

**sElectrical Conductivity**  
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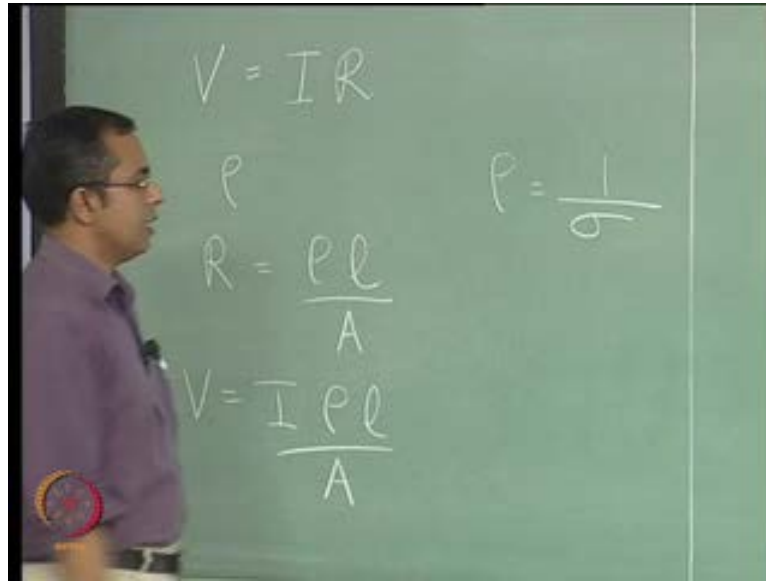
**Lecture No. #07**  
**Drude Model: Electrical conductivity**

Hello, this is the seventh class in our physics of materials lecture series. In the last few classes, we have looked at our attempt to build a model for this, for the electronic properties of materials and in particular we looked at the possibility that, we can actually use the rules that are applied to the ideal gases. And see if we can extend the same set of rules to a solid material, more specifically to the electrons present within the solid material simply, because the electrons not all the electrons, but those electrons that are supposedly free, free to roam around the extend of this solid, have characteristics which make them very similar to the molecules in an ideal gas.

So, we looked at the pros and cons of it, we looked at we put down some numbers to see whether it is reasonable or not and then assuming that it is reasonable we have proceeded forward with it. We also reviewed some of the important results of the kinetic theory of gases, because specifically those would be results that we will use as we proceed forward. Now, very specifically I just want to proceed today towards trying to put some expressions down to understanding the electronic conductivity of a metallic sample on the basis of this process, where we are looking at applying ideal gas rules to those electrons.

Before we do that, let us actually try and generalize the Ohm's law which is what we would we are all familiar with. We will write it in somewhat different manner which will then become convenient for us from the perspective of this analysis that we are attempting to do.

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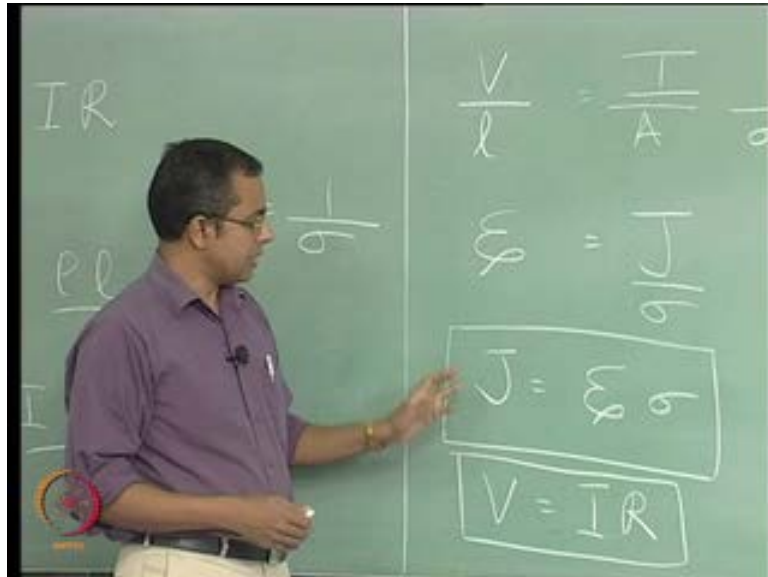


So, we are quite familiar with Ohms law written as. So,  $V$  is the voltage of potential difference that we are applying across the sample,  $I$  is the current that we are measuring and the constant of proportionality, then works out to this value which is the resistance of the sample. Now, resistance of the sample as such depends on the dimensions of the sample. In general it is found that if you take the very same sample the electronic resistance of that sample will increase, if we increase the length of the sample. Simply, because the electrons now have to travel through much a longer path and in the process undergo more of those phenomena which slow them down. At the same time, if we actually increase the diameter of the conductor. So, that gives the electrons parallel paths to travel through and there and that therefore, brings the resistance down.

So, if you if you want something that is more independent of the dimensions of the sample, then we talk in terms of resistivity which is  $\rho$ .  $\rho$  is the resistivity and the resistance then relates to  $\rho$  as the resistance then is the resistivity times the length. So, as the length goes up and the resistance goes up divided by the area. So, as the area goes up the resistance comes down which is consistent with what we just discussed. So, now we just substitute this into our equation for Ohms law and then rewrite it in manner that is convenient to us. So, this is what we have. In addition we will also take make use of the fact that, this quantity resistivity which is what we are associating as a material property is the inverse of another property which is what we are interested in which is the conductivity.

So, the resistivity  $\rho$  equals  $1/\sigma$  by conductivity which is represented by  $\sigma$ . So, this is another these are quantities that we are familiar with we are just refreshing ourselves with these quantities so, that we can use them in a more regular manner so, to speak. So, we have this expression here. So, we will just move these parameters around in a manner that makes it convenient to us.

