

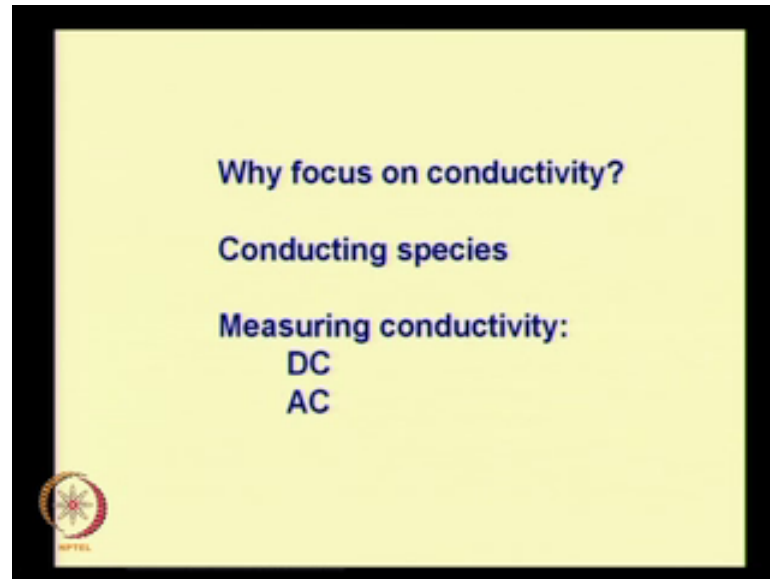
**Physics of Materials**  
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**Lecture No. # 04**  
**Measuring Electrical Conductivity: DC and AC**

Hello and welcome this fourth class on this series of lectures on physics of materials aimed at under graduate students in material science and engineering, but in a way that should be accessible to most engineering students. In our earlier lectures we have looked at why we would like to model materials, what we mean by modelling of material and what is the kind of approach that we want take to model those materials. We also looked at various common properties that materials have and what sort of the relationships those properties have with each other. And then in our very last class we looked specifically at a particular property which is the coefficient of thermal expansion or just the process of thermal expansion and we try to build a model for the material to see if we can explain from first principles or at least from the perspective of the consequent of the materials, from perspective of the atoms and electrons and electron clouds in the material and how they interact with each other, that the thermal expansion comes from.

So, these are the things that we did. Specifically, we started with thermal expansion because it was something that we could put together in a compact manner in a single class and so that what we did. From here on forward we will look at material properties which would require us to build more elaborate models and therefore, we will need to build some amount of background for it and we will do so as necessary. So, today we will spend our time on the specific property of conductivity.

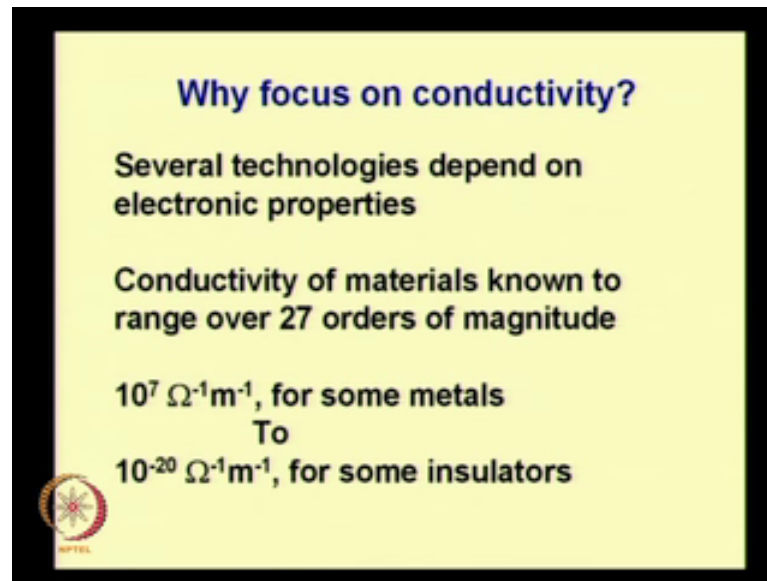
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And in today's class we will look at three things. The first is why we would like to focus on conductivity because for fair fraction of this course we will be spending our time looking at conductivity in various manners, in various ways and with different levels of details. We will build a lot of models which will specifically aim to look at conductivity. So therefore, we would first like to understand why it is necessary to spend so much time on this particular property of a material which is conductivity. We will also spend a little bit of time looking at the various conducting species that are present, that we are likely to encounter as we spend our time on material science and engineering and axis different materials and test them for various properties and so on. And finally, we will also get a sense of how we measure conductivity because it ties in a lot to what is happening in the material and therefore, what sort of things we need to look at in our models and so on.

Specifically, we will look at the D C methods for measuring conductivity as well as the A C method for measuring conductivity, D C being direct current and A C being alternative current and we will see what are some of the important differences between these two methods, what are some specific aspects that we need to be conscious about, careful about when we make those measurements. So, we will begin with our first question which is why should we focus on conductivity?

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**Why focus on conductivity?**

Several technologies depend on electronic properties

Conductivity of materials known to range over 27 orders of magnitude

$10^7 \Omega^{-1}\text{m}^{-1}$ , for some metals  
To  
 $10^{-20} \Omega^{-1}\text{m}^{-1}$ , for some insulators

 NPTL

As we see today a lot of the technologies that we use in day to day life however lot to do with electronic properties of materials, a variety of technologies have specifically used electronic properties. Some in some cases we know them upfront, in some cases we do not and so in today's world for example, mobile phone is very prevalent, all of them use electronic use materials and take advantage of their electronic properties. Any automobile you try will most likely a modern automobile typically has fairly sophisticated electronic control system which then controls how the engine functions, what sort of fuel-air ratio goes in and so on. A number of things it controls for the car optimizing it either for fuel efficiency or for power and so on.

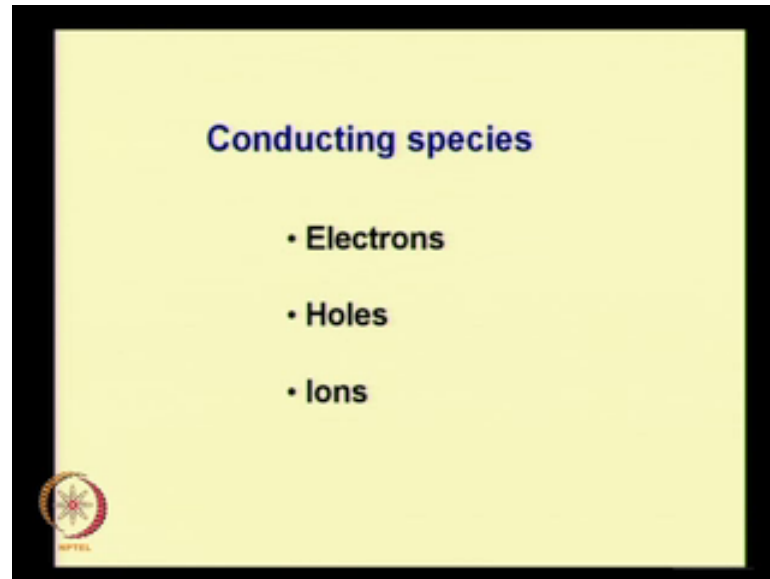
So, any consumer electronic good that you buy a television or any other any other common consumer electronic goods that you buy, many of the toys that you buy are typically having some electronic control unit in it and of course, the computers that people use. So, technologically electronic properties impact our day to day life in a very significant way. Therefore, it makes sense to spend some time to understand where those properties come from and how best we can model them. And of course, in this case conductivity is just one of the electronic properties the, of interest, but it is the one that we would like to focus on initial. So, from the, from within these electronic properties again we would like to understand why we would like to spend time on conductivity specifically.

Well as it turns out conductivity is a property where if you measure conductivity for a variety of materials that you can find, you will find that the value of conductivity that you get for all of those materials can vary over 27 orders of magnitude between the best conducting material and the worst conducting material that you can find.

