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Lecture No. # 39

Bose-Einstein Statistics

Hello, welcome to the thirty ninth class in this course the physics of materials. In our in the last several classes that we have seen we have often encountered various particles and try to understand what their behavior is what it represents in terms of material behavior.

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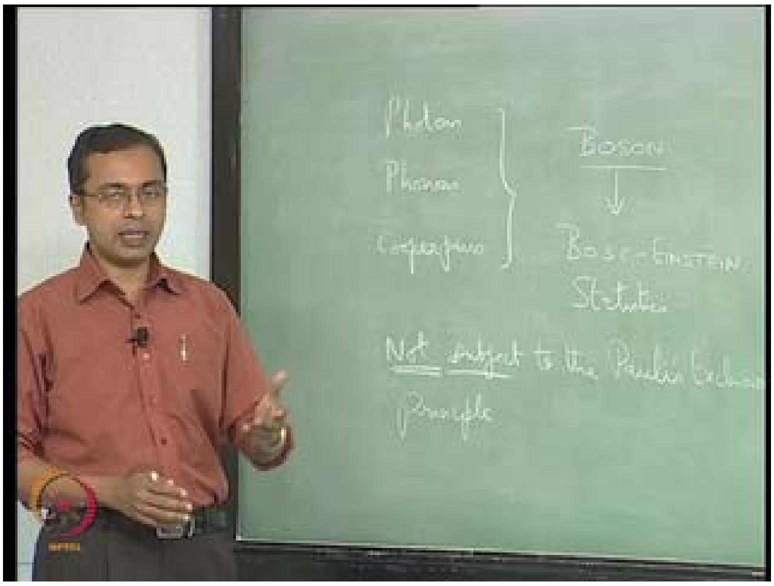

So, throughout this course at several occasions we ran into the photons. So, those are the particles of lights so to speak. When light indicates its behavior in the form of particles we call them as photons. We also looked at vibrations in the lattice and it is said that you know those are waves, but you could also look at them as particles because the manner in which they interact with respect to energy, the way they pickup energy, they look the way they lose energy that is also still quantized and so, in a sense they behave like particles, those particles of a lattice waves are referred to as phonons and in the last class we looked at particular combination of particles which basically we said that you know pairs of electrons with opposite spin and opposite wave vectors could combine together and operate in a coordinated manner and in that sense would behave like a combined particles so to speak.

All though they are quite far off from each other, but their behavior would appear like that and those would be called the cooper pairs. So, we saw these. So, this is what we saw last class and so these are all the or some of the particles that we have seen through the course. As it turns out and as I mentioned in the last class, all of these have characteristics that are common to them and which makes them a certain type of particle which is called a boson. So, this is a boson each of them is a boson and 1 of the requirements for particle to be a boson is that it has to have integer spin. So, an electron by itself has half integer spin. So, half we say plus or minus half. Spin is plus or minus half we say.

So, it has half integer spin and therefore, by itself it would actually qualify as a fermion. So, Fermi Dirac statistics is what it would follow, but cooper pairs for example, it to the extent that they behave as a combined entity, they are actually having a net spin of 0 and therefore, it now suddenly shows you an integer spin and therefore, it satisfies the requirement for it being a boson and so do phonons and photons. So, they have integers spin. So, they satisfy the criterion for being boson. So, so this is what they are. We have already seen these 3. I also mentioned that for example, when I when I spoke about these use of superconductors and the fact that they are used often in this particle accelerator and colliders. So, to speak particle colliders then we have even there for example, at CERN they are using effectively superconductors which use phonons as well as cooper pairs and therefore, or 2 bosons and they are searching for another boson called the Higgs bosons.

So, at the time of recording this lecture that is what is being done. So, it is of interest therefore, since we have discussed these in detail to see what is a boson, what is that behavior that characterizes boson, what is that distribution because we always say you know when you when you look at a set of particles we would like to know in terms of energy how are those particles distributed because that gives you a very good feel for what will that a system of such particles do, when they are subject to certain condition. So, that is why we need these kinds of information. Bosons they get their name because they are credited to Bose, Satyendranath Bose who first proposed this and essentially they follow statistical distribution referred to as the Bose Einstein statistics.

So, they follow Bose Einstein statistics and so, and hence the name bosons. So, today we will look at Bose Einstein statistics. So, that is that is our topic for today. Basically for a for 1 of the requirements for Bose Einstein statistics is we are we are dealing with a set of particles which are showing us quantum mechanical behavior, but in the specific case of Bose Einstein statistics it applies to a set of particles which are not subject to the Pauli's exclusion principle. So, the particles are not subject. So, these particles that are not subject to the Pauli's exclusion principle. They are quantum mechanical in the in their behavior, but they, but they are not subjected to the Pauli's exclusion principle. This is very important because when we did the Fermi Dirac statistical distribution this was the main principle that we incorporated.

We had the Pauli's exclusion principle there. So, it is in this fundamental sense that this distribution differs from the from what we are doing for the Fermi Dirac distribution. So, it exempts the particles from the Pauli's exclusion principle and when we say it exempts, we are basically talking of particles which do not care of I mean which are not affected by the Pauli's exclusion principle. So, that is why this statistics which is based on this idea works for that those sets of particles. So, they are not subject to the Pauli's exclusion principle meaning they are exempt from the Pauling Pauli's exclusion principle. The direct result of this is that see when we say Pauli's exclusion principle the idea that it reduces to is that you cannot have more than 1 particle in a quantum state. Once, you define all the quantum numbers at a finally, when you pick up a particular state which if it includes if that definition of the state includes the spin of the state and so on. If you put particles in their you cannot put more than 1 particle per state, that is the definition of I mean that is what it reduces to when you say Pauli's exclusion principle that is what it in terms of how you will enforce it in the system that is the rule that you will use to enforce it in the system.

Now, when you say that system is not subject to Pauli's exclusion principle, it means immediately this restriction is no longer true. It means in a given state you can put as many particles as you wish. So, the that is the big difference whereas, previously you could put only 1 particle per state now you can put as many particles as you wish in the state because they are not subject to the Pauli's exclusion principle. So, this is the general idea. On this basis we will do the statistical distribution and see what kind of a result we get. So, again we have a system where we have energy levels.