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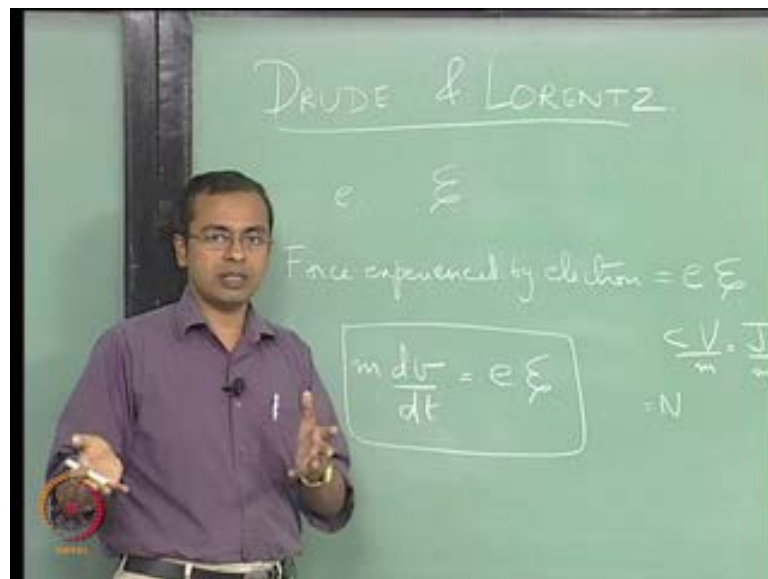
So, we can rewrite this as. So, I take  $\rho$  here and  $1/\rho$  is I mean  $\rho$  is  $1/\sigma$  and then  $I/A$ , I maintain here and  $l$  moves to the other side. So,  $V/l$  equals  $I/A$  times  $\rho$  or  $I/A$  times  $1/\sigma$ . So, that is what we have here,  $V/l$  equals  $I/A$  times  $1/\sigma$ . So, this is the new, it is an expression that we are headed towards. When you apply voltage across the sample, this same voltage could be applied across longer length of the sample or a shorter length of the sample and its impact would therefore, be somewhat different on what the electrons experienced and therefore, we are interested in this quantity  $V/l$ , which is the electric field, represented by  $E$  and this  $I/A$  is the current per unit area, which is just the current density  $J$ . So, this is electric field, this is current density and this is  $\sigma$ , the conductivity.

So, rewriting this we get  $J$  equals  $E\sigma$  or  $\sigma E$ . So, the conductivity or **I am sorry** the current density is the field times the conductivity. So, this is the equation we have and this is our original equation equals  $V = IR$ . So, both of this, both of

this equations are different ways, in which we write Ohm's law. So,  $J$  equals  $\sigma E$  or  $\epsilon \sigma$ , the way I put it down here and  $V$  equals  $IR$ , they are exactly the same, just on a small derivations to get us here. This equation up here is going to be convenient to us. So, we will use this equation in the rest of this class or towards the later part of this class. What we are going to do is, we are actually going to set this aside. Work on something associated with how electrons move when you apply a field to them and or when they experience an electric field and then we will come up with an expression where we will have current density and we will have the field and then we will have some other quantity here, that quantity we will now associate with conductivity. So, this is the general process that we are to going to do.

So, with this background, let us now come back to this model that we are trying to develop which is the, which is to extend this free electron theory into the to help us come up with an expression for the conductivity that we measure for a metal.

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Now, this approach is credited to two people, Drude and Lorentz. So, this it is often referred to as the Drude and Lorentz model or the Drude model, is what it is often called. And basically this, it is this idea that, we take this free electron present in the solid, free electrons present in the solid and treat them as molecules of an ideal gas. So, this is the approach that will use. So, and these are the people credited with it. So, with this we will begin by trying to see what happens to electrons as you apply a field across this sample.

So, we will assume for the moment, we have a 1 dimensional sample and in that the electrons are running around and then at the initial stage, they are all settings at neutral condition and then we apply a potential across a potential difference across this conductor.

So, in general, if  $e$  is the electronic charge and if you apply a field  $\epsilon$ . So, this is volts per meter, then the force experienced by the electron. So, the force experienced by the electron is the charge of the electron times the field and you can easily check the dimensionality of this, because this is in coulombs, this is volts per meter. So, coulombs volts per meter is what we have. A coulombs volt is a joule.

So, we have joules per meter and joule itself is a Newton meter. So, therefore, we have Newton's. Therefore, in terms of the units or the dimensions of this quantity we are right, this is the force experienced by the electron. So, force as we also define it is simply the mass times the acceleration, the electron has a mass  $m$ , we will assume the rate as a mass  $m$ . So, mass times  $m \, dv \, by \, dt$  equals  $ee$ ,  $m \, dv \, by \, dt$  equals  $ee$ . So, this is the mass times acceleration for the electron, this is the force that it is experiencing, both sides we have force. So, we are now left with these equations. So, let us examine this for a moment, what we see is that we have mass times acceleration is the force, at a given point in time given what we have described we have just applied a field to the sample. So, therefore, this  $\epsilon$  is actually constant quantity.

We have just specifically put some whatever, you know 5 volts or whatever 10 volts across the sample, the sample is one meter in length. So, then we have a field of 10 volts per meter. So, that is what we have. So, that quantity is fixed, we are not changing that quantity. The charge on the electron is also fixed, that is the electronic charge that we are that is  $1.6 \times 10^{-19}$  coulombs. So, that is also fixed quantities, mass of the electron is fixed  $9.1 \times 10^{-31}$  kilograms. So, that is also fixed. So, when you have all these fixed quantities, what you are left with is that the acceleration is therefore, now a fixed quantity.

So,  $dv \, by \, dt$  is a fixed quantity. So, what this means this equation means, even though we have quite straight forwardly come at arrived at this equation. What this equation means is that, once you apply the field the electrons will get accelerated, because they have to respond to that field and the acceleration is going to stay constant. So, supposing you

apply this field for a few seconds, then the electrons as per this equation will accelerate for those many seconds, if you apply for 10 minutes it will accelerate for 10 minutes.