As I have shown you some of the best conducting materials so metals for example, would have about  $10^7$  per meter and some really poorly conducting ceramic material could have  $10^{-20}$  per meter. So, this is 27 orders of magnitude variation in the value of a particular property and to our knowledge if you look at if we compare this property the range of values that this property has with the range of values that any other property has say a mechanical property or thermal property and so on. Almost no other property seems to have such a high range of values prevalent amongst the various materials that you are likely to encounter.

Therefore, from a scientific perspective it is of interest to see if we can develop models that now tell us clearly why a material could have such good conductivity and another material could have such poor conductivity. It is of scientific interest to see you know you take one extreme case of a property which shows you such a wide range of values and then if you are able to modulate, presumably you have a much better understanding of how the constituent of the materials of the material behaves. So, therefore conductivity of metals. So, both from a technological perspective as well as from a scientific perspective it is of interest to model conductivity, understand conductivity and so on. And hence in most of our course for several classes you will find that the models that we develop and the subsequent refinements of those models that we do are all aimed and focused at the conductivity, electronic conductivity of materials.

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So, now we will look at the conducting species. So, when we talk of conductivity our common perception is to associate it with electronic conductivity meaning the movement of electrons typically for example, through a wire. So, this is the one that we are most commonly familiar with. So, when we talk of conductivity we somehow always associated with the movement of electrons. In a more general sense actually when we look at conductivity what we are interested in is the movement of charge, movement of charge from one location to another represents a process where we could associate a conductivity with the process.

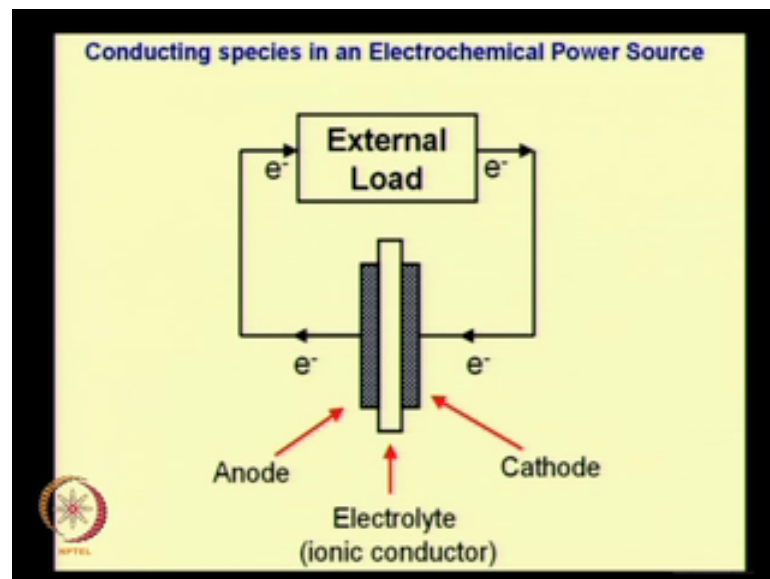
Therefore, the conducting species is also now of importance because you have a variety of species which could carry the charge and in addition to electrons the two more commonly, most commonly available conducting species that we encounter in material science and engineering are holes and ionic species. So, both of these are also present if and if you compare these three species electrons, holes and ions, there is a lot of variation between these species in terms of the charge they can carry, the mass that is associated with them and so on. And also more importantly the extent and the manner in which these species associate or interact with their surroundings. So, that then defines how well that species is able to conduct charge in a in a given circumstance.

So, if something is, if so for example, electrons and holes typically tend to move much more easily or relative to each other, relative to ions and then ions depends on the ion, it

it really depends on which ion you are looking at, it could be a  $h^+$  ion, it could be an  $o^{2-}$  ion and so on. So, it depends specifically on the ion that we select and the environment it is placed in. So, between these two you get a sense of whether or not that ion will move easily or not move easily and therefore, whether or not the conductivity will be good or otherwise. So, we can see that there are different, we would like to, I would like you to be aware that there are different conducting species.

So, when we talk of conductivity we have to also always ask ourselves the question the, it is the conductivity of which species that we are interested in? Because there are definitely circumstances where we would want one conductivity to be low while the other conductivity to be high and I will just show you an example in a moment. So, we need to know that there are different conducting species, that they can behave differently and that they can be controlled different.

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So for example, an immediate example that I will give you is on your slide here we have an electrochemical power source. We have an electrochemical power source which could be for example, a battery or a fuel cell and so that is what is shown in your slide down here. So, this is like in any other household appliance if let us assume the this is the battery, conventional battery that you would buy or this will be connected to some external circuit perhaps a light bulb of a flash light or something like that and then you are actually having some process occurring here where electricity is generated in this

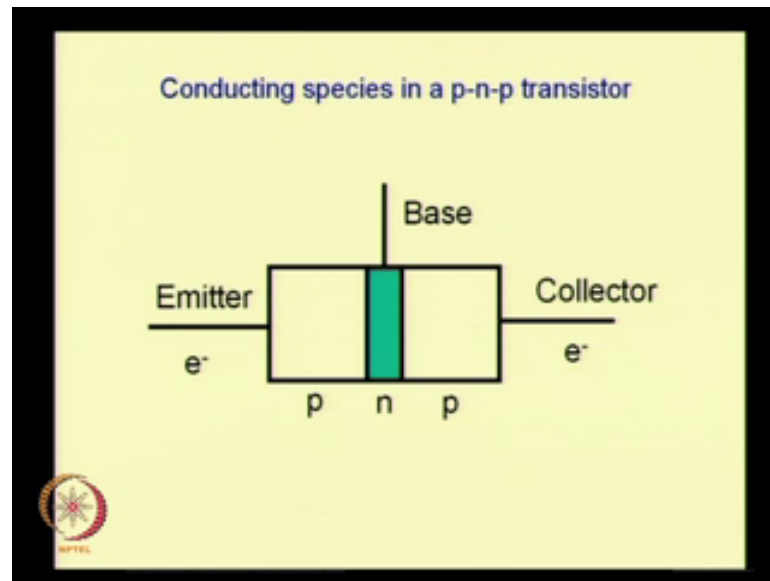
battery and is being consumed by this external load. Now, we connect these 2 using a wire and therefore, electrons flow in that wire and therefore, in this part of the circuit the conducting species is an electron as it is out here. Inside the battery we have a situation where there are 2 electrodes, an anode here, this one here and a cathode. So, this is the cathode here. So, that is what is indicated here plus we have an electrolyte which is an ionic conductor.

So, we have a situation here where we have a circuit where up to some point the current is being carried by an electron, but there is a very key component within this circuit which is the electrolyte in this case where the conduction is specifically using ions. What is even more important to note is that in this kind of a circumstance we want to be sure that the electrolyte does not have electronic conductivity, it only has ionic conductivity. In other words the electrolyte must have very high resistance to the flow of electrons, but must have very good conductivity for the flow of ions.

So, we so when we talk of conductivity he immediately here there is a outstanding example of why we have to be acutely conscious of what is that species that is the conducting the charge and whether or not it is something that we would like to encourage or discourage. So, this is an example of such a situation and incidentally in both the electrodes, the anode as well the cathode we will have the situation where both ionic conductivity as well as electronic conductivity will exist. So, there will be, there are components here where only electronic conductivity will exist, there are components where both electronic and ionic conductivity will exist and there is a component where only ionic conductivity should exist.

So, this is this is an example of such a situation.