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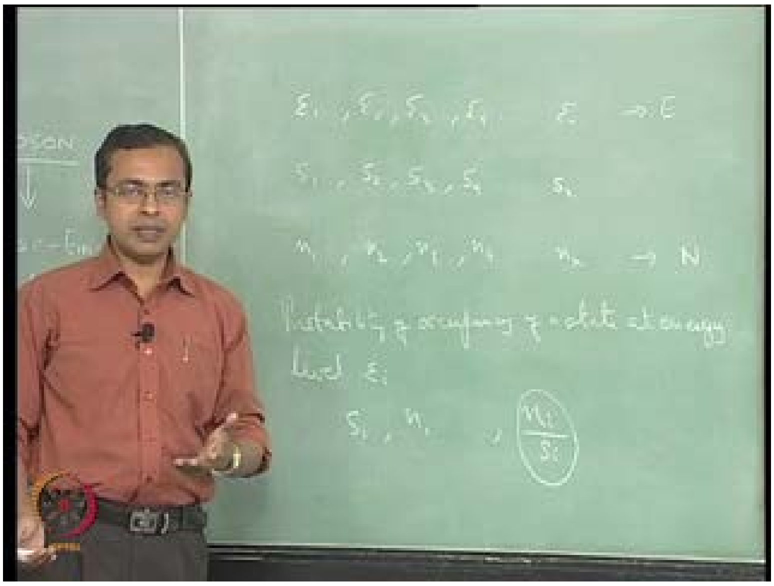

So, we let us say that the energy levels are quantized. So, you have energy levels e 1, e 2, e 3, e 4 and so on. So, some up to some e r and let us say it has a certain each of them has certain number of states. So, s 1, s 2, s 3, s 4 and so on s r. So, these are fixed for the system, given the system we it has a certain set of energy levels and certain set of states. So, again this is a constant volume system and so on we are and at equilibrium that is what we are looking at. Now, we would like know we will say that up front we do not know, but we will say that perhaps there are n 1 particles at energy level e 1, given that these are the states, n 2 at energy level e 2, n 3 at energy level e 3, n 4 at energy level e 4 and so on with a total number of particles being n and the total number of total amount of energy being capital d. So, the total energy is fixed at capital e these are the energy states that are available and the total number of particles is fixed at capital n, these are this is the way in which that capital n particles has been distributed across all these energy levels.

We have n 1, n 2, n 3, n 4 at those respective energy values. Now our problem of course, as we have done before is to find out what is the relationship that helps us figure out what is the value of n 1, what is the value n 2, what is the value of n 3 and so on. We do not know upfront what this values are, we would like to find out what these values are given that is the system subject to the rules that we have just described. And again these 2 are fixed, this is the 1 that we have freedom to change subject to this restriction the total number is n. So, given this scenario what we are interested in actually finding out is the effectively the information we are interested in we are finding out is the probability of occupancy of a state at energy level e i.

So, I said that we really want to know what are these numbers? Mathematically it reduces to this you want to find out what is the probability that a particular state at any given energy level is occupied. If you have a general expression for that probability that effectively will sort of give you all this information that we are interested in. So, then you can go to any energy level you can get an idea of what the n is at that level because that will give you because we know this. So, we have some probability of occupancy and we can sort of figure it out and there. So, we would like to get probability of occupancy of a state at energy level e i. So, if you take any energy level e i.

We basically have s 1 states and n 1 particles. I am sorry s i states and n i particles at energy level e i. So, actually the probability of occupancy of a state is simply n i by s i. So, this is all we are, this is the probability of you have s i state, you have n i particles. So, what is the chances that a particular state at this, 1 of this states is occupied its simply n i by s i. So, this is all we are interested in. So, actually. So, the problem actually reduces to finding out an expression for this.

We want an expression of for this based on the conditions that are imposed on the system. So, this is the expression we are interested in once we get an expression for this, we sort of have the answer for the information that we are looking for. So, that is that is all we are going to try and do in this class now. So, let us get on to the mathematics of how we go about it. What we will do is we are now faced with a slightly with a slightly different situation than what we are we have dealt with before and that is simply that since Pauli's exclusion principle does not apply, any number of particles could be setting at a given state.

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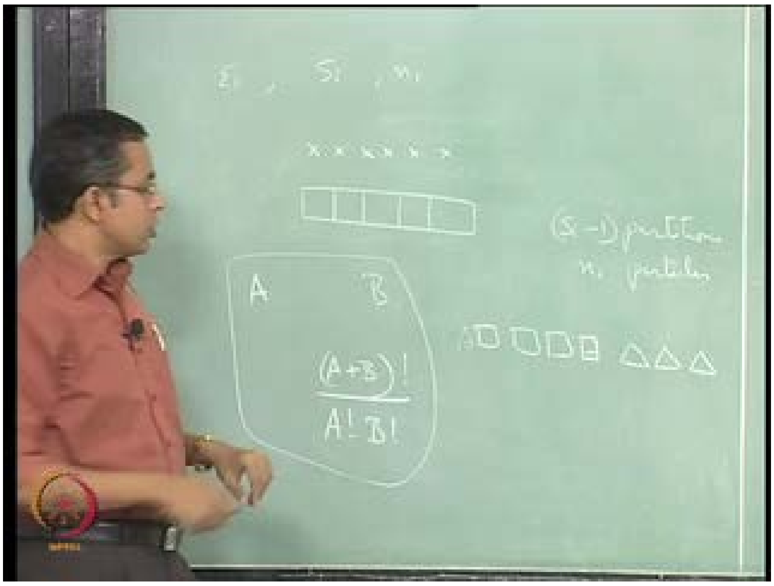

So, therefore, what we actually have is we have s 1 states and n 1, s i state and n i particles and really you can mix them of any which way you want at a given energy level e i. So, there is since any number of them could sit in 1 particular state, there is we are free to mix them up in any which way that you wish. So, we will just put it put down here we have let us say n i particles which I will mark as x with small x is here and then we have some states, some arbitrary number of states we have. These are the particles, these are the states. We would like to put this particles into the states in any which way that you want. Now, in general if you have. So, you have what is this mathematically you have s I, you have s i items of 1 kind and n i items of another kind. What is the ways in which you can mix this 2 up that is in how many ways can you arrange these given that you have s i items of 1 kind and n i items of another kind.