So, therefore, if you will put in the first law of motion and you write down  $V$  equals  $u$  plus  $at$  and let us say  $u$  is initial velocity it 0 and acceleration is now fixed  $dv$  by  $dt$ . So, as the time increases, the velocity of the electron will keep on increasing. So, and the longer you hold this potential difference across that sample the longer the velocity will keep increasing. So, the way this equation is written. It simply implies that if you wait long enough the electrons will continuously get accelerated, they will get continuously accelerated by the same quantity and indefinitely the velocity will keep going up.

So, this is what this equation implies. In reality what we find is that if they are moving faster and faster and faster and therefore, crossing the, going through the conductor faster and faster. In principle, this means that the current would in the conductor would keep on going up alright. In reality we find that this is not, this is not the case, in reality we find that once you start applying a potential at some point in time yeah, the electrons are moving, but they sort of reach some kind of steady state velocity and they keep continuing with the process. So, we have to recognize that in practicality this equation is missing something and we also try to understand from where that the missing quantity comes from.

What basically happens is that, as the electrons move, they actually collide with other electrons. Other electrons present within the sample, they are also sort of directly or indirectly influenced by all the Ionic cores that are present. So, therefore, the electrons are not moving in isolation, they actually end up bumping into each other and so on. So, they lose their velocity to some degree in this process. Now, the faster there or the higher their velocity is the sooner it is that in general as a probability the sooner it is that it will strike one of the other electrons present within the system. So, therefore, what we see is we actually recognized that there is some form of a general resistive term, which is preventing the movement of the electrons which becomes more and more prevalent or more and more visible as the velocity of the electron increases.

And if we remember when we put down these assumptions for the manner in which we will move this free electron gas theory to the, I means the ideal gas theory to the free electrons in the metal. We specifically said that between collisions, the interaction

between the electron with other electrons and with the ionic cores is not being treated in any great detail. However, we will put in some approximate terms which will sort of account for that overall interaction. So, this overall interaction and that approximate term is what we need to introduce into this equation. So, that the equations then begins to make sense. So, the way we simply do it is we will rewrite the equation here and say.

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The image shows a green chalkboard with handwritten mathematical equations. On the left side, there is a vertical label '14:27' and a small circular logo. The equations written on the board are:

$$m \frac{dv}{dt} = eE - \gamma v$$

$$eE - \gamma v = 0$$

$$\gamma = \frac{eE}{v}$$

$$m \frac{dv}{dt} = eE - \frac{eE}{v} v$$

So, we now rewrite this equation and introduce one more term here, gamma times v, where we are not really concerned about the exact nature of gamma. The exact nature of gamma is not of great interest to us, we just say that of any great immediate interest to us, but basically what we are trying to point out this, there is of accelerating force which makes the electron accelerate, and there is a general resistive force which goes up linearly with the velocity. So, as the velocity goes up there is the greater chance, that the electrons will actually bounce off some other electrons and therefore, lose their velocity and therefore, there is this term which is trying to prevent an indefinite acceleration of the electron.

So, as the velocity, since this force here is constant, the first term here is a constant and this keeps on raising the velocity goes up, what will happen is a steady state is reached when the total resistive force that is being experienced by the electron equals the total accelerating force, right, at that point, there is no further acceleration on that on the electron, it sort of reaches some steady state, some steady state velocity. So, when that

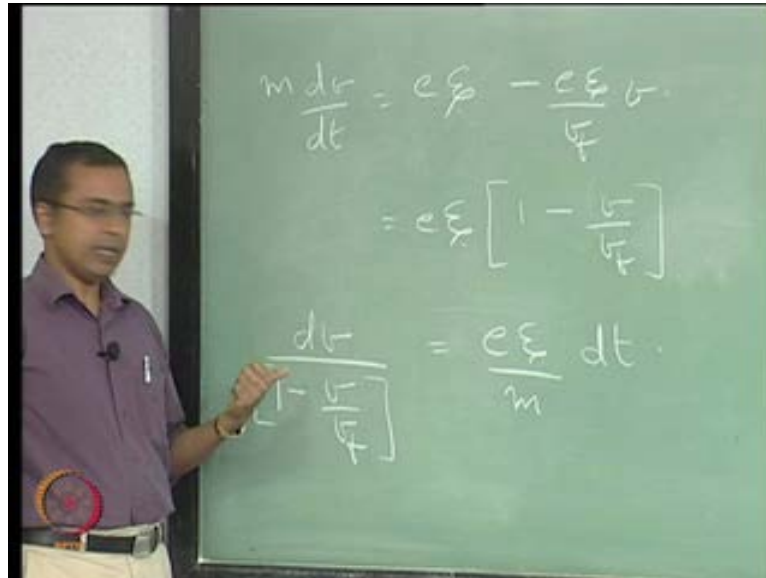
happens when the electron gains enough velocity, that we resistive term now equals the accelerating terms. We actually have  $e E - \gamma m_e \frac{dv}{dt}$ , we will put subscript f equals 0. What we mean is at that point some final velocity is being attained by those electrons, up until that time it is not a final velocity, because it is a changing velocity, the velocity is climbing up.

So, velocity increases, because of the accelerating term, but it will reach some final velocity at which point this term here now equals this accelerating term and therefore, there is no net force on the electrons. So therefore, we will get this situations. So, we will use this to rewrite it and get ourselves an expression for gamma, we get gamma equals. So,  $V_f$  is the final velocity that on average all the electrons will attain. So, when you apply a potential across an, a typical normal electronic conductor, the electrons will get accelerated and after some point in time they will reach and each of those electrons will on average reach this velocity  $V_f$ . Beyond that on average they will not accelerate.