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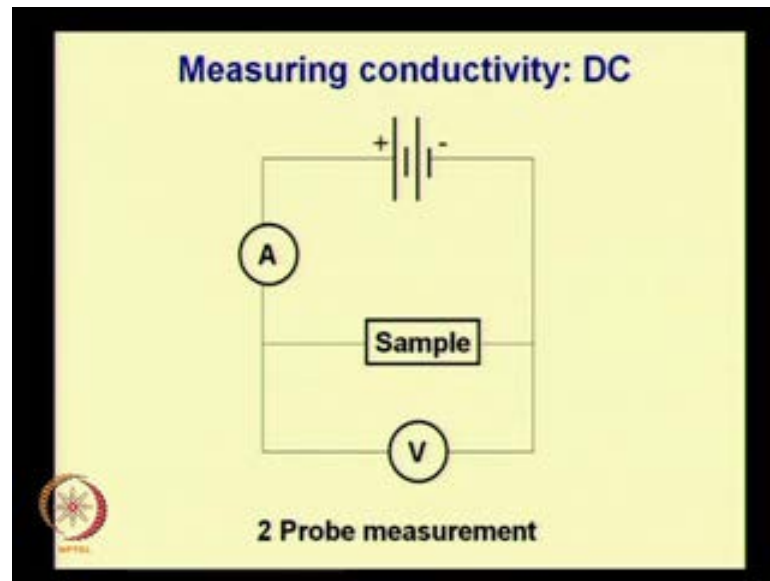
One more example which is which would come to us from the semi conductor industry is that of a transistor. So, let us assume we have a p n p transistor and again we can connect these to some external circuit. So, the wires leading up to this p n p transistor let us say the wires connecting up to the emitter or the collector are wires which would now conduct electrons. So, that is what we would see in our external circuit. Within the transistor itself we have p region here where the primary, these species that conducts charge would be a hole, so out here would be a would be a hole. Similarly, an n region where again it will be electrons and another p region where again it will be holes. So, if we go from left to right we have electrons, holes, electrons, holes and electrons.

So, we have 5 regions and 2 species spread out across these 5 regions, which then conduct charge and in fact the manner in which they are able to pass on the charge to each other, whether or not the free to move these are the phenomena that then define how the p n p transistor behaves. So, that is how the processes occurs.

So, this is yet another example of the fact that there are different conducting species and we need to aware of it and we should even take advantage of it.



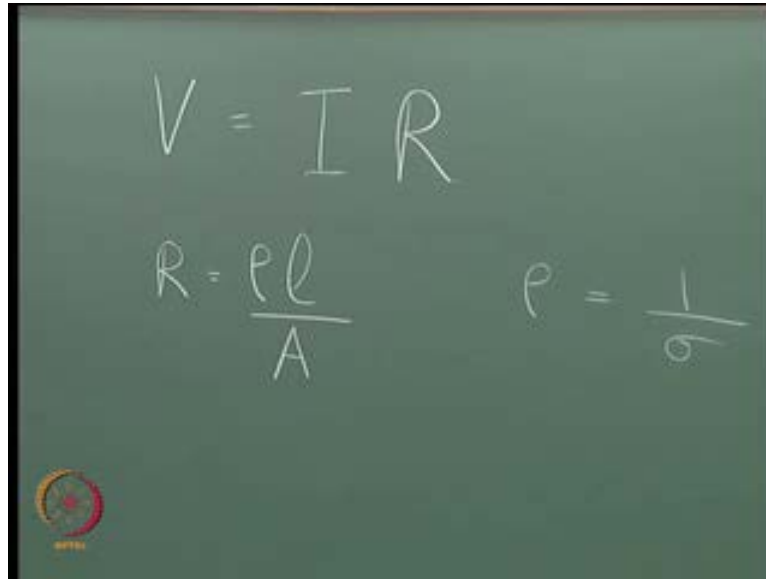
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So, here is a simple circuit which now tells us how we could go about measuring conductivity. For our initial purpose we will assume that the sample that the item marked as a sample out here is simply an electronic conductor. So, for simplicity sake we will now stick to that definition. We will write down the definition later. So, for now it is an electronic conductor and we would like to measure its conductivity. So, we now have a circuit where simply we have a source of electricity in this case a battery, connected to that is an ammeter and then connected to that it is a sample and then goes back to the battery, in parallel to the sample we have a volt meter.

So, let us for a moment just look at this circuit and understand what is going on because that will then define, that will then lead us to what we need to be careful about so to speak. The all of the current flowing through the circuit will go through the ammeter and at this junction, at this junction almost all of it will now go towards the sample. So, almost all of it at this junction will start moving towards the sample. A very tiny fraction of current will go to the volt meter. So, volt meter circuit is, a volt meter is designed to actually give you is to is to design a measure voltage with a very tiny amount of current if at all passing through it. So, almost all of the current that goes in the circuit, then goes through the sample and using this general circuit we would like to find out the conductivity of that sample. Now, before we do that let us just see what we mean by conductivity.

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$$V = IR$$
$$R = \frac{\rho l}{A}$$
$$\rho = \frac{1}{\sigma}$$

So, we are aware that we have the standard ohms law that we are aware of which is simply  $V$  equals  $I R$ . Where  $V$  is the voltage of the potential drop across particular region of the circuit,  $I$  is the current flowing through the circuit and therefore,  $R$  which is the proportionality constants is the resistance associated with that circuit. Now, this resistance if we have a wire for example, the longer the wire is more resistance it will have, the wider the wire is the less resistance it will have because the current will have alternate paths to go through. So, this is we need to get this in a in a manner that is then is in is in a manner that we need not worry about the dimensions of the sample. Therefore, we actually find that the  $R$  is proportional to the length of the sample, inversely proportional to the area of the sample and there is a proportionality constant which is  $\rho$ , this  $\rho$  is the resistivity.

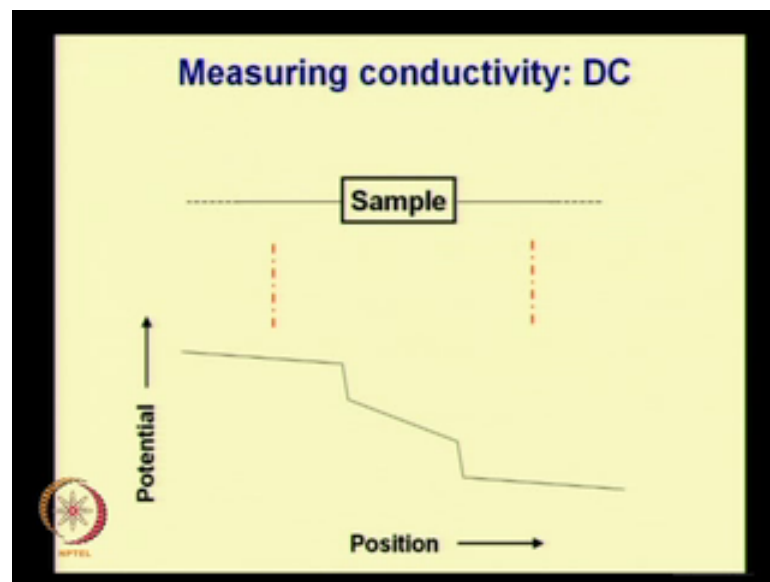
So,  $R$  is  $\rho l$  by  $A$  where  $\rho$  is the resistivity and the  $\rho$  is connected to conductivity simply as  $1$  by  $\sigma$ . So, this is the conductivity. Fine, so this is all that we are interested in and so therefore, we understand that if there is a resistance in a in a circuit as current flows through it there will be a potential drop. So, this is basically all that we are looking at. So, we will with this understanding we will now reassess the circuit that we have and what consequences it has. This kind of set up that we see here where you have a sample and you are connecting up to the sample on 2 ends of it using a wire. We would refer to this as a 2 probe measurement. It is called a 2 probe measurements simply because we connect to the sample at only 2 locations, out here and out here.

So, there are only 2 locations that we connect to in the sample and so that is why this is called a 2 probe measurement.

Now, if we take this part to the circuit which is this wire that is leading up to the sample, the sample itself and the wire that is going away from the sample and let us look at what is happening to the potential at each through this region. Let us remember that the wire itself has some finite resistance, the sample has some finite resistance which is what we would like to determine and so the again the wire on this side also has finite resistance, but importantly the location where that wire connects up to sample which is out here and similarly, out here just that region, just that contact point between the wire from the external circuit and the sample that we have that itself will have a finite resistance associated with it, which is referred to as contact resistance. There is a contact resistance out here and there is a contact resistance out here.

So, these 2 are contact resistances which are exclusively there because you have some too dissimilar materials coming in contact and there is no, there is never ever a perfect contact between the 2 of them. So, therefore, this happens.

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Now, if we look at are sample we have 2 wires leading up to the sample and let us say the volt meter is measuring the voltage between here and here. Right, we would like to measure the current and we would like to measure the voltage and the ratio would then give us the R or  $V$  equals  $I R$ ,  $V$  by  $I$  would give us the R. So, that is what we are

interested in. So, we are right now the volt meter is connecting up to these 2 points, what happens the as I mentioned the wire itself has some finite resistance so there is a drop in potential as the as you go along that wire. When you come to the location where you connect to the sample there is the contact resistance which is usually relatively high.