Normally, when you write such a problem when you have you know A items of 1 kind and B items of another kind and you would like to know in how many ways you can arrange these 2? The general answer is actually A plus B factorial by A factorial times B factorial. This is the general answer for this kind of a problem. The only issue of this kind of an answer. So, in principle we can apply this kind of an answer here, but there is 1 small detail that we have to address before we do this kind of an we apply it and that is when you do this kind of a problem you are permitted to have a situation where you have all the A items first. So, let us say A item. So, all the A items. So, let us say they are squares, A squares. So, we can have all the squares and let us say B is triangle.

So, we can have all the triangles after this. This is a valid arrangement as far as this kind of a mathematical problem is there. You can put all the squares first and then all the triangles next similarly you can put all the triangles first followed by all the squares. So, you this is allowed under this under the problem that we have defined like this in the mathematical sense and so this is. So, those are 2 arrangements that are included in this in this condition that you are looking at. Now, in the condition that we are looking at we have particles and we have states. If you have this kind of a situation here it is the same as saying you will have all the particles first and then all states or you will have all the states first and then all the particles. That creates a situation that that corresponds to what?

It corresponds to situation where all the particles are outside all of the states. So, that is not a situation that is acceptable for us. All the particles are free to be in any state they wish, but they have to been in some state you cannot have. So, this is not a valid condition to have all the particles outside all of the states, like this kind of an arrangement is not acceptable here. So, we cannot simply write we cannot use this equation in exactly this form you cannot simply say A plus B factorial by A factorial plus times B factorial. In this case you cannot simply say s i plus n i factorial by s i factorial times n i factorial, you cannot do that because that would imply that you are allowing this system, this possibility that all the particles are outside or several particles are outside that is not allowed. All the particles have to be somewhere in those states.

So, how do we do that? How do we handle that situation? A simple way to do that is to say that we will actually treat it as we will we will eliminate 1 state in our count. So, how many ever states are there we will take 1 less state and then we will arrange all the particles in the remaining states using the same kind of a formula then whenever we say that all the particles are before or after it simply means they are in the last state or in the first state. So, or alternatively if you have you know let us say for example, you have here 1, 2, 3, 4, 5. 5 states you have here, if you look at it the number of partitions between these states is 1, 2, 3 and 4. So, how many our states you have you have that many minus 1 partitions. So, we can treat the problem as though you are trying to arrange s i minus 1 partitions and n i particles in any which way that you can.

If you treat it as s i minus 1 partitions you will never have a situation where all the particles are outside the states because if all the partitions occur before the particles then you have all the particles sitting in this last state because all the four partitions are before all the particles. The other example that we have is all the four partitions are in the end, the particles are before that all the 4 partitions. So, therefore, they are in the first state. So, you can now not worry about this situation that we just described, they particles will never be outside these states, they will be stuck within this state and regardless of what possibility you look at you can always come back to this picture and you can see that they are either in the first state or in the last state or in some of the states in the middle. So, therefore, that issue is addressed. So, instead of directly just using s i and n i which can create this kind of a controversy, we will use s i minus one and n i. So, if you use that all the possibilities are addressed reasonably.

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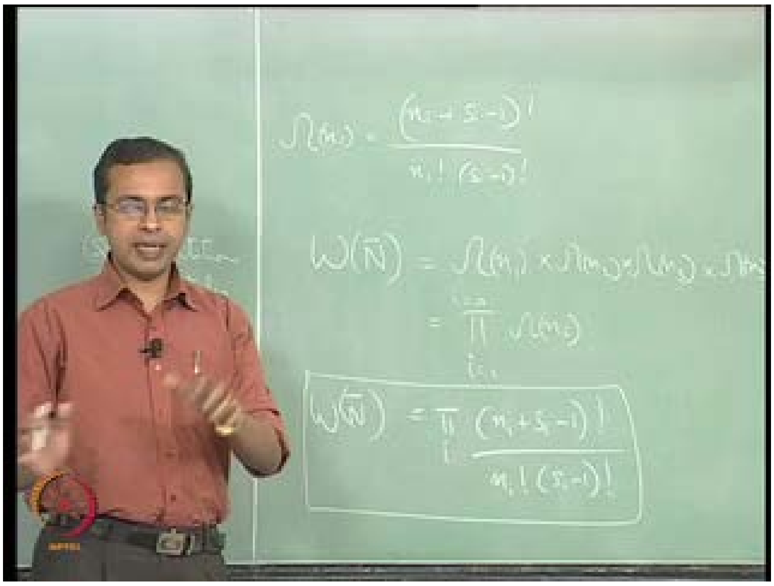

So, we will do that. So, therefore, we now have, if you use now this formula we will have n i plus s i minus 1 factorial by n i factorial times s i minus 1 factorial. This we will designate as omega n i. So, this is the number of ways in which you can attain you can arrange n i particles in s i states subject to all the conditions that the Bose Einstein statistics requires. So, this is how we get it. So, for the entire systems. So, this is a for entire system the for each energy level you can do this, for every energy level this is only an i. So, you for energy 1 you can do it, energy level 1, e 1, e 2, e 3, e 4 for every 1 of them you can do this.

So, the total number of ways in which you can accomplish a particular micro state. So, so for that micro state for example, we which will designate with capital omega. So, on particular distribution we have chosen where this distribution represents n 1 at e 1, n 2 at e 2, n 3 at e 3 and so on. The number of ways you could do that is the product of all this omega of n i n 1 times omega of n 2 times omega of n 3 times omega of n 4 and so on. For each number of particles at that particular energy level whatever e i. So, n 1 is also at e 1. So, that is the other thing we are looking at. So, if you took you write this formula down for every 1 of those energy levels and they are corresponding number of particles and number of states each 1,each of these terms will look like this and if you multiply all those terms that is a number of ways in which you can attained that micro state. So, that is that is what we are looking at.