So, now we can substitute it back in this equation and make some simplifications for ourselves. So, this is. So, we have now got this equation rewritten as this equation down here, where we have substituted we are now have some kind of expression for gamma which we have put down there. Please note in this equation this  $V_f$  is a constant. So, we have  $I$  mean based on the details of the system, some value it will come to, but it is a constant it is not a variable whereas, this  $V$  here is the variable, it could be changing at a given instant in time, it could be changing, fine. So, this is the equation we have. What we will now do is, we just integrate this equations and then see where that leads up. So, we will write those equation back here and then we will start again.

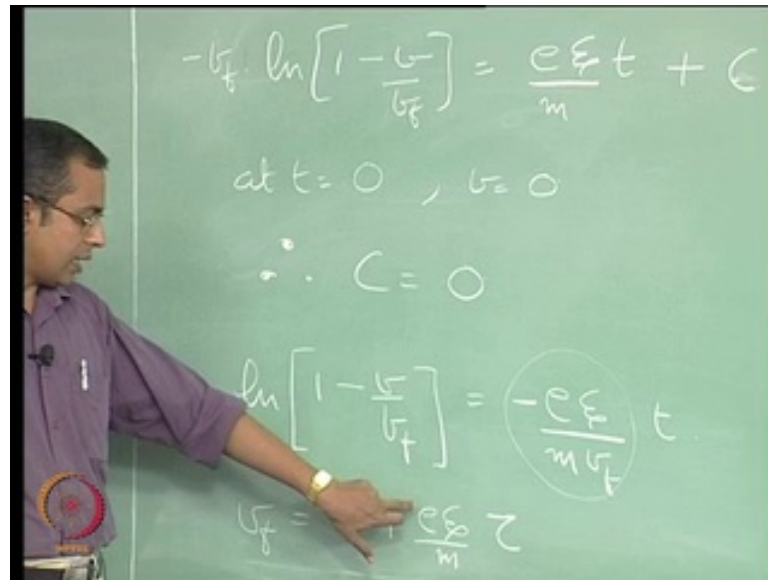


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So, let us just simplify and integrate, we will pull out  $e$  epsilon, so we will get  $1$  minus  $v$  by  $v$   $f$ . Now, we will just move the quantities around. So,  $d v$  by  $1$  minus  $v$  by  $v$   $f$  will be  $e$  epsilon by  $m$   $dt$ . So,  $dv$  by  $dt$  we now a variable to separate the variables out and then have an expressions where only velocity is on the left hand side of this equation. So, this is the equations we have. So, now we are in a position to integrate this equation. So, we will integrate it, here the integration limits will be, this will go from initially a stationary electron on average, we will say  $0$  to  $v$ , integration limits here and  $0$  to  $t$  is the integration limits here.

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So, therefore, we will get, since this is in the denominator we will get  $\left(\frac{v}{v_f}\right)$  of 1 minus v by  $v_f$  and we have a minus 1 by  $v_f$  here. So, therefore, you will have minus  $v_f$  here, this is what you will get as an integral for the left hand side of the equation. This would be equal to  $eE$  by  $m$  times  $t$ . So, 0 to  $t$  is what we will have plus  $C$ , some constant  $C$ , some constant  $C$ . Of course, this at 0 will then convert this to 0. So, therefore, that is what it is. 0 to  $v$  and 0 to  $t$  is what we have. If you want to evaluate the constant  $t$ , we just say that at  $t$  equal to 0 on average the  $v$  is equal to 0. This does not mean that the electrons are stationary.

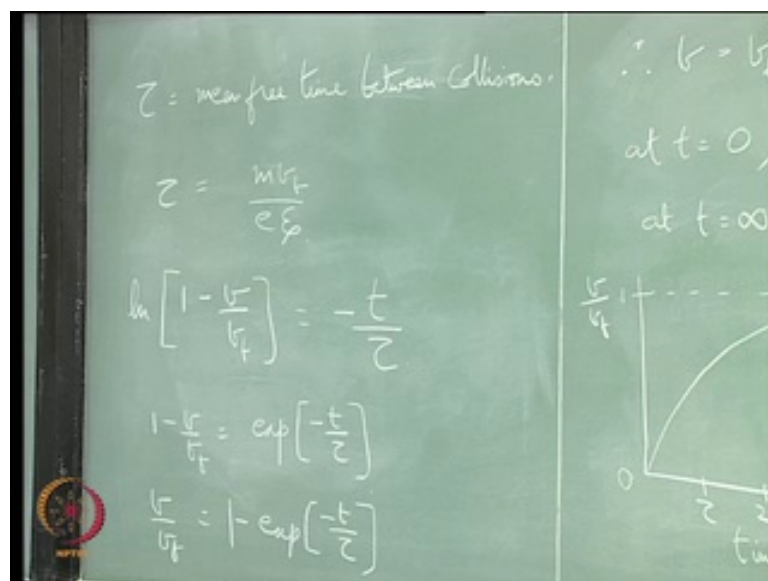
The electrons are actually moving around, we are just looking at the average velocity. On average the electrons have to be stationary in the absence of a field, because if they are moving, then you have current without the presence of a field, which is not prevalent in a standard conductor at normal room temperature. So, this is the issue here therefore, this is what we are dealing with at this stage. So, at time equal to 0 on average the velocity is equal to 0. So, therefore, if you substitute that at  $t$  equal to 0 this term will become 0 and  $v$  is equal to 0. So, this is 0. So, you have  $\left(\frac{v}{v_f}\right)$  of 1 which is 0 therefore, this entire left hand side term is also 0. So, therefore, your constant there, constant is 0. So, this is what we have.

So, therefore, we can now rewrite this a little bit, we can say  $\left(\frac{v}{v_f}\right)$  of. So, I have just taken this minus  $v_f$  and send it to the right hand side. So, this is what we have minus  $eE$  by

$mv_f$  times  $t$  and we have  $(1 - \frac{v}{v_f})$  of  $1 - \frac{v}{v_f}$  on this side. So, now let us now look at this quantity here which is in front of this variable time, time variable here. So, we have  $eE$  by  $mv_f$ .  $eE$  as we recognized is the force that is being experienced by the electron and  $m$  is its mass therefore, this is actually the acceleration, acceleration being experienced by the electron. So, if you see on average let say like I said on average, let us assume that the electron is stationary on average.