So, you see a potential drop associated with that contact resistance. Then there is a drop in potential associated with the conductivity process in the sample itself. So, this is actually the process that we are interested in. So, this is the conductivity that we would like to identify or get a numerical value for or the resistivity for which we would like to get a value for and this is what we are interested in, but on either side of it we have an I R drop as we call it. I R drop associated with the contact and other I R drop associated with the contact here. An I R drop associated with the wire leading away from that sample, similarly, an I R drop associated with the wire leading to the sample and the I R drop through the sample.

Therefore, when you do a 2 probe measurement which is like this, what we see is the voltage that you record here consists of an I R drop through this wire, an I R drop at this connection, an I R drop through this sample, again another I R drop here and an I R drop here. So, that overall drop that we are measuring is takes into account factors that we are not interested in. So therefore, at 2 probe measurement in the manner that I have shown you actually would give us an erroneous answer simply because your taking into account your adding on resistances which do not belong to the sample, which have nothing to do with the conductivity of the sample. Therefore, it is much more useful and much more appropriate that we measure the potential even though current is flowing through all of the circuit, we only measure the potential and therefore, the potential drop that is occurring within the sample.

In other words it would help if these 2 locations where we are measuring the potential are changed to these 2, instead of these 2 we would like to have these 2. Now, even though we still have a potential drop with wire in the external circuit, we still have a potential drop associated with the contact resistance again on both sides of sample. All of these are eliminated from our measurement process and as I mentioned a very tiny current, if anything goes through the volt meter so any conduct resistance associated with that or any other resistance potential drop associated with those wires is negligible. All of what we measure is simply got to do with the sample. So, this kind of a measurement

where now we would have current leads connecting to the sample on 2 positions and voltage leads connecting to the sample at 2 other positions, would be called a 4 probe measurement. So our connection would now look like this.

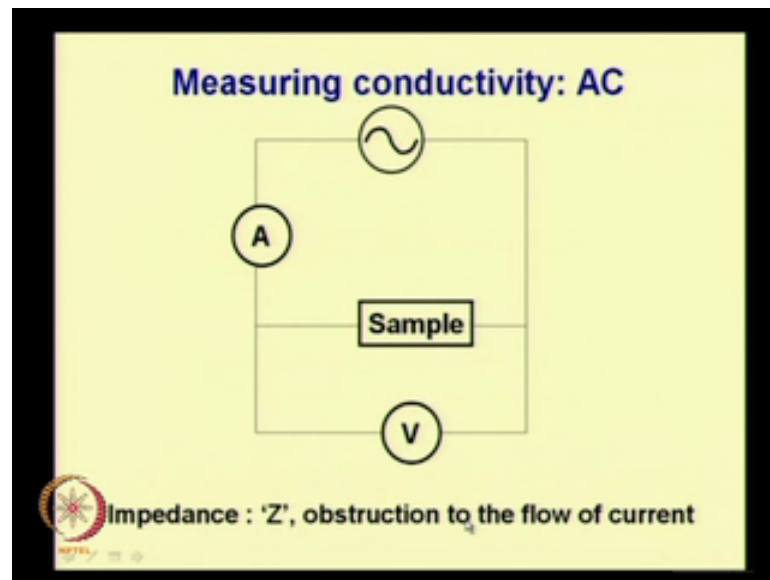
So, this is different from a 2 probe measurement that we have previously described. So, to summarize what we have discussed now on D C conductivity we find that for samples that are good electronic conductors, we can do a D C measurement a direct current measurement to identify the conductivity associated with that sample or the resistivity associated with the sample, but we need to be careful because we need to understand that we want only the potential drop that exist within that sample, anything else that we additionally measure we would have to subtract from this system. So, that we actually know what is the sample behaviour and a simple way to do this would be a 4 probe measurement. So, this is what a 4 probe measurement.

Now, we now so we already have this we have an understanding of how this process would occur and what things we need to be careful about, but as I mention when we started this discussion, for the moment we have assumed that this sample is an electronic conductor. Supposing, it were not an electronic conductor, supposing let us say it was an ionic conductor, then simply connecting it this way the manner in which this circuit is demonstrating it to you will not suffice because the ions inside the sample can only move within the confines of the sample, the circuit outside the sample only has wires which can only conduct electrons. So, the wires outside will not conduct the ions anywhere in a more general sense that I am talking about. Therefore, the ions are not confined to move within the sample.

Therefore, if we simply put D C power supply and attach it to the sample you can only send current through the sample for a small few microseconds at which point the ions will move to one side of the sample preferentially, in a manner where they oppose the potential gradient being imposed on them and therefore, the internal potential gradient will oppose the gradient that is outside the sample and it will stop the current. So, you will momentarily charge the sample that is basically all that you would do. Therefore, for you to actually study this process you need to shuttle the ions back and forth through the sample and the simplest way therefore, to do that would be to replace this direct current source with an alternative current source which therefore, would be an A C signal.

So, the circuit being largely the same we could either go with 2 probe measurement or a 4 probe measurement depend again taking into account the appropriate issues. We would just replace this power source with an alternative current power source. Therefore, it would look something like this.

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So, we would now have an alternative power source, a sample and ammeter and volt meter. The moment we do this, there are a few other aspects that we need to be aware of. So, the first thing is that the moment you say alternating current, it there it now has a frequency associated with it, normally depending on where you are in the world most usually it is either 50 hertz or 60 hertz, 50 cycles per second or 60 cycles per second. This is normally the kind of alternative current that we actually get off of the mains that we have in our rooms so as to speak of the power points that we have done, but in a more general electrical sense you can actually have any frequency associated with your alternative current.

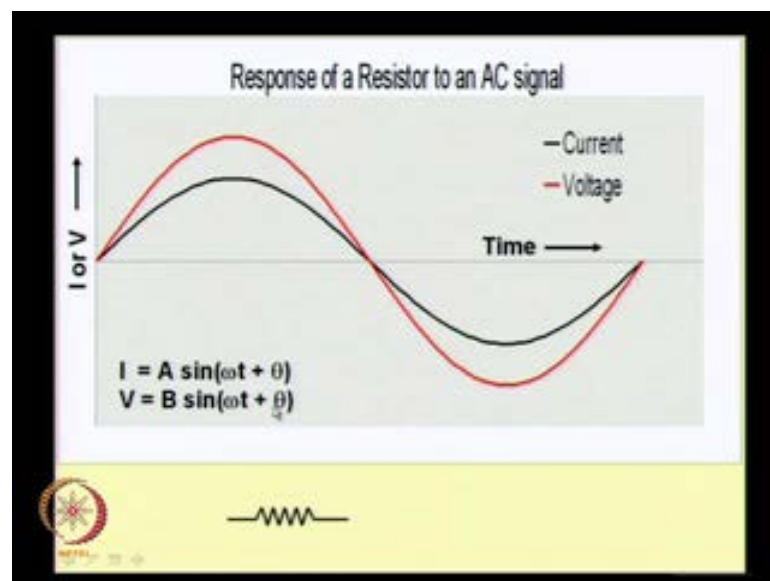
You are not restricted to 50 hertz or 60 hertz, if you go to a lab which has electrical equipment or electronic equipment that permit you to change the frequency, you could go from micro hertz to milli hertz to hertz to hundreds of hertz, kilo hertz, mega hertz and giga hertz. Just to give you an idea. So, you can change the frequency over several orders of magnitude, you are not stuck to 50 hertz to 60 hertz, alright. So, this is just one important thing that we should be aware of. The reason I am highlighting it is that there

are many components and many phenomena that occur within materials that we would that we normally studied where those phenomena are actually frequency dependent.

Therefore, When you do a D C measurement it is effectively there is only 1 frequency associated with it, if you can call it frequency which is 0 hertz, there is no change in there no cycles per second, it just a flat D C direct current. So, when we do a D C measurement the quantity that you get is a single point quantity, usually you get a single quantity out of the measurement process. When you do an A C measurement depending on your sample, it really depends on your sample not always the case, depending on your sample for every frequency that you select you can get a different response from the sample. So, this is something we should be aware of. So, where we would normally end up getting a point, we would get a line or a curve depending on what we sample behaviour is with respect to frequency. Then we measure A C conductivity so to speak.

