within this equation. So, cold to hot is N f by 2 v and E times, sorry, E of temperature T, which is a function of position; and, in this case, the position of interest is x plus v tau. So, that is the thing. So, we now have two pieces of information, energy moving from the, the hot side to the cold side, and also, energy moving from cold side to hot side. We have expressions for both. So, what we simply have to do is, we want to look at the net transfer of heat. So, we simply are going to subtract these two expressions and get ourselves that expression for the net transfer of heat, right. So, we have an expression for the transfer of heat from the hot side to the cold side, and separately, an expression for the transfer of heat from the cold side to the hot side, across that interface x. So, we will now subtract the two and give ourselves a net transfer of heat.

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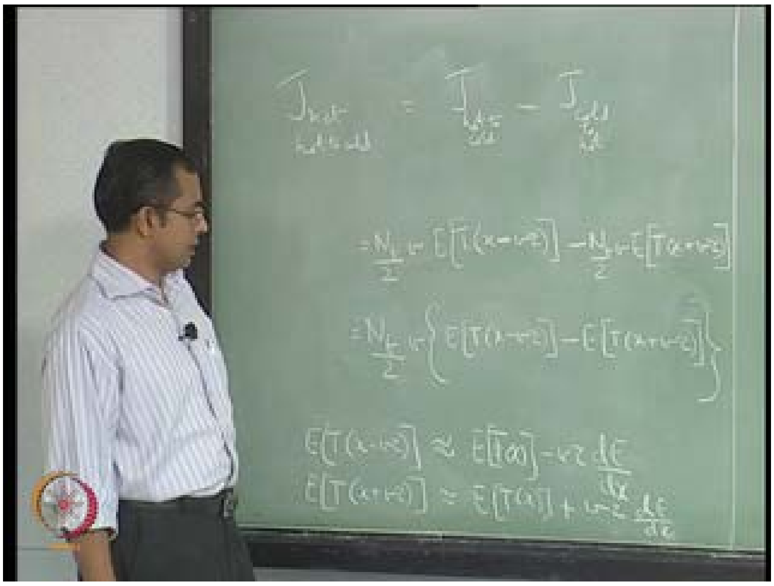

So, J net hot to cold equals, simply J hot to cold minus J cold to hot. So, this is N f by 2 v E of T of x minus v tau minus N f by 2 v E of T of x plus v tau. So, this is what we have. So, we will rewrite this in a way that is convenient to us; N f by 2 v of…So, this is what we have. So, we can write, a given E...So, we will write E T; we can write E of T of x minus v tau, as simply, approximately equal to E of T of x minus v tau d E by d x.

So, this is simply the energy at the position x; this, we are just looking at the energy at position x minus v tau, and we are sort of approximating it, for a very small variations, for very small delta x, we assume some kind of a linear variation in energy. We are saying that, at the position x minus v tau, it is that energy that is there at that position x, minus, or plus or minus whatever is that distance that you have to travel to get to that position, in this case minus v tau, minus v tau, d E by d x is the gradient, so, change in energy, as you move from position to position. So, we simply make this, say, approximation of linearity,. So, to speak and so, E of, at position x minus v tau, is simply the energy at position x, plus, plus or minus d E by d x times that position, displacement from that position, which is v tau; in this case, it is minus, because, it is x minus v tau. Similarly, E of x plus v tau would similarly be, E of T of x plus v tau d E by d x. So, plus d E by d x; plus in this case because, we have plus v tau, and minus because, we have minus v tau. So, we can make these approximations, and then, substitute back into this equation.