When you first apply the field. So, then its initial velocity is 0, its final velocity is  $v_f$ , let us say that you know it takes travel some amount of time before it has the collision. So, on average, it is going to have some time between collision and on average at the end of the time it is going to have this velocity  $v_f$ . So, these are all average quantities. So, on average it starts with  $v$  equal to 0, on average it travels for some time the mean time between collision which is  $\tau$  and at the end of that mean time between collisions it has the average velocity  $v_f$ . So, if we take and given that it has an acceleration  $eE$  by  $m$  for that same duration, we actually have the final velocity  $v_f$  equals the initial velocity 0 plus acceleration times. We will now put this quantity  $\tau$ .  $\tau$  is the mean free time between collisions, we will put that down again, we basically have  $v_f$  the final velocity is the initial velocity, these are all on average, on average final velocity is on average the initial velocity times the acceleration plus the acceleration times the time available for that process and we are defining  $\tau$  as the mean free time.

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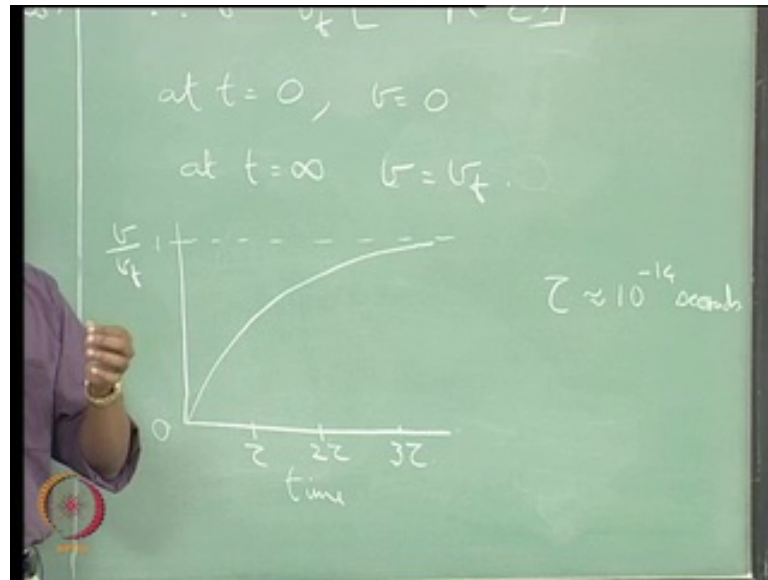


So, tau is the mean free time between collisions. So, this is on average the amount of time that an electron will have from the time it has one collision before it has the second collision. So, this is the general and again all these quantities are on average quantities. So, there is bound to be a variation between all the electrons, this going to be a distribution, some electrons may actually travel longer before they actually have that collision, some may travel for a very short period of time. So, this variation is going to be there, you are going to see a distribution, but still. So, there is a mean associated with this process. So, that is the mean that we are referring to here, mean free time between collisions. So, therefore, it is fair to say that we can define tau simply as  $\frac{m v_f}{e E}$ .

So, we are defining this tau in this manner which is the mean free time between collisions. So, if we now go back to our equation here, we have  $\left(1 - \frac{v}{v_f}\right)$  of  $1 - \frac{v}{v_f}$  is minus  $\frac{e E}{m f}$  times  $t$ , this term here  $\frac{e E}{m f}$ , the inverse of it is, what is tau? Which we just saw right. So, the inverse of this is tau. So, therefore, we can now write this equation as  $t = \tau \left(1 - \frac{v}{v_f}\right)$ , this  $t$ . So, if we again look at this equation, the  $v_f$  is a fixed quantity. On average it comes out to a certain value  $v_f$  and that is the quantity. It is a fixed quantity sort of, it is not a variable. In this system, in this particular system and for the conditions that is currently prevalent on the system this comes out to a single value.

Similarly, tau it comes out value based on all the mean of all the collision that is occurring. So, this is average quantities, but still for all processes they are no longer variable, given these conditions that we have placed on the system including the value of the field that we are place that the system.

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So, we have the situations so, we can simplify now. So, we have  $v$  by  $v_f$ ,  $v$  subscript  $f$  is  $1 - \exp(-t/\tau)$ . Therefore, the velocity at any given point in time is simply. So, we now get an expression for  $v$  which is the velocity of the electron as a function of other quantities that are prevalent in this system or are of relevant to this system. So, now what this means is simply this, we have electrons which are starting. If we look at the expression here, at time equal to 0, at  $t$  equal to 0 this quantity here work out to 1 therefore,  $v$  equal 0. So, at  $t$  equal 0,  $v$  is equal to 0, fine.