The what we the quantities have slightly different names, so resistance that we refer to in D C sense is now replaced by impedance, a term called impedance and it is represented by  $Z$  and it also represents the same idea, it represents an obstruction to the flow of current. So, impedance is an obstruction to the flow of current and in that sense it is very similar or analogues to the resistance.

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Now, let us see a little bit more about keeping in mind that you know there are frequencies and so on to be looked at. We will look at 3 commonly occurring circuit

elements, a resistor and a capacitor and an inductor and see some of the interesting aspects associated with their behaviour, how they respond to an A C signal. The reason we are doing this is not (( )) into electronics, but to keep in mind that when you actually look at materials systems and you do A C conductivity measurements or A C impedance measurements as we would call them, you will see behaviour that is very similar to that of a capacitor or an inductor or a resistor. So, understanding how each of them behaves independently helps us actually get information out of a unknown samples so to speak because it will show you very similar behaviour or aspects of it may be mimic one of these components.

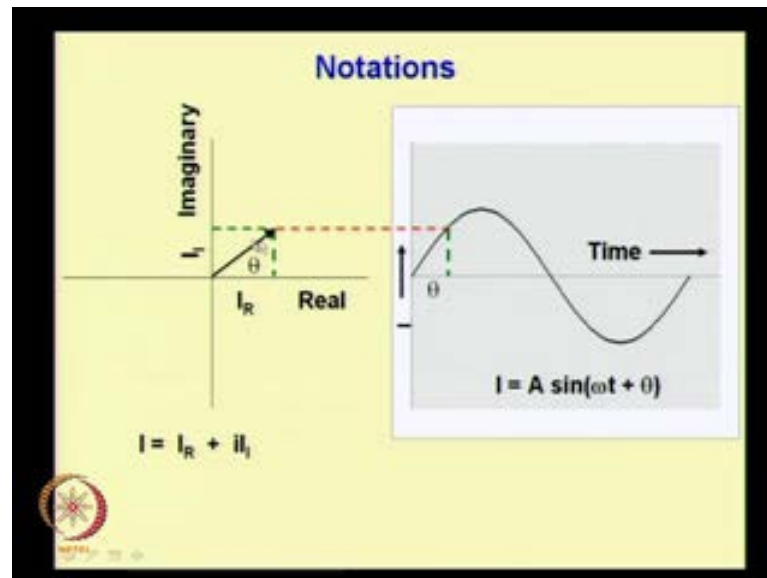
So, if we took at typical A C signal which is showing your semisoidal variation out here we will say the red line or the red curve is the voltage and the black one is the current. For a resistor both the voltage as well as the current are exactly in phase meaning with time if we look at I or V versus time, with time when both of when either current, when current is at 0, voltage is also at 0, as you go along time when in terms of space we would call this pi by 2, when current reaches pi by 2 voltage also reaches pi by 2, when current reaches pi voltage also reaches pi minus or current reaches 2 pi by 2 it reaches I am sorry they all reach the same angle, but when it is at 0 it is at 0, when it is at maximum it is at maximum. Both voltage and current are at maximum, the when it come backs to 0 at pi both voltage and current are at 0 and similarly, when it reaches an negative maximum when current reaches a negative maximum which happens to be at 3 pi by 2, voltage also reaches a maximum at 3 pi by 2 and similarly, they both reach 0 at 2 pi. So, angularly they will always be the same any way, but it is a question of whether they reach maximum at the same time or not.

In to the extend that they are in phase which is true when it is a resistor they both reach maximum at the same time point in time. So, now in a general sense we could represent I as some magnitude  $A \sin \omega t + \theta$  which would be the phase angle depends that would depend on your reference and since they are in phase, the phase angle is the same for both I and V in this case. So, there is no phase difference between the 2 of them and this is the amplitude of voltage which can be different from that of the current and  $\sin \omega t$  and this is the your symbol for a resistor. So, they are both in phase.



Now, there is another way in which we can represent this data which can become more convenient to manipulate and is what is typically used for conductivity measurements, impedance measurements and in electronics and so on which is what is shown here.

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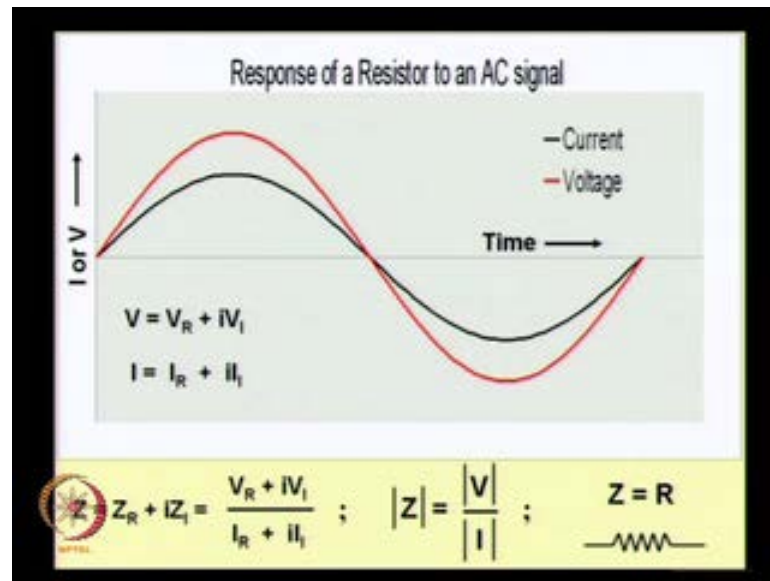


So, we can represent this same sin wave using a vector where theta now represents the angle that it makes with this horizontal axis, this vector can rotate about this origin with the same angle of frequency omega that is associated with this angle and what will happen is depending on where it is for example, it is now at this location it is now therefore, representing this point and this sin curve and the theta would then even though this is the time axis we can associate it as I said with you know, this would be 0 pi by 2 pi 3 pi by 2 and 2 pi so this is all we can also using omega t we can actually convert this to a theta axis also or associated theta with it, associated theta with it and then the horizontal component of this vector we would then call the I real and this would be I imaginary.

So, we would just convert this into 2 components and in the (( )) of electrical engineering we would call it imaginary current and real current. So, I could then be represented as I, as I real as 2 components I real plus I imaginary where this small I here is the square root of minus 1 so or in some cases they would put it as j so we would either have I R plus i times I I or I R plus I plus j times I I, it is a same concept, it just represented by either small i or small j. Therefore, the sin wave can now be represented

as 2 components as a real component, imaginary component in this form. Therefore, we will now take this notation and keep in mind that you know as this rotates for example, when this reaches the maximum we would be talking of this point here, so that is how these 2 relate to each other. When it goes back down here we would be talking about this point.

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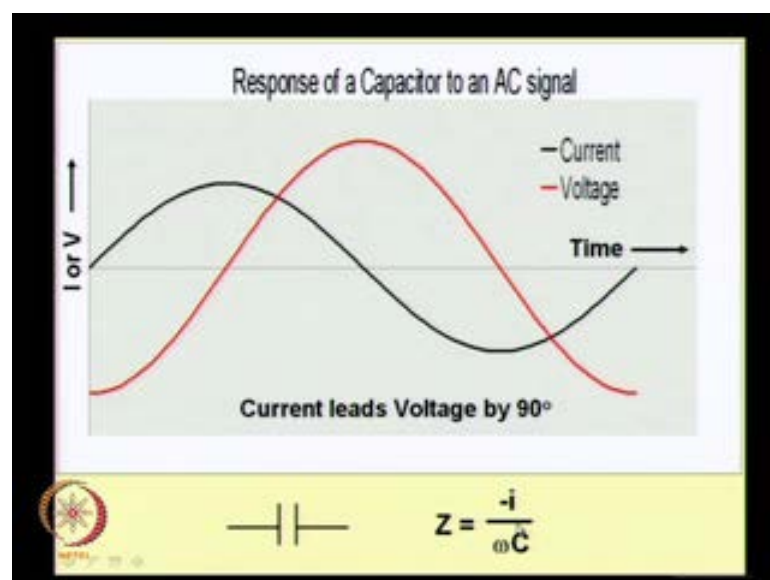
So, given that you now have a voltage and current. So, the voltage can also similarly, be written as a real component and imaginary component, current can be written as real component and imaginary component. Therefore, the impedance of the circuit is now similarly, listed as a real impedance and an imaginary impedance. So, this is what we would we would get or we can get in this analysis and in an in a manner that is analogues to what we do in a D C sense the impedance is simply the ratio of the voltage to the current.