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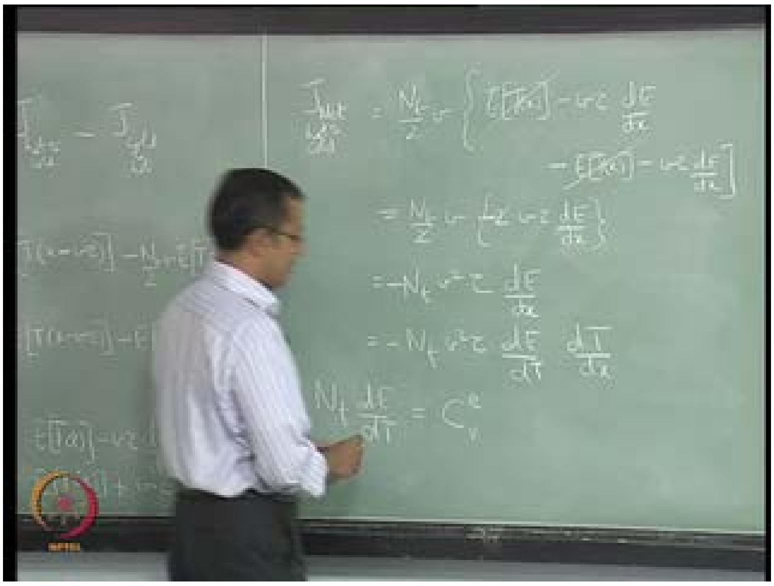

So, therefore, J net hot to cold equals N f by 2 v, you will have…We will just write it down fully,. So, that, it is clear. You have E of T of x minus v tau d E by d x minus E of T of x…So, this is actually E of T of x plus v tau d E by d x;, but we have subtracted the whole thing out; we have to put a minus sign here. Therefore, you get a minus sign in front of this entire term here. So, this minus sign would come in front of this, as well as here. So, this is what you have put down here. So, these will go. So, therefore, you have, and so…So, you have N f by 2 v times 2 v tau d E by d x. And, please note, we have put d E by d x; actually, as we said, for us to be more specific about what we have here, energy is a function of temperature, which is then a function of position. We have directly written energy, we have taken the derivative with respect to x; we should actually take it as with respect to T, which would then, again derived, differentiated with respect to x. And, we have a minus sign here; minus v, minus v tau, minus tau; we have here. So, minus 2 v tau. So, this is equal to N f v square, 2, 2 will go, tau d E by d x; minus, minus N f v square tau d E by d x.

So, writing it again, we will have minus N f v square tau d E by d T, d T by d x. So, this is what we have as the net flux of heat from the hot side to cold side. If you simplify, following everything that we have simplified, we have written it down this way. Now, if we see, in a way, this has been written for, this d E by d T, this energy at a given position and. So, on and written with respect to a single electron;, but the total number of electrons per unit volume has been taken into account, using this term N f. So, this N f d

E by d T, N f d E by d T, this is the change in energy for unit change in temperature. So, that is what it is, for the entire number of electrons per unit volume. So, this is simply your specific heat at unit volume associated with the electrons; electronic specific heat at per unit volume; number of free electrons per unit volume times the change in energy per unit change in temperature for a given electron. So, if you take that into account, this is the specific heat per unit volume, fine. So, therefore, we can put that here. So, we will do that.

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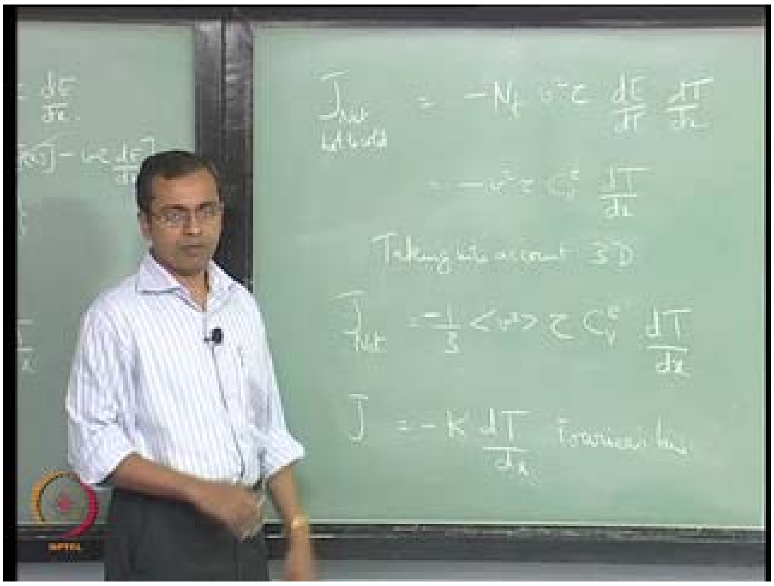