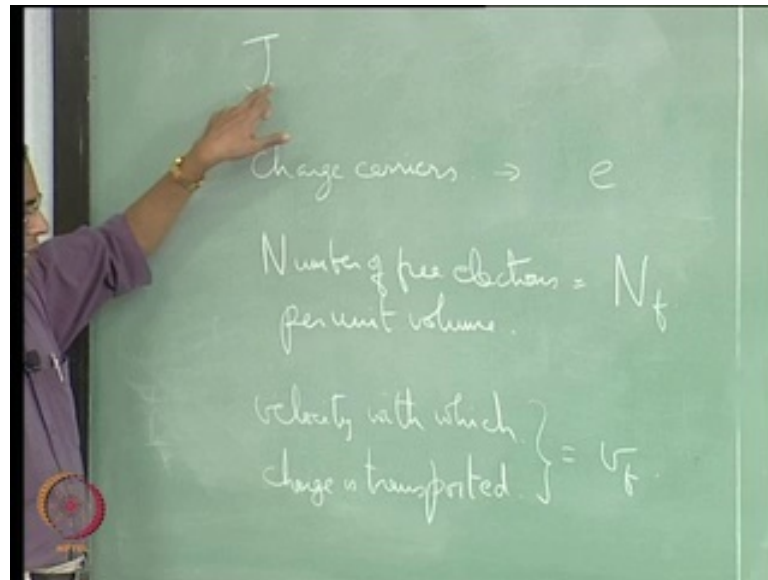
So, that is consistent with our thinking that on average, the velocity is going to be 0 as you start off. For a given electron, it is going to be 0. So, as  $t$  goes towards infinity this quantity will drop to 0 therefore, the velocity will become final velocity or as it increases basically,  $v$  will equal to  $v_f$ . So, actually if we make a plot of this you will see something like this and so on. We put  $\tau$ ,  $2\tau$ ,  $3\tau$  and so on and we are just going to plot  $v$  by  $v_f$  on this side. So, this is 0 and this is 1, what will see is that with passage of time, the velocity of the electron will actually start approaching this final velocity  $v_f$ . So, this is the general picture of everything that is occurring in terms of how the electrons are responding to the field that is being imposed on them, the electric field that is being imposed on them, they begin to move, they undergo collisions. So, on average  $\tau$  is the mean free time between collisions.

So, depending on the time of collision and what velocity of course, it started off with an initial velocity. So, with passage of time on average, this is the process that is going to occur, it is going to catch up with the final velocity that it is suppose to have. So, this value of tau is approximately of the order of the 10 power minus 14 seconds, is 10 power minus 14 seconds. So, what it means is you know when you switch on a device so to speak. When you switch on a particular, you connect a circuit you have wire and so on and you switch it on.

So, we are talking of, we from our perception the current is instantaneous, from our perception everything is working at in a instantaneously working, but if actually look at the kind of time scales that are involved here. We are looking at you know for the electrons to actually on average reach their final velocities. So, let us say on average by say 3 tau or 4 tau, we are more or less at the final velocity. So, 4 times 10 powers minus 14 seconds is the kind of time frame that is involved. When you simply you have a conductor or resistor or something and current starts flowing through it, it takes of that order of time of 4 times 10 power minus 14 seconds before things stabilize within that system. So, that you see some kind of a steady state. From our perception where even 1 second is considered as very small interval of time, this is something that we do not perceive.

So, when visit something on for us everything instantaneous. So, this gives you some insight of how things are occurring within the system, the kinds of times frame within which things are operating within the system. Alright, so, this is one aspect of how this process is going to occur, we will take this information and use it to understand more about conductivity. Now, let us look at even though, now that we have derived something up until this point, we will just halt that for a moment and step back and look at the system in more broad perspective and see if we have a different way of approaching the same problem to help us simply get, give ourselves some expressions, that we can compare with each other and then come up with some interesting values.

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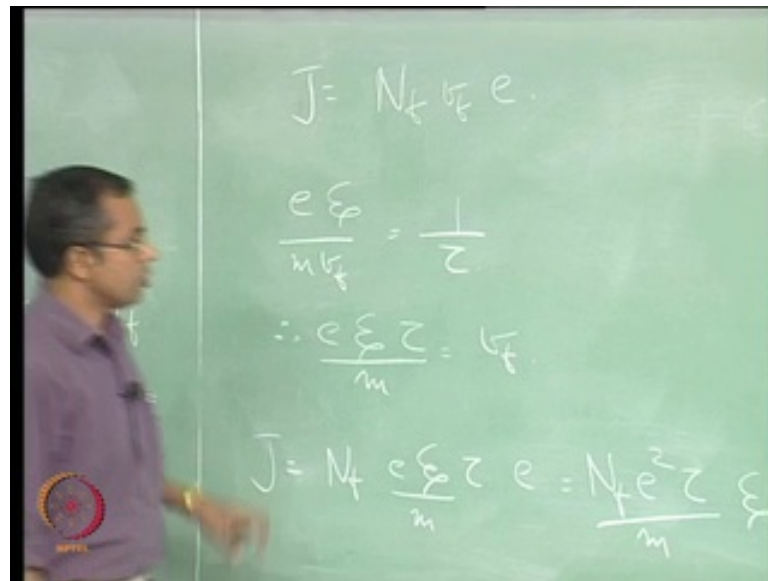


Basically, we are interested in looking at current density  $J$  as a function of the conditions that are imposed on the system. So, and in more specifically we would like to have some fundamental quantities from within the system and see what, how they impact the value of the current density that the system is demonstrating, given that it is subject to certain conditions. So, you measure a certain current density, a macroscopic sense, because you know that the conductor has a certain cross sectional area and we have a current that we are measuring. So, therefore, based on the current and the cross sectional area, we have a current density. Within the system, what do we have? We have charge carriers. So, they have a certain charge. In this case, it is electrons that are charge carriers. So, the charge is  $e$ .

So, they are moving through the system. That is the charge carrier. We would like to know, how many charge carriers are moving at a given point in time. Because that is what constitutes the current they all add up to the current, the number of charge carriers that are moving through the system. So, in our case the value of interest to us is the number of free electrons per unit volume. So, this is denoted by  $N_f$ , number of free electrons per unit volume. So, this is the number of free electron per unit volume is the quantity that is of interest to us, because that then constitutes the along with the electronic charge constitutes that amount of charge that is trying to get transported across and the speed with which they move then decides the current.

So, which in our case through the discussion that we have had so far we find that on average those electrons given that they are subject to a certain field on average they will all reach this final velocity  $v_f$ . So, the velocity with which the charge is transported is  $v_f$ . So, therefore, this current density  $J$  is now a function of the charge that is being transported, the number of those charge carriers per unit volume and the velocity with which they are moving.