So, that is the number of ways in which you can attained the micro state. So, this is simply as I using the same notation that we have done before i equal to 1 to r omega n i. So, this is simply pi over i of this, what we have just written. This is omega n bar. So, of course, So, this is the expression that we have. As we have always stated that for when we use quantum mechanical steps, I mean when we look at statistical mechanical approach of dealing with such problems which is what we are doing now. The idea is that we would like to find the conditions here which maximize this omega n bar.

So, that would represent the micro state with the maximum number of ways that it is possible to attain that micro state and that would then represent the equilibrium state of the system because as we have discussed before the most probable state is more probable then all the other states combined. We discussed that in detail before. So, will just accept it now. So, that is what we are going to do. So, we are going to try and maximize this and also in general if you look at omega n bar the it is of the form that whether you maximize this or the maximize the log of this, both of them will give you the same result. The 2 functions are such that lone omega n bar and omega n bar will behave in such a way that when you maximize omega n bar lone omega n bar maximizes. Similarly lone when lone omega n bar reaches a maximum whatever is that condition same condition omega n bar would maximize and mathematically it is easier to handle lone omega n bar.

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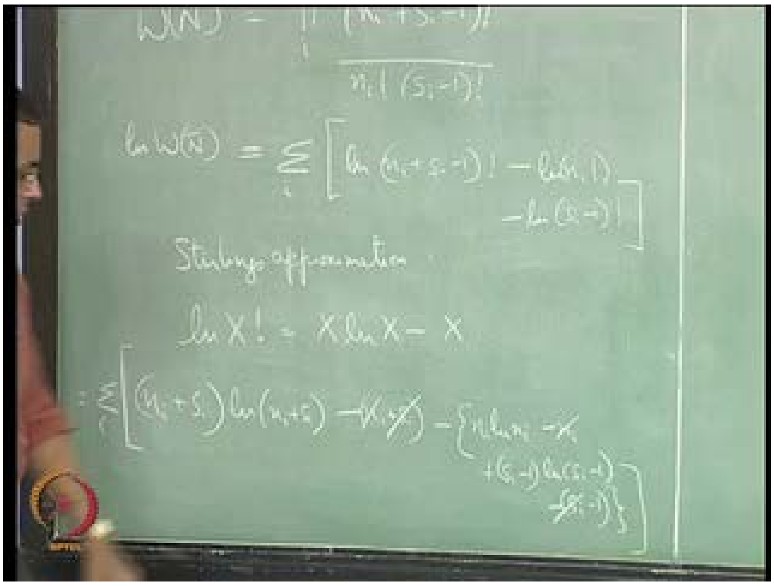

So, that is what we will do. So, rewriting it we have omega n bar is pi over I, n i plus s i minus 1 factorial by n i factorial times s i minus 1 factorial. So, if you take the lone of this it is simply, now it becomes a somewhat was product because you taken log natural log, natural logarithm of this product you get it becomes a sum, it is lone n i plus s i minus 1 factorial minus you can put this under bracket because it is sum over the same i is in for every time, it is the same i minus lone n i factorial minus lone s i minus 1 factorial. So, this is what we have and in general what is true is that given the values of n i and s i, n i plus s i is much larger than 1, in general n i plus s i will be much larger than 1. So, n i plus s i minus 1 can be reasonably approximated to n i plus s i because it is larger than minus 1. The same will not be true for this term, but for this term it will be true. So, therefore, here we will leave the minus 1, here we will remove the minus 1 because n i plus n i is the very large number generally. So, n i plus s i minus 1 is simply n I, is simply n i plus s i.

So, we can do that here. So, this is simply. So, we that is 1 thing, 1 change we will make plus we will use Stirlings approximation which simply says lone of x factorial is x lone x minus x. So, for every 1 of these term each is a lone of some factorial, for every 1 of those terms we will do this approximation and x and we will change this n i plus s i minus 1 to just simply n i plus s i. So, therefore, this is simply equal to will write that here lone of. So, this is n i plus s i. The summation is still there. So, summation is still there we are only looking at the terms within the bracket. Summation over i, n i plus s i lone n i plus s i minus n i plus s i minus you can put a bracket here curly bracket n i lone n i minus n i plus s i minus 1 lone s i minus 1 minus s i minus 1.

So, this is what we have. Now, if you look at it there is a minus n i here and there is a minus of minus n i. So, that is a plus n i. So, these n i and this n i will cancel. Again there is a minus s i here, there is a minus of minus s i here. So, this s i will cancel. So, they cancel out. So, we are just simply use Stirlings approximation to get here and substituted n i plus s i minus 1 as n i plus s i. So, those are the changes we have made.

The summation is still there, this summation that you see here that you see up here continuous to remain there. So, that is what we continue to have here. So, this is what we got. These terms go.

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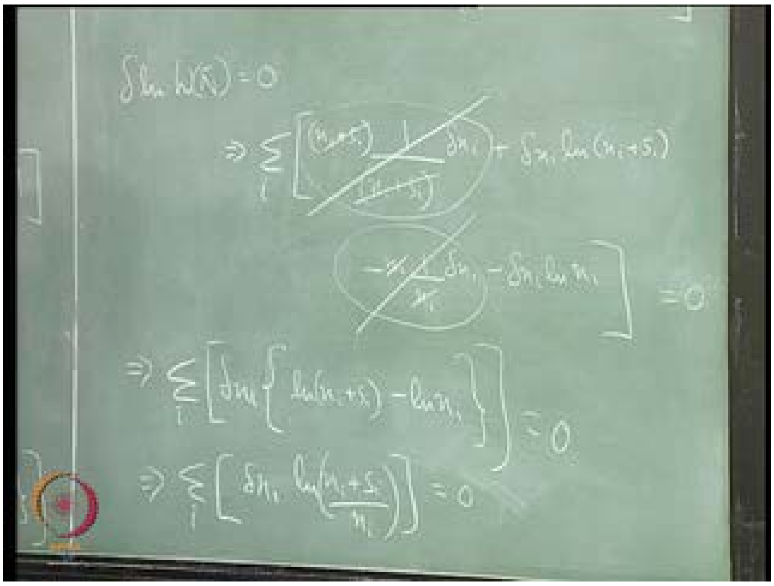

$$\delta \ln W(\bar{n}) = 0$$

$$\Rightarrow \sum_i \left[\cancel{(n_i+s_i)\frac{1}{(n_i+s_i)}\delta n_i} + \delta n_i \ln(n_i+s_i) \cancel{- n_i \frac{1}{n_i}\delta n_i} - \delta n_i \ln n_i \right] = 0$$

$$\Rightarrow \sum_i \left[\delta n_i \left\{ \ln(n_i+s_i) - \ln n_i \right\} \right] = 0$$

$$\Rightarrow \sum_i \left[\delta n_i \ln\left(\frac{n_i+s_i}{n_i}\right) \right] = 0$$