So, V real plus i times V I plus I real plus i times the imaginary current. So, this is the ratio. The magnitude of the impedance is simply the magnitude of the voltage by the magnitude of the current. So, this is what impedance is and simply because in a in a resistor the current and the voltage are exactly in phase meaning as I said when current reaches maximum voltage reaches maximum, they both reach 0 at the same time, both reach negative maximum at the same time and again reach 0 at the same time because they are exactly in phase and because of the way V by I behaves we actually have the

impedance is directly equal to the resistance in a D C sense. Now, the thing that we have to, the few points that I need to I would like you to pay attention to is that for a resistor it appears, it seems to turn out that there is no difference whether you are measuring conductivity using D C source or an A C source, you get the same value of resistance. So, the manner in which it obstructs the flow of current is the same regardless the manner and extend in which it obstructs the flow of current is the same and regardless of the source whether it is a A C source or a D C source. So, we find Z is the obstruction to the current is directly equal to R the resistance that we conventionally associate with the resistance. I would also like to draw your attention to one more aspect that is not visible here which is omega, the frequency, the angular frequency associated with this sin wave, it does not appear in this equation and the significant significance of it is that the resistance is not in any way related to the angular frequency.

So, in other words if you send 10 hertz A C current or you send 500 hertz A C current or 1 kilo hertz A C current or 1 mega hertz A C current our resistor essentially behaves in exactly in the same manner, there is no difference. So, that is the important point that you need to keep in mind. The reason you need to keep this in mind is because a capacitor and an inductor behave quite differently.

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So, here for example, we have a capacitor. In a capacitor the current and the current and the voltage are not in phase, in fact current leads voltage by 90 degrees meaning when

voltage is actually at negative maximum, current has already reached 0, when current reaches, when voltage reaches 0 current has reached positive maximum and so on. So, the current stays ahead by  $\pi$  by 2. So, whether.

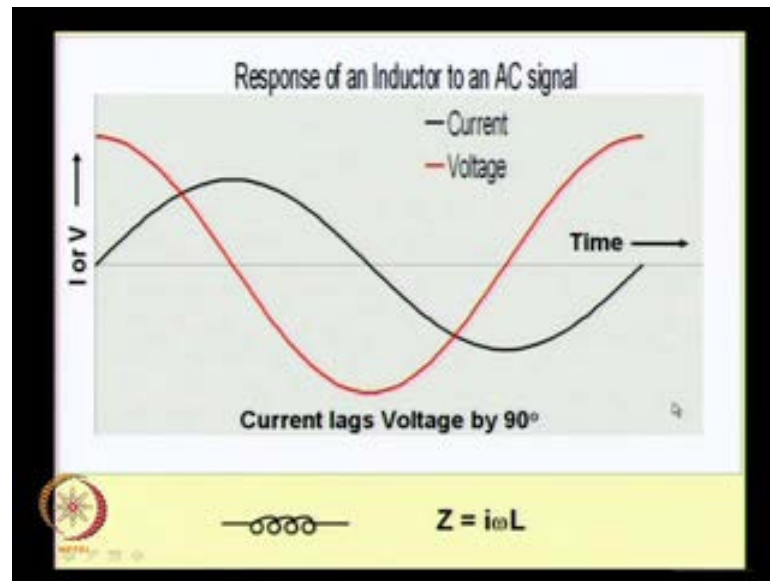
So, wherever the current is at a given point in time the voltage will be the same point in time after  $\pi$  by 2 degrees and it is exactly 90 degrees out of phase. So, given this situation if we actually run the impedance of, check the impedance of capacitor and see the manner in which it obstructs the flow of current and take into account how current voltage characteristics of capacitor behave. So, you have  $Q$  equals  $C V$  and so on.

So, if you take in take into account all that we find that the impedance of a capacitor or the manner in which a capacitor obstructs the flow of current is given by this expression. Please note in a resistor the, we got  $Z$  equals  $R$  which means it is only a real quantity. So, I mentioned that  $Z$  itself could have a real and imaginary quantity. So, we could have  $Z$  as  $Z$  real plus  $i Z$  imaginary. In the case of resistor we did not have any imaginary component, it was entirely real. In the case of a capacitor because of this  $\pi$  by 2 variation between phase difference between current and voltage, it turns out that the only component we have is imaginary, there is no real component.

So, that is the first aspect of the impedance of capacitor that we should be aware of. Second thing is we also see that  $\omega$  appears in the expression. So, therefore, we now see that the impedance of a capacitor is not only an imaginary quantity. It also depends on the frequency that we are imposing. In other words for a capacitor if you send an A C signal using 10 hertz, it will behave it will obstruct its flow in a certain way. If we send an A C signal using 1 giga hertz it will obstruct the flow of that signal in a completely different manner.

In fact it will badly obstruct that flow, if you go closer to closer because  $\omega$  is in the denominator as the frequency decreases as the frequency  $\omega$  decreases the impedance goes up, so D C, when you apply a D C source on a capacitor you will see infinite,  $\omega$  is 0 and therefore, you will see infinite impedance. It will, it will actually act as though the circuit was open and there is no current that is able to flow through it. If we send very high frequency the impedance will drop to 0 and the current will go through the capacitor as though it were just a wire. It will not even, you will not even know that there was a capacitor in this circuit.

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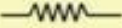


In a similar, but opposite manner so to speak we actually see that in an inductor the opposite is true. We have the current which lags behind voltage by 90 degrees. The similarity is that between a capacitor and inductor is that in both cases the impedance is entirely imaginary, there is no real component to it. Again there is also a dependence on the angular frequency  $\omega$ . So, that is also present here except that now the angular frequency is there in the numerator itself.

Therefore, for an inductor if you send 0 frequency, in other words D C, connecting D C power source to an inductor it will act as though it were just a wire, current will just flow through it smoothly, no problem. If we send a very high frequency through it so say thousands of hertz, tens of kilo hertz through it then it will, its impedance will be very high, it will prevent the flow of current through it, it will act as though it is an open circuit. So, this is what we had and that is a symbol for an inductor.

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**Measuring conductivity: AC**

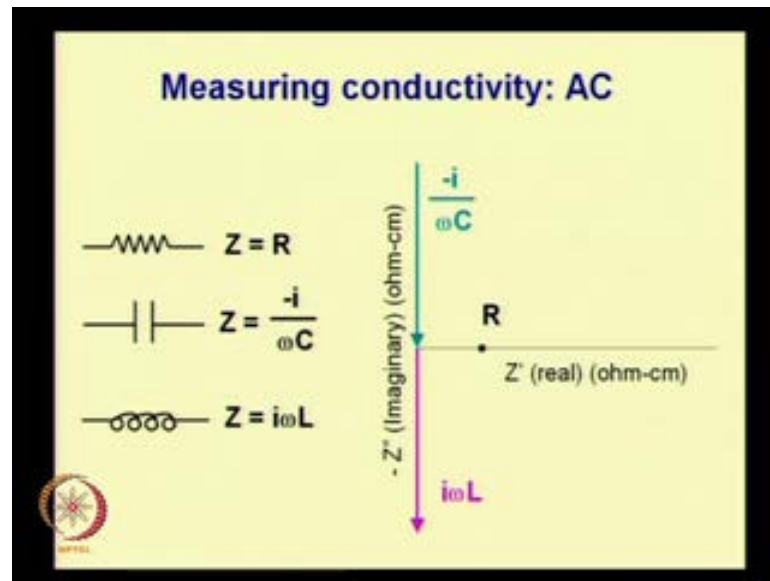
**Impedance : Obstruction to the flow of current**

	$Z = R$
	$Z = \frac{-j}{\omega C}$
	$Z = j\omega L$

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So, to summarize what we have seen here is a resistor impedance is  $R$ , capacitor impedance is  $\frac{-j}{\omega C}$ , inductor impedance is  $j\omega L$  and these 3 the resistance has no dependence on its frequency and it is a real quantity and both of these quantities depend on frequency and are imaginary quantities. And again to restate that the idea the reason I have developed this is because when we study materials and we measure their conductivity, there are aspects of material behaviour with that mimic capacitor behaviour, there are aspects of material that mimic inductor like behaviour and there are aspects that mimic pure resistance like behaviour and therefore, when we measure the conductivity we get a signal that has all these characteristics and we have to extract the information out of this signal to understand what is the conductivity we have actually measured.