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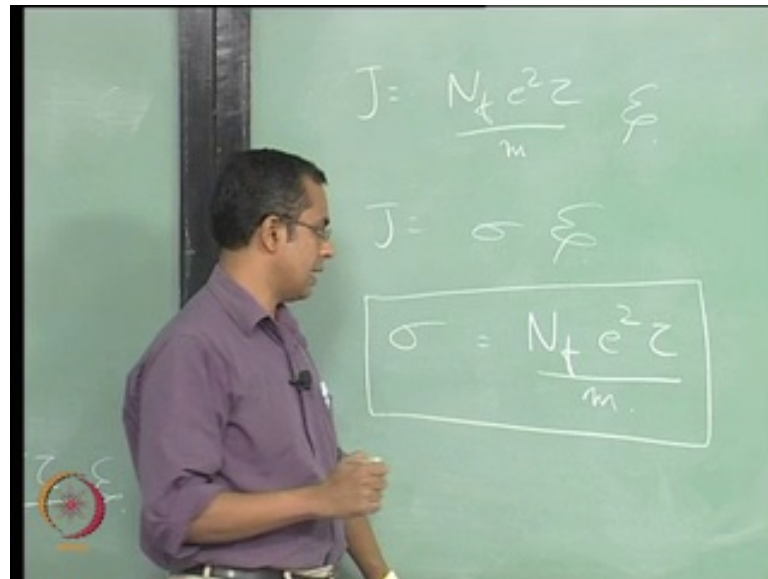
So, this is the equation. So,  $J$  is simply. So,  $J$  is simply the number of free electrons per unit volume. The velocity with which they are moving and the charge that they are carrying, this then constitutes the current density. So, all the quantities are normalized appropriately. So, you especially the number of free electrons per unit volume and therefore, we get this as the current density. So, we actually what we did is in our derivations, we had  $e e$  by  $m v_f$  equals  $1$  by  $\tau$ . So, this is the acceleration that is  $\tau$  times, so this is.

So, this is how we define the mean free time between collisions. So, we will rearrange this so, that we can give ourselves an expression from here. Therefore,  $e e \tau$  by  $m$  by  $m$  equals  $v_f$ . So, this is fixed. These are all now fixed quantities so to speak for our system, this is the charge, this is the field that charge of the electron, free field that we are applying, this is the mean free time between collisions and this is the mass of the electron.



So, we will take this expression and substitute it out here. So, therefore, we have  $J$  equals  $N f e^2 \tau$  by  $m$  times  $e$ . So, we have or rewriting this we have. So,  $J$  equals  $N f e^2 \tau$  by  $m$  times  $e$ . So, this is what we have as an expression that is available for conductivity right. So, we have an expression for conductivity or actually for the current density  $J$ .

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So, we have essentially  $J$  equals  $N f e^2 \tau$  by  $m$  times epsilon field and we also started off just a short while ago, we put down a different version. Actually, as we started the class, we put down the Ohms law written in the different manner and there we basically, had  $J$  equals sigma times epsilon, conductivity times epsilon. So, therefore, if we compare these two expressions we will basically have the conductivity is equal to  $N f e^2 \tau$  by  $m$ . So, we now have conductivity, which is now coming out to  $N f e^2 \tau$  by  $m$ . So, what we have here is on the left hand side is a parameter that we can actually measure experimentally.

Experimentally we can measure conductivity. Actually, we would measure resistivity and then the inverse of that would be conductivity and on the right hand side, we have quantities that are fundamental to the metallic system.

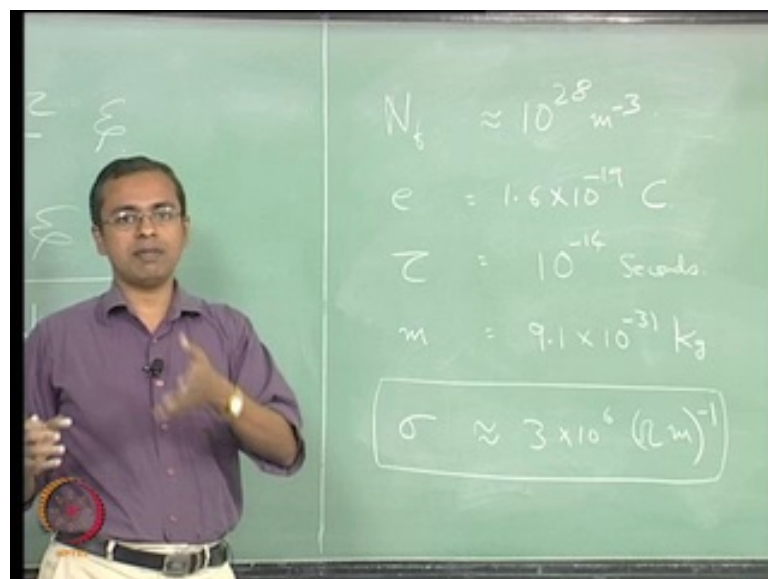
So, please remember in all these cases at least in our discussions we have, we tend to concentrate or focus on a metallic system, but in principle we could extend this to other

systems, but we are starting off with metallic systems, because they are much better defined and our idea that there is a free electron present within the system is valid for a metallic system.

So, (( )) at first glance it would meet many of the requirements of a free electron gas that, the kind of description that we give for a free electron gas would be largely true for the electrons, (( No Transcription 38:00 to 39:03 )) about  $10^{28}$  electrons per meter cube. So,  $10^{28}$  electrons per meter cube is the number of free electrons per unit volume, that is present within typical metallic system. As I mentioned then and I will mention here again, this is only the number of free electrons, which is why we have the subscript f, all the other electrons which are present within that.