So, therefore, this is. So, we have the result to be and then plus 1 will be there. Now, you will have minus of minus. So, its plus minus 1 here minus 1 there. So, they have 3 minus here minus, minus and minus. So, you have a minus 1 there. So, this is the result that we have got. Now, as we have always discussed the you know if it if you are maximizing this then effectively you know the differential of this with respect to n I, the variable you have here is only n i.

So, if you maximizing this the differential of this with respect to n i should be a 0 and it we can also write it as simply as del lone w n bar should be equal to 0 where you effectively differentiating that with respect to n i. So, if everything else is a constant here s i is a constant and of course, 1 is a constant. So, now, when you differentiate it this is. So, this is entirely a constant. 1 is a constant, all of this term is a constant. So, this entire things simply reduces to 0. So, we will only have terms coming from the first term and the second term here. This simply disappears.

So, if you write it down we get. So, this implies that sigma over i, n i plus s i that is differentiate this term. So, that will be one by n i plus s i times del n i because that that n i will be differentiated that is del n i and you can also differentiate the n i here. So, plus del n i again s i is a constant. So, that goes to 0 times lone n i plus s i. Now, you take the second term again you will have minus n i by 1 by n i del n i and also minus del n i lone n i. So, that is the. So, this will give you the 2 terms because you have 2 n i here and this also gives you 2 terms because you have 2 n i.

So, these are the 4 terms that you get, of this if you see again this reduces to simply plus del n i because this will cancel. So, that is plus Del n i and this reduces to minus del n i this will cancel. So, minus del n i. So, again this entire term will cancel with this entire term. So, this is go, this will go and so, you basically have del lone. So, this implies that you know of course, this the fact that this is equal to 0 simply implies whatever we have calculated here equals 0. So, that simply implies that the summation over i the only terms that are left are this Del n I, of lone n i plus s i minus lone n i equals 0 and since it is lone you can actually simply put that in denominator.

So, this implies sigma over i with marginal and put this just rearranging the terms, we will have or will write it write the beginning del n i lone n i plus s i by n i equals 0. So, we have got this combination. We have got simply taking the number of ways in which we can arrange this system and trying to maximize it we have found this 1 condition that needs to be satisfied which we have got as del n i lone n i plus s i by n i equals 0.

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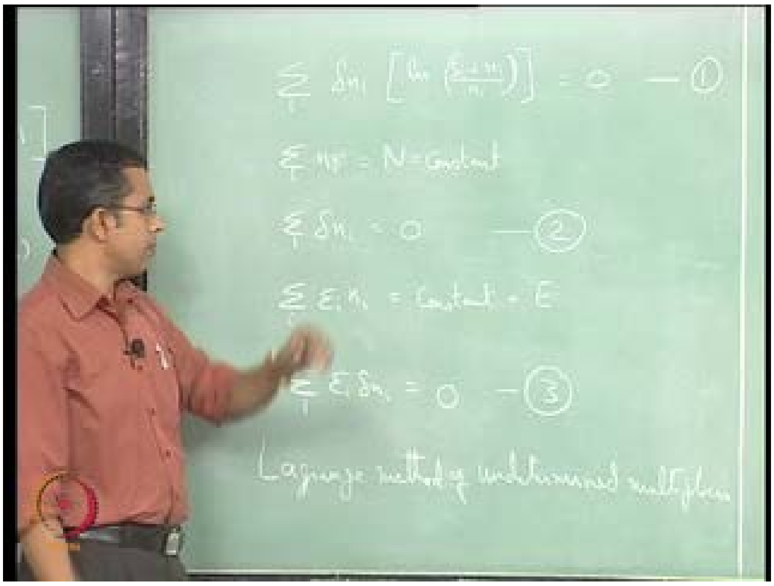

So, we will write that down here. So, that is our first equation sigma over i, del n i times lone of s i plus n i by n i equals 0. So, this is the condition that we have good got. So, this is the equation 1. As before the total energy in the system is conserved and the total number of particles in the system is conserved. Conserved meaning you cannot increase or decrease this, the number of particles is already there, you can you have no choice in it. It is not going to change. So, therefore, some of all the particles sigma over i n i.

So, n 1 plus n 2 plus n 3 plus n 4 plus n 5 etcetera equals the total number of particles equals a constant. Therefore, if you just differentiate this with respect to n i that has to be 0. So, that simply means sigma over i del n i equal 0, that is the same as saying you know you have a total number of particles, you simply remove a few particles from 1 state you distribute it amongst the other states. So, what is the change? You removed some particles. So, you have reduced. So, some minus term comes there. So, the change in particles there is some minus 5 particles let us say because you removed 5 particles, the same 5 particles you put in like 3 other states. So, therefore, you have you know plus 2, plus 2 and plus 1 let us say.

So, minus 5plus 2 plus 2 and plus 1 should give you 0 that is basically all it says. Whatever particles you remove from 1 state you have to add some other state. Therefore, this sum of all the changes has to be 0. Similarly, the energy is a constant total energy of the system is a constant. So, epsilon i n i equals constant which we designated as capital

E. So, again here the each energy level in the system is already fixed you do not have a freedom on that. So, this is already fixed for you. So, therefore, only change that you can do is here. So, again the changes in energy that you will do when you remove some particles is the energy level times the number of particles that you remove and when you add increase energy when you add it to some other state it will be the energy of that state times the number of particles that you have adding to it.