J net hot to cold equals, minus, we just put that down here, minus N f v square tau d E by d T and d T by d x; and we have identified N f d E by d T to be C v, or specific heat per unit volume, associated with electrons. So, this is simply, minus v square tau C v e d T by d x. So, we have now almost gotten to where we wish to get. We just need to generalize it a, just a little bit more, and then, we will have an expression the way we would like it to, the way, in a manner that would be useful for us. The first thing is the velocity here, we cannot, I mean, as we have done before, the velocity that is of relevance to us is the mean velocity, is square, mean of the squares of the velocities that are present in the system; because then, that would be representative of the electrons present in the system. So, whereas, we started off, all of this by just using a certain velocity; this is actually the mean of those velocities. And, the other thing is, we have actually done this, effectively in, in one dimension,. So, to speak. So, we would need to extend this to, assuming that the electrons could actually move in three different directions, and not just in one direction. So, the net transfer in any one direction would be one third; because, it is equally, randomly possible for the heat to be transferred, transported up, or down, left or right, or back, or I mean, or to the front of the board, or behind it.

So, we will take these two into account. So, taking three dimensions into account, three dimensions, we will actually have, J net equals one third, minus is there; minus one third v square; this bracket is being put in to indicate that, this is the mean that we are looking at; tau C v e d T by d x. So, we have this expression. And, we also wrote down, J equals minus K d T by d x, as the Fourier's law for heat conduction; Fourier's law for heat conduction, we wrote J equals minus K d T d x. So, first of all, we now have two expressions, where you have the heat flux on your left hand side and you have d T by d x available for you, in both cases; and then, you have K in one expression and then a bunch of other parameters here, which then, of course,, mean, imply that, they are the parameters that give us the conductivity K. I will also again, I just want to take the, take this opportunity to point out that, this minus sign comes simply because, of the way in which we define the direction of temperature gradient. Normally, any gradient will go from low to high. So, when you have a cold end, and a high, hot end, so, the temperature gradient is, the increasing direction of the temperature, temperature is from the cold to the hot; whereas, the heat flows from the hot to the cold. So, therefore, when you are looking at heat flux, it is in a direction that is opposite to the positive d T by d x. Therefore, you get this negative sign. So, that is where the negative sign comes from; because it is in, this direction is opposite to that direction. Hence, the minus sign. So, minus K d T by d x.

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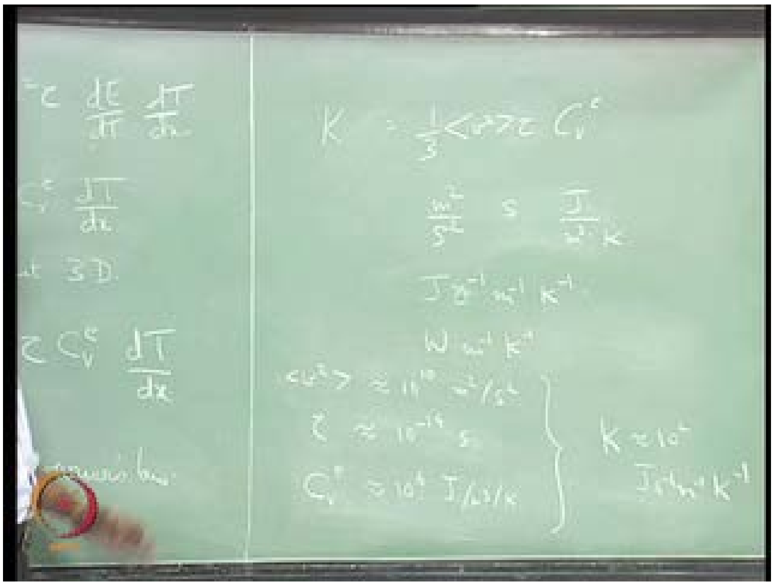