So, if the valence is whatever, if you say 30 **I am sorry** if the atomic number is 30, you have 30 electrons present of which if the valence happens to be 1, only 1 of those electrons contributes to the free electron process or the free electron volume. So, 29 electrons are not included in this in our calculation for this free electrons, number of free electrons per unit volume. So, large number of electrons in the system are not being included in this calculation, only the free electrons are included here that works out about  $10^{28}$  per meter cube. The charge on the electron  $1.6 \times 10^{-19}$  coulombs. The mean free time between collisions for most metallic systems works out to roughly about  $10^{-14}$  seconds.

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The mass of the electron, it is over  $9.1 \times 10^{-31}$  kilograms per electron is the mass. So, these are all quantities which we can, then independently I mean these are all known, some of these quantities are known, this is the new quantity that we have sort of discussed here and then we are able to obtain. So, given these values been prevalent within the system, if you actually go ahead and make this calculation of the conductivity as  $\sigma = N e^2 \tau / m$ . So, we will get the conductivity is approximately of the order of  $3 \times 10^6$  Siemens per meter. So,  $3 \times 10^6$  Siemens per meter is the kind of conductivity we will get. So, we need to look. So, this is the value now we have come up with. So, this is the value for conductivity based on fundamental values that are prevalent within this system, which we have utilized comes up this value for conductivity.

As it turns out as I mentioned in the one of our earlier classes, we looked at the conductivity. We basically said that for metallic systems the conductivity is of the order of  $10^7$  Siemens per meter, conductivity can vary from about  $10^{20}$  Siemens per meter for really poorly conducting materials to about  $10^7$  Siemens per meter for typical metallic system which could be like copper or silver and so on. And for from metal from metal then can be variation and if for examples in fact, we have not looked at all this, but if look at an alloy it is going to be little poorer conductor than a pure metal. So, even amongst metallic systems there are good conductors of electricity, there are somewhat relatively poorer conductors of electricity.

So, there is some variation available in the actual value of the conductivity, even for a typical metallic system. So, for a range of metallic systems when you compare them you will have, you will find some variations in the values. So, therefore, having of the order of the  $10^6$  to  $10^7$  Siemens per meter as conductivity is reasonable for as a number that we can associate with metallic systems in general. So, given this general understanding of how the, what kind of values you can expect for metallic system. The number that we are coming up with here  $3 \times 10^6$  Siemens per meter is actually a reasonably good value, is a reasonably good prediction so to speak, of the conductivity of the material right.

So, as I mentioned also in one of our earlier classes in some of these things our interest, is to look at the order of magnitude of the prediction to get give ourselves a sense of whether the prediction has been reasonable or is it far fetched or if not, even if it is

somewhere in the middle, how far how seriously we need to take the prediction or how much of caution we have to consider when we look at the prediction, the number that we are coming up with is actually quite reasonable, it is a very much in the range of what we are expecting for a metallic system and therefore, this kind of a prediction that has come out of this model, that we are referring to as the Drude model is actually very good.

It says that the model is actually a reasonably good model and therefore, this kind of prediction is you can consider as a success for the model, that the model has been successfully, it has been successful in making a prediction for a material property and that prediction actually matches what we are saying in the experimental data. So, to reiterate this is the order of the magnitude prediction and the order of the magnitude prediction is right and as I said for metallic system  $10^6$  and  $10^7$  siemens per meter is acceptable, these values would get impacted by any of these quantities changing. So, if your  $N_f$  number of free electrons per unit volume changes which would change, if you change from one metallic system to another and for example, we are now treated this as being some univalent material, if it were a divalent material and so on.

That it would typically assume a divalent status, then this would change, this number would change. The charges, the charge is not going to change, the masses not going to change, but depending on the other aspects associated with the system the mean free time between collisions might change, because it may have great a probability of bumping into other electrons especially, the number of electrons is going.

So, there are lot of parameters, which could impact each other in this process and therefore, for a specific systems, we would actually have to put down these numbers for that specific systems to come up with a prediction that we can then look at. So, now we have come up with a prediction for the electronic conductivity of a metallic system based on our understanding of fundamental aspects of the system and by based on the imposition of ideal gas theory processes and equations on the free electrons which are present within the system.

So, in this class, with this prediction we will we will halt our discussion, what we are going to do in our subsequent class, is in our next class is to take a similar approach and see if we can again take the same kinds of assumptions, same kinds of behavior, impose

it again on the electrons at a present within this system and see if on this basis, we are able to predict the thermal conductivity of a metallic system. So, we now have a prediction for the electronic conductivity of the metallic system, which happens to be reasonably matching with what is available as a value that you can measure. We will see if we can come up with a similar prediction for the thermal property of the same kind of a metallic system. So, the metallic system where you have some free electrons which are running across.

So, then will have two predictions. A prediction for the thermal property of the material, a prediction for the electronic property of the material both of which are made within the frame work of the Drude and Lawrence model, which is simply to use the ideal gas loss on the free electrons. So, we are now seeing that it has been successful in measure giving us predicting the electronic property, we will see how successful it is in predicting the thermal property. And I mentioned in one of our earlier classes that to one, one is to individually predict each of those properties, but it is also of interest to go to one level higher than that to see if it can predict the relationships between properties, any given model, when I say it I mean the given model. In our case it is the Drude model.

So, we would like to see if Drude model can predict electronic conductivity, predict thermal conductivity and whether or not it can also predict the relationship between the electronic conductivity and thermal conductivity. By that in this case I mean simply that we have this observation that when we have good electronic conductivity, we also seem to typically have good thermal conductivity which is what is typically true for most metallic systems. So, we will see if each of those values are individually predicted correctly and also whether it is able to predict this relationship correctly. So, when we reach this level of understanding of I mean, when we are able to compare these values and see that it is making the predictions right, it is also predicting the relationships right, then we get greater confidence in this model. So, that is the direction in which we will head and then at that stage, we will also see if there are any other properties it makes a good prediction of and also any short comings that are there present in this model. So, with this we conclude today's class. We will look at thermal conductivity in our next class.

Thank you.