So, therefore, changes in energy total change in energy is simply sum over i epsilon i del n i for all the states what is the number of particles you are adding or removing times the energy of that state that gives you the total change in energy with respect to that energy level. Sum of all those changes has to be 0 because the sum of all the energies together is any way equal to a constant. So, this is equal to 0. So, we now have 3 equations we already identified equation 1 here, this is equation 2 and this is equation 3.

So, we have three equations and what we will do is effectively as we have done before when we looked at Maxwell Boltzmann statistics and we also looked at the Fermi Dirac statistics. What we will do is we will use the Lagrange method of undetermined multipliers and then we will solve this system. So, we are going to use the Lagrange method of undetermined multipliers. So, we are actually simply going to multiply this by this by a constant alpha, this by a constant beta and add it to this equation up here. So, that is what is going to be there and there all equal to 0. So, that is sum is also going to be 0. So, that each of them is 0. So, it does not matter what you multiply it with it is going to remain 0 and we are going to add it to this.

Only thing is by convention if because the result works out more convenient to interpret instead of simply adding to this equation we add it to we add these 2 to minus of this equation and that is only a convention simply to make the interpretation easier and in a sense it makes again no difference because its anyway its equal to 0. So, whether you put a minus sign front of it or plus sign front of it makes no difference. So, we will add a minus sign here and then we will add these terms together.

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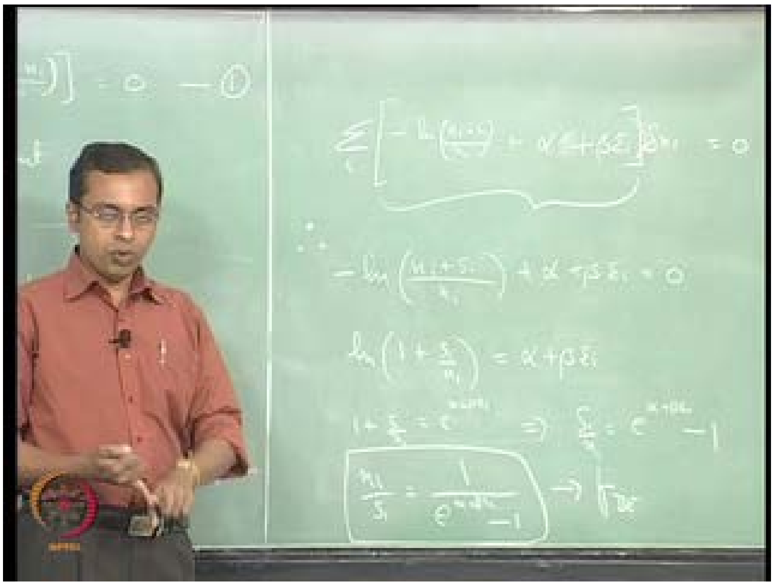

So, or when we do this would when we do what I just said when we are multiplying by 2 constants and adding them this is what we will get, summation over i. We will write the first term down this same way minus will put a bracket here minus lone n i plus s i by n i is bracket covers everything including the n i plus alpha epsilon i I am sorry plus alpha plus beta epsilon i del n i equals 0. So, we had 3 terms we had sum of epsilon i del n i equals 0 we have multiplied that by beta and brought it here.

We had as some of del n i equals 0 we have multiplied that by alpha and brought it here and I kept the original equation. So, it is equation 1 plus alpha time's equation 2 plus beta time's equation 3 which is what we have done. So, that is equal to 0 and as we discussed before the point is we have a bunch of energy levels, we have a bunch of states of those energy levels and when you maximize it regardless of what changes you are making in those states adding a little, subtracting a little the sum should always be 0. In other words this sum has be to 0 regardless of the values of these n i because you can keep making minor changes here and there still it should be a 0. So, independent in other words we have to make it independent of these del n i and the only way you can do that . So, therefore, the only way you can guarantee that the sum is 0 regardless of where you are adding, what you are adding, what minor changes you have making how the only way you can guarantee that this is 0 is to say that every individual term here within this bracket has to become I mean this whatever is within the bracket should always be 0.

Only when you do that you will you will be able to guarantee that this sum is equal to 0. So, therefore, we say lone of this is the term within the bracket as long as you guarantee that this term is 0 regardless of which energy level you are looking at i, e i you are looking at you can guarantee that this sum remains 0 otherwise there is it is difficult to guarantee because you have all sorts of various values of del n i and then it will become difficult to get. This is only way you can guarantee it 0. So, now, we have this expression we will rearrange it marginally. So, basically it says that lone of this is n i plus s i by n i. So, we this is we can simply write it as 1 plus s i by n i and we will move that other 2 terms on the other side. So, therefore, that is why this minus sign is now suddenly disappeared, is alpha plus beta epsilon i and marginally and therefore, once you simplified it further 1 plus s i by n i equals e power alpha plus beta epsilon i and this implies s i by n i equals e power alpha plus beta epsilon i minus 1.

We started off by saying we want to know the probability of occupancy of a state at energy level e i that is simply the inverse of this. We want n i by s i, we have now got s i by n i. So, that is what we want. So, we wants actually n i by s i. So, n i by s i is simply the inverse of this it is 1 by e power alpha plus beta epsilon i minus 1. So, this equation that you get here n i by s i is 1 by e power alpha plus beta epsilon i minus 1 this equation that we have got here.

This is the Bose Einstein distribution and we designated also by f which subscripts b e, this tells you for every given energy level what is a probability of occupancy of a state at that energy level. So, this is what it is. Again this can be ah this alpha and beta can be interpreted as in terms of the chemical potentials of the system and so on. So, that is not something we are immediately interested in, but this is the form of the equation that we get. This is the Bose Einstein statistical distribution. So, when you look at phonons, you look at photons, you look at cooper pairs they as long as they are to the extent that they are you know they are bosons, they are classified as bosons they are following the Bose Einstein statistical distribution which is here.