Now, comparing these two expressions, we get thermal conductivity K is now, 1 by 3 v square tau C v e. And, if you want to look at the units, this is going to be meter square per second square; tau is in seconds; this is specific heat at constant volume for the, electronic contribution to the specific heat at constant volume. So, this is Joules per constant volume is meter cube, and it is, specific heat means, per Joules per Kelvin. So, for every degree raise in temperature, what is heat that is required. So, Joules per Kelvin per meter cube is the unit that you have here. So, therefore, if you look at this now, this is then Joules per meter, Joules per second per meter per Kelvin. So, meter square, meter cube will give you meter minus 1; second, second square will give you second minus 1. So, Joules per second per meter per Kelvin. So, you could also write it as, Watts per meter per Kelvin. So, that would then be the unit for your thermal conductivity. So, now, we have an expression for thermal conductivity and if you look at values that are available to you, this v square is of the order of 10 power 10 meter square second square, meter square per second square; tau is of the order of 10 power minus 14 seconds and C v e is of the order of 10 power 6 Joules per meter cube per Kelvin. So, if you take these into account, we are looking at roughly about 10 square; we get K approximately equal to 10 square Joules per second per meter Kelvin. So, we get this value for thermal conductivity..

So, what we have done is, we have looked at the Drude model. We have developed a set of equations, that take the basic ideas of the Drude model, all those concepts, such as mean free time between collisions, the number of free electrons per unit volume, all those specific ideas that are relevant to the Drude model are taken, and with respect to those ideas, we have looked at what heat is transferred from the cold side, from the hot side to the cold side, what is being transferred from the cold side to the hot side, what is the net transfer of heat, and come up with an expression then, that has all the attributes of this Fourier's law of heat conduction; and, by comparing the two variable to pull out an expression for the thermal conductivity. And, based on values, we are able to locate parameters that are there in that expression, we have about 10 square Joules per second per meter per Kelvin. If you compare this with what is available in the literature, for good conductors of heat, especially, you know, good metallic conductors, which are then good conductors of heat; so, silver, copper and gold, for example, if you compare them, you will find that, their conductivity, thermal conductivity also comes out to the same order of the magnitude. It is of the order of 10 squared Joules per second per meter per Kelvin. Specific values vary,, but at least, order of magnitude, as I said, which is the primary parameter of interest for us, comes within the correct range. So, what we have done in this class is, we have looked at thermal conductivity, extended the Drude model to see, how well it addresses thermal conductivity, and we find that, we are able to come up with an expression for thermal conductivity, and that, the value that it is predicting, the value that it is predicting, is reasonably for some of the metallic systems that we may come across in, in the engineering and technological science. So, so, with this, we will conclude our discussion at the moment about thermal conductivity. So, what we have done in the last class, is the Drude model for the electronic conductivity of the material. Today, we have done the Drude model for the thermal conductivity of the material, and we have also placed in perspective, where this thermal conductivity is, with respect to all the heat transfer processes that are available to us; and even within the conductivity process, with respect to, say phonon based conduction of heat, versus, you know, electron based conduction of heat, what is the process that are there, what is the framework, and what is the specific region of study that we are focusing on. So, within that framework, we have come up with an expression. So, we will halt here today. In the next class, we will take up comparisons of the two conductivities that we have developed, electronic conductivity and thermal conductivity, and how these properties actually relate to each other for various materials and what does this model tell us about such a relationship; and, therefore, how reliable it is, in terms of other predictions that it makes. So, with this, we will halt for today. Thank you.