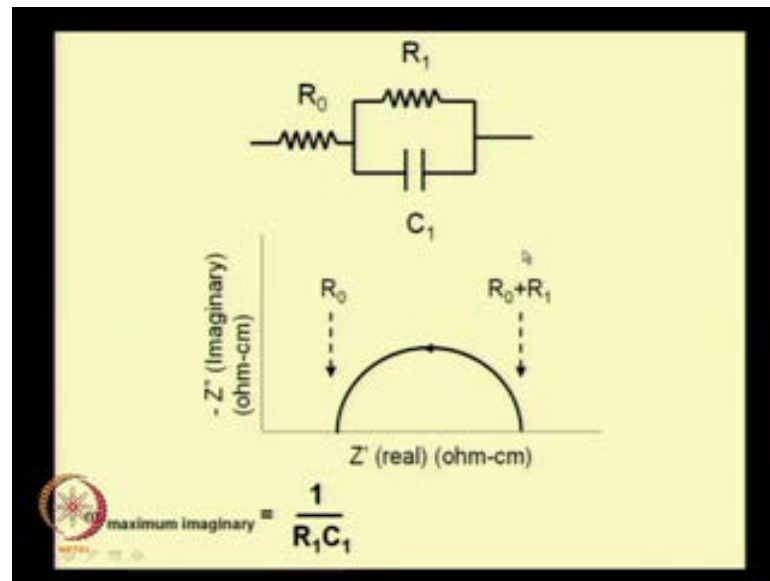
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What I just described I have shown you in a plot here, for a resistor the value of the resistance is  $R$ , what we normally plot is the imaginary impedance on the y axis, actually negative of the imaginary impedance on the y axis, that is just the convention and the real impedance on the x axis. And what we see is for a resistor you get a single point. Please remember that this is data that is collected over a wide range of frequencies. So, for a wide range frequencies at every at every frequency you can measure the impedance, at every frequency you can see what is  $V$ , what is  $i$  measure in impedance.

For a resistor it makes no difference so across if you scan you can go from you know open I mean from D C source to 10,000 hertz you will get the same point. You will get essentially this point, you will not see any difference. In for a capacitor when you are the arrow now indicates the direction of increasing frequency. So, at very low frequencies you have very high impedance, as you keep increasing the frequency you get a series of points which represent how the impedance of the capacitor which move closer and closer to the origin. All of these points lie along this imaginary axis so to speak, the vertical axis which is the imaginary axis, at very high frequency it goes to 0. For an inductor it is the opposite, it starts from 0 at very low frequencies or a D C conditions and then as you go to higher and higher frequencies the impedance goes to higher and higher values, in this in this case it is minus  $Z$  that is small guess so it is going down.

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So, in a real system like I said you will have aspects of a capacitor and an inductor and a resistor all thrown in. So, what we would normally see would be a material which would have say for example, part of it behaving like a resistor which would then behave, which would then be in series with a resistor and capacitor and parallel.

You need not worry about this exact circuits and so on, the idea we wish to convey is that when you take say a ceramic material and attach two electrodes to it, depending on what kind of reactions could occur in the electrodes, depending on what atmosphere that ceramic is sitting in, what temperature it is sitting in and so on, you can have actually several processes occurring in it including ionic conductivity occurring through the ceramic and the resultant behaviour would show you a curve that looks like that the data that looks like this and I will just walk you through one or two such examples just so you understand what this kind of an analysis means.

So, at if you look at this data and for the moment we will just assume that this circuit represents this data and we will and we will show I will just show you how that happens, at we will start at very low frequencies. In this diagram here the arrow represents this arrow represents the direction in which the frequency is increasing, so that you just keep in mind for the moment we will go back to the circuit. At very low frequencies where omega is close to 0, I mentioned that a capacitor which has an impedance given by minus  $i$  by omega  $C$ , that impedance is very high because omega is in the denominator



therefore, this acts as an open circuit, current cannot flow through this. So, therefore, all the current in your circuit actually flows through  $R_0$  and  $R_1$ . So, the resistance that you measure will be  $R_0$  plus  $R_1$  therefore, your measure a point which is here on this data. If you go to very high frequencies minus  $i$  by  $\omega C$  drops to 0, because it drops to 0 this actually acts as a short circuit, this segment of the circuit.

So, the current goes through  $R_0$ , but it takes the path of least resistance which happens to be 0 resistance in this case.  $i$  just goes through this unit here and goes out, it does not go through  $R_1$ . So, at high frequencies you only measure  $R_0$ , you do not measure anything else. So, therefore, you get a point here and as I mentioned this is the direction of increasing frequency. So, this is the high at very low frequency as you go to higher and higher frequency you get to this point. In all intermediate frequencies you actually have to get the exact value of for that  $\omega$  you get the actual value of the impedance here, this specific value of minus  $i$  by  $\omega C$  and then you have to treat these two as impedances and parallel.

So,  $1/R$  and  $1/(-i\omega C)$  will be in parallel. So, you will get impedance for this and correspondingly you will get all these points. We have seen a good justification for this point and this point, if we run through all the calculations you will get all of the rest of the points and incidentally if you know what is the frequency corresponding to this maximum point here, using this maximum  $\omega$ ,  $\omega$  corresponding to the maximum imaginary impedance that will be equal to  $1/R_1 C_1$  so to speak. So, in fact in this case it will be corresponding to that. So,  $1/R_1 C_1$  with for this circuit  $R_1 C_1$ , this value of a  $\omega$  here corresponds to  $1/R_1 C_1$ .

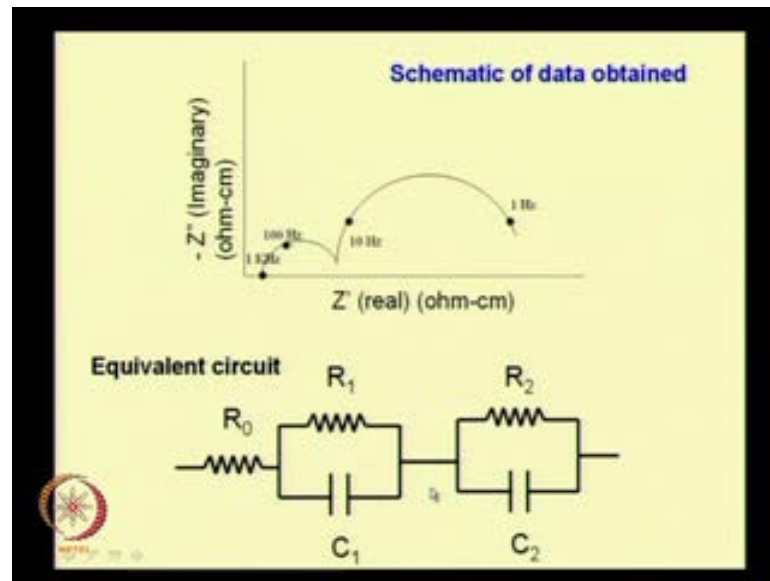
Now, actually this is the way in which you get all the values from the first intercept you get  $R_{\text{naught}}$  from the second intercept you get  $R_{\text{naught}} + R_1$  therefore, you get  $R_1$  from the maximum  $\omega$  here you get using this formula you get  $C_1$ .

So, therefore, using this data you are now been able to get  $R_0$ ,  $R_1$  and  $C_1$ . All of this information you have got.

So, if this were a real material system you could now identify the resistors resistances present within that materials and capacitance present in the material. So, for example, in that in the first example I showed you where you have a battery, often the electrodes have charge transfer process associated with them, which then behaves like an  $R C$  in

parallel, the electrolyte has an ionic conduction conductivity process associated with it which has an  $R_0$  value associated with it. So, using A C impedance we are now able to separate out all these values and find out what each of them behaves is and just to extend it in the same system for example, if you have any mass transfer process occurring it will, it could be shown as being equivalent to another R C associated with this circuit.