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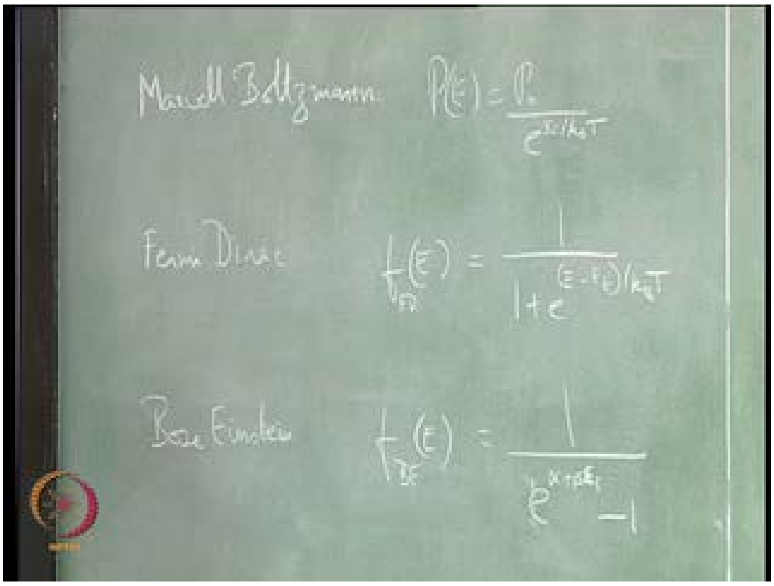

So, through this course through this course we have actually looked at 3 statistical distributions. One was the Maxwell Boltzmann distribution and that basically was that we got we could write it as P equals, P of E equals some P naught E to the power minus E i by K p T if you just write it in the denominator it will be E power E i by K p T. So, this is the Maxwell Boltzmann distribution. This applies to classical particles, we also got the Fermi Dirac distribution. This works for fermions. So, electrons would classify as fermions. There we got f of E is or f of E is 1 by 1 plus E power E minus E f by K p T. So, this is where Fermi energy and so on we have there.

So, this is what we got and now we have got the Bose Einstein distribution. So, f of. So, will call this f f d Fermi Dirac and this is Bose Einstein and that we have simply called as 1 by 1 sorry E power alpha plus beta epsilon i minus 1. So, this is a essentially a plus 1 and this is a minus 1.

So, so these are the three distributions that we have got and I mean if you did not know, only when you plot this up you realize that they are very different and or when you realize the kinds of assumption that have gone it to them they look very different. Some wave similarity is there in terms of how they layout, but they are big difference in how they behave in actual behavior and In fact, in a sense this is the Maxwell Boltzmann distribution is considered the extreme approximation of these two. So, under the right kind of conditions these kinds of particles would actually reduce to looking like classical particles under the right kind of conditions, but in general these are separate sets of particles. So, they are very distinctly different in how they behave, how they are distributed across energy and therefore, what you can expect from a system consisting of particles of any one kind when they are subject to certain experimental conditions.

Now, we have spoken about the Bose Einstein distribution because we have already encountered 3 bosons in our discussion and there are some peculiarities of it about of this bosons which is what we will familiarizes ourselves with the. As I mentioned the Bose Einstein distribution since it consist of a it is addressing a set of particles that are displaying quantum mechanical behavior, but are exempt from the Pauli exclusion principle there is no limit to the number of particles you can place in a given state. So, it turns out that given this basic idea if you actually go to very low temperatures you can expect that all the particles will keep going to lower and lower levels of energy and in fact, can condense to a single state.

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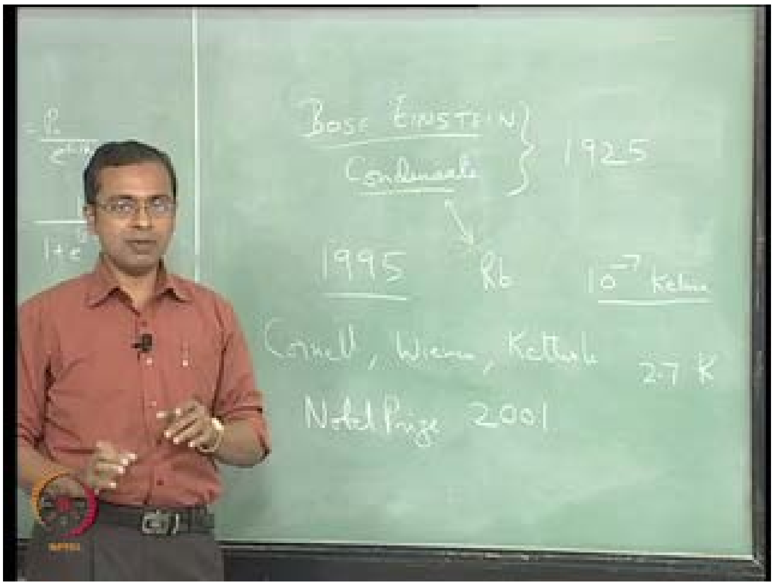

So, can condense to a single state. This kind of situation where all the particles in the system actually settle down to a single state at very low temperatures is called Bose Einstein condensate. So, this is called a Bose Einstein condensate. So, as you go to very low temperatures the particles will settle down to a single state and that is called a Bose Einstein condensate. Now, the beauty of the Bose there are several things that have very nice and interesting about the Bose Einstein condensate, the first and I would say the most startling thing about it is this was predicted in 1925. So, when the Bose Einstein statistics statistical distribution was put I mean written down and introduced to the world at that point this the possibility that such a thing could occur because of the fact that this it is allowed in that system that was propounded by Bose Einstein, Bose and Einstein. So, this is called the Bose Einstein condensate. It was predicted in 1925.

For a very long time after that experimental facilities where not available to enable people to investigate the those bosons at those very low temperatures. People were unable to investigate these bosons at extremely low temperatures to see whether or not such a condensate forms. So, this was predicted in 1925, only 70 years later in 1995, 70 years after the prediction was made that is such a such a state of matter could exist, it is considered as a new state of matter 70 years after it was predicted that such a state of matter could exist. In 1995 in a collection of rubidium atoms it was shown that when you go to very low temperatures, when you talk of very low temperatures please remember this is not I mean already talking of 2, 3 Kelvin is very low temperatures, but where they are in this kind of an experiment they are talking in terms of 10 power minus 7 Kelvin, extremely close to absolute 0.

It is a huge experimental achievement to even get a experimental set up where you can reach this temperature. So, this kind it is not a simple thing to reach this kind of a temperature. So, phenomenal piece of work to reach this kind of a temperature and in 1995 a group, I mean scientist where able to actually accomplish this temperature with and subject rubidium atoms to this kind of a temperature. When they did that they were able to get create this condensate. So, they created this Bose Einstein condensate for a set of rubidium atoms. For doing this in 1995 the 3 people involved who were Cornell, Wieman and Ketterle. Cornell, Wieman and Ketterle they got their Nobel prize in 2001 they got a Nobel prize for accomplishing this in 2001 for something that they did they managed do in 1925.

So, till 1995 70 years after this state of matter was predicted nobody was able to accomplish this, with a lot of difficulty they managed to reach this kind of a temperature this group I mean this these scientist and they were able to show in 1995 that this condensate actually exists, that it can actually be formed and for that they got the Nobel prize. The point is that. In fact, if you look at it for example, that there I would like to point out couple of things here 1 is just a matter of historical interest and sort of reflection of how sometimes things work in science. Bose Einstein predicted this in 1925, 70 years later somebody managed to actually show it, for doing this they got a Nobel prize, but strangely enough Bose who was involved in all this and who was very instrumental in this, it was this original idea strangely enough he never got a Nobel prize for the for his work. So, Satyendranath Bose somehow missed getting a Nobel prize, but people who later proved his prediction to be correct actually got a Nobel prize. So, that is 1 of these strange things about science and just wants to point that out.

The other thing is actually when you look at this kind of a temperature, I think it is important to understand what this represents. So, if you see we spoke about black body radiation, right at the beginning of a course and through the course we have touched upon it on a few occasions. Black body radiation, if you look at the universe around you people have done experiments and I mentioned this earlier where you can looked at the background radiation in the universe. Based on the background radiation in the universe you can say what is the current temperature of the universe. The current temperature of the universe is of the order of 2.7 Kelvin. So, 2.7 Kelvin is the background temperature of the universe. The current understanding of the universe is that it is started off at a very high temperature and then it has been cooling steadily.

The big bang theory that we talk of effectively implies this kind of a situation with respect to temperature, extremely high temperatures were there in the first nano seconds after the universe was formed and ever since it has been cooling and what you have this background temperature that we are talking of is the current background temperature of the universe. So, that that is in another words this is how cool the universe has become, anything else in the universe is only hotter than this. Nothing in the universe in the sense in the sense of naturally occurring parts of the universe whatever is occurred naturally in the universe over time has been cooling steadily and this is the temperature that we have in the background.

So, whenever you are doing some experiment in the lab, if you manage to reach a temperature lower than this in your experiment, if you go to a temperature lower than this which you will be doing artificially you are doing using some compressors, you are using some pumps, you are using something else, you are using much more sophisticated equipment were used here, they used all sorts of different means to slow down atoms because the energy of the atom is effectively its temperature. So, they used very interesting methods to slow down the atoms and get them and effectively lower their temperature. So, whenever you do this complicated thing you are doing it artificially, you are imposing it on the system, you have sort of forcing the system to undergo all those things. So, whenever you draw below this temperature in a sense you are dealing with a situation that does not naturally exist anywhere else in the universe.

That is a remarkable statement to make to say that in my lab I have an experimental situation which is not naturally occurring anywhere else in the universe that too we can say with confidence is whenever you are doing this kind of thing. If you are going to higher temperatures and higher pressures invariably there is you will find something in the universe which is having that kind of a temperature or pressure, but to get it to below this point you are already talking of something that does not naturally exist anywhere. I mean what it to the extent that if it exist anywhere else chances are, that means, there is intelligence life elsewhere which is managing to do this the same kind of experiment. The fact that you are actually able to accomplishing this in the lab means you have done something that nature has not accomplished of its own accord in the universe and this is just 2.7 Kelvin.

We are talking in here of 10 power minus 7 Kelvin which is remarkably lower than this and as I mentioned you know in material science and metallurgy and so on. We talk of temperatures say from say minus 100 degree c to few 1000 degree c that is a range of temperatures that we routinely access, we do not think too big about it, we just routinely access that scale of temperatures, but if you draw from about minus 100 degree c to minus 273 degrees c which is what this 0 Kelvin is. So, that is that looks that is only about 173 degrees centigrade or of the order of say you can even go to a narrower window of the order of say 100 Kelvin, liquid nitrogen will get it about 77 Kelvin. So, from there if you want to drop down it starts getting more and more difficult.

So, liquid helium will get you liquid hydrogen, liquid helium will all get you into that 10 Kelvin or there about. So, from there to drop down to 0 Kelvin the last 10 degrees starts getting to be very difficult, it is not easy to get you down the last 10 degrees. Suddenly, as you start dropping in to the last few degrees it is it is remarkable piece of work to get to that kind of temperature. That is the reason it took 70 years to go from whatever was predicted to actually demonstrate it in the lab. Simply, not being able to cool a system to that temperature that was the experimental difficulty. So, you talk of you know highly sophisticated experiments and what not, this is highly sophisticated in experiment, but what were they doing?

They were simply lowering the temperature, that is the basic that is the fundamental thing that they were doing. They were of course, lot of other things that they were doing but that is the fundamental thing that they were doing, lowering the temperature. So, lowering the temperature it itself a very phenomena likes experimental accomplishment. So, in this process they were able to show the Bose Einstein condensate. So, to sum up today what we have seen is a we have looked at a bosons in much greater detail, we have looked at the conditions that are relevant to the particles that are classified as bosons. We have looked and we have used those conditions and looked at the distribution across energy levels, that the bosons will demonstrate and so, we have therefore, worked our way through the Bose Einstein statistical distribution and we see we have also briefly seen how it looks with respect to Fermi Dirac statistics and Maxwell Boltzmann statistics and we have also looked at this phenomenal and very interesting scientific feed of some prediction that was made in 1925 and then experimentally proven 70 years later in 1995.

So, with that we will halt today. This sort of covers our discussion on the bosons and its sort of relates to all the other things that we have discussed in the last few classes. So, we will some we will halt here today at this stage and we will pick it up in the next class as we look at a just a few more interesting points before we wind up this course. Thank you.

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