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So, for example, you could have 2 R C circuits associated with this single large and then you will have 1 intercept associated with this, the really high frequency you would have  $R_0$  plus  $R_1$  plus  $R_2$  and then you will have two semicircle so to speak associated with each of the R C circuits. Therefore, this is the way in which A C impedance process is used to get very valuable information of conductivity and various other processes present within the system for often used for material systems where electrons are not the only conducting species. Therefore, it is a very powerful technique and gives you a lot of insight into material science, the physics behind how the material operates and so on.

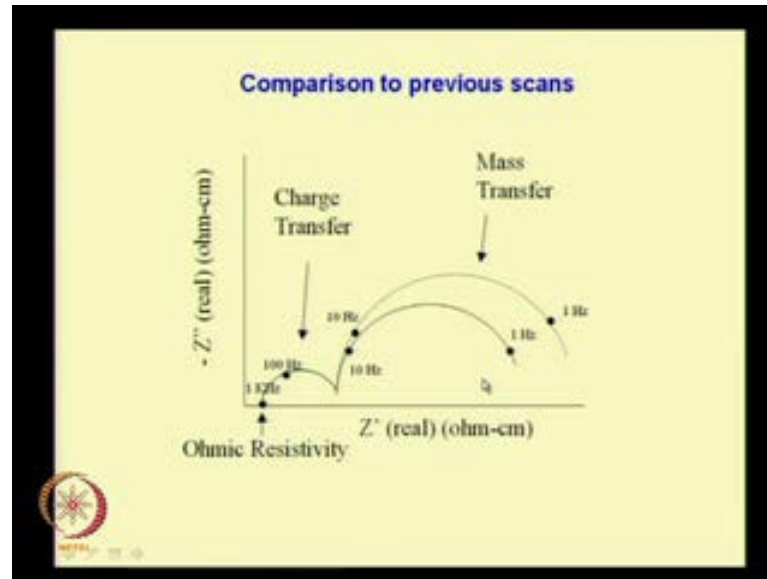
And it could be used in variety of ways as I just showed you the this kind of an approach where you get a data like this, some actual data where each point please remember I just marked some numbers here, each point here represents the impedance that was demonstrated by the sample at that specific frequency. So, throughout this curve there are thousands of points, at each point 1 particular frequency of A C signal was send through the sample and the real imaginary impedances were recorded. Therefore, you get

a complete curve like this. So and I just for information sake I will just put some values here just so that you get an idea.

So, lower frequencies are here. For example, this could be 1 hertz, this could be 10 hertz, this could be 100 hertz and this could be 1 kilo hertz. So one way would be to get this data and simulate an R C circuit which then would behave in exactly the same way, if the same signal had been send through it. Once that is done and there are programs that are that can be written or have been written using which you can, you can try out various values of resistances and capacitances which will give you a very good fit for this data. Such an approach is called an equivalent circuit approach because you are generating an equivalent circuit for this data. Our sample itself does not separately have consists of resistors and capacitors which have been wired together, but it consists of a composite set of species which behave in certain way and their overall response behaves like this.

So, by generating this equivalent circuit we associate R naught with the ionic conductivity, this R C with say the charge transfer associated with the cell and this R C with the mass transfer associated with the cell and you can do a lot of material science. Normally, we would then study these materials under various conditions and see whether anyone of these values is increasing or decreasing and therefore, get a sense of which is a better condition for the system and which is a worst condition for the system. One, other way in which we could, even if we did not have R C circuit, even if we did not wear equivalent circuit, another way we could do it simply is as I mention study the same data, same system under different operating condition.

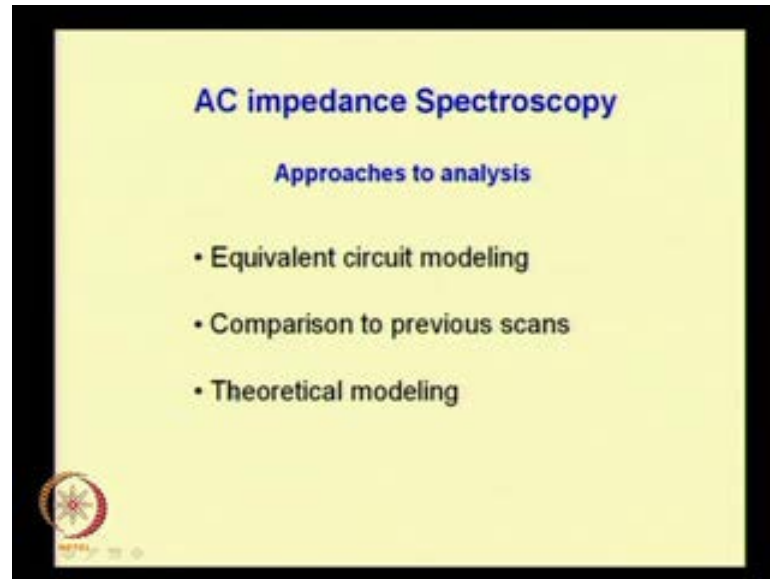
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Let just say under 2 different temperature conditions or 2 different humidity conditions and you may see that in one case, this is your A C impedance data and in another case this is your A C impedance data. So, for example, great this is a very simplified situation, but very useful information that you will get from this situation is that for the 2 experimental conditions that you have tested your system in, it turns out that the Ohmic resistance or associated with the ionic processes the and so on in the system have not changed for both the conditions. The charge transfer if we have associated this section here with charge transfer processes in the system that has also not changed.

What has really changed is the mass transfer. So therefore, we are able to use this as a very powerful diagnostic tool, in addition it gives as lot of insight into how the material is behaving, what is the physics behind the response that we get from the material, if you are done just a D C measurement you would have just got one value of a very high impedance and another value of a slightly lower impedance or resistance and you will not have been able to figure out what was what is the reason why one is better than the other or one is lower than the other. Therefore, A C impedance is a very useful technique.

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So, to summarize A C impedance spectroscopy is I have just shown you what is the manner in which you would set it up, what is the kind of reasoning that goes into understanding the data that comes out of it, how you extract the data out of it, in how many ways it differs from just simplifying a D C value for conductivity and so on from our sample, in what ways A C conductivity is better and we have seen that we could use an equivalent circuit model to get information out of this data.

And we could also simply do a comparison with previous scans or in comparison with scans, impedance scans obtained under different operating conditions. Using both of these we could get of a lot of insight into the material. Finally, there is a much more sophisticated way of taking advantage of A C impedance techniques which is to do theoretical modelling of a system and on the basis of the theoretical model, predict what A C response it would provide and compare it with the experimental data that you get and on that basis you would actually learn a lot about your system.

So, I will finish this lecture by again highlighting the fact that in this lecture, we have spend some time discussing why through this course you will focus on conductivity, why it is an important quantity because it is it has values over so many orders of magnitude and it also so technologically relevant to us. And so throughout this course we will spend fair amount of time on conductivity and in addition to whatever I have said we also have materials which are super conductors which we have not spoken about now. We will talk

about it at a much later class. So, we have a very good understanding now on why we need to focus on conductivity. We are also aware that there are different conducting species in a systems especially systems that we work with in material science and engineering.

There are holes, there are electrons and then there are ions. These species behave very differently based on the environment they are placed in, the ease with which they move or the difficulty with which they move can be very different. How they respond to the certain the environment around them can be very different. Therefore, we are now aware that such species exist and therefore, when we measure conductivity we should not just blindly measure a value, we need to try and get a greater insight into it and in this context when we when we spoke about measuring conductivity I introduced you to the 2 commonly used techniques which is the D C technique and A C technique.

I highlighted the kinds of aspect that you have to be careful about with both of these techniques and the kind of analysis that you can do with these techniques and also the kind of information that you can get out of it. So, this concludes our discussion on the on an introduction to conductivity so to speak. Like I say we have a lot ahead of us in terms of the models that we build for conductivity. So, we will get on with that in our next class. For this class we will conclude with here. Thank you.

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